# II. Generalized Parton Distributions (GPDs)

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HANUC, Jyväskylä, August 25-29, 2008

# Outline : GPDs

1) link of FF to Generalized Parton Distributions :

   nucleon "tomography"

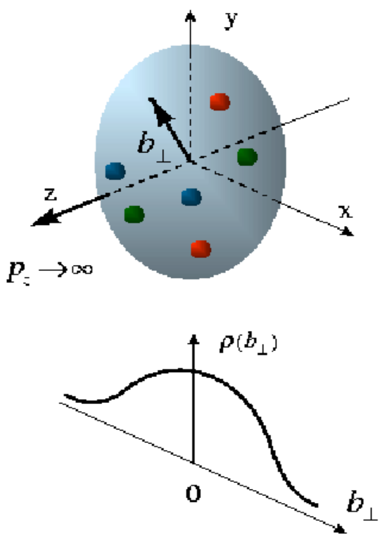
2) GPDs and spin of nucleon

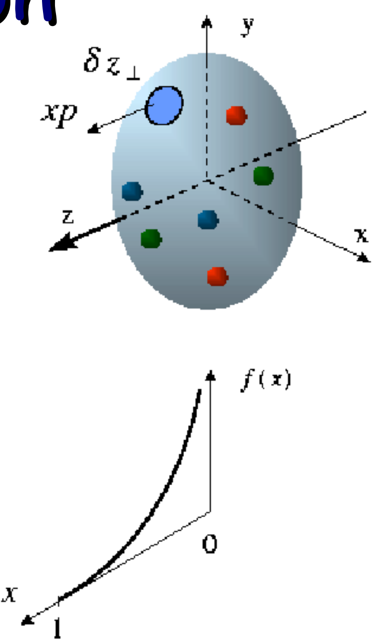
3) Hard exclusive processes : DVCS, …

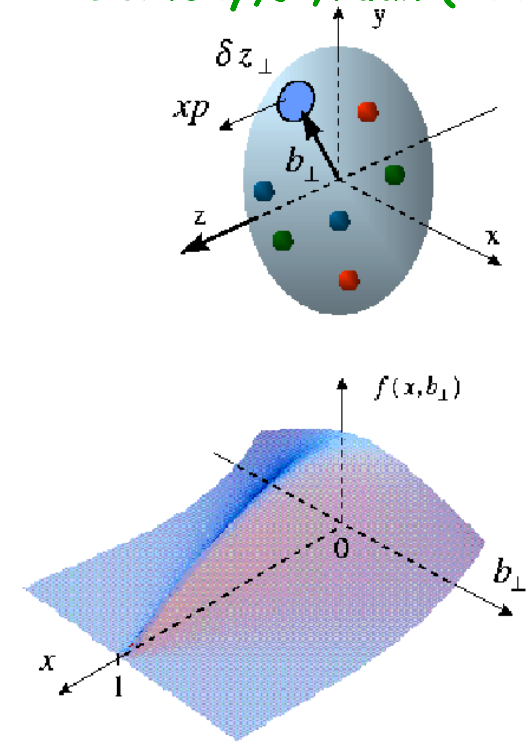
# Generalized Parton Distributions : yield 3-dim quark structure of nucleon

Burkardt (2000,2003)

Belitsky,Ji,Yuan (2004)



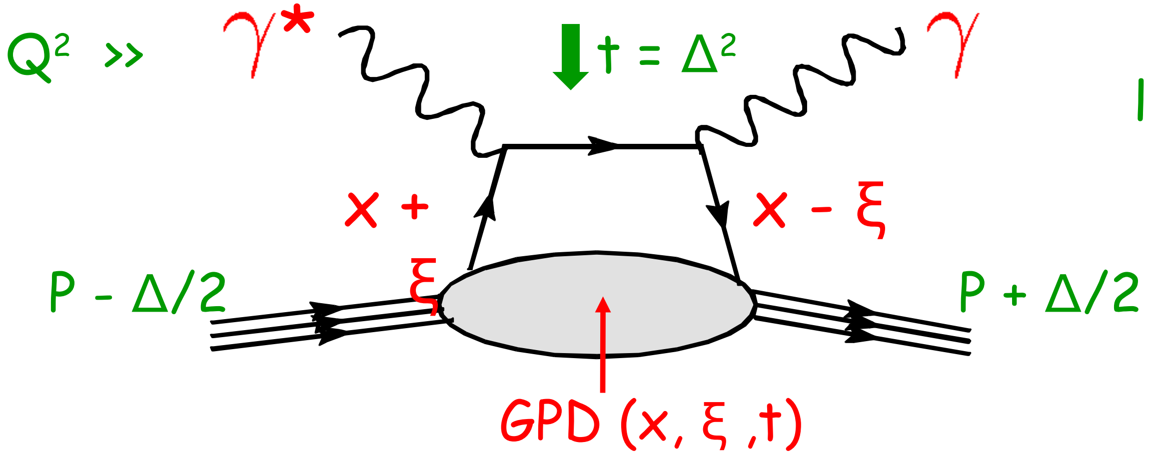




**Elastic Scattering**
transverse quark distribution in coordinate space

**DIS**
longitudinal quark distribution in momentum space

**DES (GPDs)**
fully-correlated quark distribution in both coordinate and momentum space

# Generalized Parton Distributions




low –t process : -t << $Q^2$

$\Delta^+ = -(2\,\xi)\,P^+$

light-cone dominance, $n^\mu$ (1, 0, 0, -1) / (2 $P^+$)

$$\frac{P^+}{2\pi}\int dy^-\, e^{ixP^+y^-}\,\langle P+\frac{\Delta}{2}\,|\,\bar{q}\left(-\frac{y}{2}\right)\,\gamma\cdot n\, q\left(\frac{y}{2}\right)\,|\,P-\frac{\Delta}{2}\rangle_{y^+=0,\,y_\perp=0}$$
$$=\bar{N}\left\{H(x,\xi,t)\,\gamma\cdot n\;+\;E(x,\xi,t)\,i\sigma^{\mu\nu}\frac{\Delta_\nu}{2M}n_\mu\right\}N$$

$$\frac{P^+}{2\pi}\int dy^-\, e^{ixP^+y^-}\,\langle P+\frac{\Delta}{2}\,|\,\bar{q}\left(-\frac{y}{2}\right)\,\gamma\cdot n\,\gamma_5\, q\left(\frac{y}{2}\right)\,|\,P-\frac{\Delta}{2}\rangle_{y^+=0,\,y_\perp=0}$$
$$=\bar{N}\left\{\tilde{H}(x,\xi,t)\,\gamma\cdot n\,\gamma_5\;+\;\tilde{E}(x,\xi,t)\,\gamma_5\frac{\Delta^\mu}{2M}n_\mu\right\}N$$

# known information on GPDs

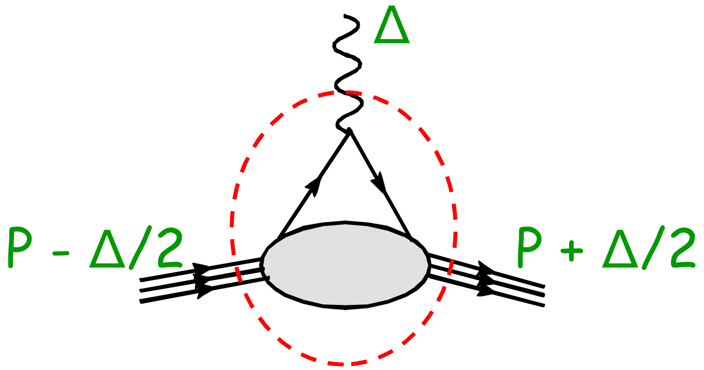
➡ forward limit : ordinary parton distributions

$$H^q(x, \xi = 0, t = 0) = q(x)$$ unpolarized quark distr

$$\tilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x)$$ polarized quark distr

$E^q$ , $\tilde{E}^q$ : do NOT appear in DIS ➡ new information

➡ first moments : nucleon electroweak form factors




ξ independence :
Lorentz invariance

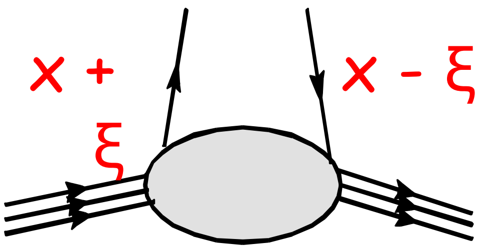
$$\int_{-1}^{1} dx\, H^q(x, \xi, t) = F_1^q(t)$$ Dirac

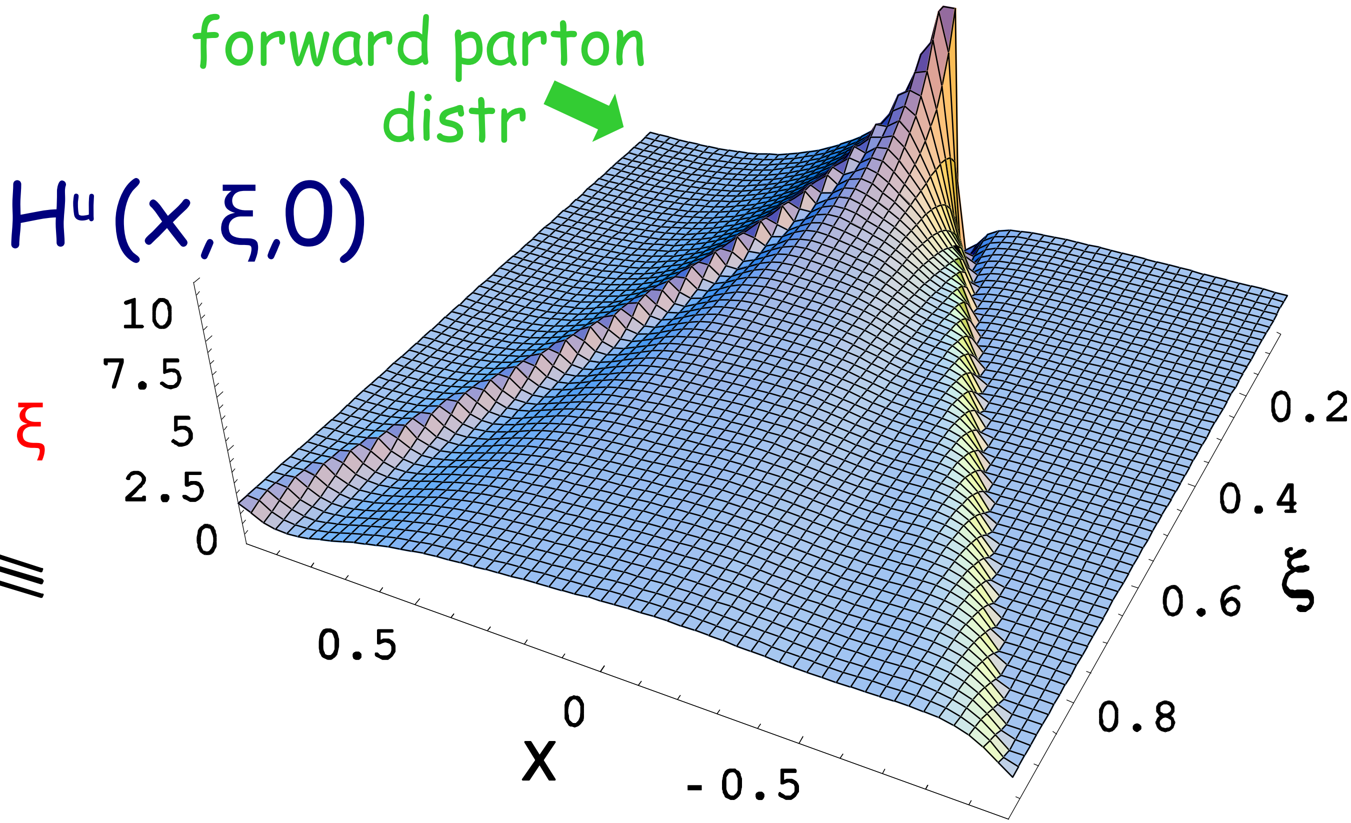
$$\int_{-1}^{1} dx\, E^q(x, \xi, t) = F_2^q(t)$$ Pauli

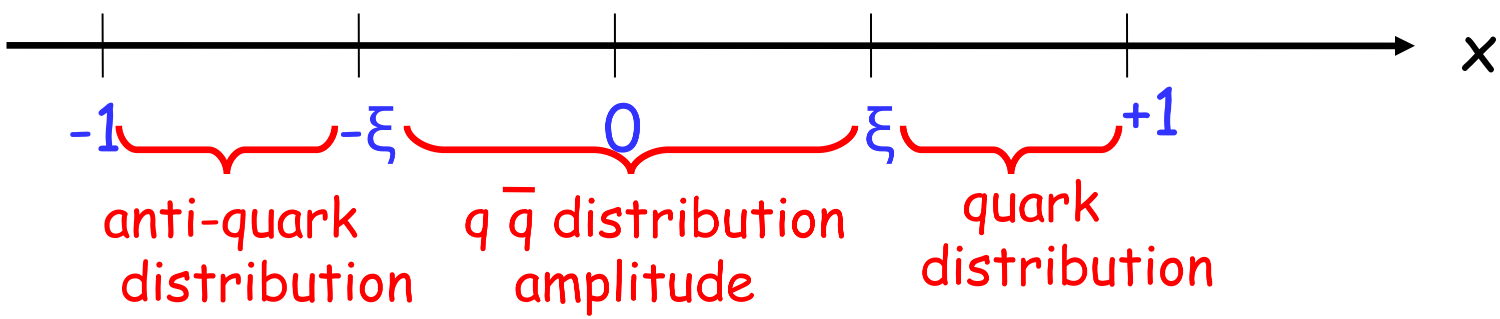
$$\int_{-1}^{1} dx\, \tilde{H}^q(x, \xi, t) = G_A^q(t)$$ axial

$$\int_{-1}^{1} dx\, \tilde{E}^q(x, \xi, t) = G_P^q(t)$$ pseudo-scalar

# GPDs : x and ξ dependence



# Why GPDs are interesting

Unique tool to explore the internal landscape of the nucleon :

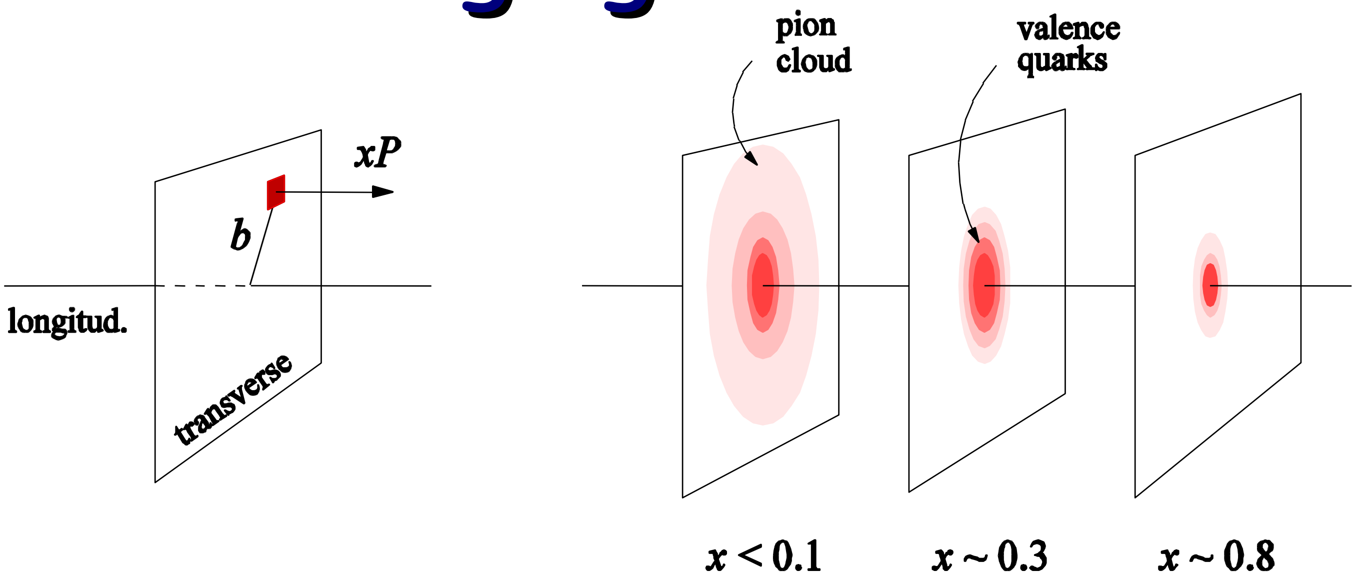
➡ 3D quark/gluon imaging of nucleon

➡ Access to static properties :

constrained (sum rules) by precision measurements of charge/magnetization

orbital angular momentum carried by quarks

# GPDs : 3D quark/gluon imaging of nucleon




Fourier transform of GPDs :

simultaneous distributions of quarks w.r.t. longitudinal momentum x P and transverse position b

➡ theoretical parametrization needed

# GPDs : t dependence

modified Regge parametrization : Guidal, Polyakov, Radyushkin, Vdh (2004)

$$H^q(x,0,t) = q_v(x)\, x^{-\alpha_1'\,(1-x)\,t}$$
$$E^q(x,0,t) = \frac{\kappa_q}{N_q}(1-x)^{\eta_q}\, q_v(x)\, x^{-\alpha_2'\,(1-x)\,t}$$

- Input : forward parton distributions at $\mu^2$ = 1 $GeV^2$ (MRST2002 NNLO)
- Drell-Yan-West relation : exp(- α´ t ) -> exp(- α´ (1 - x) t) : Burkardt (2001)
- parameters :

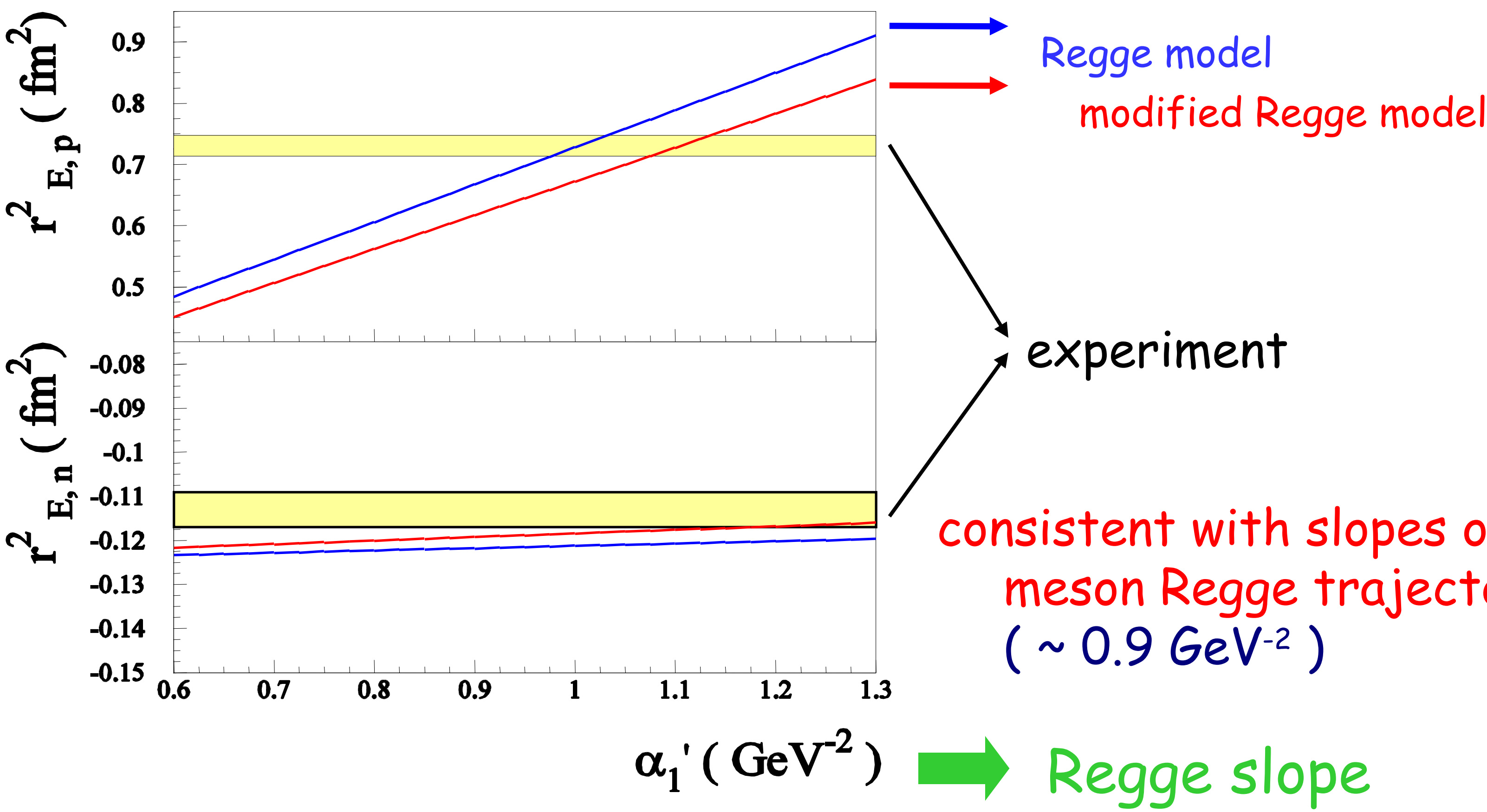
  regge slopes : $\alpha'_1 = \alpha'_2$ determined from rms radii

  $\eta_u$ , $\eta_d$ determined from $F_2$ / $F_1$ at large -t
- future constraints : moments from lattice QCD

# proton & neutron charge radii

$$r^2_{E,p} = r^2_{1,p} + \frac{3}{2}\frac{\kappa_p}{M_N^2} \qquad r^2_{1,p} = -6\,\alpha_1'\int_0^1 dx\left\{e_u\,u_v(x) + e_d\,d_v(x)\right\}\ln x$$

$$r^2_{E,n} = r^2_{1,n} + \frac{3}{2}\frac{\kappa_n}{M_N^2} \qquad r^2_{1,n} = -6\,\alpha_1'\int_0^1 dx\left\{e_u\,d_v(x) + e_d\,u_v(x)\right\}\ln x$$




consistent with slopes of meson Regge trajectories ( ~ 0.9 GeV$^{-2}$ )

# connection large $Q^2$ of FF <-> large x of GPD

$$I = \int_0^1 dx (1-x)^\nu \, e^{\alpha' Q^2 (1-x) \ln x} = \int_0^1 dx e^{\nu \ln(1-x) + \alpha' Q^2 (1-x) \ln x}$$

$$= \int_0^1 dx \, e^{f(x, Q^2)}$$

at large $Q^2$ : integral dominated by maximum of $f(x,Q^2)$, remainder region is exp. suppressed (method of steepest descent)

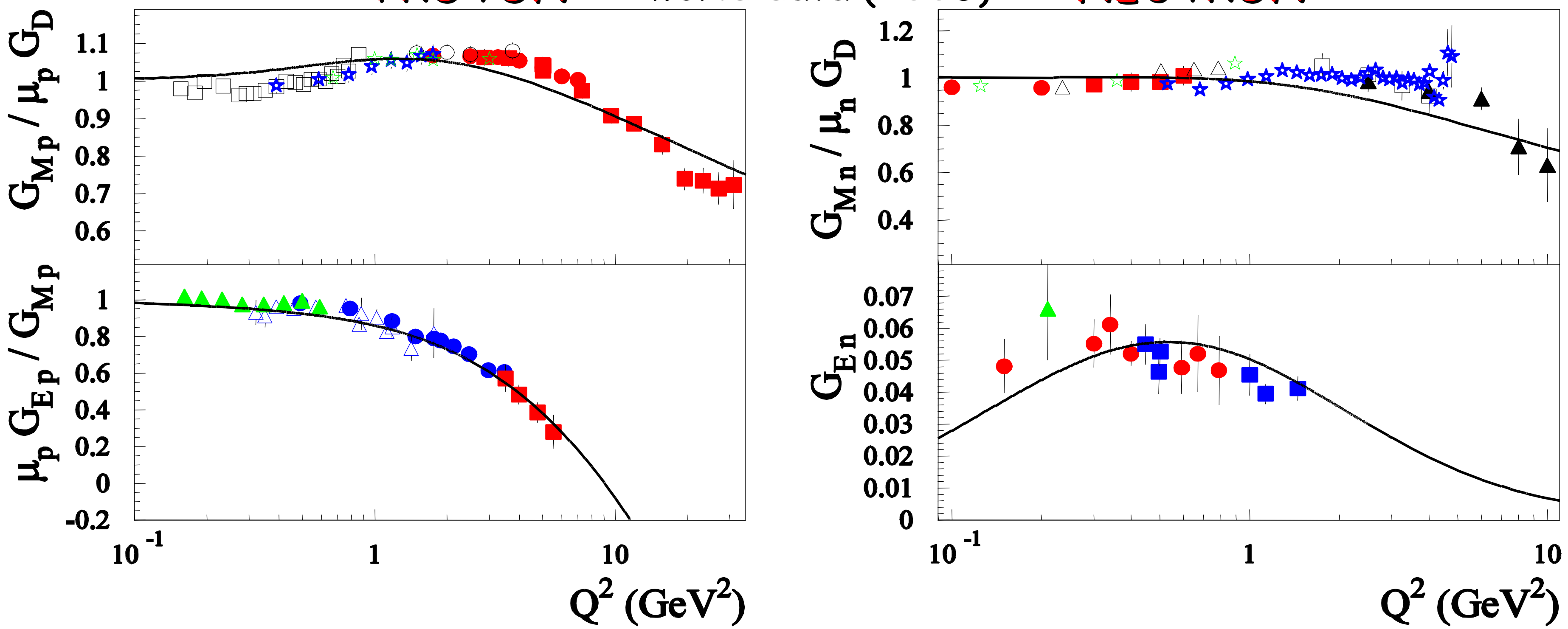
$f(x,Q^2)$ reaches maximum for : $x = x_0 \simeq 1 - \dfrac{\nu}{\alpha' Q^2}$

"Drell-Yan-West" relation for PDF/GPD :

at large $Q^2$ : I is dominated by its behavior around x -> 1

$$\boxed{I \simeq e^{f(x_0, Q^2)} \left( \frac{2}{f''(x_0, Q^2)} \right)^{1/2} \frac{\sqrt{\pi}}{2} \sim \left( \frac{1}{\alpha' Q^2} \right)^{(\nu+1)/2}}$$

# electromagnetic form factors




→ modified Regge GPD parameterization

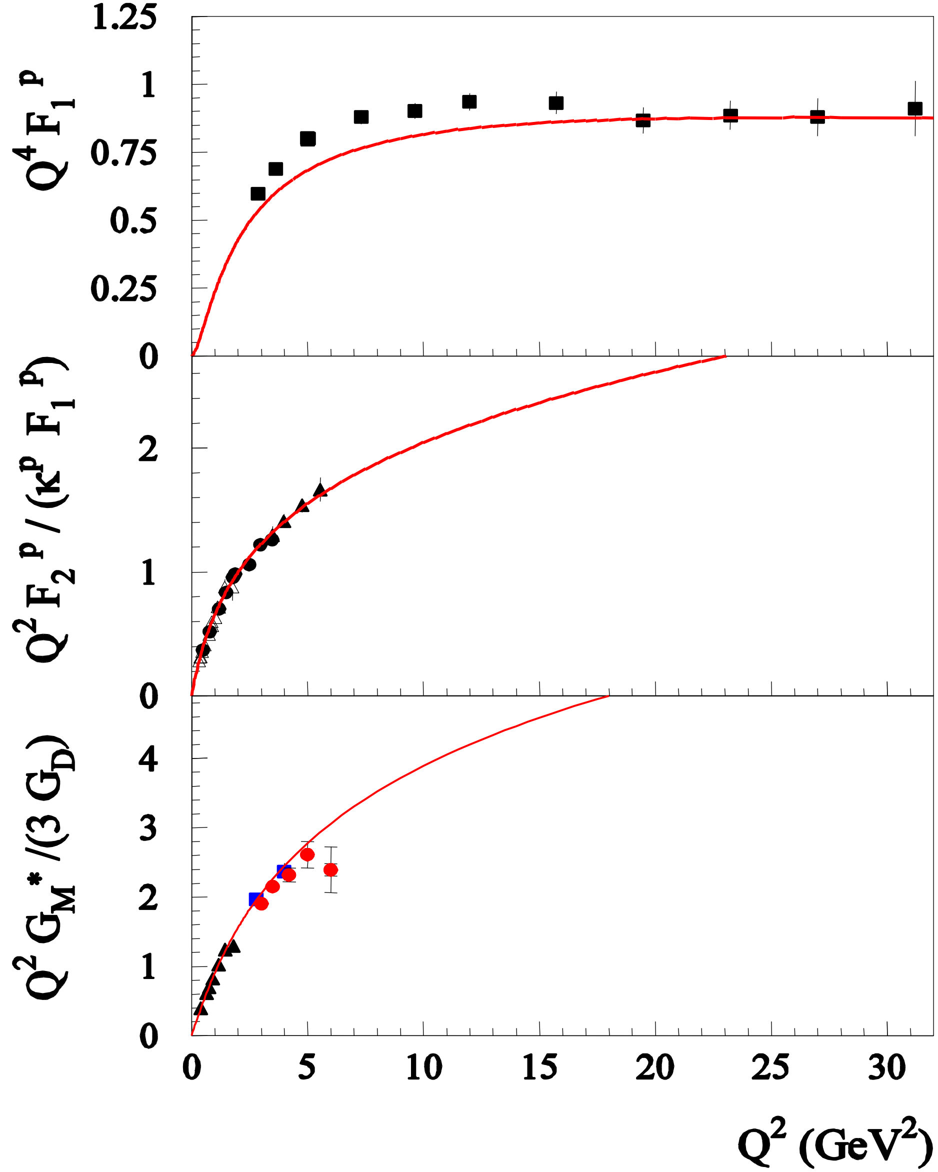
3-parameter fit
- 1 : Regge slope -> proton Dirac (Pauli) radius
- 2, 3 : large x behavior of GPD $E^u$, $E^d$ -> large $Q^2$ behavior of $F_{2p}$, $F_{2n}$

Guidal, Polyakov, Radyushkin, Vdh (2005)

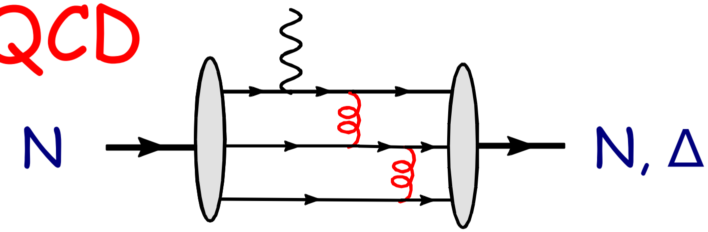
also Diehl, Feldmann, Jakob, Kroll (2005)

$\eta_u = 1.713, \quad \eta_d = 0.566$

# scaling behavior of N and N -> Δ F.F.




PQCD




+ collinear quarks

$F_1{}^p \sim 1/Q^4$

$F_2{}^p / F_1{}^p \sim 1/Q^2$

$G_M{}^* \sim 1/Q^4$

GPD ——

modified Regge model

Guidal, Polyakov, Radyushkin, Vdh (2005)

# GPDs : 3D quark/gluon imaging of nucleon

$$\text{for} \quad \xi = 0 \quad \longrightarrow \quad t = -\boldsymbol{\Delta}_\perp{}^{\mathbf{2}}$$

$$H^q(x, \mathbf{b}_\perp) = \int \frac{d^2 \boldsymbol{\Delta}_\perp}{(2\pi)^2} \, e^{i \mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp} \, H^q(x, \xi = 0, -\boldsymbol{\Delta}_\perp{}^{\mathbf{2}})$$

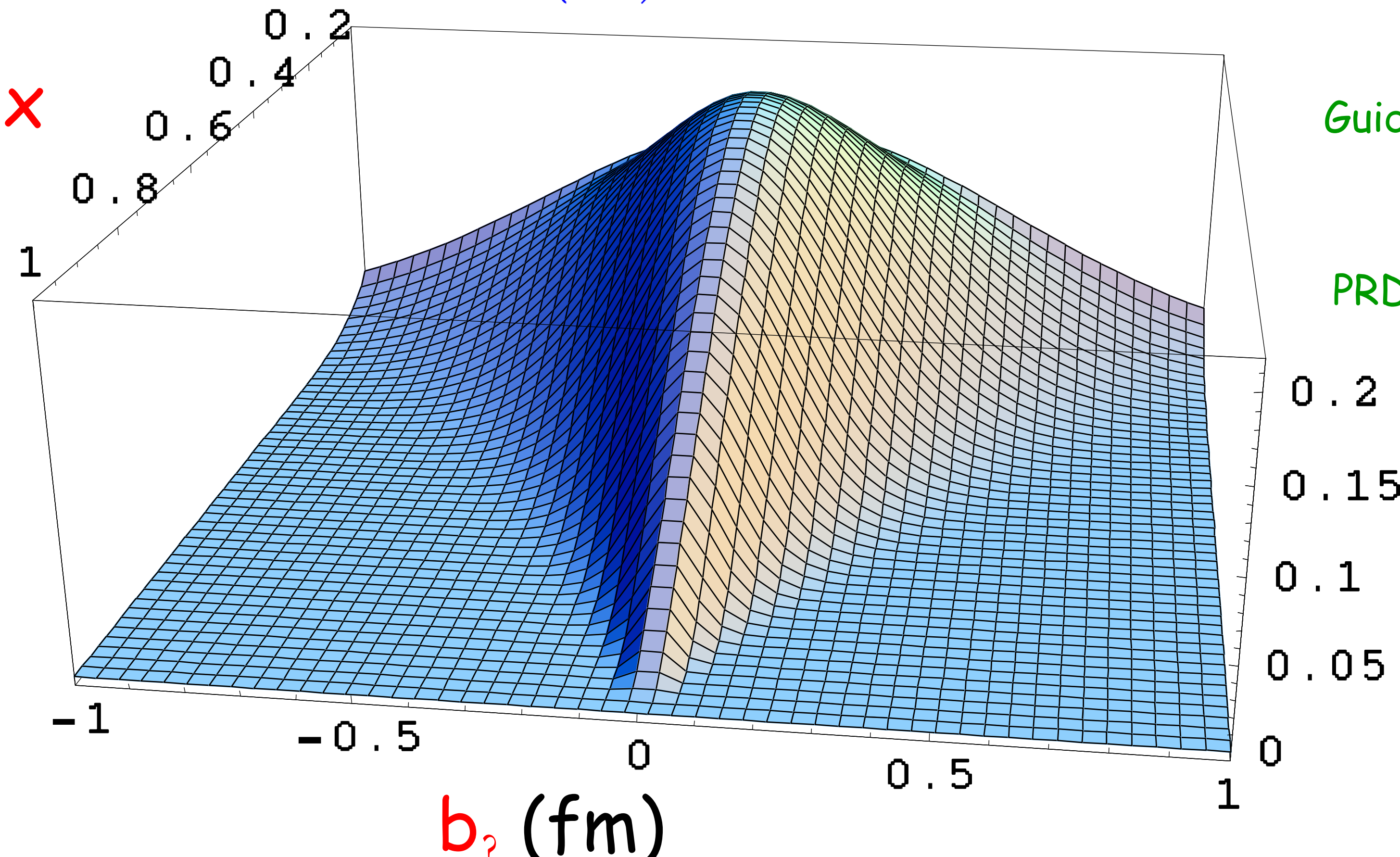
Fourier transform of GPDs :

simultaneous distributions of quarks w.r.t. longitudinal momentum x P and transverse position b

# GPDs : transverse image of the nucleon (tomography)

$H^u(x, b_?)$

$$H^q(x, \mathbf{b}_\perp) \quad = \quad \int \frac{d^2 \mathbf{\Delta}_\perp}{(2\pi)^2} e^{i \mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} H^q(x, \xi = 0, -\mathbf{\Delta}_\perp{}^2)$$

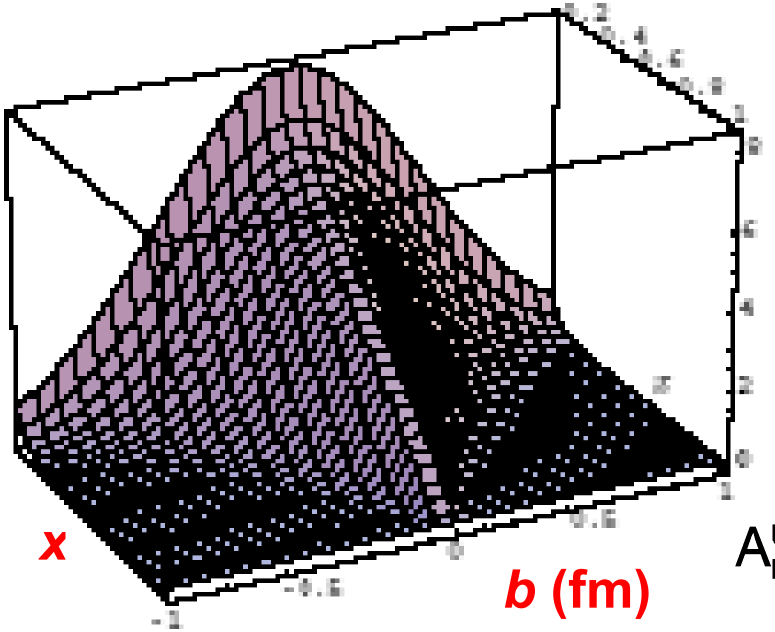



Guidal, Polyakov, Radyushkin, Vdh,

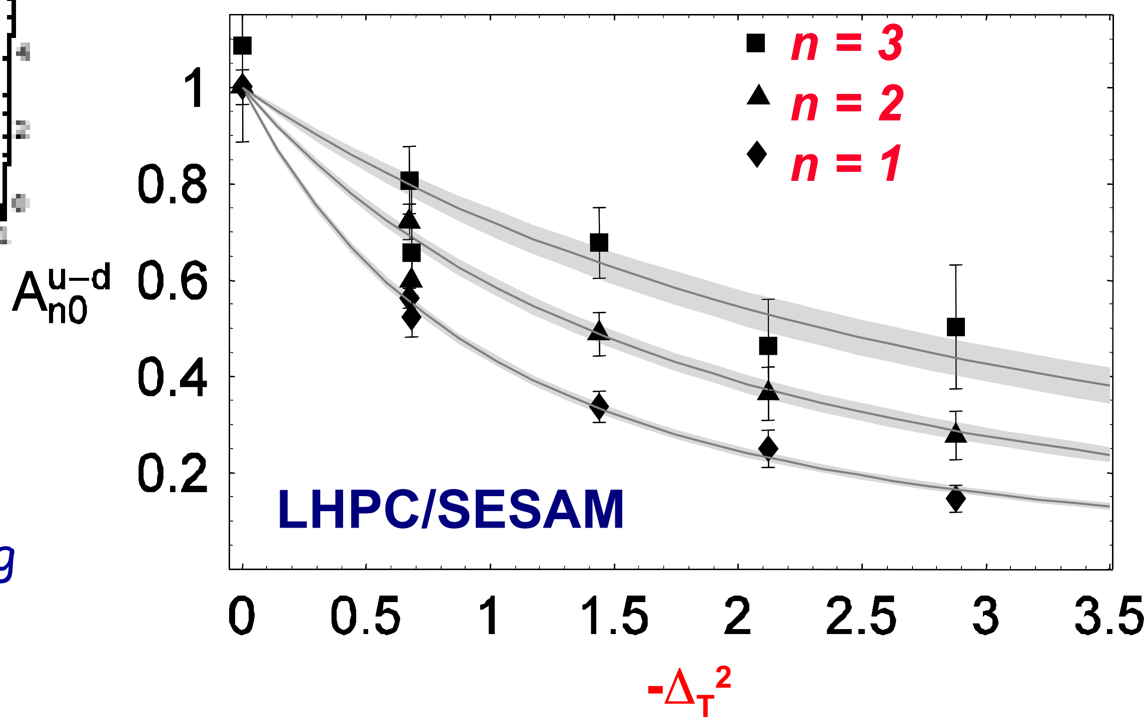
PRD 72, 054013 (2005)

# Moments of flavor-NS GPDs

$$A^q_{n0}(-\vec{\Delta}_\perp^2) = \int d^2b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)$$




Lattice results : $m_\pi$ = 740 MeV

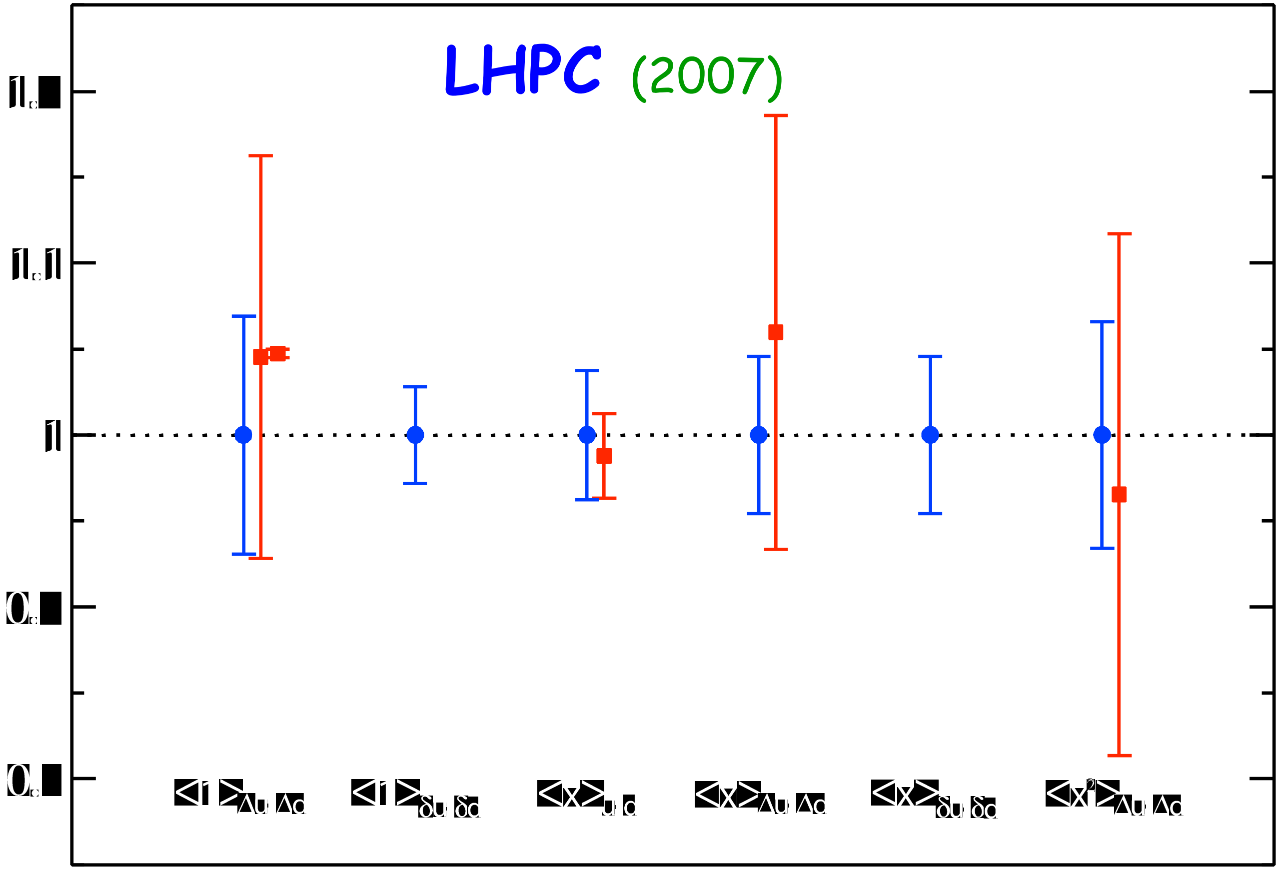



Decrease slope : decreasing transverse size as x -> 1
Burkardt

# lattice QCD for moment of PDFs : present state-of-the-art

full lattice QCD : domain wall valence quarks,

2+1 flavor staggered sea quarks




Ratio:

EXPERIMENT / Lattice

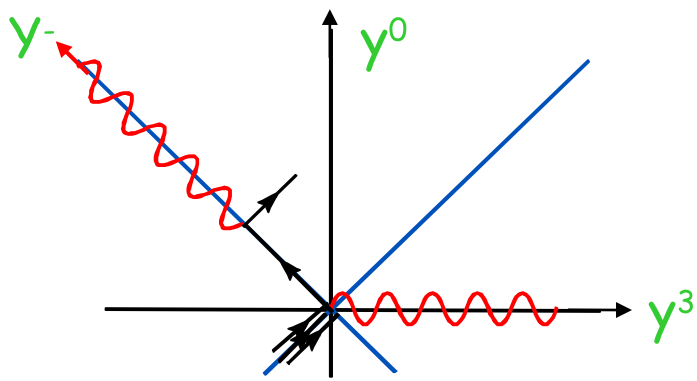
Lattice (normalized to one)

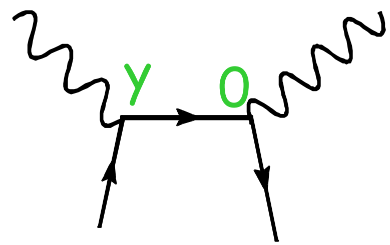
isovector quantities

u - d

(no disconnected contributions)

# Handbag (bilocal) operator :
# new way to probe the nucleon




generalized probe

$$\bar{q}(0)\,\gamma^{\mu}\,q(y) = \bar{q}(0)\,\gamma^{\mu}\,q(0) + y^{-}\,\bar{q}(0)\,\gamma^{\mu}\,\partial^{+}q(0) + \cdots$$

( y ≈ 0 )

$\gamma$ ( $W^{\pm}$, $Z^{0}$ ) probe

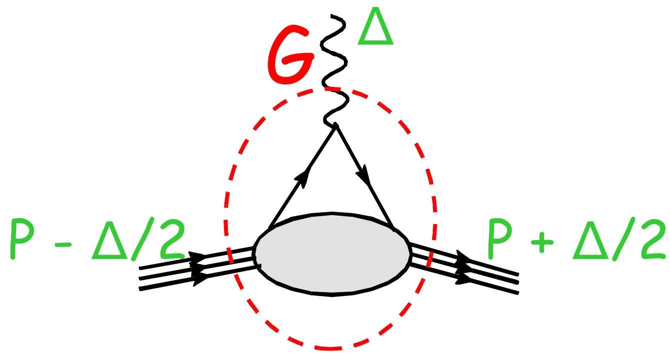
spin 2 (graviton) probe

$\langle N \,|\, \cdots \,|\, N \rangle$

electroweak form factors

energy-momentum form factors

# Energy momentum form factors




nucleon in external classical gravitational field

$\Rightarrow$ G couples to energy-momentum tensor

$$\langle P+\frac{\Delta}{2}\,|\,T^{\mu\nu}(0)\,|\,P-\frac{\Delta}{2}\rangle \qquad (\mu,\nu)\equiv\frac{1}{2}(\mu\nu+\nu\mu)$$

$$=\bar{N}(P+\frac{\Delta}{2})\Big\{A(t)\,\gamma^{(\mu}P^{\nu)}+B(t)\,P^{(\mu}i\sigma^{\nu)\alpha}\frac{\Delta_\alpha}{2\,M}$$
$$+\,C(t)\,\left(\Delta^\mu\Delta^\nu-\Delta^2 g^{\mu\nu}\right)\frac{1}{M}\Big\}N(P-\frac{\Delta}{2})$$

Gordon identity

$$=\bar{N}(P+\frac{\Delta}{2})\Big\{A(t)\,P^\mu P^\nu/M+(A(t)+B(t))\;P^{(\mu}i\sigma^{\nu)\alpha}\frac{\Delta_\alpha}{2\,M}$$
$$+\,C(t)\,\left(\Delta^\mu\Delta^\nu-\Delta^2 g^{\mu\nu}\right)\frac{1}{M}\Big\}N(P-\frac{\Delta}{2})$$

# Momentum sum rule

$$
\begin{aligned}
\langle P \,|\, \hat{P}^{\nu} \,|\, P \rangle &= \langle P \,|\, \int d^3\vec{x}\; T^{0\nu}(x) \,|\, P \rangle \\
&= \lim_{\Delta \to 0} \langle P + \frac{\Delta}{2} \,|\, \int d^3\vec{x}\; T^{0\nu}(x) \,|\, P - \frac{\Delta}{2} \rangle \\
&= \lim_{\Delta \to 0} \int d^3\vec{x}\; e^{-i\vec{x}\cdot\vec{\Delta}} \,\langle P + \frac{\Delta}{2} \,|\, T^{0\nu}(0) \,|\, P - \frac{\Delta}{2} \rangle \\
&= \lim_{\Delta \to 0} (2\pi)^3\, \delta^3(\vec{\Delta})\, \langle P + \frac{\Delta}{2} \,|\, T^{0\nu}(0) \,|\, P - \frac{\Delta}{2} \rangle \\
&= A(0)\; P^{\nu}\; \underbrace{(2P^0)\, (2\pi)^3\, \delta^3(0)} \\
&= A(0)\; P^{\nu} \qquad \langle P \,|\, P \rangle
\end{aligned}
$$

# Momentum sum rule (cont.)

$$\boxed{\langle P \,|\, \hat{P}^{\nu} \,|\, P \rangle \;=\; A(0)\, P^{\nu}\, \langle P \,|\, P \rangle}$$

- Total system : energy-momentum conservation

$$\Downarrow$$

$$A(0) = 1$$

- Physical interpretation in terms of :

quarks : $A_q(0)$

&

gluons : $A_g(0)$

$$\boxed{A_q(0) + A_g(0) \;=\; 1}$$

# Angular momentum sum rule

consider N in rest frame : $P^\mu$ (M,0,0,0) $S^\mu$ (0,0,0,1)

$$\langle P,+\frac{1}{2}\,|\,\hat{j}^{12}\,|\,P,+\frac{1}{2}\rangle \;=\; J\,\langle P,+\frac{1}{2}\,|\,P,+\frac{1}{2}\rangle$$

$$= \langle P,+\frac{1}{2}\,|\int d^3\vec{x}\;\{x^1\,T^{02}(x) - x^2\,T^{01}(x)\}\;|\,P,+\frac{1}{2}\rangle$$

$$= \varepsilon_{ij3}\lim_{\Delta\to 0}\langle P+\frac{\Delta}{2},+\frac{1}{2}\,|\int d^3\vec{x}\;x^i\,T^{0j}(x)\,|\,P-\frac{\Delta}{2},+\frac{1}{2}\rangle$$

$$= \varepsilon_{ij3}\lim_{\Delta\to 0}\int d^3\vec{x}\;x^i\,e^{-i\vec{x}\cdot\vec{\Delta}}\,\langle P+\frac{\Delta}{2},+\frac{1}{2}\,|\,T^{0j}(0)\,|\,P-\frac{\Delta}{2},+\frac{1}{2}\rangle$$

$$= \varepsilon_{ij3}\lim_{\Delta\to 0}\left[i\frac{\partial}{\partial\Delta^i}(2\pi)^3\,\delta^3(\vec{\Delta})\right] \times \langle P+\frac{\Delta}{2},+\frac{1}{2}\,|\,T^{0j}(0)\,|\,P-\frac{\Delta}{2},+\frac{1}{2}\rangle$$

# Angular momentum sum rule (cont.)

$$= \varepsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \, \delta^3(\vec{\Delta}) \left( -i \frac{\partial}{\partial \Delta^i} \right) \Big\{ [A(t) + B(t)] \; \bar{N} \, P^{(0} \, i\sigma^{j)\alpha} \frac{\Delta_\alpha}{2\,M} \, N$$

$$+ \text{terms independent of } \Delta$$

$$+ \text{terms quadratic in } \Delta \}$$

$$= \varepsilon_{ij3} \, (2\pi)^3 \, \delta^3(0) \; [A(0) + B(0)] \, \frac{1}{2\,M}$$

$$\times \, \bar{N}(P, +\tfrac{1}{2}) \left( -\frac{1}{2} \right) \{ P^0 \, \sigma^{ji} + P^j \, \sigma^{0i} \} \; N(P, +\tfrac{1}{2})$$

$M$ $0$ in rest frame

$$= (2\pi)^3 \, \delta^3(0) \; [A(0) + B(0)] \, \frac{1}{2\,M} \, M \; \underbrace{\bar{N}(P, +\tfrac{1}{2}) \, \sigma^{12} \, N(P, +\tfrac{1}{2})}_{2\,M}$$

$$= \frac{1}{2} \, [A(0) + B(0)] \, \langle P \,|\, P \rangle$$

# Angular momentum sum rule (cont.)

$$\boxed{J = \frac{1}{2}\left[A(0) + B(0)\right]}$$

- Total system : angular momentum conservation

$$A(0) + B(0) = 1 \quad \Rightarrow \quad B(0) = 0$$

- Physical interpretation in terms of quarks & gluons

$$\boxed{J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right]}$$

$$\tfrac{1}{2} = J_q + J_g = \tfrac{1}{2}\,\Delta\Sigma + L_q + J_g$$

$$\vec{J}_q = \int d^3\vec{x}\left[\underbrace{\Psi^\dagger \frac{\vec{\Sigma}}{2}\Psi}_{\frac{1}{2}\Delta\Sigma} + \underbrace{\Psi^\dagger\,\vec{x}\times\left(-i\,\vec{D}\right)\Psi}_{L_q}\right]$$

$$\vec{J}_g = \int d^3\vec{x}\;\vec{x}\times\left(\vec{E}\times\vec{B}\right)$$

# $T^{\mu\nu}$ form factors in terms of GPDs

$$\langle P + \frac{\Delta}{2} \,|\, T_q^{\mu\nu}(0) \,|\, P - \frac{\Delta}{2} \rangle \, n_\mu \, n_\nu$$

$$= \langle P + \frac{\Delta}{2} \,|\, \bar{q} \, i \, \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \, q\,(0) \,|\, P - \frac{\Delta}{2} \rangle \, n_\mu \, n_\nu$$

$$= \bar{N} \left\{ \frac{1}{M} \left[ A(t) + 4\,\xi^2 \, C(t) \right] + \left[ A(t) + B(t) \right] \, i\sigma^{\nu\alpha} \frac{\Delta_\alpha}{2\,M} n_\nu \right\} N$$

$$\frac{P^+}{2\pi} \int dy^- \, e^{ixP^+y^-} \, \langle P + \frac{\Delta}{2} \,|\, \bar{q}\left(-\frac{y}{2}\right) \, i\gamma \cdot n \, n \cdot \overleftrightarrow{D} \, q\left(\frac{y}{2}\right) \,|\, P - \frac{\Delta}{2} \rangle_{y^+=0,\, y_\perp=0}$$

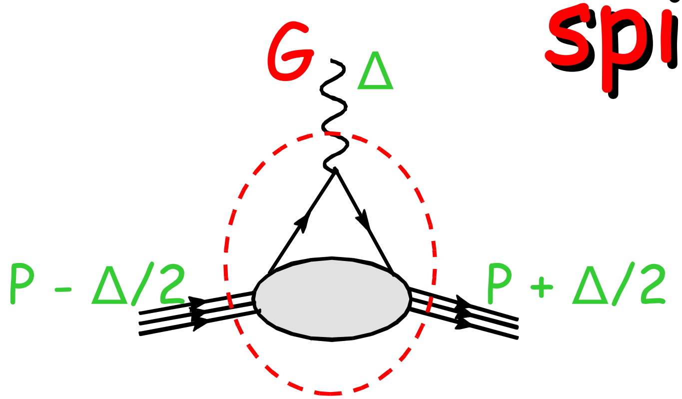
$$-i\,n \cdot \overleftrightarrow{D} = xP \cdot n = x$$

$$= x \, \frac{P^+}{2\pi} \int dy^- \, e^{ixP^+y^-} \, \langle P + \frac{\Delta}{2} \,|\, \bar{q}\left(-\frac{y}{2}\right) \, \gamma \cdot n \, q\left(\frac{y}{2}\right) \,|\, P - \frac{\Delta}{2} \rangle_{y^+=0,\, y_\perp=0}$$

$$= \bar{N} \left\{ \frac{1}{M} x \, H(x,\xi,t) \;+\; x \, \left[ H(x,\xi,t) + E(x,\xi,t) \right] \, i\sigma^{\mu\nu} \frac{\Delta_\nu}{2M} n_\mu \right\} N$$

$$\int_{-1}^{1} dx \qquad \text{of both lhs and rhs}$$

# Energy momentum form factors / spin of nucleon




nucleon in external classical gravitational field

⇒ G couples to energy-momentum tensor

$$(\mu,\nu) \equiv \frac{1}{2}(\mu\nu + \nu\mu)$$

$$\langle N \,|\, T^{\mu\nu}(0) \,|\, N \rangle$$

$$= \bar{N}\left\{ A(t)\,\gamma^{(\mu}P^{\nu)} + B(t)\,P^{(\mu}i\sigma^{\nu)\alpha}\frac{\Delta_\alpha}{2\,M} + C(t)\,\left(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu}\right)\frac{1}{M}\right\} N$$

⇒ link to GPDs :

X. Ji (1997)

$$\int_{-1}^{1} dx\, x\, H(x,\xi,t) \;=\; A(t) + 4\,\xi^2\, C(t)$$

$$\int_{-1}^{1} dx\, x\, E(x,\xi,t) \;=\; B(t) - 4\,\xi^2\, C(t)$$

⇒ SPIN sum rule

$$\boxed{\int_{-1}^{1} dx\, x\left\{H^q(x,\xi,0) + E^q(x,\xi,0)\right\} = A(0) + B(0) = 2\,J^q}$$

# quark contribution to proton spin

$$2\,J^q = \int_{-1}^{1} dx\, x \left\{ H^q(x,0,0) + E^q(x,0,0) \right\}$$

$$\boxed{J^q = \frac{1}{2}\Delta q + L^q}$$

X. Ji (1997)

$$\boxed{2\,J^q = M_2^q + \int_{-1}^{1} dx\, x\, E^q(x,0,0)}$$

with $M_2^q = \int_0^1 dx\, x\, [q(x) + \bar{q}(x)]$

parametrizations for $E^q$ : $E^q(x,0,0) = \kappa_q / N_q\, (1-x)^{\eta_q}\, q_v(x)$

| PROTON | $M_2^q$ | 2 $J^q$ valence model (GPV 01, GPRV 04) | 2 $J^q$ Lattice (QCDSF) |
|---|---|---|---|
| u | 0.37 | 0.58 | 0.66 ± 0.04 |
| d | 0.20 | -0.06 | -0.04 ± 0.04 |
| s | 0.04 | 0.04 | |
| u + d + s | **0.61** | **0.56** | **0.62 ± 0.08** |

GPD : based on MRST2002

$\mu^2$ = 2 $GeV^2$

lattice : full QCD, no disconnected diagrams so far

# orbital angular momentum carried by quarks : solving the spin puzzle

$$\boxed{2\,J^q = M_2^q + \int_{-1}^{1} dx\, x\, E^q(x,0,0)}$$ with $$M_2^q = \int_0^1 dx\, x\, [q(x) + \bar{q}(x)]$$

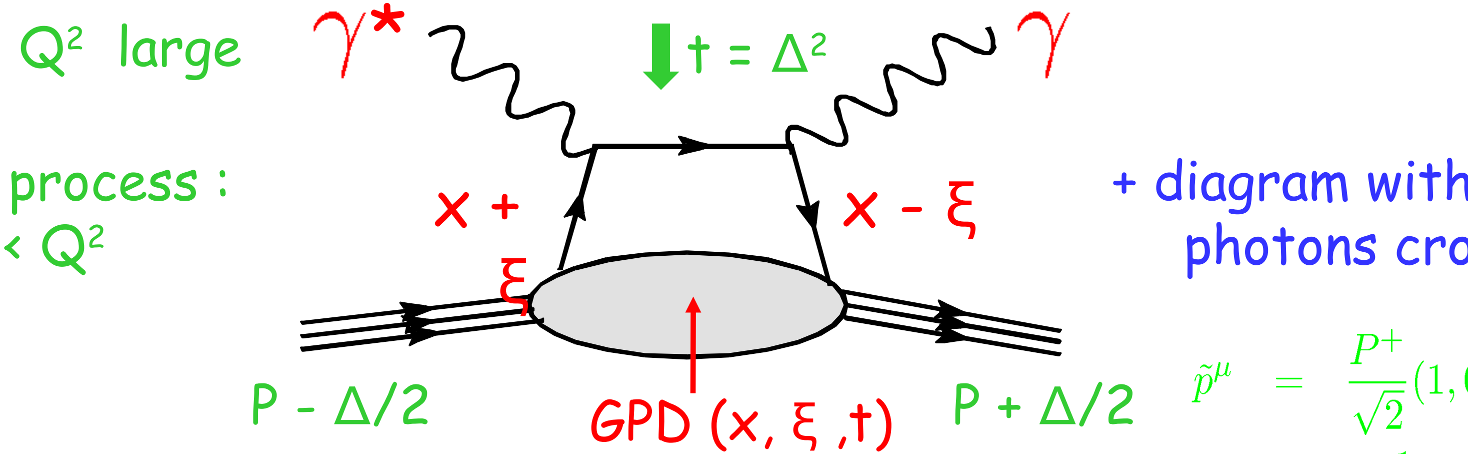
Ji (1997)

$$\boxed{J^q = \frac{1}{2}\Delta q + L^q}$$

evaluated at $\mu^2 = 2.5\ \mathrm{GeV}^2$

| PROTON | **2 $J^q$** GPD model (GPV 01) | **$\Delta q$** HERMES (1999) | **2 $L^q$** |
|---|---|---|---|
| u | 0.61 | 0.57 ± 0.04 | 0.04 ∓ 0.04 |
| d | -0.05 | -0.25 ± 0.08 | 0.20 ∓ 0.08 |
| s | 0.04 | -0.01 ± 0.05 | 0.05 ∓ 0.05 |
| u + d + s | **0.60** | **0.30 ± 0.10** | **0.30 ∓ 0.10** |

# Deeply Virtual Compton Scattering




$$\tilde{p}^\mu = \frac{P^+}{\sqrt{2}}(1,0,0,1)$$

$$n^\mu = \frac{1}{P^+\sqrt{2}}(1,0,0,-1)$$

$$\begin{aligned}
&H^{\mu\nu}_{L.O.\,DVCS} \\
&= \frac{1}{2}\left[\tilde{p}^\mu n^\nu + \tilde{p}^\nu n^\mu - g^{\mu\nu}\right] \int_{-1}^{+1} dx \left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon}\right] \\
&\quad \times \left[H^p_{DVCS}(x,\xi,t)\, \bar{N}(p')\gamma.nN(p) \;+\; E^p_{DVCS}(x,\xi,t)\, \bar{N}(p')i\sigma^{\kappa\lambda}\frac{n_\kappa\Delta_\lambda}{2m_N}N(p)\right] \\
&+ \frac{1}{2}\left[-i\varepsilon^{\mu\nu\kappa\lambda}\tilde{p}_\kappa n_\lambda\right] \int_{-1}^{+1} dx \left[\frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon}\right] \\
&\quad \times \left[\tilde{H}^p_{DVCS}(x,\xi,t)\, \bar{N}(p')\gamma.n\gamma_5 N(p) \;+\; \tilde{E}^p_{DVCS}(x,\xi,t)\, \bar{N}(p')\gamma_5\frac{\Delta\cdot n}{2m_N}N(p)\right]
\end{aligned}$$

# DVCS (cont.)

- $$H^p_{DVCS}(x,\xi,t) = \frac{4}{9} H^{u/p} + \frac{1}{9} H^{d/p} + \frac{1}{9} H^{s/p}$$
  and similarly for $\tilde{H}_{DVCS}\,, E_{DVCS}\,, \tilde{E}_{DVCS}$
- twist-2 DVCS amplitude independent of Q ➡ SCALING !
- DVCS amplitude is complex : imaginary part -> $x = \xi$
- $Q^2$, $\xi$, and $t$ are kinematic variables

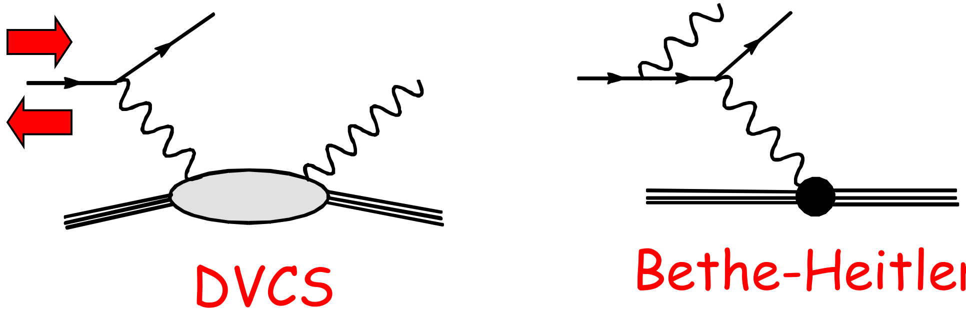
  with $\xi = \frac{x_B/2}{1 - x_B/2}$ and $x_B = \frac{Q^2}{2p \cdot q}$
- variable $x$ is integrated over ( $x \neq x_B$ ! )

  DVCS amplitude is sensitive to GPDs weighted with coefficient functions

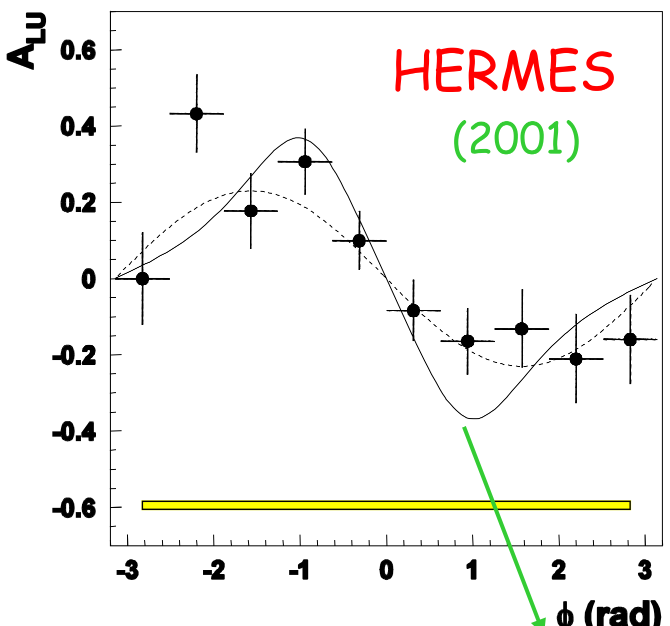
# DVCS : beam spin asymmetry

$A_{LU}$ =

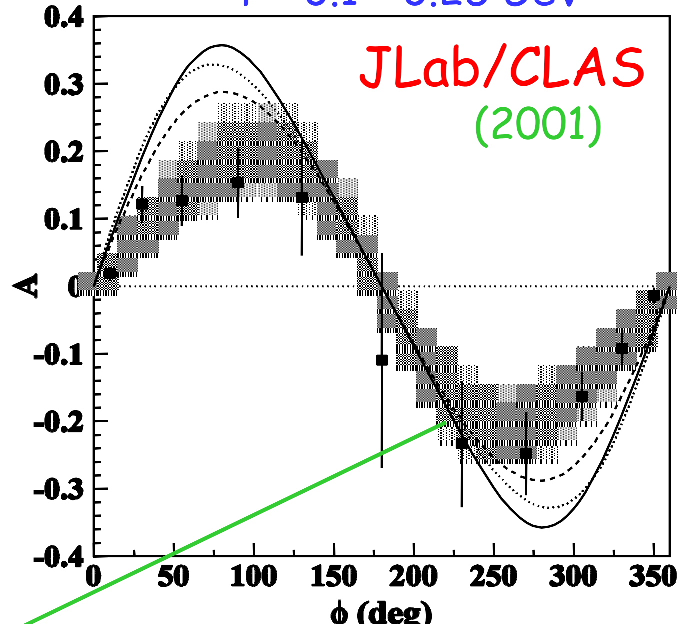
(BH) * Im(DVCS) * sin Φ




$Q^2$ = 2.6 $GeV^2$ , $x_B$ = 0.11, -t = 0.27 $GeV^2$




$Q^2$ = 1 – 1.5 $GeV^2$ , $x_B$ = 0.15 – 0.25, -t = 0.1 - 0.25 $GeV^2$

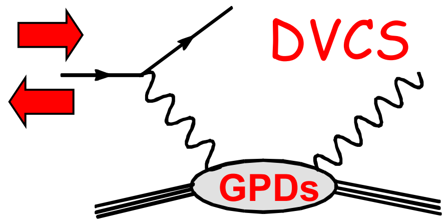



twist-2 + twist-3 : Kivel, Polyakov, Vdh (2000)

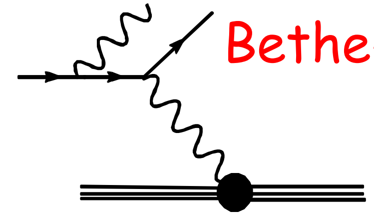
# DVCS on proton

## JLab/Hall A @ 6 GeV

(2006)

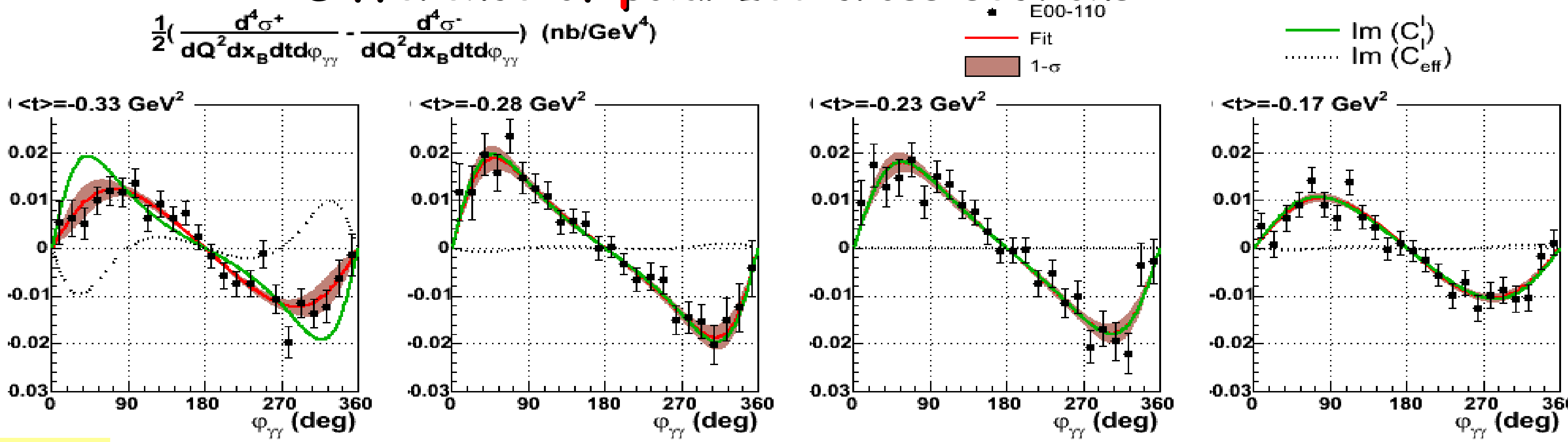



DVCS



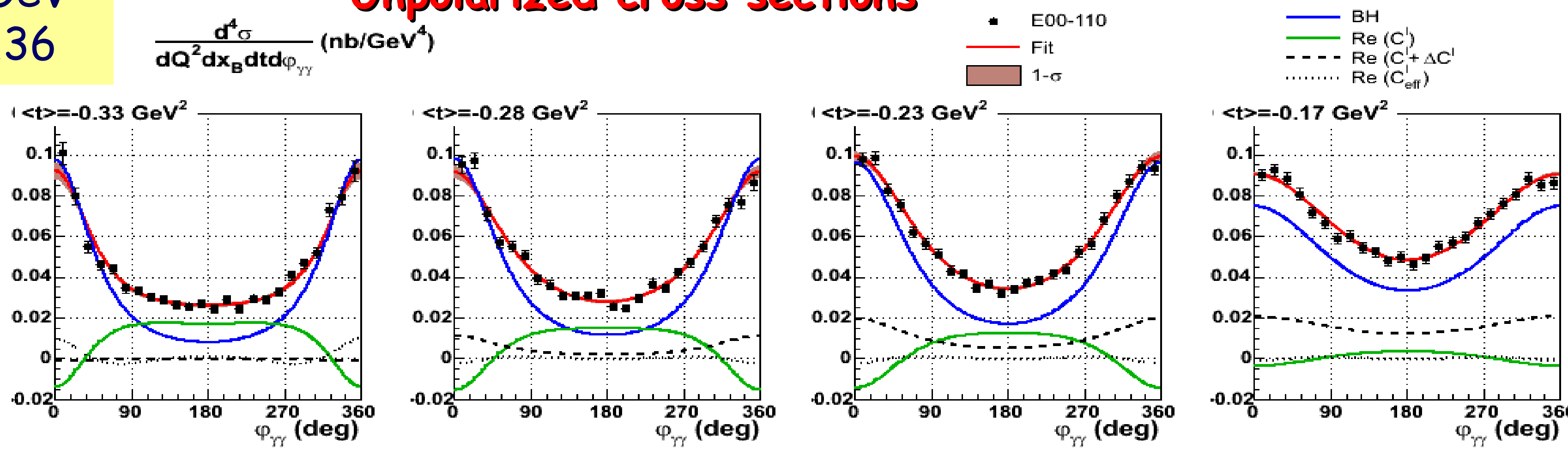
Bethe-Heitler

## Difference of polarized cross sections




$Q^2 \approx 2$ GeV$^2$
$x_B = 0.36$

## Unpolarized cross sections




also JLab/CLAS, HERMES, H1 / ZEUS

# DVCS on neutron

$$C_n^I(F) = F_1(t)\ H + \xi\ (F_1(t) + F_2(t))$$

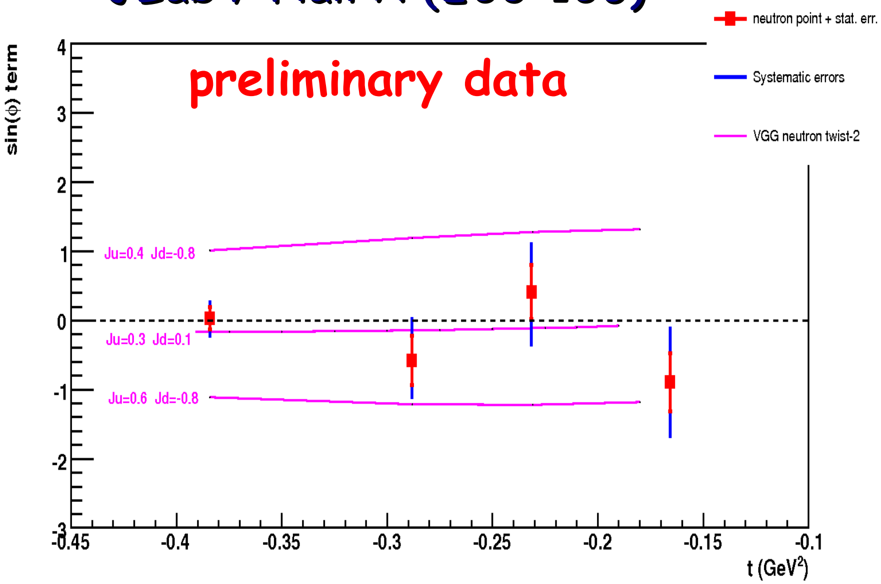
$$\frac{t}{4M^2}\ \boxed{F_2(t)\ E}$$

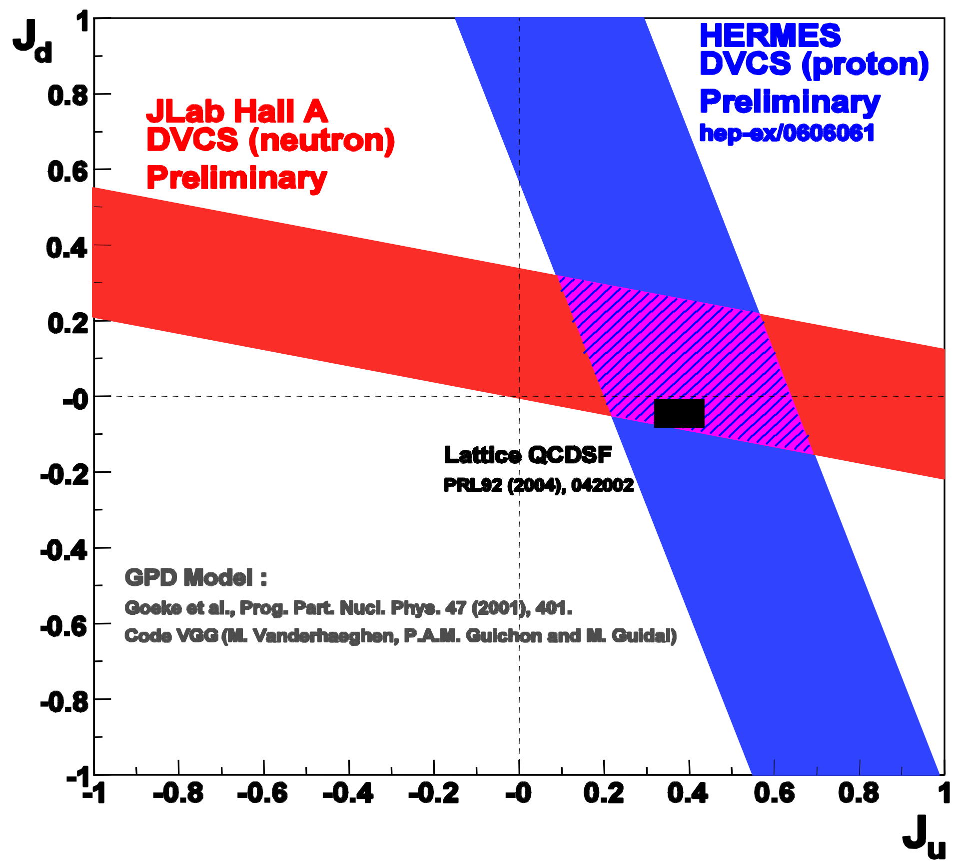
0 because $F_1(t)$ is small

0 because of cancelation of u and d quarks

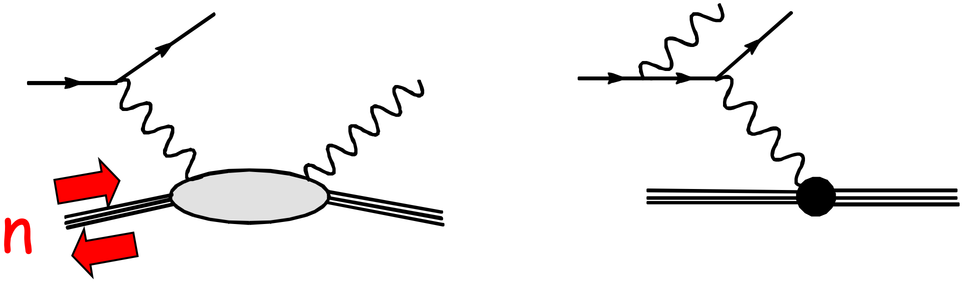
n-DVCS gives access to the least known and constrained GPD, E

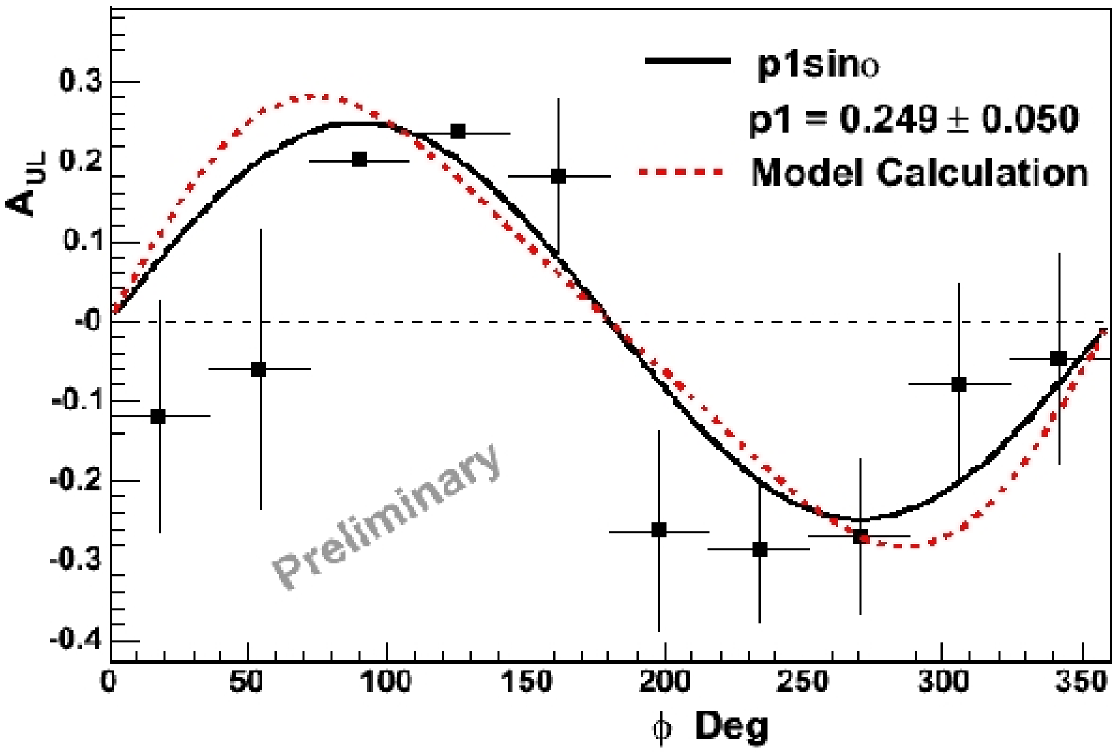
## JLab / Hall A (E03-106) :



# DVCS : target spin asymmetry




JLab/CLAS

mainly sensitive to

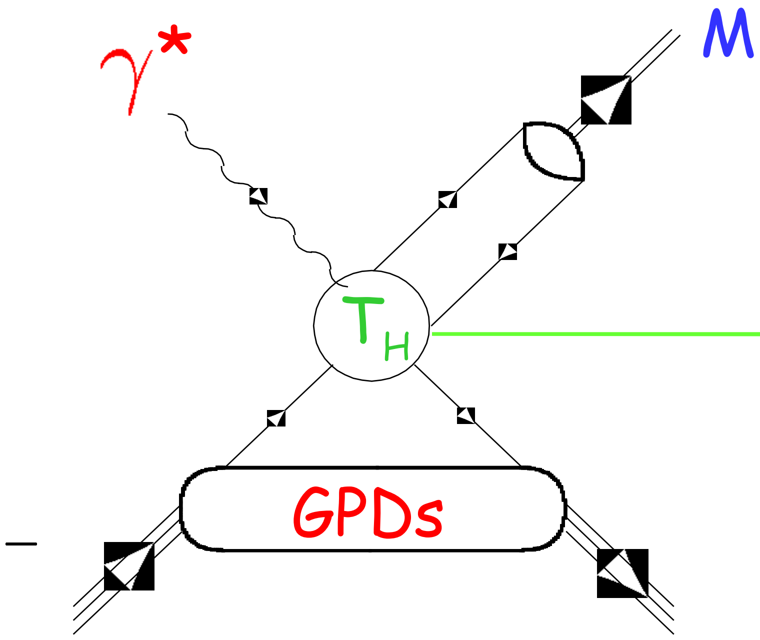
GPD $\tilde{H}$

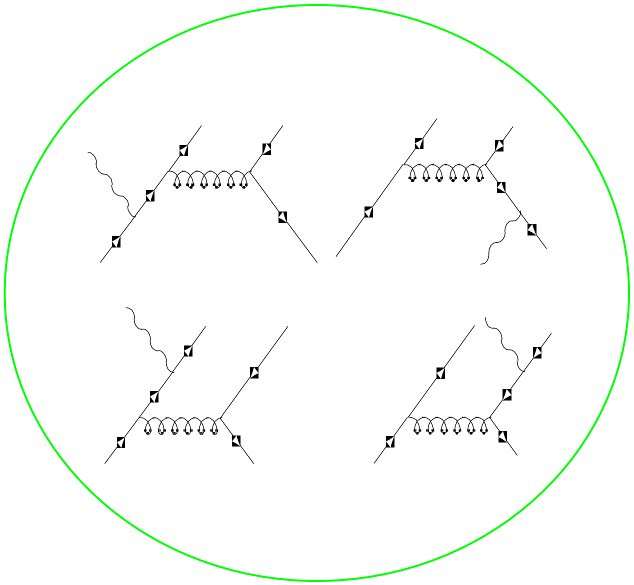
calculation :

Vdh, Guichon, Guidal

(1999)

# Hard electroproduction of mesons ($\rho^{0,\pm}$ , $\omega$ , $\phi$, $\pi$, ...)



Factorization theorem shown for longitudinal photon

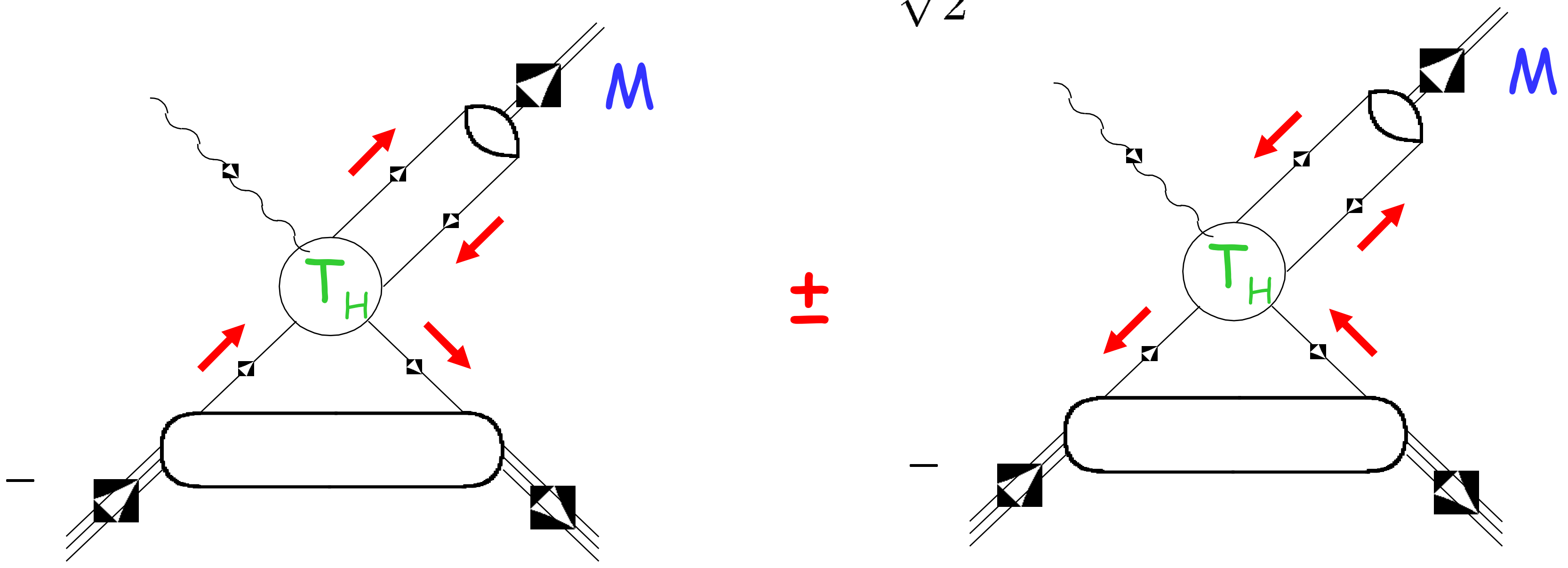
hard scattering amplitude

Collins, Frankfurt, Strikman (1997)

# Meson acts as helicity filter

longitudinally pol. Vector meson $\quad |\rho_L\rangle = \frac{1}{\sqrt{2}} | \uparrow\downarrow + \downarrow\uparrow\rangle$

PseudoScalar meson $\quad |\pi\rangle = \frac{1}{\sqrt{2}} | \uparrow\downarrow - \downarrow\uparrow\rangle$




- Vector meson : accesses unpolarized GPDs H and E
- PseudoScalar meson : accesses polarized GPDs $\tilde{H}$ and $\tilde{E}$

# Hard electroproduction of vector mesons ($\rho^{0,\pm}$ , $\omega$ , $\phi$)

➡ amplitude for longitudinally polarized vector meson

$$\mathcal{M}^L_{V_L} = -ie\,\frac{4}{9}\,\frac{1}{Q}\left[\int_0^1 dz\,\frac{\Phi_{V_L}(z)}{z}\right]\frac{1}{2}\,(4\pi\alpha_s)$$
$$\times\left\{A_{V_L N}\,\bar{N}(p')\,\gamma\cdot n\,N(p) + B_{V_L N}\,\bar{N}(p')\,i\sigma^{\kappa\lambda}\frac{n_\kappa\Delta_\lambda}{2\,m_N}\,N(p)\right\}$$

➡ leading (1 gluon exchange) amplitude depends on $\alpha_s$ goes as 1/Q

➡ dependence on meson distribution amplitude $\Phi_V$

$$\Phi_{V_L}(z) = f_V\,6\,z\,(1-z)$$

with $f_\rho = 0.216$ GeV, $f_\omega = 0.195$ GeV, from $V \to e^+e^-$

# Flavor decomposition of GPDs H and E

$\rho^0$

$$A_{\rho^0_L\, p} = \int_{-1}^{1} dx \;\; \frac{1}{\sqrt{2}} \left(e_u\, H^u - e_d\, H^d\right) \left\{ \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right\}$$

$$B_{\rho^0_L\, p} = \int_{-1}^{1} dx \;\; \frac{1}{\sqrt{2}} \left(e_u\, E^u - e_d\, E^d\right) \left\{ \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right\}$$

$\rho^\pm$

$$A_{\rho^+_L\, n} = -\int_{-1}^{1} dx \;\; \left(H^u - H^d\right) \left\{ \frac{e_u}{x - \xi + i\epsilon} + \frac{e_d}{x + \xi - i\epsilon} \right\}$$

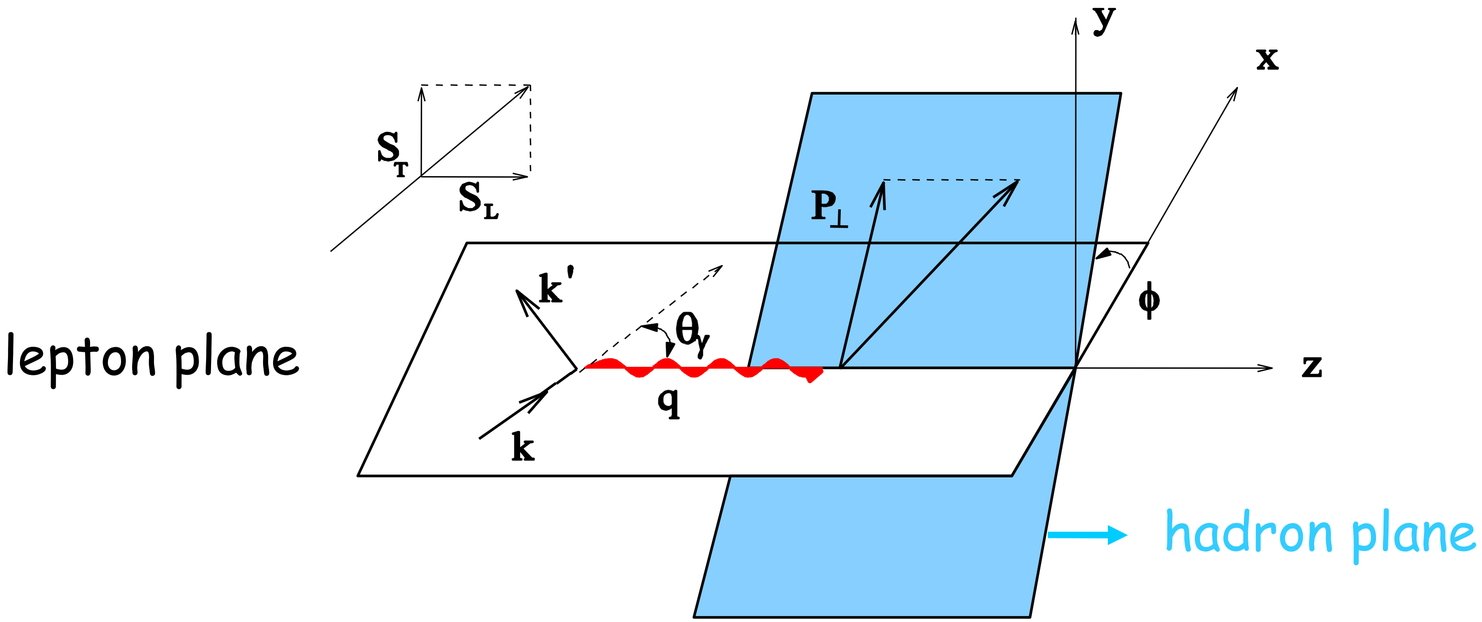
$$B_{\rho^+_L\, n} = -\int_{-1}^{1} dx \;\; \left(E^u - E^d\right) \left\{ \frac{e_u}{x - \xi + i\epsilon} + \frac{e_d}{x + \xi - i\epsilon} \right\}$$

$\omega$

$$A_{\omega_L\, p} = \int_{-1}^{1} dx \;\; \frac{1}{\sqrt{2}} \left(e_u\, H^u + e_d\, H^d\right) \left\{ \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right\}$$

$$B_{\omega_L\, p} = \int_{-1}^{1} dx \;\; \frac{1}{\sqrt{2}} \left(e_u\, E^u + e_d\, E^d\right) \left\{ \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right\}$$

# Hard electroproduction of mesons : target normal spin asymmetry




in leading order (in Q) ➡ 2 observables $\sigma = \sigma_L + P_n \; \sigma_L^n$

Target polarization normal to hadron plane

➡ Asymmetry : $\boxed{\mathcal{A} = \frac{2\,\sigma_L^n}{\pi\,\sigma_L}}$

# Hard electroprod. of vector mesons : target normal spin asymmetry

$$\mathcal{A}_{V_L N} = -\frac{2\,|\Delta_\perp|}{\pi} \times \frac{\mathrm{Im}(AB^*)\;/\;m_N}{|A|^2\,(1-\xi^2) - |B|^2\;(\xi^2 + t/(4m_N^2)) - \mathrm{Re}(AB^*)\,2\,\xi^2}$$

- A $\longrightarrow$ GPD H      B $\longrightarrow$ GPD E
- linear dependence on GPD E $\longleftrightarrow$ unpolarized cross section
- ratio : less sensitive to NLO and higher twist effects
- sensitivity to $J^u$ and $J^d$

measure of TOTAL angular momentum contribution to proton spin

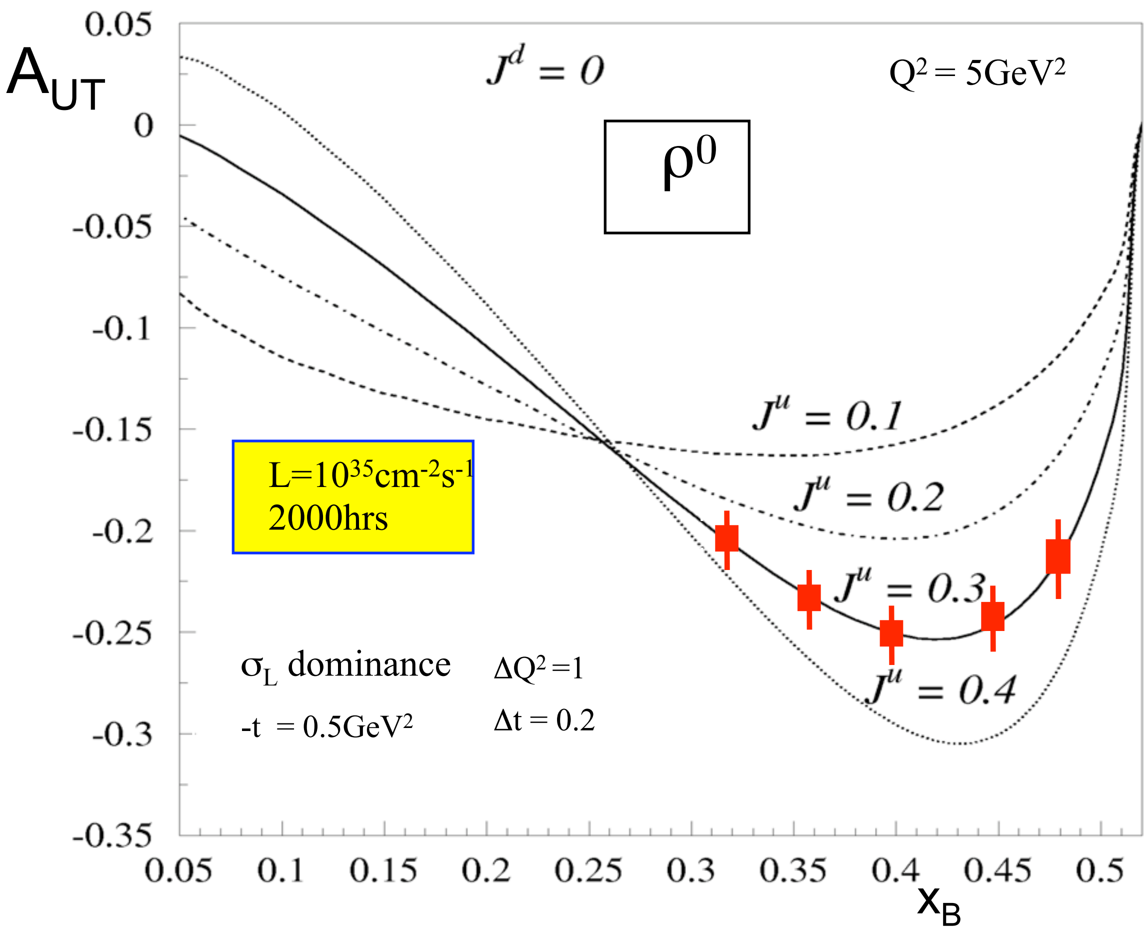
# exclusive $\rho^0$ prod. with transverse target

$$A_{UT} = - \frac{2\Delta\,(\mathrm{Im}(AB^*))/\pi}{|A|^2(1-\xi^2) - |B|^2(\xi^2+t/4m^2) - \mathrm{Re}(AB^*)2\xi^2}$$

$\boxed{\rho^0}$

$A \sim (2H^u + H^d)$

$B \sim (2E^u + E^d)$




**Asymmetry depends linearly on the GPD E, which enters Ji's sum rule.**

K. Goeke, M.V. Polyakov, M. Vdh (2001)