Interactions between Octet Baryons in the $SU_6$ Quark Model and their Applications to Light Hypernuclei *

Y. Fujiwara\textsuperscript{a}, C. Nakamoto\textsuperscript{b}, M. Kohno\textsuperscript{c}, Y. Suzuki\textsuperscript{d} and K. Miyagawa\textsuperscript{e}

\textsuperscript{a}Department of Physics, Kyoto University, Kyoto 606-8502, Japan

\textsuperscript{b}Suzuka National College of Technology, Suzuka 510-0294, Japan

\textsuperscript{c}Physics Division, Kyushu Dental College, Kitakyushu 803-8580, Japan

\textsuperscript{d}Department of Physics, Niigata University, Niigata 950-2181, Japan

\textsuperscript{e}Department of Applied Physics, Okayama Science University, Okayama 700-0005, Japan

The recent quark-model baryon-baryon interaction by the Kyoto-Niigata group is applied to the triton, hypertriton, $2\alpha\Lambda$ and $2\Lambda\alpha$ systems, in which a new three-cluster Faddeev formalism, using the 2-cluster resonating-group method (RGM) kernel, is developed for the exact treatment of the Pauli forbidden states between clusters.

1. 3-CLUSTER FADDEEV FORMALISM USING THE 2-CLUSTER RGM KERNEL

The QCD-inspired spin-flavor $SU_6$ quark model for the baryon-baryon interaction, proposed by the Kyoto-Niigata group, is a unified model for the complete baryon octet ($B_8 = N, \Lambda, \Sigma$ and $\Xi$), which has achieved very accurate descriptions of the $NN$ and $YN$ interactions. \cite{1–3} In particular, the nucleon-nucleon ($NN$) interaction of the most recent model fss2 \cite{2} is accurate enough to compare with the modern realistic meson-exchange models. These quark-model interactions can be used for realistic calculations of few-baryon and few-cluster systems, once an appropriate three-body equation is developed for the pairwise interactions described by the RGM kernel. The desired 3-cluster equation should be able to deal with the non-locality and the energy-dependence intrinsically involved in the quark-exchange RGM kernel. Furthermore, the quark-model description of the hyperon nucleon ($YN$) and the hyperon hyperon ($YY$) interactions in the full coupled-channel formalism sometimes involves a Pauli forbidden state, which excludes the most compact spatial configuration, resulting in the strongly repulsive nature of the interactions in some particular channels. We have recently formulated a new 3-cluster equation which uses two-cluster RGM kernels explicitly. \cite{4} This equation exactly eliminates 3-cluster redundant components by the orthogonality of the total wave function

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to the pairwise two-cluster Pauli-forbidden states. The explicit energy dependence inherent in the exchange RGM kernel is self-consistently determined. This equation is entirely equivalent to the Faddeev equation which uses a modified singularity-free T-matrix (which we call the RGM T-matrix) constructed from the two-cluster RGM kernel. We first applied this formalism to a 3-dineutron system and the 3α system, and obtained complete agreement between the Faddeev calculations and the variational calculations which use the translationally invariant harmonic-oscillator (h.o.) basis. [4,5] For the 3α system, the input is the 3-range Minnesota force (MN force) with the exchange mixture $u = 0.946869$, and the h.o. width parameter, $\nu = 0.257 \text{ fm}^{-2}$, is used for the $(0s)^4 \alpha$-clusters. The 2α phase shifts are nicely reproduced in the 2α RGM. We find that the 3α ground-state energies obtained by solving the present 3α Faddeev equations are only about 1.0 $\sim$ 1.7 MeV higher than those of the full microscopic 3α RGM calculations. [5]

2. TRITON AND HYPERTRITON FADDEEV CALCULATIONS

The present 3-cluster RGM formalism was applied to the Faddeev calculations of the $3N$ bound state [6] and $(2N\Lambda) - (2N\Sigma)$ system for the hypertriton, employing the off-shell $T$-matrices which are derived from the non-local and energy-dependent RGM kernel for our quark-model $NN$ and $YN$ interactions of fss2 and FSS. The model fss2 yields the triton energy $E(^3\text{H}) = -8.519 \text{ MeV}$ in the 50 channel calculation, when the $np$ interaction is employed for any $NN$ pairs in the isospin basis. [7] The charge rms radii for $^3\text{H}$ and $^3\text{He}$ are also correctly reproduced. These results are the closest to the experiments among many Faddeev calculations employing modern realistic $NN$ interactions. A characteristic description of the short range correlations in the quark model is essential to reproduce the large binding energy and the correct size of the three-nucleon bound state without reducing the $D$-state probability of the deuteron. For the hypertriton calculation, the exact treatment of the $\Lambda N - \Sigma N$ coupling and the resulting Pauli forbidden state with the $SU_3 \ (11)^*$ symmetry is very important to obtain the precise result. In the final calculation with 150 $\Lambda NN$ and $\Sigma NN$ channels included, we find $B_\Lambda(^3\text{H}) = 289 \text{ keV}$ and the $\Sigma NN$ component $P_{\Sigma NN} = 0.805\%$ for the fss2 prediction. For our previous model FSS, we obtain $B_\Lambda(^3\text{H}) = 878 \text{ keV}$ and $P_{\Sigma NN} = 1.36\%$. Since $B_\Lambda^{\exp} = 130 \pm 50 \text{ keV}$, the fss2 result is slightly overbound, which implies that the $^1S_0$ attraction of the $\Lambda N$ interaction is slightly too attractive in comparison to the $^3S_1$ attraction. From these results, we can extrapolate the desired difference of the $^1S_0$ and $^3S_1$ phase shifts at the maximum values. It turns out to be $3^\circ \sim 7^\circ$ more attractive in the $^1S_0$ state, which is consistent with the result in [8] which uses more simplified effective interactions.

3. 2αΛ FADDEEV CALCULATION FOR $^9\Lambda\text{Be}$

The formalism is now applied to the 2αΛ Faddeev calculation for $^9\Lambda\text{Be}$, by using the 2α RGM kernel and the $\Lambda\alpha$ folding potentials for various $\Lambda N$ effective forces. The effective $\Lambda N$ force, denoted by SB (Sparenberg-Baye potential) in Table 1, is constructed from the $^1S_0$ and $^3S_1$ phase shifts predicted by the $YN$ sector of the model fss2 [3], by using an inversion method based on supersymmetric quantum mechanics. [9] These are simple 2-range Gaussian potentials which reproduce the low-energy behaviour of the $\Lambda N$ phase shifts obtained by the full coupled-channel calculations. Since any central and single-
channel effective $\Lambda N$ force leads to the well-known overbinding problem of $^5\Lambda$He [10] by about 2 MeV (in the present case, it is 1.63 MeV), the attractive part of the $^3S_1\Lambda N$ potential is adjusted to reproduce the correct binding energy $E^{\text{exp}}(^5\Lambda\text{He}) = -3.12 \pm 0.02$ MeV. The odd-state $\Lambda N$ force is assumed to be zero. The partial waves up to $\ell_{\text{Max}} = 6$ are included both in the $2\alpha$ and $\Lambda\alpha$ channels. The direct and exchange Coulomb kernel between two $\alpha$-clusters is introduced at the nucleon level with the cut-off radius, $R_C = 14$ fm (central case) or 10 fm ($\ell s$ included). Table 1 shows the ground-state ($0^+$) and the excited-state ($2^+$) energies of $^9\Lambda\text{Be}$, predicted by the SB and the other various $\Lambda N$ potentials used by Hiyama et al. [11]. In the present calculations using only the central force, the SB potential with the pure Serber character can reproduce the ground-state and excited-state energies within the accuracy of 100 - 200 keV. Table 1 also shows simple estimates of the $\ell s$ splitting for the $5/2^+$ and $3/2^+$ excited states, due to the spin-orbit interaction predicted by fss2 and FSS.

4. $2\Lambda\alpha$ FADDEEV CALCULATION FOR $^6\Lambda\Lambda$He

Next, we use the $\Lambda\alpha$ T-matrix, used in the $2\alpha\Lambda$ Faddeev calculation, to calculate the ground-state energy of $^6\Lambda\Lambda$He. The full coupled-channel T-matrices of fss2 and FSS with the strangeness $S = -2$ and the isospin $I = 0$ [3] are employed for the $\Lambda\Lambda$ RGM T-matrix. We find that the Hiyama’s 3-range Gaussian $\Lambda\Lambda$ potential and our Faddeev calculation using FSS yield very similar results with the large $\Delta B_{\Lambda\Lambda}$ values (defined by $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}(^6\Lambda\Lambda\text{He}) - 2B_{\Lambda}(^5\Lambda\text{He})$) about 3.6 - 3.7 MeV, since the $\Lambda\Lambda$ phases shifts predicted by both interactions increase up to about 40°. The improved quark model fss2 yields $\Delta B_{\Lambda\Lambda} = 1.41$ MeV. (If we use the $\Lambda\Lambda$ single-channel T-matrix, this number is reduced to $\Delta B_{\Lambda\Lambda} = 1.14$ MeV.) If we use a simple 2-range Gaussian potential, $V_{\Lambda\Lambda}(\text{SB})$, derived from the fss2 $^1S_0$ $\Lambda\Lambda$ phase shift by the supersymmetric inversion method, we obtain $\Delta B_{\Lambda\Lambda} = 1.90$ MeV. We think that the 0.5 MeV difference between our fss2 result and Table 1.

The ground-state energy $E_{\text{gr}}(0^+)$ and the $2^+$ excitation energy $E_\chi(2^+)$ in MeV, calculated by solving the Faddeev equation for the $2\alpha\Lambda$ system. In the last column, $\Delta E$ is a simple estimate of the $\ell s$ splitting for the $5/2^+$ and $3/2^+$ excited states, using the $P=0$ Wigner transform of the fss2 and FSS $\Lambda N LS$ interactions. The model fss2 involves an extra $\sigma$-meson contribution, which is indicated by “+\sigma”.

<table>
<thead>
<tr>
<th>$V_{\Lambda\Lambda}$</th>
<th>$E_{\text{gr}}(0^+)$ (MeV)</th>
<th>$E_\chi(2^+)$ (MeV)</th>
<th>$\Delta E$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SB</td>
<td>-6.837</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NS</td>
<td>-6.742</td>
<td>-6.81</td>
<td>0.07</td>
</tr>
<tr>
<td>ND</td>
<td>-7.483</td>
<td>-7.57</td>
<td>0.09</td>
</tr>
<tr>
<td>NF</td>
<td>-6.906</td>
<td>-7.00</td>
<td>0.09</td>
</tr>
<tr>
<td>JA</td>
<td>-6.677</td>
<td>-6.76</td>
<td>0.08</td>
</tr>
<tr>
<td>JB</td>
<td>-6.474</td>
<td>-6.55</td>
<td>0.08</td>
</tr>
<tr>
<td>Exp’t</td>
<td>-6.62 ± 0.04</td>
<td></td>
<td>3.029(3)/3.060(3)</td>
</tr>
</tbody>
</table>
the $V_{\Lambda\Lambda}(SB)$ result is probably because we neglected the full coupled-channel effect of the $\Lambda\Lambda\alpha$ channel to the $\Xi N\alpha$ and $\Sigma\Sigma\alpha$ channels. We should also keep in mind that in all of these 3-cluster calculations the Brueckner rearrangement effect [10] of the $\alpha$-cluster with the magnitude of about $-1$ MeV (repulsive) is very important. It is also reported in [12] that the quark Pauli effect among the $\alpha$ cluster and the $\Lambda$ hyperons gives a non-negligible repulsive contribution of the order of 0.1 - 0.2 MeV for the $\Lambda$ separation energy of $^6\Lambda\Lambda$He, even when we assume a rather compact $(3q)$ size of $b = 0.6$ fm. Taking all of these effects into consideration, we can conclude that the present results by fss2 are in good agreement with the recent experimental value $\Delta B^{\exp}_{\Lambda\Lambda} = 1.01 \pm 0.20$ MeV [13] deduced from the Nagara event.

5. SUMMARY

The 3-cluster Faddeev formalism using the 2-cluster RGM kernel opens a way to solve few-baryon systems interacting by the quark-model baryon-baryon interaction without spoiling the essential features of the RGM kernel; i.e., the non-locality, the energy dependence and the existence of the pairwise Pauli-forbidden state. It can also be used for the 3-cluster systems involving $\alpha$-clusters, like the $^9\Lambda$Be system. A nice point of this formalism is that the underlying $NN$ and $YN$ interactions are more directly related to the structure of the hypernuclei than the models assuming simple 2-cluster potentials. In particular, we have found that the most recent quark-model interaction, the model fss2, yields a realistic description of many systems including the triton, hypertriton, $^9\Lambda$Be and $^6\Lambda\Lambda$He.

REFERENCES