A Note About Temperature Tests

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Abstract

The laser light heats the Helium-3 gas in the pumping chamber by an unknown amount. We derive formulas for calculating the pumping chamber temperature with the lasers on using the following arguments: (1) for those parts of the cell that are not hit by the laser beam, the gas temperature is equal to the surface temperature of the glass, (2) the temperature of the gas is inversely proportional to the density of the gas, & (3) the NMR signal is proportional to the density of the gas.

Contents

1 Pumping & Target Chamber Densities

The volumes of the pumping & target chambers are given by V_{pc} & V_{tc} respectively. If we ignore the transfer tube volume, then the total cell volume is given by:

$$V_{\text{tot}} = V_{\text{pc}} + V_{\text{tc}} \tag{1}$$

The temperatures of the pumping & target chambers are given **in Kelvin** by T_{pc} & T_{tc} respectively. The pressure is assumed to be uniform throughout the cell and, by further assuming the ideal gas behavior, can be related to the densities of the two chambers:

$$P_{\text{cell}} = n_{\text{pc}} R T_{\text{pc}} = n_{\text{tc}} R T_{\text{tc}} \tag{2}$$

where R is the ideal gas constant. The total number of particles in the cell is constant and thereby can be related to the fill density:

$$n_{\text{fill}} V_{\text{tot}} = n_{\text{pc}} V_{\text{pc}} + n_{\text{tc}} V_{\text{tc}} \tag{3}$$

Making the following substitutions:

$$v = V_{\text{pc}}/V_{\text{tc}} \tag{4}$$

$$t = T_{\text{pc}}/T_{\text{tc}} \tag{5}$$

the previous equation can be rewritten as:

$$n_{\rm fill}(v+1) = n_{\rm pc}v + n_{\rm tc} \tag{6}$$

We can solve for the cell pressure by substituting in Eqns. (2) for the densities:

$$P_{\rm cell} = n_{\rm fill}\left[\frac{1+v}{t+v}\right]RT_{\rm pc} \tag{7}$$

We can immediately put this equation for the cell pressure back into Eqns. (2) to get the densities:

$$\frac{n_{\rm pc}}{n_{\rm fill}} = \left[\frac{1+v}{t+v}\right] \tag{8}$$

$$\frac{n_{\rm tc}}{n_{\rm fill}} = t\left[\frac{1+v}{t+v}\right] \tag{9}$$

2 Ratio of NMR Signals with the Lasers On & Off

We'll assume that the gas temperature is equal to the local surface temperature, **except when the gas is being heated by interactions with the laser beam**. To calculate the pumping chamber temperature when the laser is on $T_{\rm pc}^{\rm on}$, we'll look at the size of the local NMR signal:

$$S = GP\langle n\rangle \tag{10}$$

where G is the local calibration constant and P is the local polarization. The local sensing volume averaged density $\langle n\rangle$ is related to the local density n by:

$$P_{\rm cell} = \langle n\rangle R\langle T\rangle = nRT \tag{11}$$

where T is the local temperature and $\langle T\rangle$ is the local sensing volume averaged temperature. For clarity: a local variable is the average value over the entire chamber, whereas the local sensing volume averaged variable is the average value over the part of the chamber that the pickup coil is most sensitive to. Putting this altogether gives the following for the NMR signal:

$$S = GPnT/\langle T\rangle \tag{12}$$

The ratio of local signals with the laser on & off are given by:

$$\frac{S^{\rm on}}{S^{\rm off}} = \frac{G^{\rm on}P^{\rm on}n^{\rm on}T^{\rm on}/\langle T\rangle^{\rm on}}{G^{\rm off}P^{\rm off}n^{\rm off}T^{\rm off}/\langle T\rangle^{\rm off}} \tag{13}$$

which we'll rewrite as:

$$\frac{n^{\rm on}}{n^{\rm off}} = \rho \equiv \frac{S^{\rm on}G^{\rm off}P^{\rm off}T^{\rm off}\langle T\rangle^{\rm on}}{S^{\rm off}G^{\rm on}P^{\rm on}T^{\rm on}\langle T\rangle^{\rm off}} \tag{14}$$

where ρ is the local normalized NMR signal ratio.

3 NMR in Both Chambers Simultaneously

For the case where we can measure the NMR signal in both chambers simultaneously, we can take the ratio of the pumping & target chamber normalized NMR signal ratios to get:

$$\frac{\rho_{\rm pc}}{\rho_{\rm tc}} = \frac{n_{\rm pc}^{\rm on}}{n_{\rm pc}^{\rm off}} \times \frac{n_{\rm tc}^{\rm off}}{n_{\rm tc}^{\rm on}} = \frac{n_{\rm pc}^{\rm on}}{n_{\rm tc}^{\rm on}} \times \frac{n_{\rm tc}^{\rm off}}{n_{\rm pc}^{\rm off}} = \frac{t^{\rm off}}{t^{\rm on}} \tag{15}$$

which we can easily solve for $t^{\rm on}$:

$$t^{\rm on} = t^{\rm off}\rho_{\rm tc}/\rho_{\rm pc} \tag{16}$$

Writing this equation out explicitly gives:

$$T_{\rm pc}^{\rm on} = T_{\rm tc}^{\rm on} \frac{T_{\rm pc}^{\rm off} S_{\rm tc}^{\rm on} G_{\rm tc}^{\rm off} P_{\rm tc}^{\rm off} T_{\rm tc}^{\rm off} \langle T\rangle_{\rm tc}^{\rm on} S_{\rm pc}^{\rm off} G_{\rm pc}^{\rm on} P_{\rm pc}^{\rm on} T_{\rm pc}^{\rm on} \langle T\rangle_{\rm pc}^{\rm off}}{T_{\rm tc}^{\rm off} S_{\rm tc}^{\rm off} G_{\rm tc}^{\rm on} P_{\rm tc}^{\rm on} T_{\rm tc}^{\rm on} \langle T\rangle_{\rm tc}^{\rm off} S_{\rm pc}^{\rm on} G_{\rm pc}^{\rm off} P_{\rm pc}^{\rm off} T_{\rm pc}^{\rm off} \langle T\rangle_{\rm pc}^{\rm on}} \tag{17}$$

Rearranging some things around, we get the following for the case when we can measure the NMR signal in both chambers simultaneously:

$$\langle T\rangle_{\rm pc}^{\rm on} = \langle T\rangle_{\rm pc}^{\rm off} \left[\frac{S_{\rm tc}^{\rm on} S_{\rm pc}^{\rm off}}{S_{\rm tc}^{\rm off} S_{\rm pc}^{\rm on}}\right] \left[\frac{G_{\rm tc}^{\rm off} G_{\rm pc}^{\rm on}}{G_{\rm tc}^{\rm on} G_{\rm pc}^{\rm off}}\right] \left[\frac{P_{\rm tc}^{\rm off} P_{\rm pc}^{\rm on}}{P_{\rm tc}^{\rm on} P_{\rm pc}^{\rm off}}\right] \left[\frac{\langle T\rangle_{\rm tc}^{\rm on}}{\langle T\rangle_{\rm tc}^{\rm off}}\right] \tag{18}$$

4 NMR in the Pumping Chamber Only

If we only have NMR measurement in the pumping chamber, then we must start with this equation:

$$\rho_{\rm pc} = \frac{n_{\rm pc}^{\rm on}}{n_{\rm pc}^{\rm off}} = \frac{1+v}{t^{\rm on}+v} \times \frac{t^{\rm off}+v}{1+v} = \frac{t^{\rm off}+v}{t^{\rm on}+v} \tag{19}$$

Solving for $t^{\rm on}$ gives:

$$t^{\rm on} = \frac{t^{\rm off}+v}{\rho_{\rm pc}} - v \tag{20}$$

which we can rearrange and write explicitly as:

$$T_{\rm pc}^{\rm on} = T_{\rm tc}^{\rm on} \left(\frac{V_{\rm pc}}{V_{\rm tc}}\right) \left(-1 + \left[1 + \frac{T_{\rm pc}^{\rm off} V_{\rm tc}}{T_{\rm tc}^{\rm off} V_{\rm pc}}\right] \left[\frac{S_{\rm pc}^{\rm off}}{S_{\rm pc}^{\rm on}}\right] \left[\frac{G_{\rm pc}^{\rm on}}{G_{\rm pc}^{\rm off}}\right] \left[\frac{P_{\rm pc}^{\rm on}}{P_{\rm pc}^{\rm off}}\right] \left[\frac{\langle T\rangle_{\rm pc}^{\rm off}}{T_{\rm pc}^{\rm off}}\right] \left[\frac{T_{\rm pc}^{\rm on}}{\langle T\rangle_{\rm pc}^{\rm on}}\right]\right) \tag{21}$$

Since this equation has a factor of $T_{\rm pc}^{\rm on}/\langle T\rangle_{\rm pc}^{\rm on}$, we must make some reasonable assumption about what this ratio is. It is not unreasonable to assume that it is inversely proportional to $\langle T\rangle_{\rm pc}^{\rm off}/T_{\rm pc}^{\rm off}$, which makes it very reasonable that to assume that to a very good approximation, their product is unity:

$$\left[\frac{\langle T\rangle_{\rm pc}^{\rm off}}{T_{\rm pc}^{\rm off}}\right] \left[\frac{T_{\rm pc}^{\rm on}}{\langle T\rangle_{\rm pc}^{\rm on}}\right] = 1 \tag{22}$$

Making this assumption, we get the result for the case of NMR in the pumping chamber only:

$$T_{\rm pc}^{\rm on} = T_{\rm tc}^{\rm on} \left(\frac{V_{\rm pc}}{V_{\rm tc}}\right) \left(-1 + \left[1 + \frac{T_{\rm pc}^{\rm off} V_{\rm tc}}{T_{\rm tc}^{\rm off} V_{\rm pc}}\right] \left[\frac{S_{\rm pc}^{\rm off}}{S_{\rm pc}^{\rm on}}\right] \left[\frac{G_{\rm pc}^{\rm on}}{G_{\rm pc}^{\rm off}}\right] \left[\frac{P_{\rm pc}^{\rm on}}{P_{\rm pc}^{\rm off}}\right]\right) \tag{23}$$

5 NMR in the Target Chamber Only

If we can only measure the NMR signal in the target chamber, then we must start with this equation:

$$\rho_{\rm tc} = \frac{n_{\rm tc}^{\rm on}}{n_{\rm tc}^{\rm off}} = t^{\rm on} \left[\frac{1+v}{t^{\rm on}+v}\right] \times \frac{1}{t^{\rm off}} \left[\frac{t^{\rm off}+v}{1+v}\right] = \frac{t^{\rm on}}{t^{\rm off}} \left[\frac{t^{\rm off}+v}{t^{\rm on}+v}\right] \tag{24}$$

Solving for $t^{\rm on}$ gives:

$$\frac{1}{t^{\rm on}} = \frac{1}{\rho_{\rm tc}} \left[\frac{1}{t^{\rm off}} + \frac{1}{v}\right] - \frac{1}{v} \tag{25}$$

Rearranging some things and writing it out explicitly gives:

$$T_{\rm pc}^{\rm on} = T_{\rm tc}^{\rm on} \left(-\frac{V_{\rm tc}}{V_{\rm pc}} + \left[\frac{T_{\rm tc}^{\rm off}}{T_{\rm pc}^{\rm off}} + \frac{V_{\rm tc}}{V_{\rm pc}}\right] \left[\frac{S_{\rm tc}^{\rm off}}{S_{\rm tc}^{\rm on}}\right] \left[\frac{G_{\rm tc}^{\rm on}}{G_{\rm tc}^{\rm off}}\right] \left[\frac{P_{\rm tc}^{\rm on}}{P_{\rm tc}^{\rm off}}\right] \left[\frac{\langle T\rangle_{\rm tc}^{\rm off}}{T_{\rm tc}^{\rm off}}\right] \left[\frac{T_{\rm tc}^{\rm on}}{\langle T\rangle_{\rm tc}^{\rm on}}\right]\right)^{-1} \tag{26}$$

Moving things around and substituting $V_{\rm tc} = V_{\rm tot} - V_{\rm pc}$, we can write the result in a more traditional form for NMR in the target chamber only:

$$T_{\rm pc}^{\rm on} = V_{\rm pc} T_{\rm tc}^{\rm on} \left(\left[\frac{S_{\rm tc}^{\rm off}}{S_{\rm tc}^{\rm on}} \right] \left[\frac{G_{\rm tc}^{\rm on}}{G_{\rm tc}^{\rm off}} \right] \left[\frac{P_{\rm tc}^{\rm on}}{P_{\rm tc}^{\rm off}} \right] \left[\frac{T_{\rm tc}^{\rm on}}{T_{\rm tc}^{\rm off}} \right] \left[\frac{\langle T \rangle_{\rm tc}^{\rm off}}{\langle T \rangle_{\rm tc}^{\rm on}} \right] \left[V_{\rm tot} + V_{\rm pc} \left(\frac{T_{\rm tc}^{\rm off}}{T_{\rm pc}^{\rm off}} - 1 \right) \right] - V_{\rm tot} + V_{\rm pc} \right)^{-1} \tag{27}$$

6 Some Comments About the NMR Signal

The response of the pickup coils can be affected by the local temperature which may be different when the lasers are on and off. That is why we include terms like this:

$$\left[\frac{G_{\rm tc}^{\rm off} G_{\rm pc}^{\rm on}}{G_{\rm tc}^{\rm on} G_{\rm pc}^{\rm off}} \right] \tag{28}$$

The other big issue is the polarization. First, one must correct for AFP losses. Second, there is a polarization gradient between the two chambers (see my technote “Polarization Gradients in a Two Chambered Cell”). At equilibrium (time $\to \infty$), it is given by:

$$\frac{P_{\rm tc}^{\infty}}{P_{\rm pc}^{\infty}} = \frac{1}{1 + \Gamma_{\rm tc}/d_{\rm tc}} \tag{29}$$

where $\Gamma_{\rm tc}$ is the relaxation in the target chamber and $d_{\rm tc}$ is the probability per unit time that a Helium atom leaves the target chamber and enters the pumping chamber. Finally, the polarization is evolving in time. When the laser is on, the polarizations are “spinning-up.” When the laser if off, the polarizations are “spinning-down.” Near equilibrium, which is where most temperature tests are done, the “spin-up” slope is nearly zero since the Helium polarizations aren’t changing much. However, the “spin-down” slope is very large and negative, i.e., the Helium polarization is decreasing quickly. Because the temperature test takes up a finite amount of time & the polarizations are evolving, we explicitly include terms like this:

$$\left[\frac{P_{\rm tc}^{\rm off} P_{\rm pc}^{\rm on}}{P_{\rm tc}^{\rm on} P_{\rm pc}^{\rm off}} \right] \tag{30}$$