

# HMS Matrix Elements and Kinematic Offsets

G. MacLachlan

Institute of Nuclear and Particle Physics, Ohio U., Athens, OH 45701

May 05, 2002

## Abstract

This report is an extension to a previous report regarding the optimization of the HMS delta matrix elements completed in December of 2001. Testing of the new matrix elements involved studying the reconstruction of the invariant mass of the recoiling proton observed for liquid hydrogen (LH2) data. Several effects, both physical and otherwise, conspire to give spurious results for the invariant mass and have been addressed. Kinematic offsets specific to the Hall C beam energy and the High Momentum Spectrometer (HMS) have been studied by Eric Christy[1] and the results have been adapted to the E93-038 LH2 data. Also, radiative corrections and energy loss in the target and HMS arm have been accounted for. Finally, a minor change to ENGINE must be made to correctly analyze the invariant mass for LH2 data.

## 1 Introduction

The invariant mass of a system, initially at rest, recoiling from a lepton with incident energy  $E$ , in this case an electron, is given by

$$W = \sqrt{W^2} = \sqrt{M^2 + 2M\nu - Q^2} \quad (1)$$

where  $M$  is the target mass,  $\nu = E - E'$ , and  $Q^2 = 4EE' \sin^2(\theta/2)$ . The energy and angle of the lepton after scattering is given by  $E'$  and  $\theta$  respectively. The invariant mass squared is calculated slightly differently but equivalently by the E93-038 ENGINE<sup>1</sup>. It is given by

$$\mathbf{hs\_w2} = (\mathbf{hs\_qe} + \mathbf{0.93956563}) * *2 - \mathbf{hs\_qx} * *2 - \mathbf{hs\_qy} * *2 - \mathbf{hs\_qz} * *2. \quad (2)$$

If one were to make no changes to ENGINE and were to analyze an LH2 run, e.g. 37823, the invariant mass of the proton would appear as in figure 1. For this run, which was  $Q^2 = 1.14$ ,  $E = 2.324\text{GeV}$ ,  $P_{cent} = 1.722\text{GeV}$ , and,  $\theta = 30.88^\circ$ , one would expect the invariant mass for elastic p(e,e)p scattering to be  $0.93827\text{GeV}/c^2$ . The data presented in figure 1 is subject to the condition that there be a positive charge identified in one of the front veto detectors. Further, the low energy tail in the scattered energy spectrum has also been filtered out. One sees that ENGINE identifies the invariant mass as  $0.9298\text{GeV}/c^2$  which is approximately  $8.5\text{MeV}/c^2$  too small. This error is unacceptably large given the capabilities of the HMS.

Several effects have been identified that contribute to the poor reconstruction of the invariant mass. The most trivial effect has already been mentioned and is the hard-wiring of the neutron mass into ENGINE. There are also radiative corrections and energy loss effects that occur in both the target and HMS arm that must be accounted for especially since the E93-038 experiment is the first to make use of a 15cm target in Hall C. The most difficult corrections to make are the three offsets to the beam energy, scattered electron energy, and spectrometer angle,  $\delta E$ ,  $\delta E'$ ,  $\delta\theta$  respectively.

The offset in the scattered electron energy corresponds directly to the uncertainty in the HMS dipole field setting. The E93-038 collaboration suffers from the added uncertainty that arose from the use of a backup power supply during data gathering. This effect, of course, had not been accounted for during the study[1] made by Christy.

---

<sup>1</sup>Note here that the neutron mass is hardwired into the `p_ntupleraw_keep.f` routine and ought to be changed for analyzing LH2 data.

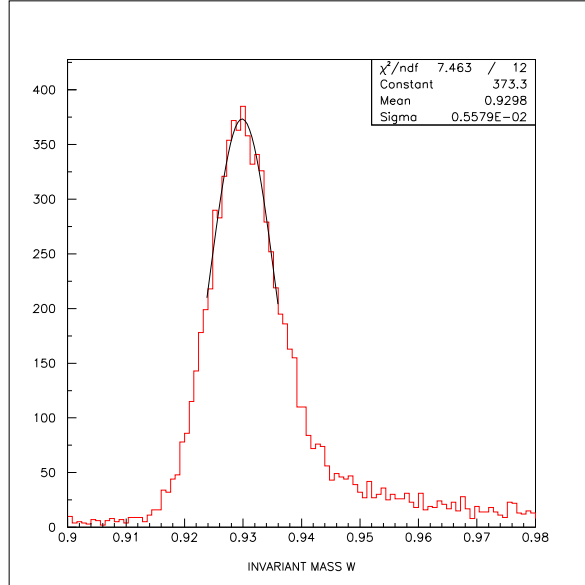


Figure 1: Invariant mass for LH2.

The offset in the spectrometer angle,  $\theta$  has been attributed to several effects such as collimator misalignment and warpage of rails that the HMS sits upon. The beam energy offset is rooted in the  $B \cdot dl$  calculations made for the Hall C arc that was supplied by the ACC[2] and which were included in our EPICS readout. Also interesting are the comparisons between the Hall A beam energy measurements and those provided by ACC. One sees that for beam energies similar to that used for E93-038 LH2 data, disagreements between ACC measurements and Hall A are known to occur on the order of 0.15 – 0.25% which is rather large given the purported 0.01% resolution obtainable via arc measurements.

It is also worthwhile to note that the nominal beam energy, 2.324 GeV, which can be found in `jan99.kinematics` file is in disagreement with the beam energy logged by the EPICS variable, `MBSY3C_energy`. The beam energy value logged by EPICS is 2.329-2.330 GeV.

## 2 First Order Corrections to W

The Taylor's expansion of  $W = W(E, E', \theta)$  is given by [3]

$$W(\vec{\xi} + \delta\vec{\xi}) = \sum_{j=0}^{\infty} \left[ \frac{1}{j!} (\delta\vec{\xi} \cdot \vec{\nabla}_{\vec{\xi}'} ) W(\vec{\xi}') \right]_{\vec{\xi}' = \vec{\xi}} \quad (3)$$

and from which the first order term is found to be

$$(\delta\vec{\xi} \cdot \vec{\nabla}_{\vec{\xi}'} ) W(\vec{\xi}') \Big|_{\vec{\xi}' = \vec{\xi}} \quad (4)$$

where  $\vec{\xi} = E\hat{i} + E'\hat{j} + \theta\hat{k}$  and  $\vec{\nabla} = \hat{i}\frac{\partial}{\partial E} + \hat{j}\frac{\partial}{\partial E'} + \hat{k}\frac{\partial}{\partial \theta}$ .

The partial derivatives of W are shown in their explicit forms below.

$$\frac{\partial W}{\partial E} = \frac{2M_p - 4E' \sin^2(\theta/2)}{2W}, \quad (5)$$

$$\frac{\partial W}{\partial E'} = -\frac{2M_p + 4E' \sin^2(\theta/2)}{2W}, \quad (6)$$

and

$$\frac{\partial W}{\partial \theta} = -\frac{4EE' \sin(\theta/2) \cos(\theta/2)}{2W}. \quad (7)$$

It is obvious that for increasing beam energy,  $W$  also becomes larger. However, for increasing scattered electron energy or spectrometer angle,  $W$  becomes smaller.

Although Christy describes the method for determining these three offsets in detail in his report, a brief summary will be informative. The problem essentially involves finding the vector  $\delta\vec{\xi}$  with non-zero magnitude such that

$$\delta\vec{\xi} \cdot \vec{\nabla}W = 0 \quad (8)$$

at points where the surface  $W$  is numerically equal to the proton mass. This vector is not unique because the problem is underconstrained. The  $\delta E'$  and  $\delta\theta$  components of  $\delta\vec{\xi}$  are held fixed and the  $\delta E$  component is allowed to vary until the condition in equation 8 is met.

For the E93-038 LH2 data, the numerical values of  $\vec{\nabla}W$  are found to be approximately

$$\frac{\partial W}{\partial E} = 1.70, \quad (9)$$

$$\frac{\partial W}{\partial E'} = -2.32, \quad (10)$$

and

$$\frac{\partial W}{\partial \theta} = -2.19 \text{ MeV/mrad} \quad (11)$$

which agree well with Christy's values[1].<sup>2</sup>

### 3 Energy Loss and Radiative Effects

The ionization energy loss of the electron through the target apparatus and HMS arm has been taken into account for the calculation of the invariant mass. Figure 2 shows data that were analyzed with `total_loss.f` both enabled and disabled. The red set of data shows the effect that adding energy loss to ENGINE has while the blue set of data are the the uncorrected set. The routine `total_loss.f` calculates the  $dE/dx$  for the electron both before and after the primary scattering vertex event by event[4]. Prior to the primary scattering vertex, `total_loss.f` calculates the energy lost by the incident beam and subtracts that amount. After the primary vertex, `total_loss.f` calculates the energy lost by the scattered electron and adds this amount back to the scattered electron's energy. A comparison between the invariant mass with energy loss included and without is made in figure 2. The overall effect of the inclusion of energy loss is a reduction of  $W$  on the order of 5.5 MeV.

Radiative effects have also been studied by simulating (e,e)p events. Electrons radiate in the presence of an electric field. The incoming and outgoing electrons can interact with the Coulomb fields of the media they pass through. Radiative interactions take place between the the incident electron and both the electron cloud as well as the nuclei of the material and these cross sections are well known for elastic scattering[4]. There are two effects associated with radiative corrections. The first is that the cross section for the scattering process is modified. The second is that the kinematics of the electron (energy, momentum, and angle) are also modified.

Two sets of simulated data were generated, one with effects included and without effects. The difference then is the offset in  $W$  introduced by real radiative effects. The shift in  $W$  for the LH2 data taken in December is found to be a further reduction in  $W$  on the order of 1.0 to 1.2 MeV. The results of the simulation for the radiative effects can be seen in figure 3.

---

<sup>2</sup>Note that here, as in Christy's report,  $\partial/\partial E \rightarrow E\partial/\partial E$  and  $\partial/\partial E' \rightarrow E'\partial/\partial E'$ .

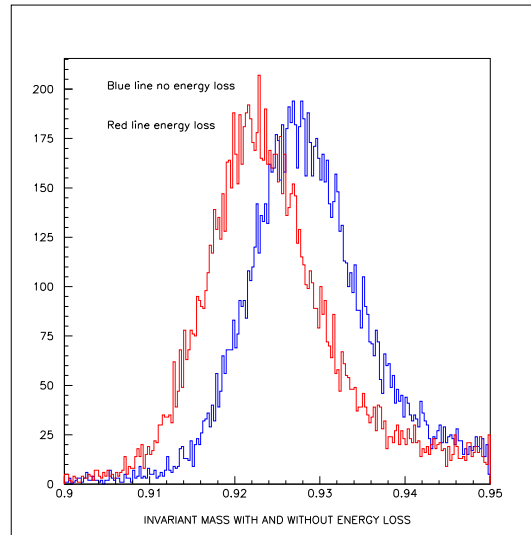


Figure 2: LH2 data with and without energy loss.

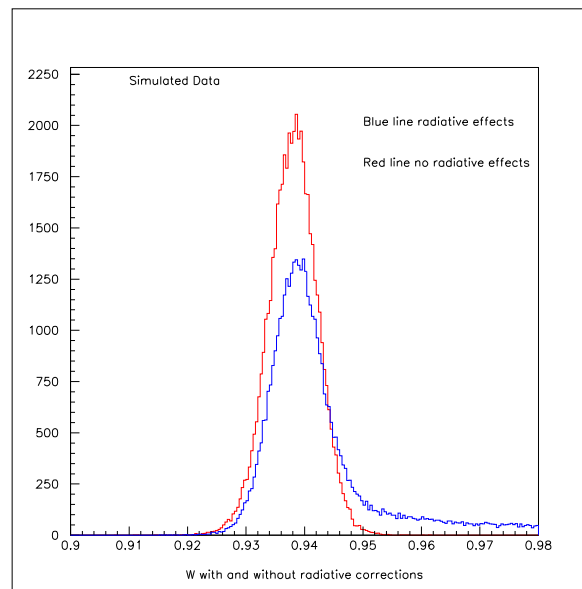


Figure 3: LH2 data with and without radiative corrections.

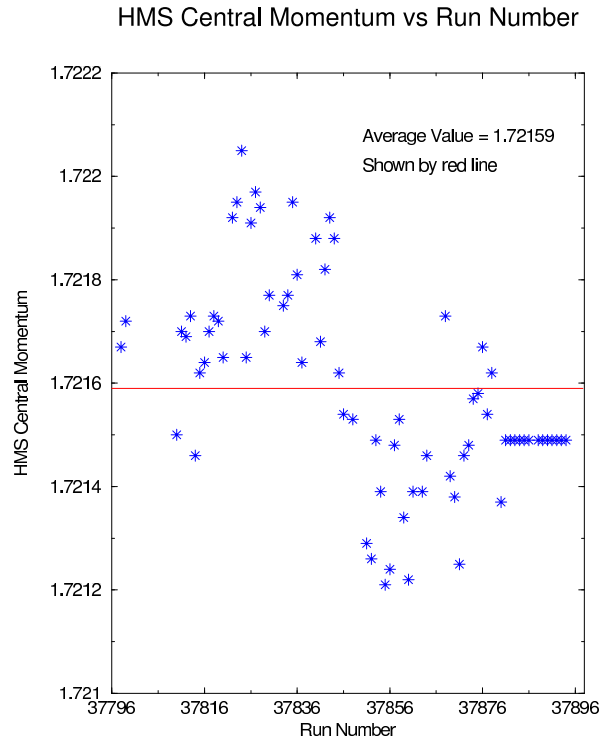


Figure 4: HMS central momentum fluctuated for much of the LH2 data gathering. Note that the central momentum stabilized toward the end of the run cycle.

## 4 HMS Dipole Field Uncertainty

Under normal operation, the HMS dipole field is thought to be stable out to at the  $5 \cdot 10^{-6}$  level. Figure 4 shows the HMS dipole field, as measured by an NMR probe and extracted from EPICS data, as a function of run number. It is plain to see that the field was not stable for most of the LH2 run cycle. The cause of this fluctuation is still under investigation however, during this period the HMS dipole was very troublesome and the usual power supply had been replaced with a back up. The stable value of the HMS central momentum seen at the end of the set of runs in figure 4 is probably due to an IOC crash. There is no mention of this in the online log.

The fluctuations give rise to a relative uncertainty in the HMS central momentum on the order of  $\Delta P_{cent} = 0.03\%$ . The fluctuations of the HMS dipole field will be followed up in a forthcoming report.

## 5 Results

After taking into account radiative effects, energy loss, and Christy's kinematic offsets, one may then determine the beam energy using the constraint of  $W = 938.3\text{MeV}$ . For the LH2 data, I find that a beam energy of  $2.329\text{GeV}$  gives the proper offset to align the invariant mass of the proton. This also coincides with the EPICS read back value stored in the log files. Again, this disagrees with the value of  $2.329\text{GeV}$  stored by ENGINE. The offsets used for these data are listed table 1. A comparison between uncorrected and corrected data is made in Figure 5. Note that the red data are uncorrected and the blue data have been corrected.

second set of data have been analyzed with  $Q^2 = 1.47\text{GeV}^2$  using the total set of offsets. The comparison between corrected and uncorrected data for this  $Q^2$  can be seen in Figure 6.

For the analysis of the LH2 data taken at  $Q^2 = 1.14\text{GeV}^2$ , the values in table 1 have been determined and are used for the analysis of LH2 data.<sup>3</sup>

<sup>3</sup>The value for the HMS central momentum will be described in a forthcoming paper that considers the instability of the HMS power supply.

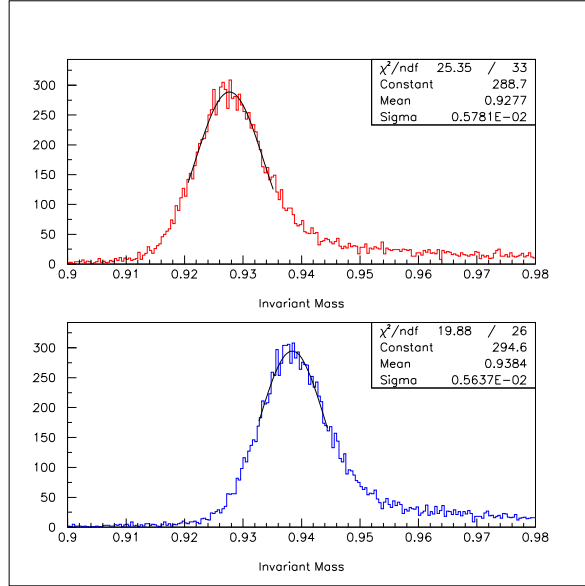


Figure 5: A comparison between data with no corrections (red) and data with all corrections included (blue). These data are the LH2 production data,  $Q^2 = 1.14 GeV^2$ .

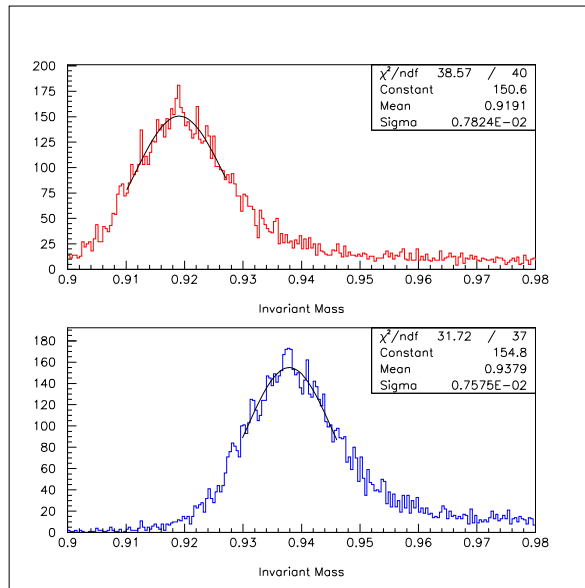


Figure 6: Another comparison between data with no corrections (red) and data with all corrections included (blue). These data are the LH2 production data,  $Q^2 = 1.47 GeV^2$ .

Table 1: Kinematic Offsets for LH2  $Q^2 = 1.14 GeV^2$

| Kinematic variable   | Value                        |
|----------------------|------------------------------|
| Beam Energy          | $E = 2.329 GeV$              |
| HMS Central Momentum | $P_{cent} = 1.7215 GeV/c$    |
| HMS Angle            | $\theta = 0.5390 rad$        |
| Beam Energy Offset   | $\Delta E = -0.055\%$        |
| HMS Momentum Offset  | $\Delta E' = -0.310\%$       |
| HMS Angle Offset     | $\Delta\theta = -1 mrad$     |
| Rad. Corrections     | $\Delta E_{hms} = -0.047\%$  |
| Energy Loss          | $\Delta E_{loss} = -0.290\%$ |

## References

- [1] Unpublished Hall C Report, [christy@jlab.org](mailto:christy@jlab.org)  
Pre-print available via [this link](#).
- [2] Tiefenback, Michael, [tiefen@jlab.org](mailto:tiefen@jlab.org)
- [3] Eric Weisstein's World of Mathematics
- [4] W.R. Leo, *Techniques for Nuclear and Particle Physics Experiments*, 2nd ed., Springer-Verlag.