Feasibility Study for Measurement of Parity Violating Asymmetry in Deep Inelastic Scattering Using Polarized Targets

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April 18, 2011

In this document we study the feasibility of measuring the parity violating asymmetry in deep inelastic scattering using an unpolarized 11 GeV electron beam and polarized targets. We will focus on the use of a polarized ^{3}He target and a large solenoid device to detect the scattered electrons. Formula used in this document were taken primiarily from Ref. [1]. In addition to the asymmetry, we will also discuss the extraction of structure function $g_3^{\gamma Z}$ and the strange parton distribution function $\Delta s + \Delta\bar{s}$.

For an unpolarized electron beam, the cross section different between scattering off a target with spin parallel and that with spin anti-parallel to the beam direction is:

$$\frac{1}{2}\left(\frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Rightarrow}}}{dx\ dy} - \frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Leftarrow}}}{dx\ dy}\right) + \frac{1}{2}\left(\frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Rightarrow}}}{dx\ dy} - \frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Leftarrow}}}{dx\ dy}\right) \equiv \frac{d^2\sigma^{\Rightarrow}}{dx\ dy} - \frac{d^2\sigma^{\Leftarrow}}{dx\ dy}$$

$$\begin{aligned} \approx\ & 16\pi ME\frac{\alpha^2}{Q^4}\ \Big\{(1-y)\left[g_V\eta^{\gamma Z}(g_3^{\gamma Z} - g_4^{\gamma Z}) + g_A^2\eta^Z(g_3^Z - g_4^Z)\right] \\ & + xy^2\left[g_V\eta^{\gamma Z}\ g_5^{\gamma Z} + g_A^2\eta^Z\ g_5^Z\right] + xy(2-y)g_A\eta^{\gamma Z}g_1^{\gamma Z}\Big\}, \end{aligned} \tag{1}$$

where for electron scattering, $g_V = g_V^e = -\frac{1}{2} + 2\sin^2\theta_W$ and $g_A^e = -\frac{1}{2}$. For positron scattering, one only need to replace g_A^e by $g_A^{e^+} = -g_A^e$ in the equation above. Other variables involved are

$$\eta^\gamma = 1 \tag{2}$$

$$\eta^{\gamma Z} = \left(\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\right)\left(\frac{Q^2}{Q^2 + M_Z^2}\right), \tag{3}$$

$$\eta^Z = \left(\eta^{\gamma Z}\right)^2 \tag{4}$$

with $G_F = 1.166 \times 10^{-5}$ (GeV)$^{-2}$, $M_Z = 91.2$ GeV, $\frac{G_F M_Z^2}{2\sqrt{\pi}\alpha(Q^2+M_Z^2)} \approx \frac{G_F}{2\sqrt{2}\pi\alpha} \approx 180$ ppm and $\eta^{\gamma Z} = Q^2 \times 180$ ppm/(GeV)^2. At medium energies relavant to JLab, one has Q^2 a few

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GeV^2 hence $\eta^Z \ll \eta^{\gamma Z}$. In the naive parton model, the structure functions involved on the RHS of Eq.(1), as well as other frequently used ones, are

$$F_1^\gamma = \frac{1}{2}\sum_f e_{q_f}^2 (q_f + \bar{q}_f) \qquad F_2^\gamma = 2xF_1^\gamma \tag{5}$$

$$g_1^\gamma = \frac{1}{2}\sum_f e_{q_f}^2 (\Delta q_f + \Delta\bar{q}_f) \qquad g_2^\gamma = 0 \tag{6}$$

where $q_f = u, d, s, c$ with u stands for the number density of the u quark and so on. The interference contribution ($i = \gamma Z$) is:

$$F_1^{\gamma Z} = \sum_f e_{q_f} (g_V)_{q_f} (q_f + \bar{q}_f) \qquad F_2^{\gamma Z} = 2xF_1^{\gamma Z} \tag{7}$$

$$F_3^{\gamma Z} = 2\sum_f e_{q_f} (g_A)_{q_f} (q_f - \bar{q}_f) \tag{8}$$

$$g_1^{\gamma Z} = \sum_f e_{q_f} (g_V)_{q_f} (\Delta q_f + \Delta\bar{q}_f) \tag{9}$$

$$g_2^{\gamma Z} = g_4^{\gamma Z} = 0 \tag{10}$$

$$g_3^{\gamma Z} = 2x\sum_f e_{q_f} (g_A)_{q_f} (\Delta q_f - \Delta\bar{q}_f) = 2xg_5^{\gamma Z} \tag{11}$$

and the purely weak interaction ($i = Z$) leads to:

$$F_1^Z = \frac{1}{2}\sum_f (g_V^2 + g_A^2)_{q_f} (q_f + \bar{q}_f) \qquad F_2^Z = 2xF_1^Z \tag{12}$$

$$F_3^Z = 2\sum_f (g_V g_A)_{q_f} (q_f - \bar{q}_f) \tag{13}$$

$$g_1^Z = \frac{1}{2}\sum_f (g_V^2 + g_A^2)_{q_f} (\Delta q_f + \Delta\bar{q}_f) \tag{14}$$

$$g_2^Z = -\frac{1}{2}\sum_f (g_A^2)_{q_f} (\Delta q_f + \Delta\bar{q}_f) \tag{15}$$

$$g_3^Z = 2x\sum_f (g_V g_A)_{q_f} (\Delta q_f - \Delta\bar{q}_f) = 2xg_5^Z \tag{16}$$

$$g_4^Z = 0\,. \tag{17}$$

The weak neutral couplings for the electron and quarks can be calculated using

$$\sin^2\theta_W = 0.232\ ,$$

which gives

$$g_V^e = -\frac{1}{2} + 2\sin^2\theta_W = -0.036 \tag{18}$$

$$g_V^u = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W = 0.191 \tag{19}$$

$$g_V^d = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W = -0.345 \tag{20}$$

$$g_V^s = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W = -0.345 \tag{21}$$

And the axial couplings are

$$g_A^e = -\frac{1}{2}, \quad g_A^u = \frac{1}{2}, \quad g_A^d = -\frac{1}{2}, \quad g_A^s = -\frac{1}{2}$$

Since $g_3^{\gamma Z}$ is involved in the polarized cross section of Eq. (1), one can see that the value of measuring g_3 structure functions is to provide information on the polarized valence quark distributions $(\Delta q_f - \Delta\bar{q}_f)$. Compared to semi-inclusive DIS measurements, using unpolarized beam - polarized target DIS can avoid dealing with hadron fragmentations and thus can provide a cleaner information on Δq_V. From the target single-spin asymmetry one can also extract $g_1^{\gamma Z}$. Because the $(\Delta q + \Delta\bar{q})$ in $g_1^{\gamma Z}$ are weighted by different couplings from that in the "regular" spin structure function g_1^{γ} (usually called g_1), by measuring three out of the four structure functions: $g_1^{\gamma,n}$, $g_1^{\gamma,p}$, $g_1^{\gamma Z,n}$ and $g_1^{\gamma Z,p}$, and assuming isospin symmetry between the proton and the neutron, one can separate $(\Delta u + \Delta\bar{u})$, $(\Delta d + \Delta\bar{d})$ and $(\Delta s + \Delta\bar{s})$. If we measure all six structure functions: $g_1^{\gamma,n}$, $g_1^{\gamma,p}$, $g_1^{\gamma Z,n}$, $g_1^{\gamma Z,p}$, $g_3^{\gamma Z,n}$ and $g_3^{\gamma Z,p}$, then all six PDFs (Δq, $\Delta\bar{q}$ for u, d and s) can be extracted.

To measure the $g_1^{\gamma Z}$ or the $g_3^{\gamma Z}$ structure functions, we can measure the target single-spin asymmetry. To find out the size of this asymmetry, we devide Eq.(1) by the sum of the two polarized cross sections, which equals to twice the unpolarized cross section (see *p*.195 of Ref. [2]):

$$\left(\frac{d^2\sigma}{dxdy}\right) = \frac{2\pi\alpha^2}{Q^4} s\left[1 + (1-y)^2\right]\sum_f e_{q_f}^2 x q_f(x) \tag{22}$$

$$= \frac{4\pi\alpha^2 ME}{Q^4}\left[1 + (1-y)^2\right]\sum_f e_{q_f}^2 x q_f(x) \tag{23}$$

where $s = 2k \cdot p = 2ME$ is used for fixed target scatterings. The asymmetry is thus

$$A = \frac{16\pi ME\frac{\alpha^2}{Q^4}}{2 \times 4\pi ME\frac{\alpha^2}{Q^4}} \times \tag{24}$$

$$\frac{\left\{(1-y)\left[g_V\eta^{\gamma Z}(g_3^{\gamma Z} - g_4^{\gamma Z}) + g_A^2\eta^Z(g_3^Z - g_4^Z)\right] + xy^2\left[g_V\eta^{\gamma Z}\ g_5^{\gamma Z} + g_A^2\eta^Z\ g_5^Z\right] + xy(2-y)g_A\eta^{\gamma Z}g_1^{\gamma Z}\right\}}{[1 + (1-y)^2]\sum_f e_{q_f}^2 x q_f(x)}$$

using $g_2^{\gamma Z} = g_4^{\gamma Z} = g_4^Z = 0$ and $g_5^{\gamma Z} = g_3^{\gamma Z}/(2x)$ to simplify, one has

$$A = \frac{\left\{2(1-y)\left[g_V\eta^{\gamma Z}(g_3^{\gamma Z}) + g_A^2\eta^Z(g_3^Z)\right] + y^2\left[g_V\eta^{\gamma Z}\ g_3^{\gamma Z} + g_A^2\eta^Z\ g_3^Z\right] + 2xy(2-y)g_A\eta^{\gamma Z}g_1^{\gamma Z}\right\}}{[1 + (1-y)^2]\sum_f e_{q_f}^2 x q_f(x)} \tag{25}$$

Now, although $g_3^{\gamma Z}$ and g_3^Z are both sensitive to the polarized valence pdf, g_3^Z is weighted by η^Z while $g_3^{\gamma Z}$ is weighted by $\eta^{\gamma Z}$ in the numerator of the asymmetry. Since $\eta^Z \ll \eta^{\gamma Z}$, the asymmetry is much more sensitive to $g_3^{\gamma Z}$ than to g_3^Z. Therefore one has approximately

$$A = \frac{\left\{2(1-y)\left[g_V \eta^{\gamma Z}(g_3^{\gamma Z})\right] + y^2\left[g_V \eta^{\gamma Z}\ g_3^{\gamma Z}\right] + 2xy(2-y)g_A \eta^{\gamma Z} g_1^{\gamma Z}\right\}}{[1+(1-y)^2]\sum_f e_{q_f}^2 x q_f(x)} \tag{26}$$

$$= \eta^{\gamma Z}\frac{(2-2y+y^2)g_V g_3^{\gamma Z} + 2xy(2-y)g_A g_1^{\gamma Z}}{[1+(1-y)^2]\sum_f e_{q_f}^2 x q_f(x)} \tag{27}$$

For an 11 GeV electron beam, typical x and Q^2 ranges are $0.2 < x < 0.8$ and $2 < Q^2 < 10$ (GeV)^2. The size of the asymmetry, the percent-contribution from g_3 and $\Delta s + \Delta\bar{s}$ are calculated for both the proton and the neutron, see Fig. 1. The polarized PDF used are those from Bluemlein and Boettcher [3] and the unpolarized cross sections were calculated using the classical NMC95 fit [4]. In addition, using a large solenoid device possible for Hall A in the future, rates for a polarized ^{3}He target have been simulated with a luminosity of 10^{36} ^{3}He/(cm^{2}sec) [5]. Time needed for a 100% measurement of the neutron asymmetry (which take into account a 80% beam and a 50% target polarizations as well as nuclear corrections) has also been estimated. Figure 1 shows these results.

From Fig. 1 one can see that a reasonable measurement of g_3 (in the sense that it can help us to furthur understand the polarized parton distributions and thus the nucleon spin structure) requires measurement of the asymmetry to at least a couple of % level. But the current rate and luminosity do not allow us to do so. The situation for the proton is better for the neutron (the contribution from g_3 is relatively large), but the luminosity of polarized proton targets currently available is much lower than that of the polarzied 3He target, thus the beam time needed would be not realistic.

References

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Figure 1: Target spin-flip asymmetry using an unpolarized 11 GeV electron beam for the proton (left) and the neutron (right). The rate and time needed for a 100% measurement on the neutron asymmetry were estimated using a large solenoid device and a polarized ^{3}He target with a luminosity of 10^{36} cm^{-2}s^{-1}.