A Note About Beam Depolarization

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1 Basic Mechanism of Beam Depolarization

Ionizing radiation increases the nuclear spin relaxation in the target chamber. Also known as "beam depolarization," it is essentially a two step process. First, the beam ionizes an ^{3}He atom which results in an free electron and an atomic ion $^3\mathrm{He}^+$. There is also the possibility that the atomic ion bonds with an neutral ^{3}He atom to form an molecular ion $^3\mathrm{He}_2^+$. Second, interactions with ^{3}He ions induce ^{3}He nuclear spin flips. Therefore, the total relaxation rate due to ionization by the beam is given by:

$$\begin{aligned}
\Gamma_{\mathrm{beam}} &= \left[\begin{array}{c}\text{ionization rate}\\ \text{per target chamber atom}\end{array}\right]\cdot\left[\begin{array}{c}\text{mean number of nuclear spin flips}\\ \text{per atomic ion}\end{array}\right] && (1)\\
&= \left[\left(\begin{array}{c}\text{electrons}\\ \text{per unit time}\end{array}\right)\cdot\left(\begin{array}{c}\text{atomic ions created}\\ \text{per electron}\end{array}\right)\cdot(\text{atoms in tc})^{-1}\right]\cdot(n_a+n_m) && (2)\\
&= \left[\left(\frac{I}{e}\right)\cdot\left(\frac{\text{total energy lost}}{\text{mean energy per ion}}\right)\cdot\left(\frac{1}{V_{\mathrm{tc}}[\mathrm{He}]_{\mathrm{tc}}}\right)\right]\cdot(n_a+n_m) && (3)\\
&= \left[\left(\frac{I}{e}\right)\cdot\left(\frac{\left[\frac{1}{\rho}\frac{dE}{dx}\right]L_{\mathrm{tc}}[\mathrm{He}]_{\mathrm{tc}}}{E_i}\right)\cdot\left(\frac{1}{V_{\mathrm{tc}}[\mathrm{He}]_{\mathrm{tc}}}\right)\right]\cdot(n_a+n_m) && (4)\\
&= \left(\frac{I}{e}\frac{1}{E_i}\left[\frac{1}{\rho}\frac{dE}{dx}\right]\frac{1}{A_{\mathrm{tc}}}\right)\cdot(n_a+n_m) && (5)\\
&= \Gamma_{\mathrm{ion}}\cdot(n_a+n_m) && (6)
\end{aligned}$$

where I is the electron beam current, E_i is the mean energy for ion-electron pair creation, A_{tc} is the mean cross sectional area of the target chamber, Γ_{ion} is the ionization rate per ^{3}He atom in the target chamber, and n_a & n_m are the average number of spins lost per atomic ion created due to interactions with atomic & molecular ions respectively.

2 Beam Energy Lost to Ionizing Interactions

The energy lost to collisions per unit density per unit length is given by the celebrated Bethe-Bloch formula and, for an electron beam, it is [1]:

$$\begin{aligned}
\left[\frac{1}{\rho}\frac{dE}{dx}\right]_{\mathrm{c}} &= 2\pi r_{\mathrm{e}}^2 m_{\mathrm{e}}c^2\frac{Z}{\beta^2}\left[\log\left(\left[\gamma-1\right]^2\left[\gamma+1\right]\right)-\delta+2\log\left(\frac{m_{\mathrm{e}}c^2}{I_{\mathrm{BB}}}\right)-F(\gamma)-2\frac{C_s}{Z}\right] && (7)\\
2\pi r_{\mathrm{e}}^2 m_{\mathrm{e}}c^2 &= 6.85\ \mathrm{eV/amagat/cm} && (8)\\
F(\gamma) &= \left[1+\frac{2}{\gamma}-\frac{1}{\gamma^2}\right]\log(2)-\frac{1}{8}\left[1-\frac{1}{\gamma}\right]^2-\frac{1}{\gamma^2} \quad \& \quad \gamma=\frac{1}{\sqrt{1-\beta^2}}=\frac{E_{\mathrm{beam}}}{m_{\mathrm{e}}c^2} && (9)
\end{aligned}$$

where Z is the target atomic number, $\beta(=v/c)$ is the electron velocity relative to the speed of light, I_{BB} is the mean excitation potential of the target material, δ is the density correction, and C_s is the shell correction.

parameter	value	comments
Z	2	atomic number
$I_{\rm BB}$	41.8 eV	mean excitation potential
C_s	0	for shell correction
δ_0	0	for density correction (1 atm & 20 °C)
Y_a	5.5697	
Y_0	5.0696	
Y_1	8.3174	
m	5.8347	
$[^3{\rm He}]_0$	0.93141 amg	

Table 1: Bethe-Bloch Formula Parameters for Electron-Helium Interactions. All values taken from [2].

The shell correction is significant only when the incident electron velocity is roughly equal to or slower than the bound electron orbital velocity. For JLab beam energies, this is not the case; therefore the shell correction will be neglected ($C_s = 0$). The density correction δ is given by [1, 2]:

$$\delta(Y) = \left\{ \begin{array}{ll} \delta_0 \exp\left[2\left(Y - Y_0'\right)\right] & Y \le Y_0' \\ 2\left(Y - Y_a'\right) + \left[\delta_0 - 2\left(Y_0' - Y_a'\right)\right] \left[\frac{Y_1' - Y}{Y_1' - Y_0'}\right]^m & Y_0' < Y \le Y_1' \\ 2\left(Y - Y_a'\right) & Y_1' < Y \end{array} \right\} \tag{10}$$

$$Y = \log\left(\beta\gamma\right) \quad \& \quad Y_{a,0,1}' = Y_{a,0,1} - \log\sqrt{[^3{\rm He}]/[^3{\rm He}]_0} \tag{11}$$

where Y_a, Y_0, Y_1, m, and $[{\rm N}]_0$ depend on the target material at 1 atm & 20 °C and for ^{3}He are listed in Tab. (1). For a ^{3}He density of 8.3 amg or higher, the equivalent beam energy for $Y = Y_1'$ is 700 MeV or less. Therefore for typical ^{3}He experiments at JLab, we get:

$$\left[\frac{1}{\rho}\frac{dE}{dx}\right]_{\rm c} = 4\pi r_{\rm e}^2 m_{\rm e} c^2 \left[\log\left(\frac{E_{\rm beam}}{1\ {\rm GeV}}\right) - \log\left(\frac{[{\rm N}]}{10\ {\rm amg}}\right) + 34.6\right] \tag{12}$$

$$4\pi r_{\rm e}^2 m_{\rm e} c^2 = 510\ {\rm keV}\cdot{\rm barn} = 13.70\ {\rm eV/amagat/cm} \tag{13}$$

3 Mean Energy for Helium Ion-Electron Pair Creation

The mean energy per ion-electron creation has been measured in helium a number of times, see Tab. (2). The early measurements found about 32 eV per pair. As later authors noted on more than one occasion [3, 4, 5, 6], these early measurements were performed on insufficiently pure helium samples. Later measurements, which took great care to purify the helium sample, obtained results about 10 eV per pair higher. We need to know the value for pure He because we are interested in knowing how many He ions are created. Consequently, we use a weighted average of five "modern" measurements that went to great lengths to purify their He sample. As a side note, the mean energy per ion-electron creation E_i is entirely different than the mean excitation potential $I_{\rm BB}$. It is merely a coincidence that they have nearly the same value for He. We are finally in a position to calculate the mean ionization rate per atom:

$$\Gamma_{\rm ion} = \left(\frac{1}{eE_i}\left[\frac{1}{\rho}\frac{dE}{dx}\right]_{\rm c}\right)\frac{I}{A_{\rm tc}} = \beta\frac{I}{A_{\rm tc}} \tag{14}$$

where e is the elementary charge, I is the beam current, $A_{\rm tc}$ is the mean cross sectional area of the target chamber, and β^{-1} is tabulated in Tab. (3) for various beam energies. The dependence of β on the beam energy is soft; consequently the mean ionization rate per atom within 5 percent over all JLab energies is:

$$\Gamma_{\rm ion} = \left(0.0095\ \frac{{\rm cm}^2}{\mu{\rm A}\cdot{\rm hr}}\right)\frac{I}{A_{\rm tc}} = \left(\frac{1}{21\ {\rm hrs}}\right)\cdot\left(\frac{I}{10\ \mu{\rm A}}\right)\cdot\left(\frac{2.0\ {\rm cm}^2}{A_{\rm tc}}\right) \tag{15}$$

E_i(eV)	**year**	**comments**	**ref.**
26.2	1925	purified in charcoal at liquid air temps, possible double ionization of He?	[7]
31	1927	purified in charcoal at liquid air temperatures	[8]
31.0	1944	value listed in [9] and [10]	[11]
29.9	1951	tank He at 99.95% purity with traces amounts of N_2 and O_2	[12]
30.9	1952	cited in [4, 13]	[14]
(32.5 ± 0.5)	1952	He/Ar/CH_4 mixture	[15]
29.7	1952	He with 0.13% Ar	[3]
41.3		purified with charcoal at liquid air temperatures	
(26.0 ± 1.6)	1953	was purified, but not pure enough?	[16]
$(42.7 \pm 0.2)^*$	1953	purified with charcoal at liquid air temperatures	[4]
33.8	1954	tank He with less than 0.02% N_2	[17]
$(44.2 \pm 0.9)^*$	1954	purified with Ca-Mg chips at 470°C	[13]
$(46.0 \pm 0.5)^*$	1954	two sets of He samples with different purification methods	[5]
$(42.3 \pm 0.3)^*$	1955	purified with charcoal at liquid air temperatures	[18]
$(40.3 \pm 0.8)^*$	1956	purified with charcoal at liquid air temperatures	[19]
55,60 (±5%)	1957	used He-ethylene mix, but applied an "impurity" correction	[20]
29.9/35.2	1954	theoretical calculation for impure He sample	[6]
41.1		theoretical calculation for pure He	
42.7,42.3	1964	sensitivity to impurities discussed, but no original sources listed	[21]
41	1994		[1]
		E_i(weighted mean) $= (43.2 \pm 0.1)$ eV	

Table 2: Mean Energy per Ion-e^- Pair Creation in He Gas. Only measurements performed on carefully purified samples (*) are used in the calculation of the weighted mean. The different measurement techniques and their respective sensitivities to impurities are discussed in the 1958 review article by Valentine and Curran [22].

E_{beam} (GeV)	$\left[\frac{1}{\rho}\frac{dE}{dx}\right]_{\text{c}}$	η (%)	β^{-1} $\left(\text{hr}\cdot\mu\text{A/cm}^2\right)$
0.7	0.97	0.1	110.1
1.0	0.98	0.2	109.0
2.0	1.00	0.3	106.8
4.0	1.02	0.5	104.8
8.0	1.04	1.0	102.8
16.0	1.06	1.8	100.9
32.0	1.08	3.6	99.04
64.0	1.10	7.0	97.26

Table 3: Variation of Ionizing Energy Loss Parameters with Electron Beam Energy. The second column is the energy lost to collisions relative to the value at 2 GeV. The maximum relative ionization contribution from radiation, η, is estimated assuming a ^{3}He density of 10 amg and a target chamber length of 40 cm.

4 Spin Relaxation Due to Atomic and Molecular Helium Ions

Atomic ions contribute to polarization loss due to a hyperfine interaction between the ^{3}He nucleus and the unpaired electron in the atomic ion. Because charge exchange occurs readily, electrons from highly polarized neutral atoms jump to lowly polarized atomic ions. The newly formed atomic ion partially depolarizes until it undergoes charge exchange and so on. The cumulative effect is at most one nuclear spin flip [23]. In addition to this process, molecular ions also lose polarization to the rotational degrees of freedom via a spin-rotation interaction [24].

Before estimating the number of spin flips induced by both processes, it is useful to first estimate the fraction of ions of both types and their typical lifetimes. First we write down the rate equations for the fractionof atomic ions h_a and molecular ions h_m (in the target chamber), where we have assumed $h_a, h_m \ll 1$:

$$\begin{array}{lll} dh_a/dt = +\Gamma_{\rm ion} - h_a/\tau_a & \& & \tau_a^{-1} = k_n[{\rm N_2}]_{\rm tc} + k_m[{\rm He}]_{\rm tc}^2 \\ dh_m/dt = +k_m h_a[{\rm He}]_{\rm tc}^2 - h_m/\tau_m & \& & \tau_m^{-1} = [{\rm N_2}]_{\rm tc}\,(k_n' + k_n''[{\rm He}]_{\rm tc}) \end{array} \tag{16}$$

where k_m, k_n, k_n', & k_n'' are the rate constants for molecular formation, atomic ion charge exchange, binary molecular charge exchange, and three body molecular charge exchange, see Tab. (4). The mean atomic and molecular ion lifetimes are τ_a and τ_m. The equilibrium fractions are obtained from setting the rates to zero and give:

$$h_a^\infty = \Gamma_{\rm ion}\tau_a = \frac{\Gamma_{\rm ion}}{k_n[{\rm N_2}]_{\rm tc} + k_m[{\rm He}]_{\rm tc}^2} \tag{17}$$

$$h_m^\infty = k_m[{\rm He}]_{\rm tc}^2\tau_m h_a^\infty = \frac{\Gamma_{\rm ion}}{[{\rm N_2}]_{\rm tc}\,(k_n' + k_n''[{\rm He}]_{\rm tc})}\left(1 + \frac{k_n[{\rm N_2}]_{\rm tc}}{k_m[{\rm He}]_{\rm tc}^2}\right)^{-1} \tag{18}$$

Under our conditions, we find $\tau_a, \tau_m \approx 100$ ps and $h_a^\infty, h_m^\infty \approx 10^{-15}$, which justifies our previous assumption that there are very few ions.

The presence of a foreign gas such as N_2 greatly limits the lifetime of molecular ions. Whereas molecular ions have the potential to depolarize many nuclei, their effect is greatly reduced because they are so short lived. Relaxation due to molecular ions is discussed in [24] and they derive an expression for n_m of the following form:

$$\Gamma_{\rm ion} n_m = \left\langle \frac{\gamma_m N}{h} \right\rangle h_m^\infty Q_m \rightarrow n_m = \left\langle \frac{\gamma_m N}{h} \right\rangle \left(\frac{h_m^\infty}{\Gamma_{\rm ion}}\right) Q_m \tag{19}$$

where $\gamma_m N/h = 29$ MHz is the molecular spin-rotation coupling constant and Q_m is the unitless relative relaxation rate that depends on the magnitude of the magnetic field and the density of ^{3}He. Since Q_m can be at most 1, the maximum value for n_m is given as:

$$n_m \leq \frac{\left\langle \frac{\gamma_m N}{h} \right\rangle}{[{\rm N_2}]_{\rm tc}\,(k_n' + k_n''[{\rm He}]_{\rm tc})}\left(1 + \frac{k_n[{\rm N_2}]_{\rm tc}}{k_m[{\rm He}]_{\rm tc}^2}\right)^{-1} \approx 0.002 \tag{20}$$

According to [23] the mean number of spin flips due to an atomic ion n_a when the atomic charge exchange rate $\tau_{\rm ex}^{-1}$ is much slower than the hyperfine precessional frequency A_a/h,

$$n_a \approx \frac{\tau_{\rm ex} + \tau_{\rm a}}{2\tau_{\rm ex} + \tau_{\rm a}} \qquad \& \qquad \tau_{\rm ex}^{-1} = k_{\rm ex}[{\rm He}]_{\rm tc} \tag{21}$$

where $k_{\rm ex}$ is the binary atomic charge exchange rate constant.

Under our conditions, $\tau_{\rm ex}^{-1} \approx 150$ GHz is much faster than $A_a/h = 8.66$ GHz. Physically, this means that electron and nucleus have very little time to interact before the election is exchanged to another atom. This has the effect of suppressing the the chances of a nuclear spin flip and therefore reduces the overall relaxation rate. Over a ^{3}He density range of 9 amg to 12 amg and a N_2 to ^{3}He density ratio range of 0.5% to 2%, the following parameteriation reproduces the more general calculation from [23] to better than 3%:

$$n_a = 0.50618 - [0.62409 - 0.05691\cdot(\rho - 1)]\cdot(h - 1) - 0.075812\cdot(\rho - 1) \tag{22}$$

where h and ρ are given by:

$$h = [^3{\rm He}]_{\rm tc}/10\ {\rm amg} \qquad \& \qquad \rho = 100\cdot[{\rm N_2}]_{\rm tc}/[^3{\rm He}]_{\rm tc} \tag{23}$$

reaction	type	binary	3-body	ref
$He^+ + He \rightarrow He + He^+$	charge exchange	15 ± 5	-	[25]
$He^+ + N_2 \rightarrow He + N_2^+$	charge transfer	27 ± 8	-	[26]
$He^+ + 2He \rightarrow He + He_2^+$	molecular formation	-	0.060 ± 0.012	[27]
$He_2^+ + He \rightarrow He + He_2^+$	charge exchange	6 ± 3	-	[24]
$He_2^+ + (0,1)He + N_2 \rightarrow (2,3)He + N_2^+$	charge transfer	30 ± 3	9.8 ± 1.4	[28]

Table 4: Atomic and Molecular Ion Reaction Rate Constants. Binary rate constants are in GHz/amg and 3-body rate constants are in GHz/amg^2. All values are assumed to be measured at 300 K and to have negligible temperature dependence within the quoted uncertainties.

References

[1] William R. Leo. *Techniques for Nuclear and Particle Physics Experiments: A How-to Approach*. Springer-Verlag, Berlin, second revised edition, 1994.

[2] R.M. Sternheimer, M.J. Berger, and S.M. Seltzer. Density Effect for the Ionization Loss of Charged Particles In Various Substances. *Atomic Data and Nuclear Data Tables*, 30(2):261–271, March 1984.

[3] William P. Jesse and John Sadauskis. Alpha-Particle Ionization in Mixtures of the Noble Gases. *Phys. Rev.*, 88(2):417–418, Oct 1952.

[4] William P. Jesse and John Sadauskis. Alpha-Particle Ionization in Pure Gases and the Average Energy to Make an Ion Pair. *Phys. Rev.*, 90(6):1120–1121, Jun 1953.

[5] T.E. Bortner and G.S. Hurst. Ionization of Pure Gases and Mixtures of Gases by 5-Mev Alpha Particles. *Phys. Rev.*, 93(6):1236–1241, Mar 1954.

[6] G.A. Erskine. The Effect of Contaminating Gases on the Energy per Ion Pair in Helium. *Proceedings of the Physical Society. Section A*, 67(7):640–642, 1954.

[7] R.W. Gurney. Ionisation by Alpha-Particles in Monoatomic and Diatomic Gases. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 107(742):332–340, February 1925.

[8] J.F. Lehmann. The Absorption of Slow Cathode Rays in Various Gases. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 115(772):624–639, August 1927.

[9] Hans H. Staub. Detection Methods. In E. Segrè, editor, *Experimental Nuclear Physics, Volume I*, pages 1–165, New York, 1953. John Wiley and Sons.

[10] Bruno Rossi. *High-Energy Physics*. Prentice-Hall, Incorporated, New York, 1952.

[11] L.H. Gray. *Proceedings of the Cambridge Philosophical Society*, 40:72, 1944.

[12] C.J. Bakker and E. Segrè. Stopping Power and Energy Loss for Ion Pair Production for 340-Mev Protons. *Phys. Rev.*, 81(4):489–492, Feb 1951.

[13] Lloyd O. Herwig and Glenn H. Miller. Alpha-Particle Ionization Yields in a Gridded Chamber. *Phys. Rev.*, 94(5):1183, Jun 1954.

[14] W. Haberli, P. Huber, and E. Baldinger. *Helv. Phys. Acta*, 25:467, 1952.

[15] J.M. Valentine. Energy per Ion Pair for Electrons in Gases and Gas Mixtures. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 211(1104):75–85, February 1952.

[16] Robert H. Frost and Carl E. Nielsen. The Specific Probable Ionization of Electrons Observed with a Wilson Cloud Chamber. *Phys. Rev.*, 91(4):864–870, Aug 1953.

[17] Leon M. Dorfman. Absorption of Tritium Beta Particles in Hydrogen and Other Gases. *Phys. Rev.*, 95(2):393–396, Jul 1954.

[18] William P. Jesse and John Sadauskis. Ionization in Pure Gases and the Average Energy to Make an Ion Pair for Alpha and Beta Particles. *Phys. Rev.*, 97(6):1668–1670, Mar 1955.

[19] Jerome Weiss and William Bernstein. Energy Required to Produce One Ion Pair in Several Noble Gases. *Phys. Rev.*, 103(5):1253, Sep 1956.

[20] W.G. Stone and L.W. Cochran. Ionization of Gases by Recoil Atoms. *Phys. Rev.*, 107(3):702–704, Aug 1957.

[21] Emilio Segrè. *Nuclei and Particles: An Introduction to Nuclear and Subnuclear Physics*. W.A. Benjamin, INC., New York, 1964.

[22] J.M. Valentine and S.C. Curran. Average energy expenditure per ion pair in gases and gas mixtures. *Reports on Progress in Physics*, 21(1):1–29, 1958.

[23] K.D. Bonin, D.P. Saltzberg, and W. Happer. Relaxation of gaseous spin-polarized ^{3}He targets due to creation of $^3He^+$. *Phys. Rev. A*, 38(9):4481–4487, Nov 1988.

[24] K.D. Bonin, T.G. Walker, and W. Happer. Relaxation of gaseous spin-polarized ^{3}He targets due to ionizing radiation. *Phys. Rev. A*, 37(9):3270–3282, May 1988.

[25] Donald Rapp and W.E. Francis. Charge Exchange between Gaseous Ions and Atoms. *The Journal of Chemical Physics*, 37(11):2361–2645, Dec 1962.

[26] J. Heimerl, R. Johnsen, and Manfred A. Biondi. Ion-Molecule Reactions, $He^+ + O_2$ and $He^+ + N_2$, at Thermal Energies and Above. *The Journal of Chemical Physics*, 51(11):5041–5048, Dec 1969.

[27] J.D.C. Jones, D.G. Lister, D.P. Wareing, and N.D. Twiddy. The temperature dependence of the three-body reaction rate coefficient for some rare-gas atomic ion-atom reactions in the range 100-300 K. *J. Phys. B: Atom. Molec. Phys.*, 13:3247–3255, 1980.

[28] J.M. Pouvesle, A. Bouchoule, and J. Stevefelt. Modeling of the charge transfer afterglow excited by intense electrical discharges in high pressure helium nitrogen mixtures. *The Journal of Chemical Physics*, 77(2):817–825, Jul 1982.