August 16, 2005

RSS Analysis Formula

v. 0.1

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1 Experimental Data

1.1 Asymmetries

We begin with a raw NH3 asymmetry which has to be corrected for dilution factor.

From the extracted proton asymmetry A we form polarized cross section differences using an unpolarized cross section model:

$$\Delta\sigma = 2\Sigma A$$

Here, Σ is the unpolarized cross section from E. Christy's 2005 fit [2].

1.2 Structure Functions

The structure funtions are formed from the polarized cross section differences $\Delta\sigma$ via the following relation [1]:

$$\begin{aligned} g_1 &= \frac{MQ^2}{4\alpha^2}\frac{y}{(1-y)(2-y)}\left[\Delta\sigma_{\parallel} + \tan\frac{\theta}{2}\Delta\sigma_{\perp}\right] \\ g_2 &= \frac{MQ^2}{4\alpha^2}\frac{y^2}{2(1-y)(2-y)}\left[-\Delta\sigma_{\parallel} + \frac{1+(1-y)\cos\theta}{(1-y)\sin\theta}\Delta\sigma_{\perp}\right] \end{aligned} \tag{1}$$

where $y = \nu/E_0$

The following formulation gives identical numerical results:

$$\begin{aligned} g_1 &= \frac{MQ^2}{4\alpha^2}\frac{\nu E}{E'}\frac{1}{E+E'}\left[\Delta\sigma_{\parallel} + \tan\frac{\theta}{2}\Delta\sigma_{\perp}\right] \\ g_2 &= \frac{MQ^2}{4\alpha^2}\frac{\nu^2}{2E'(E+E')}\left[-\Delta\sigma_{\parallel} + \frac{E+E'\cos\theta}{E'\sin\theta}\Delta\sigma_{\perp}\right] \end{aligned} \tag{2}$$

The d_2 matrix element is defined as:

$$2\int_0^1 x^2(g_1 + 1.5g_2)dx \tag{3}$$

1.3 Spin Asymmetries

For A_1 and A_2 I follow Oscar's technote [3] eq. (10):

$$\begin{aligned} A_1 &= \frac{C}{D}\Big(A_{||} - dA_{\perp}\Big) \\ A_2 &= \frac{C}{D}\Big(c'A_{||} + d'A_{\perp}\Big) \end{aligned} \tag{4}$$

The kinematic factors are as follows:

$$
\begin{aligned}
C &= 1/(1+\eta c') \\
\eta &= \varepsilon\sqrt{Q^2}/(E-\varepsilon E') \\
c' &= \eta(1+\varepsilon)/(2\varepsilon) \\
D &= (1-\varepsilon E'/E)/(1+\varepsilon R) \\
d' &= 1/\sqrt{2\varepsilon/(1+\varepsilon)} \\
d &= \eta d' \\
\varepsilon &= \frac{1}{1+2(1+\nu^2/Q^2)\tan^2(\theta/2)}
\end{aligned}
$$

Note that I am not correcting at all for the out of plane scattering angle ϕ.

1.4 Virtual Photon Cross Sections

The virtual photoabsorption cross sections can be formed from the experimentally measured g_1 and g_2 via eq (5) of Drechsel *et al.* [4]:

$$
\begin{aligned}
\sigma'_{LT} &= \frac{4\pi^2\alpha}{MK}\gamma(g_1+g_2) \\
\sigma'_{TT} &= \frac{4\pi^2\alpha}{MK}(g_1-\gamma^2 g_2)
\end{aligned}
\tag{5}
$$

Here M is the proton mass, and the virtual photon flux factor is defined according to the Hand convention:

$$
K = \nu - Q^2/2M \tag{6}
$$

2 Polarized Models

In order to radiatively correct the RSS data we need polarized cross sections at lower incident energy. In the absence of actual data (especially in the transverse configuration) we resort to using a model as input.

The two models used in this analysis are the MAID model [5], and the fits to world data of S. Khun *et al.* [6] known as the Hall B model.

2.1 MAID

The MAID model outputs the five virtual photoabsorbtion cross sections for the proton (or the neutron if desired): σ_T, σ_L, σ_{LTy}, σ'_{TT}, and σ'_{LT}.

All results are for single pion production only. The results are at constant Q^2 and need to be interpolated to constant energy. From the virtual photoabsorbtion cross sections we can reconstruct the desire experimental quantities.[7]

$$\begin{aligned} A_1 &= \frac{\sigma'_{TT}}{\sigma_T} \\ A_2 &= \frac{\sigma'_{LT}}{\sigma_T} \end{aligned} \tag{7}$$

The unpolarized structure functions can be evaluated as[4]:

$$\begin{aligned} F_1 &= -\frac{MK}{4\pi^2\alpha}\sigma_T \\ F_2 &= \frac{2x(1+R)}{1+\gamma^2}F_1 \\ R &= \frac{\sigma_L}{\sigma_T} \end{aligned}$$

Inverting Eq. 5 above, we can solve for g_1,g_2 in terms of σ'_{TT}, and σ_{LT}.

$$\begin{aligned} g_1 &= \frac{MK}{4\pi^2\alpha}\frac{1}{1+\gamma^2}(\sigma_{TT} + \gamma\sigma_{LT}) \\ g_2 &= \frac{MK}{4\pi^2\alpha}\frac{1}{1+\gamma^2}(-\sigma_{TT} + \frac{\sigma_{LT}}{\gamma}) \end{aligned} \tag{8}$$

To obtain $A_\parallel$ and $A_\perp$ from MAID we use the relation:

$$\begin{aligned} A_\parallel &= D(A_1 + \eta A_2) \\ A_\perp &= d(A_2 - \zeta A_1) \end{aligned} \tag{9}$$

$$\begin{aligned} D &= (1 - \varepsilon E'/E)/(1 + \varepsilon R) \\ d &= D\sqrt{2\epsilon/(1+\varepsilon)} \\ \eta &= \varepsilon\sqrt{Q^2}/(E - \varepsilon E') \\ \zeta &= \eta(1+\varepsilon)/(2\varepsilon) \end{aligned}$$

To obtain $\Delta\sigma_{\parallel}$ and $\Delta\sigma_{\perp}$ from the maid output I invert Eq. 1 to obtain:

$$\Delta\sigma_{\parallel} = \frac{4\alpha^2}{MQ^2}\frac{(1-y)(2-y)}{y}\left(\frac{2}{y}\right)\frac{\tan\theta/2\ g_2 - \frac{1+(1-y)\cos\theta}{(1-y)\sin\theta}\frac{y}{2}\ g_1}{-\tan\theta/2 - \frac{1+(1-y)\cos\theta}{(1-y)\sin\theta}} \tag{10}$$

$$\Delta\sigma_{\perp} = \frac{4\alpha^2}{MQ^2}\frac{(1-y)(2-y)}{y}\left(\frac{2}{y}\right)\frac{-g_2 - \frac{2}{y}g_1}{-\tan\theta/2 - \frac{1+(1-y)\cos\theta}{(1-y)\sin\theta}} \tag{11}$$

2.2 Hall B model

Who knows?

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3. O. Rondon. RSS technote 2003-01. spin.phys.virginia.edu/∼or/e01006/analysis/aparaper.pdf
4. D. Drechsel, S. S. Kamalov and L. Tiator, Phys. Rev. D **63**, 114010 (2001) [arXiv:hep-ph/0008306].
5. D. Drechsel, O. Hanstein, S. S. Kamalov and L. Tiator, Nucl. Phys. A **645**, 145 (1999) [arXiv:nucl-th/9807001]. http://www.kph.uni-mainz.de/MAID/maid2003/total.html
6. Hall B model. S. Kuhn, private communication.
7. K. Abe *et al.* [E143 collaboration], Phys. Rev. D **58**, 112003 (1998) [arXiv:hep-ph/9802357].