Some Considerations Regarding Pumping Chamber Diameter

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Abstract

Quantitative examples of two chambered cell polarization dynamics are presented. Specifically, the effect of the pumping chamber diameter on the maximum polarization, the size of polarization gradients, and the sensitivity to beam induced relaxation in ^{3}He target cells are discussed.

1 A Simple Example

It is useful to first explore a simple example that retains all of the qualitative features of the "full" calculation without having to worry about all the important but obfuscating details. Suppose that we are comparing a 2.5" diameter pumping chamber cell to a 3.5" diameter pumping chamber cell for which the:

1. target chambers are identical
2. alkali polarizations (P_A) are the same
3. spin exchange rates (γ_{se}) in the pumping chamber are the same
4. spin relaxation rates (Γ_{pc}) in the pumping chamber are the same
5. spin relaxation rates (Γ_{tc}) in the target chamber are the same

We would like to know which of the two cells has:

1. a higher equilibrium polarization and by how much?
2. a smaller polarization gradient and by how much?

Let's take a step back and ask ourselves what the answers would be if these two cells were *single chambered* spherical cells:

1. Both cells would have the same equilbrium polarization given by:

$$P = P_A \left[\frac{\gamma_{\mathrm{se}}}{\gamma_{\mathrm{se}} + \Gamma} \right] \tag{1}$$

2. Since these are single chambered cells, there would be no polarization gradient.

Now let's go back to our two chambered cells (ignoring the transfer tube volume). In the limit that the diffusion rate between chambers is much faster than all other rates, then it is straightforward to show that the equilibrium polarizations in the pumping and target chambers are:

$$\frac{P_{\rm pc}}{P_A} = \frac{\langle\gamma_{\rm se}\rangle}{\langle\gamma_{\rm se}\rangle + \langle\Gamma\rangle} \tag{2}$$

$$\frac{P_{\rm tc}}{P_{\rm pc}} = 1 \tag{3}$$

$$\langle\gamma_{\rm se}\rangle \equiv f_{\rm pc}\gamma_{\rm se} \tag{4}$$

$$\langle\Gamma\rangle \equiv f_{\rm pc}\Gamma_{\rm pc} + f_{\rm tc}\Gamma_{\rm tc} \tag{5}$$

where $\langle\gamma_{\rm se}\rangle$ & $\langle\Gamma\rangle$ are the "volume" averaged spin exchange and spin relaxation rates respectively. The parameters $f_{\rm pc}$ & $f_{\rm tc}$ are the fraction of ^{3}He nuclei in the pumping and target chambers respectively and are related to the pumping chamber to target chamber volume and temperature ratios:

$$f_{\rm pc} = \frac{v}{v+t} \tag{6}$$

$$f_{\rm tc} = \frac{t}{v+t} \tag{7}$$

$$v = \frac{V_{\rm pc}}{V_{\rm tc}} \tag{8}$$

$$t = \frac{T_{\rm pc}}{T_{\rm tc}} \tag{9}$$

$$\tag{10}$$

where we have applied the ideal gas law at thermal equilibrium. Under these conditions, the answers become:

1. The larger pumping chamber cell has a higher equilibrium polarization, because a greater fraction of ^{3}He nuclei are in the pumping chamber.

2. Neither cell has a polarization gradient between the pumping and target chambers, because the diffusion rates are assumed to be much faster than the spin exchange and spin relaxation rates.

Finally let's relax the condition that the diffusion rates are much faster than the spin exchange and spin relaxation rates. This leads us to the general solution for a two chambered cell (assuming thermal equilibrium):

$$\frac{P_{\rm pc}}{P_A} = \frac{\langle\gamma_{\rm se}\rangle}{\langle\gamma_{\rm se}\rangle + \langle\Gamma\rangle - f_{\rm tc}\Gamma_{\rm tc}\left[\frac{\Delta}{1+\Delta}\right]} \tag{11}$$

$$\frac{P_{\rm tc}}{P_{\rm pc}} = \frac{1}{1+\Delta} \tag{12}$$

$$\Delta \equiv \frac{\Gamma_{\rm tc}}{d_{\rm tc}} \tag{13}$$

where the diffusion rate $d_{\rm tc}$ is the probability per unit time per nucleus in the target chamber that it will exit the target chamber and enter the pumping chamber and Δ is the ratio of the spin relaxation rate in the target chamber to the diffusion rate. It can be shown that the diffusion rate can be written as:

$$d_{\rm tc} = \left(0.54\ {\rm hrs}^{-1}\right)\left(\frac{A_{\rm tt}}{0.55\ {\rm cm}^2}\right)\left(\frac{10\ {\rm cm}}{L_{\rm tt}}\right)\left(\frac{80\ {\rm cm}^3}{V_{\rm tc}}\right)\left(\frac{11\ {\rm amg}}{n_{\rm tc}}\right)\left(\frac{\Upsilon\left(T_{\rm pc}, T_{\rm tc}\right)}{4/3}\right) \tag{14}$$

where Υ is a dimensionless factor with a relatively soft temperature dependence.

As a reminder we would like to know which of the two cells has:

1. a higher equilibrium polarization and by how much?

parameter	description	2.5" PC value	3.5" PC value	units
P_A	alkali polarization	0.75	0.75	-
$\gamma_{\rm se}$	spin exchange rate	1/3	1/3	hrs^{-1}
$\Gamma_{\rm pc}$	pumping chamber spin relaxation rate	1/8	1/8	hrs^{-1}
$\Gamma_{\rm tc}$	target chamber spin relaxation rate	1/20	1/20	hrs^{-1}
$T_{\rm pc}$	pumping chamber temperature	260	260	$^{\rm o}$C
$T_{\rm tc}$	target chamber temperature	40	40	$^{\rm o}$C
t	temperature ratio	1.70	1.70	-
$V_{\rm pc}$	pumping chamber volume	134	368	cm^3
$V_{\rm tc}$	target chamber volume	80	80	cm^3
v	volume ratio	1.68	4.60	-
$f_{\rm pc}$	fraction of ^{3}He nuclei in pumping chamber	0.50	0.73	-
$f_{\rm tc}$	fraction of ^{3}He nuclei in target chamber	0.50	0.27	-
$\langle\gamma_{\rm se}\rangle$	volume averaged spin exchange rate	1/6	1/4.1	hrs^{-1}
$\langle\Gamma\rangle$	volume averaged spin relaxation rate	1/11.4	1/9.5	hrs^{-1}
$L_{\rm tt}$	transfer tube length	10.3	9.0	cm
$n_{\rm tc}$	target chamber operating density	11	11	amg
$A_{\rm tt}$	transfer tube area	0.5	0.5	cm^2
$d_{\rm tc}$	diffusion rate	0.51	0.54	hrs^{-1}
Δ	target chamber spin relaxation to diffusion rate	0.098	0.093	-
$1 - P_{\rm tc}/P_{\rm pc}$	relative polarization gradient	8.9	8.5	pct.
$P_{\rm pc}$	pumping chamber equilibrium polarization	0.50	0.53	-
$P_{\rm tc}$	target chamber equilibrium polarization	0.46	0.48	-

Table 1: Values for a Simple Example. For the values chosen, the difference in absolute polarization between the two cells is much more significant than the difference in the relative size of the polarization gradients. We have also assumed that the transfer tube length is constrained by the pumping chamber center to target chamber center distance.

2. a smaller polarization gradient and by how much?

In this last scenario, the answers are:

1. Again, the larger pumping chamber cell has a higher equilibrium polarization, because a greater fraction of ^{3}He nuclei are in the pumping chamber.

2. Since the cells are assumed to have the same spin relaxation rate in the target chamber, the cell with the faster diffusion rate will have a smaller polarization gradient between the pumping and target chambers. From Eq. (14), we see that the diffusion rate mainly depends on the dimensions of the transfer tube, the volume of the target chamber, and the operating density of ^{3}He in the target chamber. Since we have already stipulated that the target chambers are identical (and let's say have the same operating density), it comes down to the transfer tube. If the transfers tubes are the identical, then the relative polarization gradient will also be identical. If the distance from the center of the pumping chamber to the center of the target chamber is fixed, then the larger pumping chamber will have a smaller polarization gradient.

Finally, to get a quantitative feel for the size of the differences, let's calculate these quantities for typical values, see Tab. (1). Since the spin exchange rate is essentially fixed by the temperature, the two parameters that we can tune to match the experimental results for the ^{3}He polarization are the alkali polarization P_A and the pumping chamber spin relaxation rate $\Gamma_{\rm pc}$. If we decrease the pumping chamber spin relaxation rate in both cells, then the equilibrium polarization for the large pumping chamber cell will increase by a larger amount than that for the small pumping chamber cell.

2 Requirements and Assumptions

1. We'll assume the K:Rb alkali ratio that was made for Edna (about 5 to 1 in the vapor phase at 230 °C and about 25 to 1 in the solid alloy).

2. We'll assume there is a 5 percent by mole fraction impurity in the hybrid alloy that depresses the alkali vapor pressure. This assumption is motivated by the pressure broadening measurements.

3. We'll assume a maximum X factor relaxation rate that is dependent on the surface to volume ratio of the pumping chamber.

4. To better match the calculated polarizations with measured polarizations, we assume a fixed alkali polarization of 0.75 for all cells regardless of pumping chamber diameter and operating temperature. We do this because we don't know for sure what the alkali polarizations are under operating conditions for target cells of this type.

5. The target oven can accept pumping chambers as large as 3.5 inches diameter.

6. The distance between the center of the pumping chamber to the center of the target chamber is fixed at the G_E^n value of $5\frac{5}{8}$ inches.

7. The wall relaxation is assumed to be independent of both the surface to volume ratio and temperature. We'll choose this value to be $\Gamma_{\rm wall}^{-1} = 75$ hours. This is a typical rate at room temperature for a 40 hour lifetime cell.

8. The operating temperature of the pumping chamber is assumed to be $T_{\rm pc} = 260$ °C, which is low compared to G_E^n. Because of the variability and importance of this parameter, we will also plot results corresponding to $T_{\rm pc} = 230$ & 290 °C as well.

9. The operating temperature of the target chamber is assumed to be $T_{\rm pc} = 40$ °C, which is a little high compared to G_E^n, but low compared to other experiments.

10. The operating density in the target chamber is $n_{\rm tc} = 11$ amg. This implies that the fill density must be adjusted accordingly with varying pumping chamber size.

11. The ratio of N_2 to ^{3}He denity will be assumed to be 1/100. This is lower than G_E^n and more typical of past experiments.

12. The remaining target chamber dimensions will be assumed to be the same as those of typical 40 cm cells used in past experiments.

3 Discussion

First we'll review the hybrid mechanism. Spin exchange efficiency between an alkali metal atom and a noble gas atom is defined as:

$$\frac{\text{rate of spin exchange to the noble gas nucleus}}{\text{total rate of alkali spin relaxation including spin exchange}} \tag{15}$$

In this sense, K-^{3}He spin exchange is more efficient than Rb-^{3}He spin exchange. Therefore the same amount of laser intensity can result in either:

1. the same density of alkali metal at a higher polarization

2. a greater density of alkali metal at nearly the same polarization

parameter	description	value	units
P_A	alkali polarization	0.75	-
$X_{\max}\left(1\ \mathrm{cm}^{-1}\right)$	maximum X factor when $S/V = 1\ \mathrm{cm}^{-1}$	0.4	-
l_{p2t}	pumping chamber center to target chamber center	14.2875	cm
A_{tt}	inner transfer tube area	0.5	cm^2
A_{tc}	inner target chamber area	2.0	cm^2
L_{tc}	inner target chamber length	40.0	cm
T_{tc}	target chamber temperature	40	$^{\circ}$C
n_{tc}	operating target chamber density	11	amg
f_{K}	K mole fraction in alloy	0.900	-
f_{Rb}	Rb mole fraction in alloy	0.045	-
f_{Na}	Na mole fraction in alloy	0.0	-
I	beam current	15	μA
E_{beam}	beam energy	6000	MeV
T_{room}	room temperature	23	$^{\circ}$C
τ_{wall}	wall relaxation time constant	75	hrs
$[\mathrm{N}_2]/[^3\mathrm{He}]$	N_2 to ^{3}He density ratio	0.01	-

Table 2: Summary of input values.

One eventually reaches a regime where there is more to be gained from increasing the density of alkali metal than from trying to fully saturate the alkali polarization (for example from $P_A = 0.90$ to $P_A = 0.99$). In this limit, one raises the cell oven temperature to increase the density of alkali metal. This consequently increases the alkali-^{3}He spin exchange rate. If the relaxation rate of ^{3}He stays the same or increases by a lower fraction than the spin exchange rate, then the ^{3}He polarization will increase. One would assume that if the temperature is increased indefinitely and enough laser intensity is supplied, then the ^{3}He polarization would eventually saturate at unity. This is not true from experience. In fact, [**Babcock et al, PRL 96, 083003 (2006)**] have shown that there is an additional ^{3}He spin relaxation mechanism that seems to depend on the alkali density and the surface to volume ratio of the cell:

$$0.15 \leq X \leq X_{\max} \approx (0.4\ \mathrm{cm})\,\frac{S}{V} \tag{16}$$

Experimentally this would manifest itself as a saturation of ^{3}He polarization as the cell oven temperature is increased up to some threshold value (assuming enough laser intensity is being supplied). This behavior *has* been observed numerous times with different hybrid cells. These observations are very suggestive but do not necessarily imply that we are in the "plenty of laser intensity" regime.

Therefore, although the pumping chamber diameter has a significant effect on the amount of laser power needed to polarize a given amount of ^{3}He, we will focus only on polarization dynamics instead. The two main parameters that dictate the effect of varying pumping chamber diameters on the polarization in the target chamber are the transfer tube length and the fraction of nuclei in the pumping chamber. A larger pumping chamber results in:

1. a shorter transfer tube length because the pumping chamber center to target chamber center distance is fixed.
$$L_{\mathrm{tt}} = l_{\mathrm{p2t}} - R_{\mathrm{pc}} - R_{\mathrm{tc}} \tag{17}$$
2. a larger fraction of ^{3}He nuclei in the pumping chamber mainly because of the volume imbalance with the target chamber.

The main consequences are:

1. a faster diffusion rate and consequently a smaller polarization gradient.
2. higher polarizations because more ^{3}He nuclei are in direct contact with the polarization source (the polarized alkali vapor).

3. the polarization gradient difference is smaller than the difference in the equilibrium polarizations.

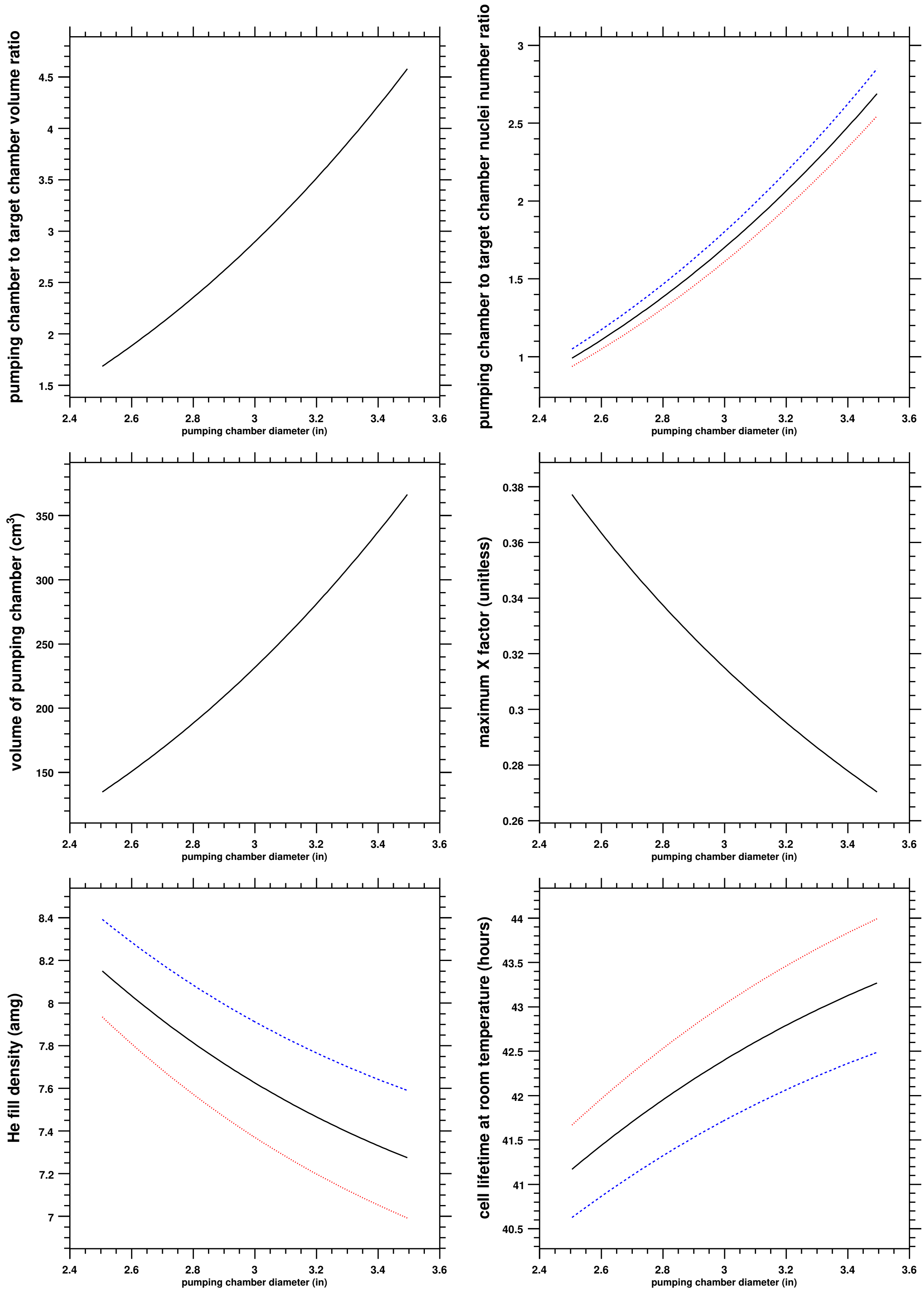
We have reproduced the all of the qualitative results of the "simple" example given in the first section. However, why is the difference in absolute polarization even larger in the full calculation? In this case, the most relevant difference between the two calculations is the inclusion of the maximum X factor. In the simple example, we assumed that the spin relaxation rates in the pumping chambers were equal. In the full calculation, we incorporated the fact that the maximum size of the X factor scales with surface to volume ratio. Therefore, the larger pumping chamber cells have a relatively lower spin relaxation rate in the pumping chamber. The empirical data we have for hybrid cells is inconclusive because of the variablity of many factors. However, assuming commensurate experimental conditions, identical alkali polarizations, and incorporating the X factor into our calculations, a larger pumping chamber cell will outperform a smaller pumping chamber cell. Although one can argue convincingly that the average equilibrium polarization of large pumping chamber cells is higher than those for small pumping chamber cells under similar conditions, the potentially large variability of many factors may essentially "wash" out any advantage.

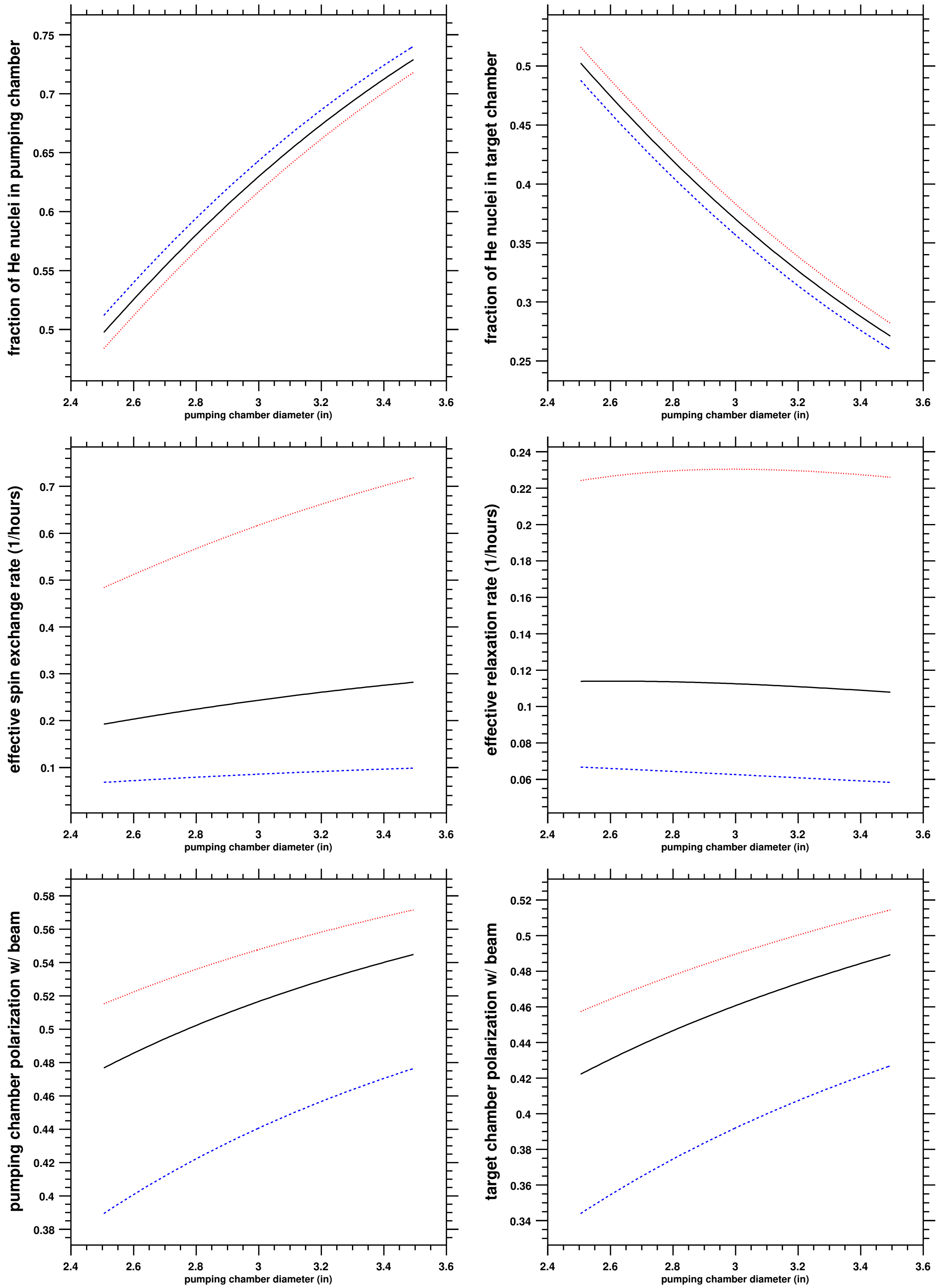
The following plots are meant to illustrate the quantitative size of these effects for representative input parameters. All of the parameters are calculated from the "Polarization Gradient in a Two Chambered Cell" technote. The pumping chamber diameter is the *inner* diameter.
The dashed blue curves are for a pumping chamber operating temperature of 230 °C.
The solid black curves are for a pumping chamber operating temperature of 260 °C.
The dotted red curves are for a pumping chamber operating temperature of 290 °C.

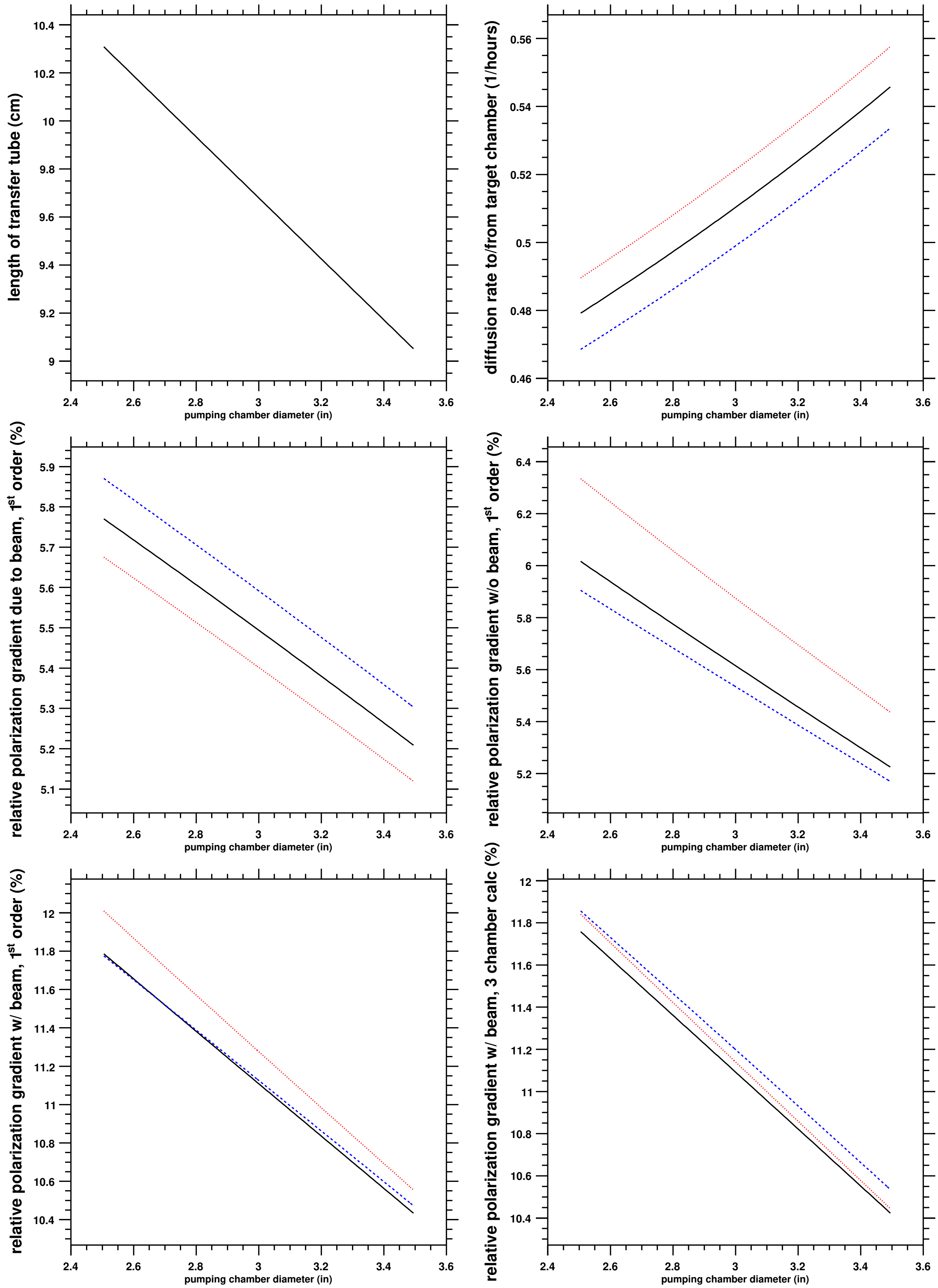
To generate plots for a different set of input parameters, see:
`http://www.jlab.org/~singhj/codes/make_pcd_plots.macro`

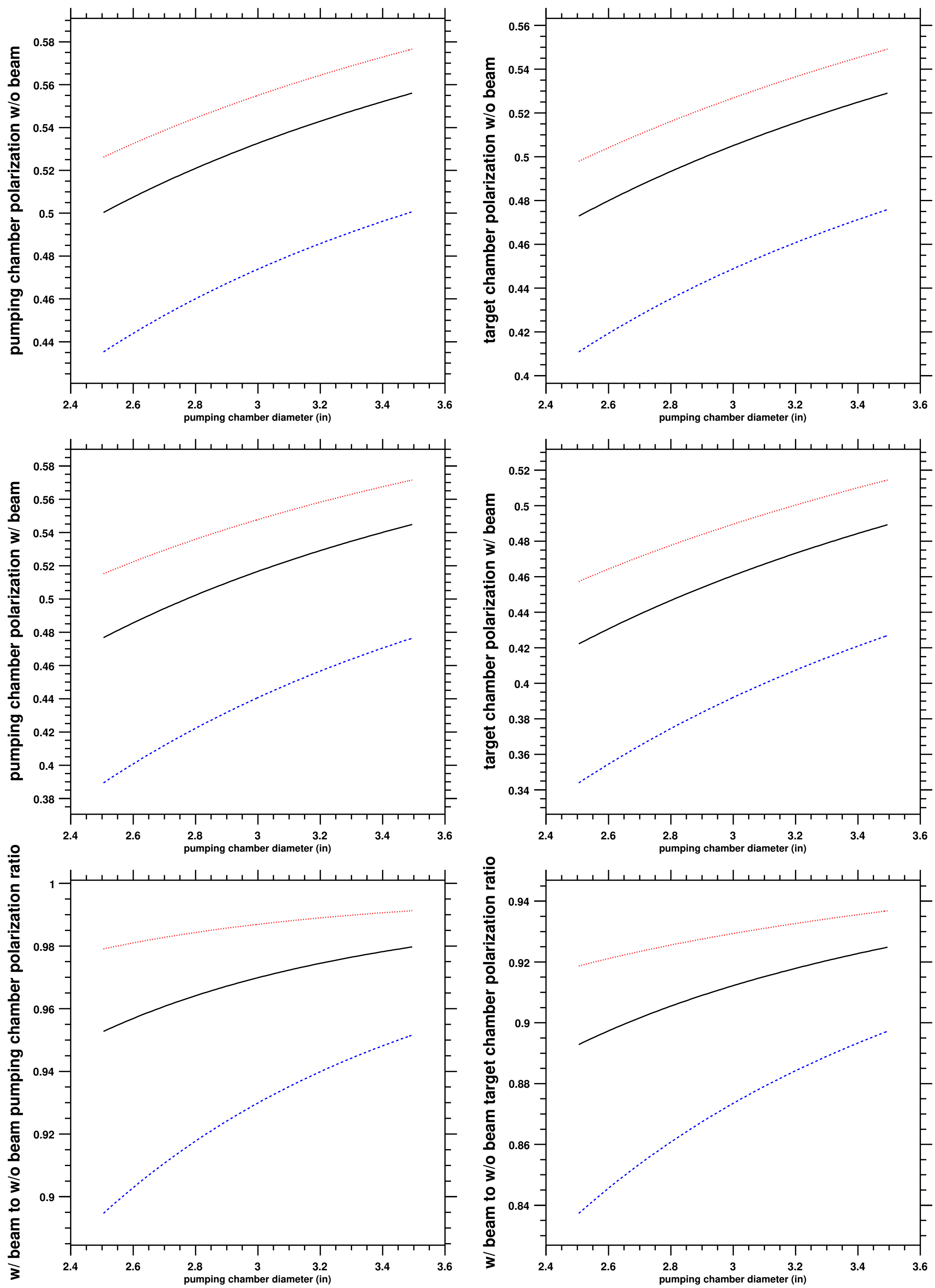
For more details about how various quantities are calculated, see:
`http://www.jlab.org/~singhj/docs/polgrad137.ps.gz`

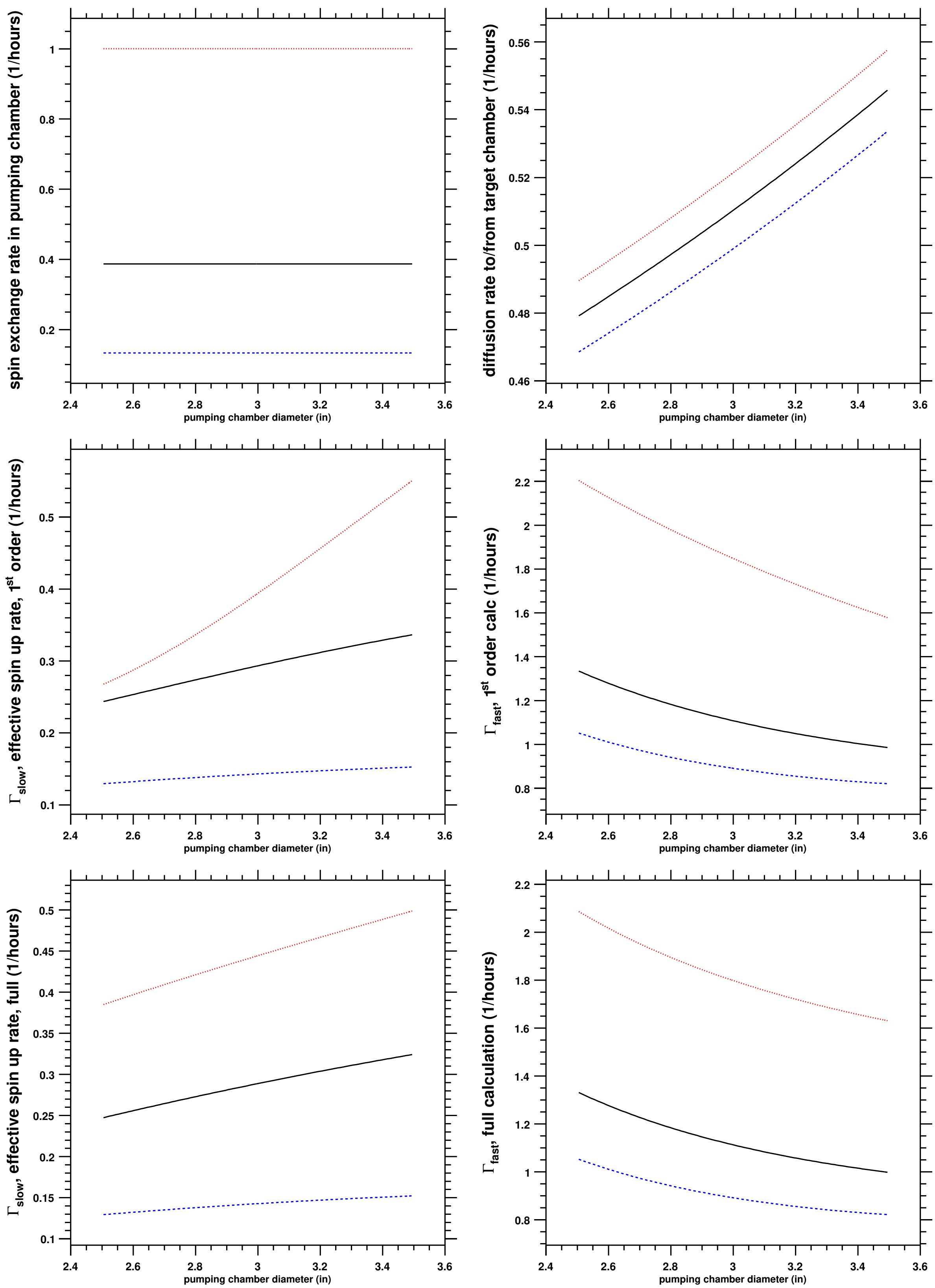
4 Various Plots


pumping chamber to target chamber volume ratio
1.5
2
2.5
3
3.5
4
4.5
pumping chamber diameter (in)
2.4
2.6
2.8
3
3.2
3.4
3.6
pumping chamber to target chamber nuclei number ratio
1
1.5
2
2.5
3
volume of pumping chamber (cm³)
150
200
250
300
350
maximum X factor (unitless)
0.26
0.28
0.3
0.32
0.34
0.36
0.38
He fill density (amg)
7
7.2
7.4
7.6
7.8
8
8.2
8.4
cell lifetime at room temperature (hours)
40.5
41
41.5
42
42.5
43
43.5
44


fraction of He nuclei in pumping chamber
0.75
0.7
0.65
0.6
0.55
0.5
2.4
2.6
2.8
3
3.2
3.4
3.6
pumping chamber diameter (in)
fraction of He nuclei in target chamber
0.5
0.45
0.4
0.35
0.3
0.25
2.4
2.6
2.8
3
3.2
3.4
3.6
pumping chamber diameter (in)
effective spin exchange rate (1/hours)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2.4
2.6
2.8
3
3.2
3.4
3.6
pumping chamber diameter (in)
effective relaxation rate (1/hours)
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
2.4
2.6
2.8
3
3.2
3.4
3.6
pumping chamber diameter (in)
pumping chamber polarization w/ beam
0.58
0.56
0.54
0.52
0.5
0.48
0.46
0.44
0.42
0.4
0.38
2.4
2.6
2.8
3
3.2
3.4
3.6
pumping chamber diameter (in)
target chamber polarization w/ beam
0.52
0.5
0.48
0.46
0.44
0.42
0.4
0.38
0.36
0.34
2.4
2.6
2.8
3
3.2
3.4
3.6
pumping chamber diameter (in)


length of transfer tube (cm)
9
9.2
9.4
9.6
9.8
10
10.2
10.4
diffusion rate to/from target chamber (1/hours)
0.46
0.48
0.5
0.52
0.54
0.56
relative polarization gradient due to beam, 1st order (%)
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8
5.9
relative polarization gradient w/o beam, 1st order (%)
5.2
5.4
5.6
5.8
6
6.2
6.4
relative polarization gradient w/ beam, 1st order (%)
10.4
10.6
10.8
11
11.2
11.4
11.6
11.8
12
relative polarization gradient w/ beam, 3 chamber calc (%)
10.4
10.6
10.8
11
11.2
11.4
11.6
11.8
12
pumping chamber diameter (in)
2.4
2.6
2.8
3
3.2
3.4
3.6


pumping chamber polarization w/o beam
target chamber polarization w/o beam
pumping chamber polarization w/ beam
target chamber polarization w/ beam
w/ beam to w/o beam pumping chamber polarization ratio
w/ beam to w/o beam target chamber polarization ratio
pumping chamber diameter (in)



spin exchange rate in pumping chamber (1/hours)
diffusion rate to/from target chamber (1/hours)
Γslow, effective spin up rate, 1st order (1/hours)
Γfast, 1st order calc (1/hours)
Γslow, effective spin up rate, full (1/hours)
Γfast, full calculation (1/hours)
pumping chamber diameter (in)