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TARGET MASS EFFECTS IN POLARIZED DEEP INELASTIC SCATTERING

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The target mass effects in polarized DIS have been studied. It was demonstrated that taking into account the first-order target mass corrections to g_1 a very good approximation of the exact formula is achieved. It was also shown that their magnitude in the pre-asymptotic DIS region is small except for $x > 0.65$, where their large effect is partially suppressed by the large values of Q^2 due to the cut $W^2 > 4$ GeV^2. The difference between the size of the target mass and higher twist corrections is illustrated.

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1. Introduction

One of the features of polarized DIS is that a lot of the present data are in the pre-asymptotic region ($Q^2 \sim 1$–5 GeV^2, 4 GeV$^2 < W^2 < 10$ GeV^2). While in the unpolarized case we can cut the low Q^2 and W^2 data in order to minimize the less known higher twist effects, it is impossible to perform such a procedure for the present data on the spin-dependent structure functions without losing too much information. This is especially the case for the HERMES, SLAC and Jefferson Lab experiments. So, to confront correctly the QCD predictions with the experimental data and to determine the *polarized* parton densities, special attention must be paid to higher twist (powers in $1/Q^2$) corrections to the nucleon structure functions. The latter are nonperturbative effects and cannot be calculated without using models. That is why a *model-independent* extraction of the dynamical higher twists from the experimental data is important not only for a better determination of the polarized parton densities but also because it would lead to interesting tests of the nonperturbative QCD regime and, in particular, of the quark–hadron duality.

Before one can properly obtain information on the higher twist contribution, it is important to take into account in the analysis the so-called target mass corrections (TMCs) arising from purely kinematic effects associated with finite values of the quantity $4M^2x^2/Q^2$. These are also powers in $1/Q^2$ corrections, however formally related to the twist-two operators and therefore, unlike the higher twist ones, can be calculated without using models. In this note we present numerical results which illustrate the main features of the TMCs to the spin structure function g_1 valid in the pre-asymptotic DIS region. We consider that their knowledge is useful and important in the QCD analyses of the present and future data on polarized DIS at moderate energies.

2. Target Mass Corrections

We will follow the method proposed by Georgi and Politzer[1] in the case of unpolarized structure functions.[a] According to this method the target mass corrections to the spin structure function g_1 have the following form[4,5]

$$g_1^{\mathrm{TMC}}(x,Q^2) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn\, x^{-n} \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \times \frac{n(n+j)!}{j!(n-1)!(n+2j)^2} M_{n+2j}(Q^2; M=0)\,, \tag{1}$$

where $M_n(Q^2; M=0)$ are the Cornwall–Norton (CN) moments of g_1

$$M_n(Q^2; M=0) = \int_0^1 dx\, x^{n-1} g_1(x,Q^2; M=0) \tag{2}$$

calculated in the perturbative QCD when the mass terms $\mathcal{O}(M^n/Q^n)$ are neglected, or equivalently, the nucleon mass M is putting equal to zero, $M = 0$. As seen from (1) TMCs are expressed by the higher moments of $g_1(x,Q^2; M=0)$. If one performs the infinite sum over j in (1) in order to take into account all TMCs, g_1 can be written in the following form[5]

$$\begin{aligned} g_1^{\mathrm{TMC}}(x,Q^2) = {} & \frac{x g_1(\xi,Q^2; M=0)}{\xi(1+4M^2x^2/Q^2)^{3/2}} \\ & + \frac{4M^2x^2}{Q^2}\frac{(x+\xi)}{\xi(1+4M^2x^2/Q^2)^2}\int_\xi^1 \frac{d\xi'}{\xi'} g_1(\xi',Q^2; M=0) \\ & - \frac{4M^2x^2}{Q^2}\frac{(2-4M^2x^2/Q^2)}{2(1+4M^2x^2/Q^2)^{5/2}}\int_\xi^1 \frac{d\xi'}{\xi'}\int_{\xi'}^1 \frac{d\xi''}{\xi''} g_1(\xi'',Q^2; M=0)\,, \end{aligned} \tag{3}$$

[a] About the correspondence of this method with the Nachtmann approach[2] see the recent paper Ref. 3 and references therein.

where $g_1(x, Q^2; M = 0)$ is the well-known pQCD expression for g_1 (logarithmic in Q^2) obtained in LO or NLO approximation *neglecting* the target mass corrections, and

$$\xi = \frac{2x}{1 + (1 + 4M^2x^2/Q^2)^{1/2}} \tag{4}$$

is the Nachtmann variable. In Eqs. (1)–(3) we have dropped the nucleon target label N.

Let us now discuss the numerical results on the target mass corrections to the spin structure function g_1. In our analysis we will concentrate on their features in the pre-asymptotic DIS region (the invariant mass $W > 2$ GeV and moderate values of Q^2 : 1–10 GeV^2). In the calculations of the target mass effects we have used for $g_1(x, Q^2; M = 0)$ our recent NLO results from "g_1/F_1" fit presented in Ref. 6. For further discussion it is useful to rewrite (1) in the form:

$$\begin{aligned} g_1^{\rm TMC}(x, Q^2) = g_1(x, Q^2; M = 0) &+ \frac{M^2}{Q^2} g_1^{(1){\rm TMC}}(x, Q^2) \\ &+ \left(\frac{M^2}{Q^2}\right)^2 g_1^{(2){\rm TMC}}(x, Q^2) \\ &+ \left(\frac{M^2}{Q^2}\right)^3 g_1^{(3){\rm TMC}}(x, Q^2) + \mathcal{O}(M^8/Q^8)\,, \end{aligned} \tag{5}$$

where

$$g_1^{(j){\rm TMC}}(x, Q^2) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn\, x^{-n} \frac{n(n+j)!}{j!(n-1)!(n+2j)^2} M_{n+2j}(Q^2; M = 0)\,, \quad j = 1, 2, \ldots\,. \tag{6}$$

The questions we address here are (i) How large are the target mass effects in the pre-asymptotic DIS region and (ii) How fast the series (1) converges, or in other words, how large is the difference between $g_1^{\rm TMC}$ calculated by (3) and $g_1^{{\rm TMC},Q^2}$ obtained when the TMCs are calculated only up to the first order in (M^2/Q^2) (the first two terms in (1) or equivalently, in (5)). We will focus on the proton target, but in the end of our discussion we will also mention our results concerning the neutron spin structure function $g_1^n(x, Q^2)$. Note that it is of interest from a theoretical point of view to examine the convergence of the TMCs irrespective of the size of the dynamical HT. For practical purposes, however, there is no sense to use the exact formula (3) in the analysis of the experimental data if the dynamical HT are not negligible to those of the TMCs. In this case the target mass and dynamical higher twist corrections have to be taken into account up to the same finite order in $\mathcal{O}(1/Q^2)$.

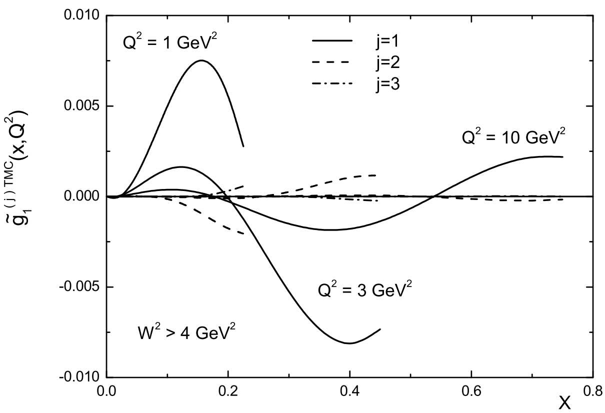

Fig. 1. Target mass corrections $\tilde{g}_{1,p}^{(j)\mathrm{TMC}}(x, Q^2)$ at fixed orders in $(M^2/Q^2)^j$ for three different values of Q^2 (see text).

The target mass corrections at fixed order in (M^2/Q^2) in the expansion (5) are illustrated in Fig. 1 where the notations

$$\tilde{g_1}^{(j)\mathrm{TMC}}(x, Q^2) = \left(\frac{M^2}{Q^2}\right)^j g_1^{(j)\mathrm{TMC}}(x, Q^2)\,, \qquad j = 1, 2, \ldots \tag{7}$$

are used. The TMCs are given outside the resonance region, i.e. $W^2 > 4$ GeV^2, where $W^2 = M^2 + Q^2(1 - x)/x$, at different fixed values of $Q^2 : Q^2 = 1$ GeV^2, 3 GeV^2 and 10 GeV^2. One can see from Fig. 1 that the series (1), or equivalently (5), is an alternating series and the sizes of TMCs higher than the first order in (M^2/Q^2) are small, especially the target mass corrections of third order in (M^2/Q^2), $\tilde{g_1}^{(3)\mathrm{TMC}}(x, Q^2)$, which are smaller than $6 \times 10^{-4}(Q^2 = 1$ GeV$^2)$; $2.5 \times 10^{-4}(Q^2 = 3$ GeV$^2)$; $5 \times 10^{-6}(Q^2 = 10$ GeV$^2)$.

In order to estimate the target mass effects in the pre-asymptotic DIS region we demonstrate also the relative changes of g_1^p taking into account TMCs up to order $\mathcal{O}(M^2/Q^2)$, $\mathcal{O}(M^6/Q^6)$ and all orders of M^2/Q^2 [Eq. (3)]. These effects are presented for $Q^2 = 1$ GeV^2 (Fig. 2), $Q^2 = 3$ GeV^2 and $Q^2 = 10$ GeV^2 (Fig. 3), respectively. The notations used in these figures are as follows:

$$\begin{aligned} g_1^{\mathrm{TMC},Q^2} &= g_1(M=0) + \frac{M^2}{Q^2} g_1^{(1)\mathrm{TMC}}\,, \\ g_1^{\mathrm{TMC},Q^6} &= g_1(M=0) + \sum_{j=1}^{3} \left(\frac{M^2}{Q^2}\right)^j g_1^{(j)\mathrm{TMC}}\,, \\ g_1^{\mathrm{TMC,tot}} &= g_1^{\mathrm{TMC}} \quad [\text{Eq. (3)}]\,. \end{aligned} \tag{8}$$

Note that to calculate $g_1^{\mathrm{TMC,tot}}(x, Q^2)$ we have used the prescription given in Ref. 5 that $g_1(\xi, Q^2; M = 0)$ on the R.H.S. of Eq. (3) vanishes for $\xi > \xi_0$, where $\xi_0 = \xi(x = 1)$.

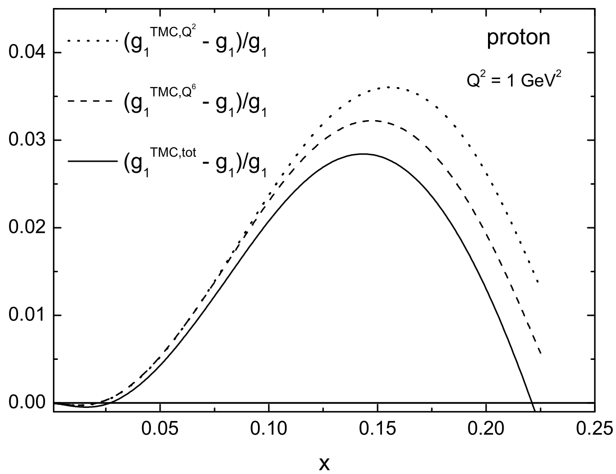

Fig. 2. The proton target mass effects at $Q^2 = 1$ GeV^2 (see text).

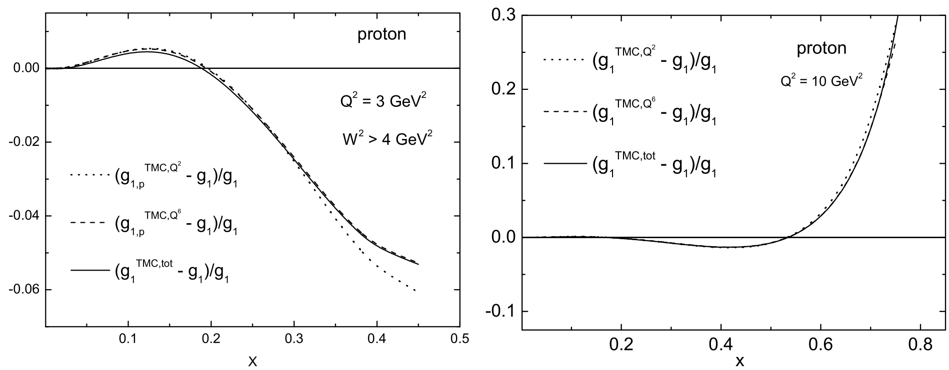

Fig. 3. The proton target mass effects at $Q^2 = 3$, 10 GeV^2 (see text).

In Figs. 2 and 3 the dotted, dashed and solid curves correspond to the change of g_1 proton when the first, third and all orders in (M^2/Q^2), respectively, are taken into account. As seen from these figures, the sign of the target mass corrections depends on the x region and the maximum change of the magnitude of g_1 due to the first-order TMCs is 3.6% at $Q^2 = 1$ GeV^2, 6% at $Q^2 = 3$ GeV^2 and 28.6% at $Q^2 = 10$ GeV^2.[b] The TM effects are large in the large x and small Q^2 region. However, because of the cut $W^2 > 4$ GeV^2, the large x region at small Q^2 is outside of the pre-asymptotic DIS region and their effects are much smaller than those in the resonance one. One can see also from Figs. 2 and 3 that taking into account a first-order TMCs to g_1 a good approximation of the exact equation (1) is already

[b]The fact that in the polarized case the target mass corrections at first order in M^2/Q^2 are generally small was first established in Ref. 4.

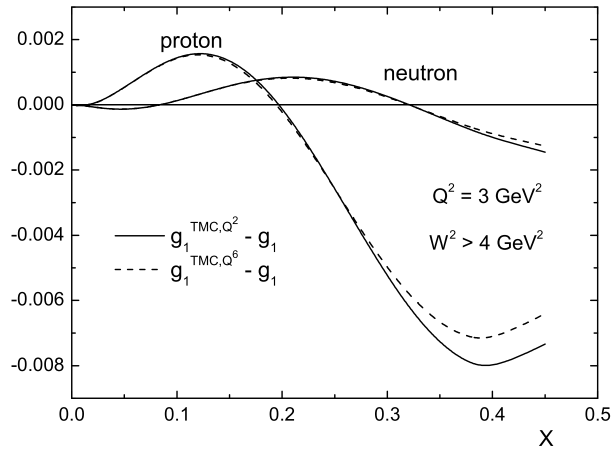

Fig. 4. Comparison between the target mass corrections to g_1^n and g_1^p at $Q^2 = 3\ \mathrm{GeV}^2$.

achieved. The deviation of this approximation of g_1 from $g_1^{\mathrm{TMC,tot}}(x, Q^2)$, is not more than 1.5% for $Q^2 = 1\ \mathrm{GeV}^2$, 0.8% for $Q^2 = 3\ \mathrm{GeV}^2$ and 1.8% for $Q^2 = 10\ \mathrm{GeV}^2$. The dashed curves, which correspond to g_1^{TMC,Q^6} (the TMCs are taken into account up to a third order in M^2/Q^2), practically coincide with $\Delta g_1^{\mathrm{TMC,tot}}/g_1$ (solid curves), especially in the cases of $Q^2 = 3\ \mathrm{GeV}^2$ and $Q^2 = 10\ \mathrm{GeV}^2$. This observation is a consequence of the fact we have already discussed above that the series of the target mass corrections (1) is an alternative sign one.

In Fig. 4 we compare the TMCs to neutron spin structure function $g_1^n(x, Q^2)$ with those to g_1^p. As seen from Fig. 4 its magnitude is much smaller than that in the proton case. This is due to the fact that the negative g_1^n crosses zero at $x \approx 0.4$ and becomes positive at x higher than 0.4.[7] As a result, the values of the moments of g_1^n on the R.H.S. of (1) are much smaller than those of g_1^p. The main conclusion about the proton TMCs, namely, that the first-order TMCs is a good approximation of their total account, holds for the neutron target too.

Finally, we would like to compare the size of the TMCs with that of the higher twist power corrections to the spin structure function g_1. Taking into account only the first-order corrections in $1/Q^2$, g_1 has the following form:

$$g_1(x, Q^2) = g_1(x, Q^2; M = 0) + \frac{M^2}{Q^2} g_1^{(1)\mathrm{TMC}}(x, Q^2) + \frac{h(x, Q^2)}{Q^2} + \mathcal{O}(1/Q^4)\,, \quad (9)$$

where $h(x, Q^2)$ are the dynamical higher twist ($\tau = 3$ and $\tau = 4$) corrections to g_1, which are related to multi-parton correlations in the nucleon. The latter are nonperturbative effects and cannot be calculated without using models.

In Fig. 5 we compare the first-order TMCs, $\tilde{g}_1{}^{(1)\mathrm{TMC}}(x, Q^2)$ [see Eq. (7)], with the values of the HT corrections, $h(x, Q^2)/Q^2$, extracted in a model-independent way from the world data on polarized DIS.[6] One can see from Fig. 5 that: (i) In the neutron case the target mass contribution in g_1 is negligible compared to the higher

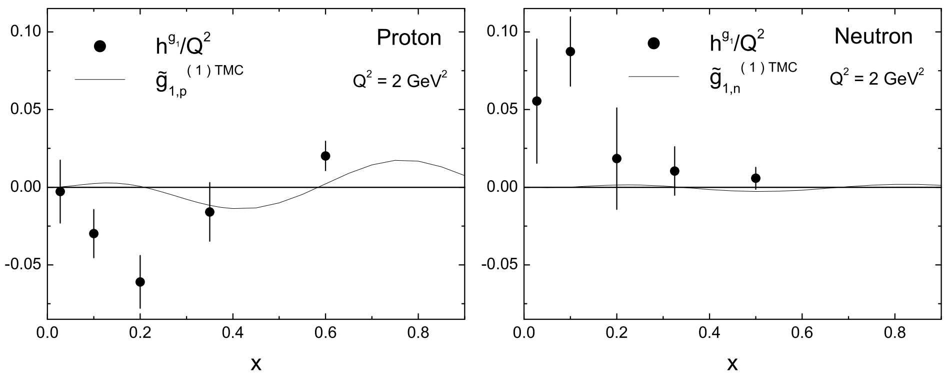

Fig. 5. Comparison between the first-order target mass corrections to g_1 (solid curves) and the HT (twist-3 and 4) corrections at $Q^2 = 2$ GeV^2.

twist effects and (ii) in the proton case the TMCs are essential at $x > 0.3$ while for $x < 0.3$ their magnitude is much smaller than that of the HT corrections. Note also the different shapes of TM and HT corrections. As a result of this analysis we conclude that in the pre-asymptotic DIS region the higher twist contribution in g_1 becomes more and more important, so the TMCs cannot be trusted alone.

3. Summary

We have studied the target mass effects in polarized DIS scattering which would be important in pre-asymptotic region. It was demonstrated that accounting for the first-order target mass corrections to g_1 a very good approximation of the exact formula (3) is achieved. It was also shown that the size of the TMCs in the pre-asymptotic DIS region is small ($< 6\%$) except for $x > 0.65$ where their effect is partially suppressed by the large values of Q^2 due to the cut $W^2 > 4$ GeV^2. Compared to the higher twist effects the contribution of target mass corrections to the spin structure function g_1 in the pre-asymptotic DIS region is insignificant except for $x > 0.3$ in the proton case. Nevertheless, to extract correctly the unknown high twist effects from the experimental data, the calculable target mass corrections to the spin structure functions should be taken into account.

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