Polarization observables for single and double charged pion photo-production with polarized HD target

Peng Peng
Houma, Shanxi, China

M. S., Xi’an Jiaotong University, 2007
B. S., Xi’an Jiaotong University, 2004

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Abstract

A complete picture of hadron spectroscopy is important for the understanding of the degrees of freedom inside the hadron in the medium energy region. This study will bridge the gap between the hadron/pion picture in the low-energy region and the quark/gluon picture in the high-energy region. However, while only the low-energy resonances have been well identified, the high-energy excited states are broad and overlapping with each other, which can only be disentangled with Partial Wave Analysis (PWA). The present PWA still has ambiguities due to the lack of experimental data, especially the data using a polarized neutron target.

The G14 experiment in Hall B of Jefferson Lab was designed and conducted to measure polarization observables from both polarized proton and neutron targets. The reaction channels studies in this thesis are the single pion ($\gamma n \rightarrow p\pi^-$) and double pion ($\gamma p \rightarrow p\pi^+\pi^-$, and $\gamma n \rightarrow n\pi^+\pi^-$) photo-production. The E asymmetry in the single pion channel has good agreement at lower energy with the prediction from two PWA groups: SAID from George Washington University and Bonn-Gatchina from University of Bonn. At higher energy, there is a big discrepancy between the experimental result and PWA prediction. This result is significant since the “missing resonances” are at this higher energy region. The polarization observables ($I^\circ, P_z, P^\circ_z$) in the two-pion channel with polarized neutron target are the first results that have been measured, and these three observables ($I^\circ, P_z, P^\circ_z$) in the two-pion channel with polarized proton target are compared with a previous CLAS experiment (G9a). The results are essential for the study of high-energy excited states since this channel has a bigger cross section than the single pion channel in the high-energy region.
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Chapter 1

Introduction

The science of Physics we know so far tells us there are four fundamental forces in nature: gravitational, electromagnetic, strong and weak. Depending on the scale and the speed of the system we are studying, there are four realms of mechanics: classical mechanics, relativistic mechanics, quantum mechanics, and quantum field theory. The hadron spectroscopy studies the system with quarks and gluons, which are small and fast, thus it falls under the category of quantum field theory. The quarks carry color charges, and interact with each other by strong force, the strong interaction is mediated by gluons, the theory for this mechanism is referred as quantum chromodynamics (QCD). The classification of quarks, and how quarks are binding into different hadrons are described by the quark model.

1.1 The Quark Model

The quark model can be described in several levels: from the simple dynamic-free picture of particles as bound states of quarks and anti-quarks, to models with dynamics described by phenomenology theories, to models derived directly from QCD. For the simplest case, the properties of hadrons are derived from the additive quantum numbers of quarks. Table 1.1 shows the properties for the six flavors of quarks.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Symbol</th>
<th>Down</th>
<th>Up</th>
<th>Strange</th>
<th>Charm</th>
<th>Bottom</th>
<th>Top</th>
</tr>
</thead>
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<td>+2/3</td>
<td>-1/3</td>
<td>+2/3</td>
<td>-1/3</td>
<td>+2/3</td>
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<tr>
<td>Mass (MeV/c²)</td>
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<td>1.5 - 3.0</td>
<td>95 ± 25</td>
<td>1250 ± 90</td>
<td>4200 ± 70</td>
<td>1.725 ± 0.023 × 10³</td>
<td></td>
</tr>
<tr>
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<td>1/2</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
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<td>1/2</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>C (charm)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>B (bottomness)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>T (topness)</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: Properties for the six flavors of quarks.

Quarks are spin 1/2 fermions with positive parity, and each quark has a baryon number
1/3. There is a formula relates the quantum numbers for quarks, which is referred as Gell-Mann-Nishijima formula:

\[ Q = I_z + \frac{B + S + C + B + T}{2}, \]  

where B is the baryon number. There is also a "hidden" color quantum number for quarks, which has three values: red, green and blue. It is "hidden" in the sense that all the particles or quark bound states are colorless (color singlets). Each quark has its antiquark with the quantum numbers having the opposite sign. In the quark model, the bound states of quarks are referred as hadrons, and there are two kinds of hadrons: meson and baryon, in which meson has two quark configuration with baryon number B = 0 and baryon has three quark configuration with baryon number B = 1 [1].

**Mesons**

Mesons are bound states of quarks and antiquarks (qq̄). They are classified using \( J^{PC} \) multiplets, where \( J = L + S \) is total angular momentum, \( P = (-1)^{l+s} \) is the parity, \( C = (-1)^{l+s} \) is the C-parity. Since quarks are spin 1/2 particles, \( 1/2 \otimes 1/2 = 0 \oplus 1 \) gives states with total spin \( S = 0 \) or \( S = 1 \). For orbital angular momentum \( l = 0 \), there are two states: pseudo-scalars (0−−) and vectors(1−−) as in Figure 1.1. \( l = 1 \) has scalars (0+++), axial vectors (1+++ and 1+−−), and tensors (2++−). Mesons with exotic quantum numbers are predicted by QCD, which takes into account of the gluon excitations. The 'GlueX' experiment conducted in Hall D at Jefferson Lab is designed to search for these states.

![Figure 1.1: Spin states of mesons.](image)

Considering only the three lightest quarks: up, down and strange, the three flavors form an approximate SU(3) group. Mesons can be classified in the Eightfold Way using the following decomposition:

\[ 3 \otimes 3 = 8 \oplus 1. \]  

The nine states (nonet) is made of a octet and a singlet, as shown in Figure 1.2.

**Baryons**

Baryons are bound states of three quarks. The total state functions should be antisymmetric, since baryons are fermions:

\[ |qqq>_A = |color>_A \times |space, spin, flavor>_S, \]  

\[ (1.3) \]
where the subscripts S and A are indicators for symmetric or antisymmetric functions. The color part is an SU(3) singlet (antisymmetric), this makes the | space, spin, flavor > part symmetric. When decomposing baryons in Eightfold way, it’s more clear to use the flavor-spin SU(6) group as below:

\[
6 \otimes 6 \otimes 6 = 56_S \bigoplus 70_M \bigoplus 70_M \bigoplus 20_A, \tag{1.4}
\]

where subscripts S, M, A represents symmetric, mixed-symmetric, or antisymmetric functions. For ground state particles (l = 0), the space part is symmetric, this makes | spin, flavor > part symmetric, so only 56_S states in the above equation is allowed, the 70_M and 20_A states require nonzero orbital angular momenta. 56_S can be further decomposed into flavor SU(3) multiplets as:

\[
56 = 10^{3/2} \bigoplus 8^{1/2}, \tag{1.5}
\]

where \(10^{3/2}\) is spin 3/2 baryon decuplet, \(8^{1/2}\) is spin 1/2 baryon octet as in Figure 1.3.

So far, we only looked at the classification of the ground states of the hadrons (mesons and baryons), there is no information about their excited states and the mass values for these states in the simple quark model. These information can be obtained in the more advanced quark models, which consider the dynamics between quarks. The quark model considering the dynamics will be introduced in the next section, this model is often referred as the constituent quark model (CQM).

### 1.2 Baryon Spectroscopy

Experiment results from deep inelastic scattering tell us that the nucleon has a more complicated structure than the three quark configuration. Other than the three valence quarks, sea quarks (quark-antiquark pair) and gluon fields also play important roles.
natural question to ask is: what is the effective degrees of freedom for this complicated system? As in atomic physics, the degrees of freedom for the system can be revealed in the spectrum of the excited states.

The spectrum of strongly-interacting particles is predicted by the QCD Lagrangian:

\[ L_{QCD} = \frac{\hbar}{2} \phi_i [i(\gamma^\mu D_\mu) - m \delta_{ij}] \phi_j - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} , \]  

where \( \phi_i(x) \) is the quark field, and \( G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \) is the gluon field strength tensor, \( A^a_\mu \) are the gluon fields, \( m \) and \( g \) are the quark mass and coupling constant. In the renormalized theory, the QCD coupling decreases as energy increases as in Figure 1.4, this phenomenon is referred to as 'asymptotic freedom'.

The consequence of "asymptotic freedom" is that only at very high energy, the Lagrangian equation can be solved in a perturbative way. In the energy region of nucleon resonances, there is no analytical solution using first principle calculation. Two methods are developed to solve this problem. One method is to use phenomenological models, whose ingredients are abstracted from QCD, or from low-energy limit of QCD (chiral Lagrangians). The most successful phenomenological models in making predictions on hadron properties are constituent quark models (CQMs). The other method is to use numerical method (Lattice QCD).

### 1.2.1 Constituent Quark Models

In constituent quark models, the three constituent quarks are the only relevant degrees of freedom in the hadron. The constituent quarks can be viewed as a low momentum current quarks dressed with a cloud of low momentum gluons, it has a effective mass of about one third of the nucleon mass (350 - 400 MeV). Based on different models to describe the effective interaction between the constituent quarks, several versions of the constituent quark model have been proposed. An overview of different quark models for baryons can...
be found in [1] and [2]. Most models include two kinds of interaction. The first interaction is a spin-independent confining interaction, such as harmonic or linear confinement. The second interaction is a spin-dependent force, the Isgur-Karl (IK) model uses a color-magnetic flavor-independent interaction modeled in terms of the gluon exchange in QCD [3], the instanton model uses a flavor-dependent short-range quark forces from instanton effects [8], other models use quark dynamics, such as: algebraic [5], hyper-central [6], Goldstone Boson Exchange [7].

In the Isgur-Karl (IK) model [3], the non-relativistic three valence quarks are dressed quarks with effective mass of 200-300 MeV for u and d quarks, the gluon fields provide a confining potential between quarks. The potential has a spin independent part $V_{ij}$ and a spin dependent part $V_{ij}^{\text{hyp}}$. $V_{ij}$ can be written as the sum of a harmonic oscillator potential $K r_{ij}^2/2$, and an anharmonicity potential $U_{ij}$, which is treated as a perturbation. The spin dependent potential $V_{ij}^{\text{hyp}}$ is the sum of a contact term $S_i \cdot S_j \delta(r_{ij})$ and a tensor term, it is a hyperfine interaction, which causes the $\Delta - N$ and $\Sigma - \Lambda$ splittings. The Hamiltonian is written as:

$$H = \sum_i (m_i + p_i^2/(2m_i)) + \sum_{i<j} (V_{ij} + V_{ij}^{\text{hyp}}).$$  \hspace{1cm} (1.7)

The wave function is the product of a antisymmetric color wave function and a sum form of $\sum \psi \chi \phi$, in which $\psi$, $\chi$, $\phi$ are the spatial, spin and flavor wave functions. The spatial wave functions $\psi$ are the harmonic-oscillator eigenfunctions $\psi_{NLM}(\rho, \lambda)$ in the zeroth order of the perturbation calculation, in which $\rho = (r_1 - r_2)/\sqrt{2}$ and $\lambda = (r_1 + r_2 - 2r_3)/\sqrt{6}$, $N = 2(n_{\rho} + n_{\lambda}) + l_{\rho} + l_{\lambda}$. The ground states (N=0 band, N(938) and $\Delta(1232)$) have positive parity ($l_{\rho} = l_{\lambda} = 0$). The N=1 excited resonances are 'P waves' ($l_{\rho} = 1$ or $l_{\lambda} = 1$) with negative-parity. The N=2 band includes positive-parity excited states with radial excitations in one of the two oscillators ($n_{\rho} = 1, n_{\lambda} = 0$ or $n_{\rho} = 0, n_{\lambda} = 1$) or $l_{\rho} + l_{\lambda} = 2$. The Schrödinger equation is solved for first-order perturbation for the anharmonic term.

Figure 1.4: QCD coupling constant as a function of gluon momentum transfer Q.
$U_{ij}$ and $V_{hyp}^{ij}$. This model is quite successful in describing the low-lying baryon resonances. However, the baryons are strongly bound systems, the approximation of non-relativistic kinematics and dynamics is not proper.

In the relativized model, the Hamiltonian is given by:

$$H = \sum_i \sqrt{p_i^2 + m_i^2} + V,$$

where $V = V_{\text{string}} + V_{\text{Coul}} + V_{\text{hyp}} + V_{\text{so(cm)}} + V_{\text{so(Tp)}}$. $V_{\text{string}}$ is the potential generated by adding the lengths of the gauge-invariant string configuration and multiplying by the meson string tension. $V_{\text{Coul}}, V_{\text{hyp}}, V_{\text{so(cm)}}, V_{\text{so(Tp)}}$ are color-Coulomb, color-hyperfine, color-magnetic spin-orbit, and Thomas-precession spin-orbit potentials [4]. The spectroscopy derived from the relativized model is comparable to that of the Isgur-Karl model. However, the relativized model is more tightly constrained with less fitting parameters than those in the Isgur-Karl model. The wave functions from the relativized model are also very different from those of the Isgur-Karl model, due to the more realistic treatment of the spin-independent potential and inclusion of the configuration mixings.

The most recent CQM predictions for hadron spectroscopy are three papers published in 2001 [8], a relativistic quark model with instanton-induced quark forces was used in these papers. The instantaneous force is flavor-dependent, and was originally designed to solve the $\pi - \eta - \eta'$ puzzle, which exists in quark models of mesons based on one-gluon exchange. An expansion of the Euclidean action around single-instanton solutions of the gauge fields assuming zero-mode dominance in the fermion sector leads to an effective contact interaction between quarks, which acts only if the quarks are in a flavor antisymmetric state.

In this model, quarks are confined by a linearly rising string potential, which is a three-quark string potential $V^{(3)}$, in addition, 't Hooft’s instanton-induced two quark interaction is chosen as residual two-body force $V^{(2)}$. The non-strange baryon spectrum is calculated on the basis of the three-fermion Bethe-Salpeter equation. Figure 1.5 shows the comparison of calculated values with the experiment results for $\Delta$ (isospin $T=3/2$) and $N^*$ (isospin $T=1/2$) resonances.

Overall, the model predictions are in good agreement with the experimental results. However, there are two problems: the assignment problem, and the missing resonance problem.

The assignment problem refers to the discrepancy between the model predicted values and experiment identified states up to the fourth shell ($0\hbar\omega$, $1\hbar\omega$, $2\hbar\omega$ and $3\hbar\omega$) in the harmonic oscillation basis. Most states in the first three shells predicted by the model have been identified, but for some states, the predicted values are outside the error bars of the experimental measurements, such as $\Delta_{3/2}^{-}(1700)$, $\Delta_{3/2}^{+}(1600)$, $N_{1/2}^{1/2}+(1440)$, $N_{1/2}^{-}(1530)$ (as seen in Figure 1.5). Moreover for the $3\hbar\omega$ shell in the $\Delta$ spectrum, three states $\Delta_{1/2}^{-}(1900)$, $\Delta_{3/2}^{-}(1940)$ and $\Delta_{5/2}^{-}(1930)$ from the experiment measurements have values significant lower than the model predicted values. The assignment of experimentally observed states is only complete and well established up to $N = 1$ band, the assignment for $N = 2$, $N = 3$ bands and beyond are still tentative.

The missing resonance problem refers to the situation that there are many more excited states predicted by the modern quark models than experimentally observed. A detailed discussion for the missing resonance problem will be presented in the next section.
Figure 1.5: Calculated hadron spectrum from instanton-induced quark mode (black lines on the left side of each column), and the experiment measured resonances (black lines with shaded box on the right side of each column). The resonances in each column are denoted by $J^\pi$, where $J$ is total spin and $\pi$ is parity, a detailed description of how $J^\pi$ is determined can be seen in Figure 1.6. Another way to denote the states is $X_{2J+1,2I+1}$, where $X = S, P, D, ...$ is the orbital angular momentum $l = 0, 1, 2, ...$ for the pion-nucleon system (for the reason that most old experiment data are from pion nucleon scattering). $I$ is the total isospin, which can be 1/2 or 3/2, and $J = |l \pm \frac{1}{2}|$. For the experiment results, the lines indicate the position of the resonance, the shaded boxes are the uncertainties, and the stars under the lines indicate the status of the resonances, more stars correspond to better establishment. Image source: [8]
<table>
<thead>
<tr>
<th>N</th>
<th>sym</th>
<th>( L^P )</th>
<th>( S )</th>
<th>( N(I = 1/2) )</th>
<th>( \Delta(I = 3/2) )</th>
</tr>
</thead>
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<tr>
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<td>M</td>
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<td>3/2(^+)</td>
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<td>M</td>
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<td>M</td>
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<td>S</td>
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<td>3/2(^+)</td>
<td>1/2(^+)</td>
<td>3/2(^+) 5/2(^+) 7/2(^+) (\Delta(1910)) (\Delta(1920)) (\Delta(1905)) (\Delta(1950))</td>
</tr>
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</tr>
<tr>
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<td>1/2(^+) N(1440)</td>
<td></td>
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<tr>
<td>1</td>
<td>M</td>
<td>1(^-)</td>
<td>3/2(^-)</td>
<td>1/2(^-) 3/2(^-) 5/2(^-) N(1650) N(1700) N(1675)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>M</td>
<td>1(^-)</td>
<td>1/2(^-)</td>
<td>1/2(^-) 3/2(^-) N(1535) N(1520) (\Delta(1620)) (\Delta(1700))</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>S</td>
<td>0(^+)</td>
<td>3/2(^+)</td>
<td></td>
<td>3/2(^+) (\Delta(1232))</td>
</tr>
<tr>
<td>0</td>
<td>S</td>
<td>0(^+)</td>
<td>1/2(^+)</td>
<td></td>
<td>(\Delta(938))</td>
</tr>
</tbody>
</table>

Figure 1.6: \( N \) and \( \Delta \) states for the \( N = 0,1,2 \) harmonic oscillator bands from PDG [1]. A denotes the states with antisymmetric spatial wave function, S is for symmetric states, M is for mixed symmetric states. \( L^P \) is for angular momentum and parity. \( S \) is the three-quark spin.
1.2.2 Missing resonances

The constituent quark models predict a large number of baryon resonances. However, as shown in the previous subsection, in the mass region above 1.8 GeV, only a few have been observed in experiment. The discrepancy between the theoretical predictions and experimental results needs more work from both the theory side and the experiment side.

On the theory side, a possible explanation for the fewer observed resonance states is the 'qurak-diquark' model [9]. This model assumes a tightly bound di-quark, thus only two degrees of freedom are relevant at low energies, and fewer resonances are predicted. However, the Lattice QCD calculation of the hadron spectroscopy gives results in agree with the traditional Constituent Quark Models, thus the 'qurak-diquark' model has been considered not being the solution for the missing resonance problem.

On the experiment side, most data for the study of nucleon resonances are from the \( \pi N \) elastic scattering, this will bias the results for resonances coupling weakly to the \( \pi N \) channel. Some quark models have predicted several unobserved resonances to have large decay branching ratios for the emission of mesons other than pions. The use of photon beam provides a way to study the resonances, which couples strongly to other branching channels.

Moreover, as shown in figure 1.7, the nucleon resonances have broad width in the cross section plot, which makes the method of peak hunting impossible to identify all the resonances. Thus using the right way to decode the experimental results is crucial for the test of the theoretic models. The method used most often to identify the nucleon resonances is partial wave analysis (PWA). In the framework of PWA, the cross section and other experimentally measured quantities, such as polarization observables, are fitted using minimal set of partial waves (resonances), the discovery of a new resonance requires that the inclusion of the new state improves the fitting results in a significant way. The next section will present an introduction to the partial wave analysis for photo-production reactions, and introduce the methods from three PWA groups.

Figure 1.7: Total photonabsorption cross section with proton and neutron target. Points are measured data, curves are fitting results of Breit-Wigner shapes for nucleon resonances \( P_{33}(1232) \), \( D_{13}(1520) \), \( S_{11}(1535) \), \( F_{15}(1680) \) (only for proton), \( F_{37}(1950) \) and a smooth background. Image source: [10]
1.3 Partial Wave Analysis

The reaction between the photon beam and the nucleon involves two processes, one process is the electro-magnetic excitation of the nucleon, the other process is the strong decay in the s and u channels, or a meson exchange in the t channel. The tree level Feynman diagrams for the single-pion photo-production for this process are shown in Figure 1.8.

![Feynman diagrams](image)

Figure 1.8: Tree level Feynman diagrams for single pion photoproduction.

The process of photo-production of a pseudo-scalar meson from a nucleon can be described by four complex amplitudes, which are most commonly parameterized in terms of the Chew-Goldberger-Low-Nambu (CGLN) amplitude[11]:

\[
F = iF_1 \cdot \hat{\sigma} \cdot \hat{e} + F_2 (\hat{\sigma} \cdot (\hat{k} \times \hat{e})) + iF_3 (\hat{\sigma} \cdot \hat{k})(\hat{q} \cdot \hat{e}) + iF_4 (\hat{\sigma} \cdot \hat{q})(\hat{q} \cdot \hat{e}),
\]

where \(\hat{k}, \hat{q}\) are unit vectors for the momentum of the photon and meson, \(\hat{e}\) is the polarization vector for a real photon, and \(\hat{\sigma}\) are the nucleon’s spin matrices. The differential cross section expressed in terms of the CGLN-amplitudes in the center of momentum frame is:

\[
\frac{k}{q} \frac{d\sigma}{d\Omega} = |F_1|^2 + |F_2|^2 + \frac{1}{2} |F_3|^2 + \frac{1}{2} |F_4|^2 + Re(F_1 F_4^*)
\]

\[
+ [Re(F_3 F_4^*) - 2Re(F_1 F_2^*)] \cos(\theta_{cm})
\]

\[
- \frac{1}{2} |F_3|^2 + \frac{1}{2} |F_4|^2 + Re(F_1 F_4^*) + Re(F_2 F_3^*) \cos^2(\theta_{cm})
\]

\[
- [Re(F_3 F_4^*)] \cos^3(\theta_{cm}),
\]

where \(q, k\) are momenta for meson and photon in center of momentum frame, \(\theta_{cm}\) is cm polar angle for the meson.

The four complex amplitudes can also be expressed in terms of helicity amplitudes. There are 8 helicity amplitudes, which comes from the combination of two helicity states \(\lambda_\gamma = \pm 1\) for real photon, two spin states \(\nu_i = \pm 1/2, \nu_f = \pm 1/2\) for the initial and final state nucleons respectively. The 8 matrix elements can be reduced into four by parity conservation:

\[
H_1 = H_{1/2, 3/2} = H_{-1/2, -3/2},
\]

\[
H_2 = H_{1/2, 1/2} = -H_{-1/2, -1/2},
\]

\[
H_3 = H_{-1/2, 3/2} = -H_{1/2, -3/2},
\]

\[
H_4 = H_{1/2, -1/2} = -H_{-1/2, 1/2},
\]

(1.11)
The relationship between the CGLN and helicity amplitudes is given in [12]. The differential cross section expressed in terms of the helicity amplitudes has a very simple form:

$$\frac{k \, d\sigma}{q \, d\Omega} = \frac{1}{2} (H_1^2 + H_2^2 + H_3^2 + H_4^2)$$  \hspace{2cm} (1.12)

Using the selection rules and parity conservation, we can relate the intermediate resonance to the initial states and final states of the reaction.

For the initial state, the photon has total angular momentum \( \vec{L}_\gamma = \vec{l} + \vec{s}_\gamma \), where \( \vec{l} \) is the orbital angular momentum of photon relative to the target nucleon, and spin \( \vec{s}_\gamma \) (s=1) is the spin of the photon, the photon has parity \( P_\gamma = (-1)^L \) for EL- and \( P_\gamma = (-1)^{L+1} \) for ML- multipoles of the photon field. The initial nucleon has spin \( J_N = 1/2 \), and parity \( P_N = 1 \). For a resonance with spin \( J_N^* \) and parity \( P_N^* \), the selection rules and parity conservation gives:

$$|L_\gamma - J_N| = |L_\gamma - 1/2| \leq J_N^* \leq |L_\gamma + 1/2| = |L_\gamma + J_N|,$$
$$P_N^* = P_N \cdot P_\gamma = P_\gamma.$$  \hspace{2cm} (1.13)

For the strong decay process of the resonance, the intermediate and final states have the following relations:

$$|L_{meson} - J_N| = |L_m - 1/2| \leq J_N^* \leq |L_{meson} + 1/2| = |L_{meson} + J_N|,$$
$$P_N^* = P_N \cdot P_{meson} \cdot (-1)^{L_{meson}} = (-1)^{L_{meson}+1}.$$  \hspace{2cm} (1.14)

From the above four equations, we can relate the initial photon field multipoles (EL- and ML-) and the multipoles for meson photo-production (\( E_{l\pm} \) and \( M_{l\pm} \)) to the resonance states (\( N^* \)), the result for several low order multipole amplitudes are shown in Table 1.2. From the table, we can see that if we can determine the components of the photo-production multipoles in the reaction, we can identify the corresponding nucleon resonances.

The CGLN amplitudes can be decomposed into the partial wave multipoles as 1.15:

$$F_1 = \sum_{l=0}^{\infty} \{ (lM_{l+} + E_{l+}) P'_{l+1}(\theta_{cm}) + [(l+1)M_{l-} + E_{l-}] P'_{l-1}(\theta_{cm}) \},$$
$$F_2 = \sum_{l=0}^{\infty} [(l + 1)M_{l+} + lM_{l-}] P'_l(\theta_{cm}),$$
$$F_3 = \sum_{l=0}^{\infty} [(E_{l+} - M_{l+}) P''_{l+1}(\theta_{cm}) + (E_{l-} + M_{l-}) P''_{l-1}(\theta_{cm})],$$
$$F_4 = \sum_{l=0}^{\infty} (M_{l+} - E_{l+} - M_{l-} - E_{l-}) P''_l(\theta_{cm}),$$  \hspace{2cm} (1.15)

where the \( P'_l, P''_l \) are derivatives of Legendre polynomials. From this equation, we know that if we can determine the partial wave multipoles in the reaction amplitude \( F_i \) for the reaction channel, the resonances that couple to this channel can be determined. The determination for the four complex amplitudes \( F_1, F_2, F_3, F_4 \) need eight independent measurements. For the single and double meson photo-productions, there are 64 polarization
Table 1.2: Relating the initial states and final states to the intermediate resonance states. The resonances $N^*$ are shown in two different representations as in the fifth and sixth columns. The corresponding photon multipoles and photo-production multipoles are shown in the first and fourth columns. For the photo-production multipoles $(E/M)_{l\pm}$, $l$ is the relative angular momentum for the meson-nucleon system $L_{\text{meson}}$, ± indicates whether the spin 1/2 of the nucleon should be added to or subtracted from $l$ to give the total angular momentum $J_{N^*}$ for the resonances.
\[ \rho_f I_0 = I_0 \left\{ (1 + \vec{\Lambda} \cdot \vec{P} + \vec{\sigma} \cdot \vec{P}' + \Lambda_i^o \sigma^{\beta'} O_{\alpha \beta'}) \\
+ \delta_c \left( I_c + \vec{\Lambda} \cdot \vec{P}_c + \vec{\sigma} \cdot \vec{P}'_c + \Lambda_i^o \sigma^{\beta'} O_{\alpha, \beta'}^c \right) \\
+ \delta_l \left[ \sin 2\beta (I^s + \vec{\Lambda} \cdot \vec{P}^s + \vec{\sigma} \cdot \vec{P}'^s + \Lambda_i^o \sigma^{\beta'} O_{\alpha, \beta'}^s) \\
+ \cos 2\beta (I^c + \vec{\Lambda} \cdot \vec{P}^c + \vec{\sigma} \cdot \vec{P}'^c + \Lambda_i^o \sigma^{\beta'} O_{\alpha, \beta'}^c) \right] \right\}, \]

where I is the reaction rate, \( \Lambda \) is the degree of polarization for the target, \( \rho_f = (1 + \vec{\sigma} \cdot \vec{P}')/2 \) is the density matrix of the recoiled nucleon, \( \delta_c \) is the degree of circular polarization for the photon beam, \( \delta_l \) is the degree of linear polarization for the photon beam. There are 9 single polarization observables: \( \vec{P} \) is the polarization observables relate to the the target nucleon polarization (\( P_x, P_y, P_z \) give three observables for target asymmetries), \( \vec{P}' \) is the polarization observables relate to the recoiled nucleon polarization (\( P'_x, P'_y, P'_z \) give three observables for recoil asymmetries), \( I_c, I^s, I^c \) are photon beam asymmetries. There are 27 double polarization observables: \( \vec{P}_c, \vec{P}_s, \vec{P}^c \) are beam target asymmetries, \( \vec{P}'_c, \vec{P}'^s, \vec{P}'^c \) are beam recoil asymmetries, \( O_{\alpha, \beta'} \) is the target-recoil asymmetries. There are 27 triple polarization observables: \( O_{\alpha, \beta'}^c, O_{\alpha, \beta'}^s, \) and \( O_{\alpha, \beta'}^c \). Adding \( I_0 \) which is the unpolarized differential cross section gives a total of 64 observables. There are 28 relations relates their amplitude, and 21 equations relates their phase, which gives 15 independent quantities. A complete set of experiments for the determination of the reaction amplitude will need measurements of the differential cross section and the single, double and triple polarization observables.

However, the complete set of experiments is often out of reach for most reaction channels. The extraction of the multipoles will have to rely on model dependent analysis. There are three groups updating their solutions for multipoles regularly with the inclusion of the most recent experiment results: Bonn-Gatchina [13], MAID [14], and SAID [15].

The three groups are using different phenomenological models to parametrize the multipoles. In the Bonn-Gatchina method, a combined analysis is used to analyze large number of reactions at the same time, which include the interference between different states. In the MAID method, the unitary isobar model contains both a common background, which is unitarized according to the K-matrix prescription, and 13 resonance terms, which are assumed to have the Breit-Wigner forms for the resonance shape. In the SAID method, the multipoles are parametrized in the form: \( M = (\text{Born} + A)(1 + iT\pi_N) + BT\pi_N \), where \( T\pi_N \) is the associated elastic pion-nucleon T matrix, A and B are purely phenomenological polynomials with correct threshold properties. Figure 1.9 shows the comparison of the multipoles calculated from these three models. The positions of the poles of the curve indicate the resonance energy of the multipoles, for example for the \( M_1^+ \) multipole, the curve peaks at around 1232 MeV, which is the prominent \( P_{33}(1232) \) peak as shown in Figure 1.7 for the cross section for single pion production reaction.

### 1.4 Previous Experiment Measurements

For the last two decades, with the advent of the high duty accelerator and advances in the nearly 4\( \pi \) detectors (CLAS at JLab, Crystal Ball and TAPS at MAMI and ELSA), a lot of
data have been collected for the reaction of single and double meson photo-production. Table 1.3 shows the observables and number of kinematic data points that have been included in the Bonn-Gatchina PWA model. For the reaction of double pion photo-production: \( \gamma p \rightarrow p \pi^+ \pi^- \), three more data sets have been acquired recently in CLAS at Jefferson lab. The first is the circular beam helicity asymmetry \( I^\odot \) measurement [20] using a circularly polarized photon beam and unpolarized proton target (CLAS g1c). The second is the linear beam helicity asymmetries \( I^s \) and \( I^c \) using a linearly polarized photon beam and unpolarized proton target (CLAS g8b). The third is the beam helicity asymmetry \( I^\odot \), target asymmetry \( P_z \), and beam target asymmetry \( P_{z^\odot} \) using a circularly polarized photon beam and longitudinal polarized proton target (CLAS g9a). From table 1.3, we notice that most of the data are taken on a proton target, and the data from double meson reaction are much less than those from single meson reaction. In this study, we will report the first measurement of beam target asymmetry \( E \) for the reaction: \( \gamma n \rightarrow p \pi^- \) with a circularly polarized photon beam and longitudinal polarized neutron target. Moreover, we will compare the asymmetries \( I^\odot \), \( P_z \) and \( P_{z^\odot} \) with the result from CLAS g1c and g9a for the reaction \( \gamma p \rightarrow p \pi^+ \pi^- \), and then report the first measurement of \( I^\odot \), \( P_z \) and \( P_{z^\odot} \) for the reaction \( \gamma n \rightarrow p \pi^+ \pi^- \).
Table 1.3: Polarization observables in the photo-production of mesons used in Bonn-Gatchina PWA model. Image Source: [26]

<table>
<thead>
<tr>
<th>Observable</th>
<th>$N_{data}$ (Institute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\sigma/d\Omega(\gamma p \to p\pi{}^0)$</td>
<td>1106 (CB-ELSA), 861 (GRAAL), 592 (CLAS), 1692 (TAPS@MAMI)</td>
</tr>
<tr>
<td>$\Sigma(\gamma p \to p\pi{}^0)$</td>
<td>540 (CB-ELSA), 1492 (SAID db)</td>
</tr>
<tr>
<td>$E(\gamma p \to p\pi{}^0)$</td>
<td>140 (A2-GDH)</td>
</tr>
<tr>
<td>$P(\gamma p \to p\pi{}^0)$</td>
<td>607 (SAID db)</td>
</tr>
<tr>
<td>$T(\gamma p \to p\pi{}^0)$</td>
<td>389 (SAID db)</td>
</tr>
<tr>
<td>$H(\gamma p \to p\pi{}^0)$</td>
<td>71 (SAID db)</td>
</tr>
<tr>
<td>$O_+(\gamma p \to p\pi{}^0)$</td>
<td>7 (SAID db)</td>
</tr>
<tr>
<td>$O_-(\gamma p \to p\pi{}^0)$</td>
<td>7 (SAID db)</td>
</tr>
<tr>
<td>$d\sigma/d\Omega(\gamma p \to n\pi{}^0)$</td>
<td>484 (CLAS), 1583 (SAID db), 408 (A2-GDH)</td>
</tr>
<tr>
<td>$\Sigma(\gamma p \to n\pi{}^0)$</td>
<td>899 (SAID db)</td>
</tr>
<tr>
<td>$E(\gamma p \to n\pi{}^0)$</td>
<td>231 (A2-GDH)</td>
</tr>
<tr>
<td>$P(\gamma p \to n\pi{}^0)$</td>
<td>252 (SAID db)</td>
</tr>
<tr>
<td>$T(\gamma p \to n\pi{}^0)$</td>
<td>661 (SAID db)</td>
</tr>
<tr>
<td>$H(\gamma p \to n\pi{}^0)$</td>
<td>71 (SAID db)</td>
</tr>
<tr>
<td>$G(\gamma p \to n\pi{}^0)$</td>
<td>86 (SAID db)</td>
</tr>
<tr>
<td>$O_+(\gamma p \to n\pi{}^0)$</td>
<td>7 (SAID db)</td>
</tr>
<tr>
<td>$O_-(\gamma p \to n\pi{}^0)$</td>
<td>7 (SAID db)</td>
</tr>
<tr>
<td>$d\sigma/d\Omega(\gamma p \to \Lambda K^+)$</td>
<td>1320 (CLAS 09)</td>
</tr>
<tr>
<td>$P(\gamma p \to \Lambda K^+)$</td>
<td>1270 (CLAS 09), 84 (GRAAL)</td>
</tr>
<tr>
<td>$C_x(\gamma p \to \Lambda K^+)$</td>
<td>160 (CLAS)</td>
</tr>
<tr>
<td>$C_z(\gamma p \to \Lambda K^+)$</td>
<td>160 (CLAS)</td>
</tr>
<tr>
<td>$\Sigma(\gamma p \to \Lambda K^+)$</td>
<td>66 (GRAAL), 45 (LEP)</td>
</tr>
<tr>
<td>$T(\gamma p \to \Lambda K^+)$</td>
<td>66 (GRAAL 09)</td>
</tr>
<tr>
<td>$O_+(\gamma p \to \Lambda K^+)$</td>
<td>66 (GRAAL 09)</td>
</tr>
<tr>
<td>$O_-(\gamma p \to \Lambda K^+)$</td>
<td>66 (GRAAL 09)</td>
</tr>
<tr>
<td>$d\sigma/d\Omega(\gamma p \to \Sigma^0 K^+)$</td>
<td>1590 (CLAS)</td>
</tr>
<tr>
<td>$P(\gamma p \to \Sigma^0 K^+)$</td>
<td>344 (CLAS)</td>
</tr>
<tr>
<td>$C_x(\gamma p \to \Sigma^0 K^+)$</td>
<td>94 (CLAS)</td>
</tr>
<tr>
<td>$C_z(\gamma p \to \Sigma^0 K^+)$</td>
<td>94 (CLAS)</td>
</tr>
<tr>
<td>$\Sigma(\gamma p \to \Sigma^0 K^+)$</td>
<td>42 (GRAAL), 45 (LEP)</td>
</tr>
<tr>
<td>$d\sigma/d\Omega(\gamma p \to \Sigma^+ K^0)$</td>
<td>48 (CLAS), 72 (CB-ELSA 10)</td>
</tr>
<tr>
<td>$P(\gamma p \to \Sigma^+ K^0)$</td>
<td>24 (CB-ELSA 10)</td>
</tr>
<tr>
<td>$\Sigma(\gamma p \to \Sigma^+ K^0)$</td>
<td>15 (CB-ELSA 10)</td>
</tr>
<tr>
<td>$d\sigma/d\Omega(\gamma p \to p\pi^0\pi^0)$</td>
<td>1.4 GeV, 3.2 GeV (CB-ELSA)</td>
</tr>
<tr>
<td>$E(\gamma p \to p\pi^0\pi^0)$</td>
<td>16 (MAMI)</td>
</tr>
<tr>
<td>$\Sigma(\gamma p \to p\pi^0\pi^0)$</td>
<td>128 (GRAAL)</td>
</tr>
<tr>
<td>$d\sigma/d\Omega(\gamma p \to n\pi^0\eta)$</td>
<td>3.2 GeV (CB-ELSA)</td>
</tr>
<tr>
<td>$\Sigma(\gamma p \to n\pi^0\eta)$</td>
<td>180 (GRAAL)</td>
</tr>
<tr>
<td>$I_c, I_s(\gamma p \to p\pi^0\eta)$</td>
<td>3.2 GeV (CB-ELSA)</td>
</tr>
</tbody>
</table>
Chapter 2

Experiment Setup: CEBAF, Hall B and HD-ice Target

The data in this study were taken in the G14 run period from Nov. 19, 2011 to May 17, 2012 at the Thomas Jefferson National Accelerator Facility (Figure 2.1), also named as Jefferson Lab in Newport News, Virginia. This experiment used a tagged photon beam to hit on an HD-ice target, and the final state particles were detected by the Hall B CLAS(CEBAF Large Acceptance Spectrometer) detector. The photon beam was produced by the production of bremsstrahlung from polarized electrons from the CEBAF accelerator using a radiator with the Hall B photon tagging system. Either circularly or linearly photon beam could be produced by using different radiators. The HD-ice target was composed with solid hydrogen deuteride, with one hydrogen atom and one deuterium atom in each molecule. The HD was polarized longitudinally along the beam direction. CLAS had quasi $4\pi$ solid angle coverage($\sim 3\pi$), and was optimized to study exclusive channels with multiple final state particles.

Figure 2.1: Jefferson Lab: The racetrack shape is the CEBAF accelerator, and the three mounds on the right are the three experimental halls A, B and C. Image Source: [27]
2.1 CEBAF: Continuous Electron Beam Accelerator Facility

Figure 2.2: The Continuous Electron Beam Accelerator Facility (CEBAF): The polarized electron beam is generated in the Injector, it gains 1.2 GeV in the two LINACs in each pass, and reaches 6 GeV after 5 passes. Image Source: [28]

CEBAF is an electron accelerator that is able to deliver up to 6 GeV electron beams to the three experimental halls simultaneously. Superconducting radio frequency (SRF) cavities are used for electron acceleration. The SRF cavities allow the accelerator to reach a full-time duty factor without down-time for cooling the conduction elements of the accelerator. The nearly zero energy lost of the SRF cavities provide three times higher efficiency in terms of RF power than non-superconducting RF cavities. As shown in Figure 2.2, the accelerator consists of an injector and a pair of superconducting RF linear accelerators (LINACs). The two LINACs are connected by two arc sections, which have four circulating arcs at the west end and five at the east end. The electrons gain 1.2 GeV for each pass. After five successive passes, its energy reaches 6 GeV. Beams from different passes can be extracted for different experimental halls, which gives each hall the ability to choose different beam energies.

2.1.1 Injector

The injector has a polarized photo-emission electron gun with three pulsed lasers. The three lasers, one for each experimental hall, are pulsed independently and differ by 120° in phase. The three lasers are synchronized and combined to strike a strained GaAs photocathode at the third subharmonic (499 MHz) of the accelerating cavity frequency (1497 MHz) [30]. The electron gun produces a polarized 100 KeV electron beam, this beam then pass through three-hole chopper to cleanly separate the bunches for the different halls. Electrons are then accelerated to 67 MeV in 2 1/4 cryomodules (18 SRF cavities) before sending them to the CEBAF recirculating LINACs. This arrangement allows each hall
2.1.2 Linear accelerators (LINACs)

Each of the two LINACs contains 20 cryomodules (Figure 2.4). Inside each cryomodule, there are eight SRF cavities. Figure 2.5 shows a pair of 5 cell SRF cavities. During the run, these SRF cavities are cooled down to 2K with liquid Helium to become superconducting. An RF standing wave in the cavities is locked in phase with the electron bunches, as shown in Figure 2.6, so that the electrons always experience an accelerating electrical field. Each LINAC can increase the electron energy up to 600 MeV. After five passes, the energy of the electron beam can reach up to 6 GeV.
Figure 2.4: Cryomodules. Image Source: [28]

Figure 2.5: A pair of CEBAF’s 5 cell SRF cavities. Image Source: [28]

Figure 2.6: Accelerating RF field inside the SRF cavities. Image Source: [28]
2.1.3 Recirculation Arcs and RF Separators

Since the electron beams need to recirculate in the accelerator and pass the two LINACs 5 times in order to reach the maximum energy of 6 GeV, electron beams from different passes coexist in the accelerator. In order to bend them around to recirculate in the LINACs, a configuration as shown in Figure 2.7 is used. Figure 2.7 is a photo of the eastern recirculation area, where electron beams with five different energies enter from the north LINAC, and are separated by a group of dipole magnets. The beams pass through the five arcs with different magnetic fields to be bent to the south LINAC. In the western recirculation area, a similar arrangement is used, but only four arcs are needed for the first four pass beams, since the fifth pass will go to the RF separators and be directed to the halls.

The RF separators are used to extract the electron beams for different halls. As shown in Figure 2.8, there are two working modes for the separator, a 2-way and a 3-way beam split. For the 2-way beam split, the phase of RF wave is tuned with the electron bunches so that a transverse force is applied to the electron bunch to deflect the beam to the experiment hall. Meanwhile, another transverse force in the opposite direction is applied to the other two electron bunches, forcing them back to the recirculating loops. The 2-way beam split is used for the pass 1-4 beams. For the pass 5 beam, the RF wave exert a transverse force on the first electron bunch, leave the second electron bunch alone, and exert an opposite transverse force on the third electron bunch. The 3-way beam split separates the electron beams into three beams. From the description above, we notice that the pass 1-4 beams can be used in one experimental hall at a time, while the pass 5 beam can be used in all three experimental halls simultaneously as needed.
Figure 2.8: RF Separators. Image Source: [32]
2.2 Hall B

Hall B in Jefferson Lab can run experiments using either electron or photon beams. This hall used a nearly $4\pi$ detector, the CEBAF Large Acceptance Spectrometer (CLAS), which is ideal for the study of exclusive reactions with multiple particles in the final state detected. Figure 2.9 shows the beam-line layout in Hall B, which includes the photon tagging system, the CLAS detector, and some beam-line devices for diagnostic studies.

2.2.1 Photon Tagging System

Photon beams are produced by the electron bremsstrahlung reaction, where an electron with energy $E_0$ is scattered by the electromagnetic field of a nucleus, and emits a photon with energy $E_\gamma$. By detecting the energy of the scattered electron $E_{e'}$, the photon energy $E_\gamma$ is 'tagged' by energy conservation: $E_\gamma = E_0 - E_{e'}$. The Hall B system can tag photon energies ranging from 20% to 95% of the incident electron energy. As shown in Figure 2.10, the tagging system has five main parts: the radiator, the tagger magnet, the hodoscope, the readout, and the collimation system. Three modes of photon beams can be generated: a non-polarized mode, a circularly polarized mode, and a linearly polarized mode, as determined by the polarization of the electron beam and the type of photon radiator in use[33].

The Radiator

There are two kinds of photon radiators used in Hall B: amorphous or diamond crystals. The amorphous radiator is a thin Gold/Carbon foil, which is $5 \times 10^{-5}$ to $3 \times 10^{-4}$ radiation lengths of gold plated on a thin carbon support foil. The bremsstrahlung process for the amorphous radiator is incoherent and the generated photon beam has a $1/E_\gamma$ energy spectrum. Non-polarized or circularly polarized photon beams are generated with this radiator depending on the electron beam polarization. The circular polarization of the photon beam comes from polarization transfer from the longitudinally polarized electron beam. The polarization transfer relation is given by Eq. 2.1[34].

$$P_{\text{circ}} = P_{el} \frac{4x - x^2}{4 - 4x + 3x^2}$$  \hspace{1cm} (2.1)

where $x = E_\gamma/E_0$ is the ratio of the photon energy ($E_\gamma$) over the electron energy ($E_0$), $P_{\text{circ}}$ and $P_{el}$ are the degree of polarization for the circularly polarized photon beam and the longitudinally polarized electron beam. For high energy photons, $x \approx 1$, $P_{\text{circ}} \approx P_{el}$, the polarization transfer is almost 100%. For low energy photons, such as $x = 0.5$, $P_{\text{circ}} \approx 0.6P_{el}$.

The other kind of radiator is a thin diamond crystal, which is 20-50 $\mu$m thick. Linearly polarized photon beams are produced via coherent bremsstrahlung from the well-ordered crystal lattice. The photon beam produced by the diamond crystal radiator contains both the coherent linearly polarized photons and the incoherent background, in which the incoherent part is due to the lattice vibrations. The angular cone of incoherent bremsstrahlung varies slowly with photon energy, while the emission angle of coherent bremsstrahlung is sharply peaked[35]. By passing the photon beam through a tight collimator, the relative contribution of coherent bremsstrahlung can be enhanced. Figure
Figure 2.9: Hall B beamline. Image Source: [29]
Figure 2.10: Hall B photon tagging system. Image Source: [33]

Figure 2.11: Calculated photon spectrum and polarization for a photon beam produced by incident an electron beam of 5.8 GeV on a 20 µm diamond crystal. Image Source: [35]
2.11 shows simulated results for the photon spectrum and its polarization before and after using a collimator. The blue dashed lines show that by using a collimator, the polarization of the photon beam is enhanced in a window of 200 MeV near 2.0 GeV. (We refer to this window as the coherent peak.) Since the energy of the coherent bremsstrahlung is related to the crystal orientation, by rotating the crystal, the position of the coherent peak can be changed gradually to cover all of the desired energy region. The polarization of the linearly polarized beam is determined by fitting the photon spectrum from the diamond radiator with respect to the photon spectrum from an amorphous radiator to a model that takes into account all known contributions of coherent and incoherent bremsstrahlung, multiple scattering, mosaic spread and beam divergence[36].

The Tagger Magnet

The tagger magnet is a single uniform dipole. It has a full-energy radius of curvature of 11.80 meters and deflection angle of 30°. It’s length is 6.06 m along the open chord, and there is a gap of 5.7 cm wide in the middle for the electron beam to pass through (Figure 2.12). The magnet bends the electron beam out of the beamline, which bends the energy-degraded electrons to the hodoscope, and the full-energy electrons to the beam dump. The requirement of $10^{-3}E_0$ in energy resolution requires good field uniformity. For the originally upper limit of the CEBAF beam of 4 GeV, the required magnetic filed of 1.13T has good uniformity. For the upgraded 6 GeV beam, the tagger magnet needs to provide a magnetic field of 1.75T to bend the full-energy beam to the beam dump. This high field has some degraded field uniformity, but by modifying the shape of the return yoke, the field could achieve a satisfactory uniformity.

![Figure 2.12: The transverse section of the tagger magnet. The laminar structure of the yoke is completely open in the middle for the electron beam passage. Image Source: [33]](image-url)
The Hodoscope

The focal-plane hodoscope is used to measure the momentum and timing of the recoiling electrons. It consists of two separate planes of scintillator detectors, the E-plane and T-plane. Since electron trajectories cross the focal plane at different angles ranging from $9.5^\circ$ for high momentum electrons to $25^\circ$ for low momentum electrons, each detector needs to be arranged with its working surface normal to the electron trajectory (Figure 2.13),

![Figure 2.13](image1.png)

Figure 2.13: A section of the hodoscope showing the arrangement of the scintillator detectors in E-plane and T-plane. The parallel lines indicate the direction of the electron trajectories. Image Source: [33]

The E-plane lies on the optical focal plane of the magnet (Figure 2.15). This plane is used for the momentum definition of the recoiling electrons. The momentum of the recoiling electron are then used to determine the energy of the bremsstrahlung photon. The requirement of an energy resolution of $10^{-3}E_0$ results in a plane with a high degree of segmentation. For a focal plane of around 9 m in length, there are 384 scintillators. These scintillators are 20 cm long and 4 mm thick. Their widths (along the dispersion

![Figure 2.14](image2.png)

Figure 2.14: The relative positions for the E-plane and T-plane. Image Source: [33]
direction) vary from 6 to 18 mm, which covers approximately equal size momentum intervals of 0.003 $E_0$. The scintillators are arranged to optically overlaps their neighbors by one-third of their width, this arrangement increases the effective number of photon energy bins to 767 and gives an energy resolution of 0.001 $E_0$.

The T-plane lies 20 cm downstream of the E-plane. This plane is used to make accurate timing measurements for the recoiling electrons. It contains 61 counters, each counter is 2 cm thick, and can provide a timing resolution of 110 ps. The T-counters are separated into two groups. The first group has 19 narrower counters, and covers the photon energy from 75% to 95% of the incident electron energy. The second group of 42 counters covers the range from 20% to 75%. The widths of the T-counters vary to compensate for the $1/E_\gamma$ distribution of the bremsstrahlung cross section, and make the counting rate approximately the same within each group. Because the widths of the counters in first group are narrower, the counting rate in the first group is about 1/3 of the second group. (This design is for the experiments that are only interested in high energy photons, the relatively low counting rate allows for operation at higher tagged-photon rates for the high energy photons.) The lengths of the T-counters also varies from 20 cm at the high-momentum end to 9 cm at the low-momentum end in order to compensate the the bremsstrahlung characteristic angle distribution. The timing information extracted from the T-counter is used to associate photons with events in the CLAS detectors.

The Readout

![Figure 2.15: Schematic of the tagger readout setup. Image Source: [33]](image)

The signals produced by the scintillators are read out by photomultiplier tubes (PMTs). Each E-counter has one phototube attached at one end by optical fiber light guide, each
T-counter has two phototubes attached by solid light guides at both ends. T-counter signals are fed to constant-fraction discriminators (CFDs) first, and are then used in two ways. First, the signals from the discriminator are sent to the FASTBUS TDCs (LRS 1872), operating in the common-start mode with a timing resolution of 50 ps/channel. The TDCs are used to extract the timing information from the T-counters and to count the total number of hits recorded in the tagger. The other CFD output establishes a coincidence with a fast AND logic unit between the two PMTs from the same T-counter. Then the output of the AND gate is sent to a Master OR, which sets the trigger for CLAS events. The output of each E-counter PMT is sent to a signal-amplifier, discriminator, multiplexer and logic module (ADML). The outputs from the ADMLs are fed to the FASTBUS TDCs (LRS 1877), running in the common-stop mode with a timing resolution of 500 ps/channel. The TDCs are used to determine which of the E-counters were hit during the time bucket of the trigger. The rest of the signals from the discriminator are used to establish a hardware coincidence between each T-counter and the group of E-counters. The output of the E-T logic unit is fed to one of the inputs of the AND gate for each T-counter for the MASTER OR trigger. However the output gate width of the E-T logic unit is set to be much longer than the outputs from the T-counter discriminators, so the timing of the event is still determined by the T-counter signal.

During the offline analysis, the timing information from both the E- and T- counters is used to establish the hit pattern and to make tight timing coincidences between the counters to remove background events. The timing information is also used to determine the event start time. The time resolution of the tagger is sufficient to identify the RF beam bucket associated with each photon. By calculating the propagating time of the photon from the radiator to the event interaction vertex, the event vertex time can be determined.

The Collimation System

The collimator downstream the tagger magnet is used to remove the halos of the photon beam, and restricts the size of the photon beam on target. A magnet is placed just downstream from the collimator (a 25 cm long nickel block) to clean up any charged particles created in the collimator walls. The collimator is placed about 14 m downstream the radiator, and trims the diameter of the photon beam so that it is entirely within the target. In G14 experiment, for the circularly polarized beam, a collimator with 2.6 mm diameter hole has been used. For the linearly polarized beam, a 2.0 mm diameter collimator has been used.

2.2.2 The CLAS Detectors

The CEBAF Large Acceptance Spectrometer (CLAS) is used to detect the final state particles in G14 experiment. It composes a toroidal magnet and several layers of sub-detectors. The main CLAS subsystems are the start counter for event triggering and start time measurement for time-of-flight measurement, drift chambers to track the trajectories of charged particles, Cherenkov counters for electron identification, TOF scintillators for time-of-flight measurement, and electromagnetic calorimeters for neutral particle detection. The Cherenkov counters is used in electro-production experiments for recoiled
electron triggering and identification, and will not be discussed here. The polar angle coverage of CLAS for charged particles is $8^\circ < \theta < 142^\circ$, and for neutral particles, it is $8^\circ < \theta < 45^\circ$. The azimuthal angles are mostly covered except for the 6 gaps where the torus magnets are. Figure 2.16 is a schematic diagram showing all the sub-detectors of CLAS, and a photo of it in real life.

![Figure 2.16: Left plot is a schematic diagram for the CLAS detector, right plot is a photo of the CLAS in Hall B during a maintenance period. Image Source: [28]](image)

**Torus Magnet**

The Torus Magnet is the heart of the CLAS detectors. Its magnetic field gives good momentum resolution for charged particles, provides large angle coverage, and keeps the target region magnetic field free for the use of polarized targets. It consists of six kidney-shaped superconducting coils arranged in a toroidal way around the beam-line, each separated in the azimuthal direction by $60^\circ$. This arrangement generate a filed mainly pointing in the $\phi$ direction around the beam-line. Figure 2.17 shows the magnetic field in the middle plane between two coils, and in a plane transverse to the beam and centered on the target. The maximum design current of the torus coils is 3860A, during G14 runs, two torus currents have been used, which are $+1918A$ and $-1495A$.

**Start Counter**

The Start Counter is a six sector scintillator detector (Figure 2.18) [37]. Each sector has four scintillator paddles. Each paddle is a single piece scintillator with a straight section of 502 mm long, 29 mm wide and 2.15 mm thick with a bend at each end. The upstream bend couples to an acrylic light guide and then to the PMTs, the downstream bend couples to a tapered end to form the "nose".

The Start Counter is used for two purposes. One is for particle identification, this is done by measuring the time-of-flight of the scattered particles from the interaction vertex to the TOF or EC. The interaction time is determined from identification of the photon
Figure 2.17: (A) Magnetic field of the CLAS torus magnet in the middle plane between two coils; (B) Magnetic field vectors of the CLAS torus magnet in the plane transverse to the beam and centered on the target. Image Source: [29]

Figure 2.18: Start Counter. Image source: [37]
beam bucket that produced the hadronic interaction. Since the photon beam buckets have 2 ns intervals, a sub-nanosecond coincidence of the Start Counter with the Tagger T-counter is required for this purpose. For a confidence interval of 99% and a Gaussian time distribution, the Start Counter need to have resolution less than 388 ps. With good identification of the photon bucket, the RF time gives the start time of the scattering particle trajectory a resolution less than 25 ps.

The other use of the Start Counter is to define the trigger for photon runs. As shown in Figure 2.19, the Start Counter signals are used in two parts of the Level 1 trigger: the synchronous leg, and the asynchronous leg. In the synchronous leg, for each of the six sectors, 4 discriminated PMT signals and the corresponding TOF, EC PMT signals are fed into a synchronous memory lookup unit. If the required topology is found, a trigger signal is generated. In the asynchronous leg, the 24 Start Counter signals from the CAMAC C207 discriminators are combined in an OR circuit, the output of the OR circuit and the MOR signal from the tagger are ANDed to form the fast asynchronous (ASYNC) signals. The trigger signals from the synchronous leg and the ASYNC signals from the asynchronous leg are fed into a final memory lookup unit. If the desired logic are met, the trigger supervisor will generate a signal to initiates the digitization and readout of the event.

**Drift Chambers**

The Drift Chambers are used to reconstruct the trajectories of the charged particles emerging from the target with momenta greater than 200 MeV/c. When the charged particles pass through the gas inside the drift chamber, they ionize the gas and produce

![Figure 2.19: Start Counter trigger logic.](image)
free electrons and ions, the static electric field in the detector drift the electron cloud to the Anode, and produce a signal. The goals for the track resolution for 1 GeV/c charged particles are $\frac{\delta p}{p} \leq 0.5\%$, and $\delta \theta, \delta \phi \leq 2$ mrad. In order to achieve these goals, the tracks are measured at three locations in the three 'Regions' (R1, R2, R3) of the Drift Chambers. Since the Torus Magnet coils separate the detector into six independent sectors, in total there are 18 separate drift chambers locate at three radial locations in the six sectors. The six 'Region One' chambers are inside the torus coils, where the magnetic field are low. The six 'Regions Two' chambers are placed between the torus coils in an area of high magnetic field, where the maximum curvature of the particle’s trajectory is. The six 'Region Three' chambers are situated outside the torus coils. This setup provides a polar angle coverage from $8^\circ$ to $142^\circ$, and an azimuthal angle coverage up to $80\%$. A schematic view of the three regions of the Drift Chambers is shown in Figure 2.20.

![Figure 2.20: Horizontal cut and Vertical cut of the Drift Chambers. Image Source: [29]](image)

In each of the 18 Drifter Chambers, the wire layers are grouped into two 'super-layers'. In one group, the wires are aligned along the magnetic field, and in the other group, the wires are tilted at a $6^\circ$ stereo angle around the radius to provide azimuthal information. Inside each super-layer, the wire positions are shifted by half the nominal spacing in successive layers with the repeating pattern of two filed-wire layers and one sense-wire layer, this arrangement results in a quasi-hexagonal cell as shown in Figure 2.21, each cell has one sense wire in the middle, surrounded by six field wires. The distance between the field and sense wires increases uniformly with increasing distance from the target. The average distance between the field and sense wires is 0.7 cm in R1, 1.5 cm in R2, and 2.0 cm in R3. Each R2 and R3 sector consists of 12 layers of sense wires, the 6 axial layers are at smaller radius, and 6 stereo layers are at larger radius. For R1 sectors, there are only 10 layers because of the limited space, and the arrangement of stereo and axial layers is opposite to R2 and R3, with the 4 stereo layers at smaller radius and 6 axial layers at larger radius. A 88% argon and 10% CO$_2$ mixture gas is chosen for the chamber gas. This mixture is non-flammable, and has reasonably low multiple scattering, good efficiency, and short collection times. To keep constant pressure in the chamber, an active feedback system is used to make small adjustments to the out-flow.
Time-of-flight counters

The TOF system is used for particle identification, CLAS Level 1 trigger, and energy-loss measurements. It covers an area of 206 $m^2$, which fills all 6 sectors of the CLAS in the azimuthal direction, and from 8° to 142° in the polar direction. The scintillators are positioned between the Cherenkov detectors and the forward electromagnetic calorimeter, as shown in Figure 2.16. The system is composed of 342 scintillation counters with nominal thickness of 2 in. (5.08 cm), the forward-angle system has 15-cm wide and 32 to 376 cm long scintillators, the large-angle system has 22-cm wide and 371 to 445 cm long scintillators. As shown in Figure 2.22, the 57 scintillators are grouped in four panels in each of the six CLAS sectors. Counters 1-22 in panel 1 are called the forward-angle system, they cover the polar angle up to 45°. Counters 23-57 in panel 2,3,4 are refereed as large-angle system. The high segmentation of the TOF system is for achieving excellent timing resolution, the time resolution of the system determined by cosmic-rays is shown in Figure 2.23.

Forward Electromagnetic Calorimeter

The forward electromagnetic calorimeter (EC) is used in G14 experiment for detection of photons with energies above 0.2 GeV and neutrons. The detected photons are used to reconstruct $\pi^0$ and $\eta$ from the $2\gamma$ decays, and the discrimination between photons and neutrons is from the $\beta$ value from the time of flight measurement. The mass separation of $\pi^0$ and $\eta$ require the detection of photons has an energy resolution $\sigma/E \leq 0.1/\sqrt{E(\text{GeV})}$ and a position resolution $\delta r \approx 2$ cm at 1 GeV. To distinguish photons from neutrons
Figure 2.22: Four panels of the TOF scintillators in one CLAS sector. The forward-angle scintillators have short stubby light guide and 2 inch PMTs, the large-angle scintillators, except for the last four, has bent light guides and 3 inch PMTs. Image Source: [38]

Figure 2.23: The timing resolution as determined from cosmic-ray tests for all counters. Image Source: [38]
requires the time resolution $\approx 1$ ns.

The EC covers the $\theta$ range up to $45^\circ$. This system uses a sampling calorimeter made of alternating layers of scintillator strips and lead sheets. The ratio of the thickness of the lead to scintillator is 0.24, with 39 cm thickness of scintillator and 8.4 cm thickness of lead.

There are six EC modules to match the hexagonal shape of the CLAS, each module is contained in a volume with the shape an equilateral triangle. There are 39 layers in each module, which consisting of a 10 mm thick scintillator followed by a 2.2 mm thick lead sheet. Along the direction pointing away from the CLAS target, the area of each successive layer increases, this "projective" geometry minimizes the the shower leakage at the edges of the active volume. Each scintillator layer is made of 36 strips parallel to one side of the triangle, with the orientation of the strips rotated by $120^\circ$ in each successive layer as shown in Figure 2.24. This arrangement results in three views (labeled U, V and W), and provides stereo information of the location of energy deposition. Each stereo view contains 13 layers, which are further subdivided into an inner (5 layers) and outer (8 layers) set, each set is connected to an PMT, as shown in Figure 2.25. Thus each module needs $36 \times 3 \times 2 = 216$ PMTs, and all six modules need 1296 PMTs. The reconstruction of a EM shower event requires a peak in all three views.

Figure 2.24: The U-, V- and W-plane of the electromagnetic calorimeter. Image Source: [39]

U, V and W. An algorithm is used to check the peak position, and the peak energy to conform a hit of a particle on the EC. A reconstructed EM shower events is shown in Figure 2.26.

2.2.3 Beam-line Devices

Beam Position Monitors: BPMs

The Beam Position Monitors (BPMs) are used to measure continuously the position of the electron beam entering Hall B. The BPM is made of three RF cavities. For G14 experiment, two BPMs are used at two different positions. The 2C21A BPM is placed
Figure 2.25: EC light readout system. PMT-photomultiplier, LG-light guide, FOBIN-fiber optic bundle inner, FOBOU-fiber optic bundle outer, SC-scintillators, Pb-2.2 mm lead sheets, IP -inner plate(closest to target). Image Source: [39]

Figure 2.26: The diagram of an event reconstruction in EC. Image Source: [39]
36 m upstream of the target, the 2C24C is placed 24.6 m upstream of the target. The BPMs provide the electron beam position (x,y) and intensity, these information is used continuously in the feedback loops to keep the electron beam centered, and the position and current information is also written in the data stream every 20 seconds.

**Beam Profile Monitors: Harps**

The Beam Profile Monitors measure the electron beam profile with moving thin wires through the beam. As the crossed wires (horizontal and vertical) move across the beamline along the 45° angle with respect to the horizontal axis, the scattered electrons are measured by the Cherenkov light in the glass windows of the PMTs. Figure 2.27 shows the results of a harp scan during the G14 experiment. The harp scan intercepts the beam, so this procedure occurs when CLAS is not taking data and are performed after any major change to the electron beam.

![Figure 2.27: The electron beam profile from a harp scan during the G14 experiment.](image)

**Beam Polarization: Møller polarimeter**

The polarization of the electron beam used in Hall B is measured by a Møller polarimeter. The polarimeter is located in the beam-line upstream of the bremsstrahlung tagging system (Figure 2.9). The schematic diagram of the system is shown in Figure 2.28. The asymmetry in the elastic electron-electron (Møller) scattering is measured by the two detectors, which gives the polarization of the electron beam.
Photon Flux: Total Absorption Shower Counter and Pair Spectrometer

The Total Absorption Shower Counter (TASC) at the end of the beam-line is used for measuring real photon flux. The TASC is a lead glass scintillator array, with close to 100% photon detection efficiency at low photon flux rate. The reason the TASC is placed at the end of the beam-line is that not all the tagged photons arrives at the target, some of them are lost due to the collimator and light dispersion. The tagging ratio $\epsilon = \frac{N_T \otimes TASC}{N_T^T}$ is used to calculate the number of photons reach the target, in which $N_T \otimes TASC$ is the number of "good" hits in the tagger in coincidence with the photons measured by the TASC, and $N_T^T$ is the total number of "good" hits in the tagger. The tagging ratio is only calculated in the normalization runs, which have very low flux rate (beam currents up to 100 pA).

The TASC must be retracted from the beam-line in the production runs. Thus, secondary monitors, with absolute efficiency of only a few percent but linear in flux over a wide range, are cross-calibrated against the TASC at low rates and used to determine the flux at higher rates. Two devices are used as the secondary monitors, the Pair Spectrometer (PS) and the Pair Counter (PC), they are placed in front of the TASC as shown in Figure 2.29. The Pair Spectrometer consists of a thin conversion foil, a large aperture dipole magnet and eight scintillator counters. A photon hitting on the thin converter produces $e^+e^-$ pairs, which are diverged by the magnetic field and then detected in the eight scintillator counters. The pair counter is a much simpler device, and is used as a backup intensity monitor.

2.2.4 Trigger and Data Acquisition System

Trigger System

Each of the CLAS subsystem is actively run and generating signals. But not all signals are worth recording, background signals such as the signals from the cosmic rays or electronic noise should be rejected. CLAS uses a two-level hierarchical trigger system to acquire
events of interest. This system consists of three parts: Level 1 trigger, Level 2 trigger and Trigger supervisor.

The Level 1 trigger is deadtime-less, and uses any or all available information from PMT channels to select the desired events. The information used in the Level 1 trigger includes the signals from the Starter Counters, the TOF Counters and the tagger Master OR output. Preloaded trigger definitions are programmed in the hardware memory, and the signals are processed through a 3-stage memory lookup to generate an event trigger.

The Level 2 trigger uses the track information from the Drift Chambers. It is designed to reject events from the cosmic-rays that can set the Level 1 trigger but lack matching tracks in the Drift Chambers. If no track candidates are found, a fast-clear signal is issued.

The Trigger supervisor (TS) takes the Level 1 and Level 2 triggers as inputs and produces the common signals, busy gates, and resets for the detector electronics. The TS can be programmed in two configurations. The first configuration (CLASS 1) requires that the event passes only the Level 1 trigger, the second configuration (CLASS 2) requires both a Level 1 and Level 2 confirmation. After the event pass the TS requirement, it will be placed on the readout queue for asynchronous readout.

Data Acquisition System

The Data Acquisition System (DAQ) is used to manipulate and record data from various CLAS subsystems. The data flow is shown in Figure 2.30. Firstly the data are digitized in 24 FASTBUS and VME crates in the experiment hall and collected by the Readout Controllers (ROC1 to ROC24). Then the data arrays are buffered and transferred to the CLAS online acquisition computer (CLON10) in the control room via fast Ethernet cables. The CLON10 computer carries out three primary processes, the Event Builder (EB), Event Transport (ET), and Event Recorder (ER). The EB assembles the data arrays, or event fragments, into complete events, which are labeled by a run and event number, and event
type, and trigger bits that are all contained in a header bank. The completed events from EB will then be passed to shared memory (ET1). ET1 can be accessed by several processes, the ER takes all events for permanent storage, part of the events from ET1 are transferred to remote ET systems (ET2 and ET3) for raw data checks and online reconstruction. The ER writes the data in a single stream to an array of local magnetic media (RAID disks), the data files are then transferred to the remote tape silo by a fiber link from the local RAID in the control room to the computer center a kilometer away.

![Data flow diagram for CLAS DAQ](image-source.jpg)

Figure 2.30: The data flow diagram for CLAS DAQ. Image Source: [29]

### 2.3 HD-ice Target

HD-ice is a frozen spin target of solid hydrogen deuteride. It has many attractive features, such as low Z, high dilution factor (all protons and neutrons in HD are polarizable), long spin-relaxation time, and freedom of transferring polarization between H and D. Figure 2.31 shows what the target looks like. The cell walls are made from $C_2ClF_3$ (poly-chloro-tri-fluor-enthalylene), which is a hydrogen-free polymer that minimizes the NMR backgrounds for H. Aluminum wires are put in the target to extract the heat from HD. The aluminum wires are soldered to a copper ring, which is thermally anchored to a cold bath. The target has 0.4 moles of HD, which at 1K occupies a cylindrical volume of 1.5cm in diameter and 5.0cm in length. The composition of a standard HD target is: 77% HD,
17% Al(750 × 50µ Al wires) and 6% C₂ClF₃.

Figure 2.31: HD-ice target cell used in G14 experiment. Image source: [40].

2.3.1 Basic HD physics

Polarized target is essential in nuclear physics, since it provides access to study the spin structure of the nucleon. One often measured observable is the double spin asymmetry:

\[
A_{\text{exp}} = \frac{N(\rightarrow \Rightarrow) - N(\leftarrow \Rightarrow)}{N(\rightarrow \Rightarrow) + N(\leftarrow \Rightarrow)} = P_B P_T f A_{\text{phys}}
\]

\[
\frac{\delta A_{\text{exp}}}{A_{\text{exp}}} = \frac{1}{\sqrt{2 N P_B P_T f A_{\text{phys}}}}
\]

where \(\rightarrow\) is the direction of beam polarization, \(\Rightarrow\) is the direction of target polarization, \(P_B\) is beam polarization, \(P_T\) is target polarization, \(f\) is dilution factor, \(N\) is number of events. In order to get big signal for \(A_{\text{exp}}\) and small \(\frac{\delta A_{\text{exp}}}{A_{\text{exp}}}\), we need big \(P_B\), \(P_T\) and \(f\). HD has the highest content of polarizable nucleons(\(f\)), compared with other solid polarized targets such as the butanol(\(C_4H_9OH\)), ammonia(\(NH_3, ND_3\)), Lithium deuteride(\(^6LiD\)). The HD molecule has no limitation of symmetry, both its proton and deuteron can be polarized by aligning their spins with a magnetic field.

At its lowest energy state with no phonon coupling in solid HD crystal, it’s spin-relaxation time is very long(years). But it also means to polarize pure HD would need a long time. This problem can be solved by adding a small amount of \(H_2\) and \(D_2(\sim 10^{-4})\) in the target. The \(H_2\) and \(D_2\) molecules have two spin isomers with spins either parallel (ortho-\(H_2\), para-\(D_2\)) with total molecular nuclear spin \(I_{\text{molecular}} = 1\) or antiparallel (para-\(H_2\), ortho-\(D_2\)) with total molecular nuclear spin \(I_{\text{molecular}} = 0, 2\). The name para or ortho is determined by the number of spin degeneracy(\(g_I\)) of the species, the one with larger spin degeneracy is named ortho as in Table 2.1.
Table 2.1: Nuclear spin states and rotational states for hydrogen and deuterium. $I_N$ is spin of nucleon, $I_{mol}$ is molecular nuclear spin, $J$ is rotational state, $g_I$ is nuclear weight. AS(S) is abbreviation for antisymmetric (symmetric). $\Phi(r), \Phi(J), \Phi(I_{mol})$ are the vibrational, rotational and nuclear wave functions. $\Phi_{total}$ is the total wave function, which is a product of vibrational, rotational and nuclear wave functions. Since the nucleons of $H_2$ and $D_2$ are identical, their wave functions are restricted by special symmetries. Since proton(deuteron) is fermion (boson), the total wave function of $H_2(D_2)$ is AS(S). Table source [43].

The polarization of H in HD uses both species of $H_2$. At room temperature, the normal-$H_2$ has both para and ortho species. The thermal equilibrium between para- and ortho-$H_2$ is governed by Boltzmann’s law. From Table 2.1, we know that para-$H_2$ has even and ortho-$H_2$ has odd rotational quantum numbers, the ratio ($\beta$) can be expressed by Equation 2.3 [44].

$$\beta = \frac{[p - H_2]}{[o - H_2]} = \frac{\sum_{J=\text{even}} (2J + 1)e^{-E_J/kT}}{3\sum_{J=\text{odd}} (2J + 1)e^{-E_J/kT}}$$

At high temperatures limit ($E_J << kT$), $\beta = 1/3$. Since the transition from the ortho state to the para state is forbidden, if the $H_2$ is cooled down to solid state, almost all the para-$H_2$ fall into the $J = 0$ state and the ortho-$H_2$ fall into the $J = 1$ state, and these two rotational states have an energy gap of 172 K ($\frac{E_J = 1 - E_J = 0}{k_B}$) in the temperature units. In the solid state ($T = 4$ K), the interactions between molecules can cause conversion from ortho-$H_2$ to para-$H_2$ with a decay constant of 151 hours.

The process of the polarization can be described in four steps as in the Figure 2.32.

![Figure 2.32: Polarization mechanism for HD-ice target. Image source: [40].](image-url)

Step 1: HD is cooled down to solid state ($T = 4$ K), almost all population of $H_2$ falls to the molecular states $I = 1, J = 1$(ortho-$H_2$, 75%) and $I = 0, J = 0$(para-$H_2$, 25%), in
which ortho-$H_2$ has very short $T_1$ and can be polarized by external field very fast.

Step 2: ortho-$H_2$ molecule generates an external field $B_{J=1}$, through the interaction $B_{J=1} \cdot S_H$, ortho-$H_2$ exchanges spin of $H$ with an adjacent HD molecule.

Step 3: Since HD is quantum solid, the wave function of adjacent HD molecule overlap each other, spin of $H$ will diffuse inside HD, in this way, the HD molecules far away from the ortho-$H_2$ also get polarized.

Step 4: Since the nuclear spin interaction between the nucleons in two different atoms is weak, the decay of ortho-$H_2$ to para-$H_2$ is forbidden, which provide enough time for ortho-$H_2$ to polarize the HD. After several half times, most ortho-$H_2$ molecules decay to para-$H_2$. Para-$H_2$ is $J = 0$ state, which is symmetric and generate no external field. There is no dipole interaction between para-$H_2$ and HD, leaving the HD target in a frozen-spin state.

The $H_2$ impurity plays a role of 'switch' for the frozen spin state of HD. Ortho-$H_2$ shortens $T_1^H$ for HD, which speeds up ('turns on') the polarization for $H$ in HD. As ortho-$H_2$ decays to para-$H_2$, $T_1^H$ grows longer for HD, which slows down ('stops') the polarization process, and HD becomes frozen spin state. In practice, the commercial HD gas already contains about 1.5% of $H_2$ and 0.5% of $D_2$, the HD gas must be purified to the desired level of $10^{-4}$.

The $D$ in HD can be polarized in a similar way. The equilibrium $H$ and vector-$D$ polarization is described by the Brillouin function (Eq 2.4):

$$P_{TE}(x, I) = \frac{2I + 1}{2I} \coth\left(\frac{2I + 1}{2I} \frac{x}{2T}\right),$$

where $x = \frac{\mu B}{k_BT}$, $I$ is nuclear spin (1/2 for $H$, 1 for $D$). Since $I_D = 1$, deuteron has both vector polarization ($P_V(D)$) and tensor polarization ($P_T(D)$):

$$P_V(D) = \frac{N(I_z = 1) - N(I_z = -1)}{N(I_z = 1) + N(I_z = -1) + N(I_z = 0)},$$
$$P_T(D) = \frac{N(I_z = 1) + N(I_z = -1) - 2N(I_z = 0)}{N(I_z = 1) + N(I_z = -1) + N(I_z = 0)}. $$

In thermal equilibrium, the population of $I_z = 0$ is the geometric mean of ±1 states, which gives $P_T(D) = 2 - \sqrt{(4 - 3(P_V(D))^2)}$. Current setup can reach $B = 15T$ and $T = 12mK$, which can produce ~ 90% polarized $H$ and ~ 30% polarized vector $D$.

However, the decay of Ortho-$H_2$ to para-$H_2$ generates heat, which makes temperature higher than 12$mK$ during polarization, and $T_1^H$ becomes very long as most of Ortho-$H_2$ decay, ideal thermal equilibrium will not be achieved, so the final polarization of $H$ and $D$ is lower than the above values, polarization of ~ 60% for $H$ and ~ 15% for $D$) have been achieved in HD-ice lab as in Figure 2.33.

The polarization for $D$ can be further increased by transferring polarization from $H$ to $D$ via RF transition [45, 46]. The transferring mechanism is shown in Figure 2.34. The transitions between the mixed state $|m_H, m_D>$ are referred as Forbidden Adiabatic Fast Passage (FAFP). It is 'Forbidden', because the transitions move polarization from $H$ in one molecule to $D$ in a neighboring molecule via the inter-molecular dipole-dipole coupling, and it is a two-photon process with RF frequency equal to the difference of the $H$ and $D$ Larmor frequencies $\nu(H-D) = 36.0416$ MHz/Tesla. The intra-molecular
Figure 2.33: HD polarization in theory and in reality. For magnetic field of 15 T and temperature of 12 mK, polarization of $\sim 90\%$ for H and $\sim 30\%$ for D is expected in theory, $\sim 60\%$ for H and $\sim 15\%$ for D is achieved in reality.

dipolar coupling of H and D in the same HD molecule averages to zero, because at this low temperature, almost all HD molecules are in the $J=0$ rotational state. The term "Adiabatic" refers to the time scale for the process is relative long with respect to the Larmor frequencies, and "Fast" is with respect to the spin-relaxation times. The 'Forbidden' transitions require very high power to flip the spins, this would raise the temperature of the dewar and result in a loss of frozen-spin polarization. The method the HD-ice group uses is to saturate the RF transition over a long period of time ($\sim 15$ min) with low-power RF, which equalize, instead of reverse, the populations of the $m_H = +1/2; m_D = -1$, 0 and $m_H = -1/2; m_D = 0, 1$ states. This method has boosted the deuteron vector polarization ($P_V(D)$) from about 15\% to 27\%.

2.3.2 Cryostats for HD target

The production and use of the polarized HD target is more complicated than that of the conventional polarized nuclear targets. In the lifetime of a HD target, it goes through 5 different cryogenic systems: production dewar (PD), transfer cryostat (TC), dilution refrigerator (DF), storage dewar (SD), in-beam cryostat (IBC).

Production dewar

The PD (production dewar) is a stock Janis cryostat, which has pumpable varitemp system in a liquid helium bath with a liquid nitrogen space and shield (Figure 2.38). Within the varitemp space is a custom-build insert holding the HD target and NMR coils. PD is used as the cryostat for condensing the HD and calibrating the NMR signals for polarization calculation. It has a 2 T magnet and the varitemp space can be cooled.
below 2 K. After the condensation and NMR calibration, the HD target will be moved to the dilution refrigerator with the transfer cryostat.

**Transfer cryostat**

The TC (transfer cryostat) is a tool to relocate the target from one cryostat to another. It has a left-handed threaded coldhead on the end of a liquid helium tube, which can be extended into the cryostat, and be used to capture the HD target with the matching threads between the TC and the target cell (Figure 2.36). The liquid helium volume of TC can be pumped to cool the coldhead below 2 K. The cylindrical LHe tube is separated by vacuum from the surrounding LN$_2$ toroidal volume, and then surrounded by an insulating vacuum space formed from highly compressible bellows (Figure 2.37). A cylindrical permanent magnet array (Halbach magnet) produces a magnetic field of 0.1 T for keeping the polarization of the target. Since HD target has the shortest $T_1$ in TC compared with the other 4 cryostats (the smallest B/T), minimizing the time for target inside TC is preferred. With the failure rate less that 5% per transfer, the overall success rate exceeds 75% for using the HD target, which requires 5 transfers.

**Dilution refrigerator**

The DF system consists of an Oxford Instruments dilution refrigerator and a large magnet system. The dilution refrigerator has a base temperature of about 8 mK without energizing the magnet, and a base temperature below 10 mK with the magnetic field energized up to 15 T. The DF is able to polarize up to 3 targets at the same time.
Figure 2.35: Schematic plot for the production dewar. Image source: [48].

Figure 2.36: Threads on the copper target ring for relocating target. The interior left-handed threads match the threads on coldhead of TC, the exterior right-handed threads match the threads of the cryostats, such as PD, DF, SD, IBC. Image source: [47].
Figure 2.37: The schematic plot (left) and photo (right) of the TC (transfer cryostat). Image source: [47].

Figure 2.38: Schematic plot for the dilution refrigerator. Image source: [48].
Storage dewar

The SD (storage dewar) is also a stock Janis cryostat, which has a bigger magnetic field (7 T), and lower temperature (1.6 K) than PD. The sample space has thread positions for holding up to 3 targets. The SD has a longer liquid helium holding time and portable, and is used to move the polarized target from the HD-ice lab to the experiment hall.

In-beam cryostat

The IBC (In-beam cryostat) is a dilution refrigerator and magnet system for holding the polarized HD target during the experiment. The base temperature of the dilution refrigerator is 50 mK. And it has two magnet systems. The solenoid magnet creates a field of 1 T along the beamline, which is also the polarization direction of the longitudinal polarized target. The saddle coil magnet creates a field of 0.075 T perpendicular to the beamline, which is used during the spin rotation to change the polarization direction of the longitudinal polarized HD target. A backup coil with a field of 0.01 T is used during the run, in case there is a quench of the main solenoid magnet.

The HD target life cycle

The life of the HD target starts with the condensation of HD gas into PD, where a NMR measurement is done for calibrating the thermally equilibrium polarization of the target. Then the solid HD target is transferred to DF for polarization. After 3-6 month of polarizing and aging of the target, it will be relocated from DF to PD for another NMR measurement to determine the polarization of the target, this step can be bypassed if a good cross-calibration between the PD and the IBC NMR systems is known. After the NMR measurement, HD will be transferred from PD to SD for storage and then it is transported to the experiment hall. In the experiment hall, HD is transferred from SD to IBC for the experiment. Inside the IBC, periodic NMR measurements are performed for monitoring the target polarization during the experiment. Moreover, several spin manipulations for H and D are conducted, which includes using SFP (saturated forbidden adiabatic fast passage) to transfer polarization from H to D, reversing the z-direction of the polarization of H or D. After the experiment is finished, the HD is transferred from IBC to PD for another NMR measurement in PD for cross-calibration between PD and IBC NMR circuits, and then HD is warmed up to be collected for gas chromatography measurement and Raman scattering measurement to determine the $H_2$ and $D_2$ level in the HD gas. Figure 2.39 is a schematic plot for the life cycle of a HD target. Figure 2.40 is a schematic plot for the HD target transfer process.

2.3.3 NMR analysis for target polarization

Definition of target polarization

A nucleon with non-zero spin $I$ has $2I + 1$ spin states for its ground state. These states are degenerate in the absence of a magnetic field, but will be separated into different energy levels in the magnetic field (Zeeman effect). The energy levels for the spin states relative to the ground state is: $E = -\mu \cdot B = -\gamma I \cdot B = -\gamma (m\hbar)B$, where $m = -I, \ldots, I$. 

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Figure 2.39: Schematic plot for the life cycle of a HD target. The label from 1 to 5 for the arrows represents the 5 times use of TC for transferring the HD target.

Figure 2.40: The schematic plot for the HD target transfer process. Image source: [40].
\( \mu = \gamma I \) is the magnetic moment for the nucleon, \( \gamma = gq/2m \) is the gyromagnetic ratio, \( g \) is the g-factor.

The vector polarization for a target is the normalized difference between the number of spins parallel to the magnetic field and those anti-parallel to the magnetic field, as in Equation 2.6.

\[
P = \frac{N_{m=I} - N_{m=-I}}{\sum_{m=-I}^{I} N_{m}} \tag{2.6}
\]

In thermal equilibrium, the spin state population follows Boltzmann distribution. The thermal equilibrium polarization for spin \( I = 1/2 \) (proton) and spin \( I = 1 \) (deuteron) can be shown in Equation 2.7 and 2.8. These two equations are equivalent to Equation 2.4 in the previous section.

\[
P_{TE}(H) = \frac{e^{\frac{\gamma B}{2kT}} - e^{-\frac{\gamma B}{2kT}}}{e^{\frac{\gamma B}{2kT}} + e^{-\frac{\gamma B}{2kT}}} = \tanh\left(\frac{\gamma hB}{2kT}\right), \tag{2.7}
\]

\[
P_{TE}(D) = \frac{e^{\frac{\gamma hB}{2kT}} - e^{-\frac{\gamma hB}{2kT}}}{e^{\frac{\gamma hB}{2kT}} + 1 + e^{-\frac{\gamma hB}{2kT}}} = \frac{4\tanh\left(\frac{\gamma hB}{2kT}\right)}{3 + \tanh\left(\frac{\gamma hB}{2kT}\right)}, \tag{2.8}
\]

**Principles for NMR measurement**

The polarizations for H and D in the target are measured by the NMR (Nuclear Magnetic Resonance) polarimetry. Figure 2.41 is a cartoon for a NMR setup. In the plot, the main static magnetic field \( B_0 \) is along z-axis, the transmitter (RF drive coils) generates an oscillating magnetic field \( B_1 \) along x-axis, the receiver (pick-up coils) measure the induced oscillation of the target magnetism along y-axis. The induced RF signals in the receiver is proportional to the polarization of the target.

The principle of the NMR measurement can be described in the following way. Take the spin half proton \( (I = \hbar/2) \) as an example, in an external magnetic field \( B_0 \), the nucleons’ spin states are separated to different energy levels, which follows the Boltzmann distribution. More spins are aligned parallel to \( B_0 \) \( (I_z = -\hbar/2) \) than anti-parallel \( (I_z = +\hbar/2) \), resulting a net magnetization \( M_0 \) along \( B_0 \) (z-axis). The RF drive coils produce an oscillating RF field \( B_1 \) along the x-axis, when the frequency of RF satisfies the resonance condition: \( h\omega = \Delta E = \gamma hB_0 \), magnetization \( M_0 \) will be tipped away from the z-axis and obtains a component in the x-y plane. The oscillation of \( M_y \) will induce a current in the pick-up coils, which is the NMR signal. \( \Delta E \) is the difference of the energy levels for the two spin states of proton in a magnetic field, \( \omega = \gamma B_0 \) is referred as Larmor frequency. There is a proportional relationship between the NMR signal \( S_{NMR} \) and the polarization of the target \( P_t \) as in Equation 2.9, in which \( M_y \) and \( S_{NMR} \) are the magnitude of the oscillating signals.

\[
P_t \propto M_0 \propto M_y \propto S_{NMR}. \tag{2.9}
\]

There are two methods to reach the resonance condition \( \omega = \gamma B_0 \), one is to vary the RF frequency with fixed magnetic field \( B_0 \), another is to vary \( B_0 \) with fixed RF frequency. The G14 experiment uses the second method. Figure 2.42 shows the down-sweep and up-sweet of the magnetic field to search for the NMR resonance peak for deuteron. The sweep range of the magnetic field is 300 Gauss, which is from 3004 G to 2704 G in 31
seconds, stay at the low field for 1 second, back to 3004 G in 31 second, wait in high field for 1 second. The RF frequency is fixed to 1774.00 kHz. The finite width of the peak is caused by the fact that nucleons experience slightly different magnetic field in solid state environment.

The polarization of the target is proportional to the integrated area under the NMR peak: \( P_t = c \cdot I_{NMR} \), where \( I_{NMR} \) is the integration of \( S_{NMR} \) over time. It is noted that there is a linear relationship between the magnetic field and time, thus the integration has more physical meaning if it is integrated over the changing magnetic field: \( I_{NMR} = \int S_{NMR} dB \). The determination of the constant overall factor(\( c \)) is done by a cross calibration for the target in frozen spin mode (FS) and in thermal equilibrium (TE). In these two conditions, we have \( P_{FS} = c_{FS} \cdot I_{FS} \), \( P_{TE} = c_{TE} \cdot I_{TE} \), combining the two equations gives

\[
P_{FS} = \frac{c_{FS}}{c_{TE}} \cdot \frac{P_{TE}}{I_{TE}} \cdot I_{FS} = c_{overall} I_{FS},
\]

in which \( P_{TE} \) can be calculated using the Equation 2.4. The ratio \( \frac{c_{FS}}{c_{TE}} \) relates to the conditions of NMR measurements as in Equation 2.11.

\[
\frac{c_{FS}}{c_{TE}} = \frac{f_{TE} \cdot 10^{\frac{pwr_{TE}}{20dBm}} \cdot Gain_{TE}}{f_{FS} \cdot 10^{\frac{pwr_{FS}}{20dBm}} \cdot Gain_{FS}}.
\]

in which \( f_{TE}, I_{FS} \) are the RF frequency used in the two measurement, \( pwr_{TE}, pwr_{FS} \) are the RF power in the units of dBm, \( Gain_{TE}, Gain_{FS} \) are the gains of the RF circuit. Table 2.2 gives the overall factor \( c_{overall} \) for different frozen spin HD targets used in G14 experiment.
Figure 2.42: Magnetic field sweeps (black line) through $B_0$ twice searching for the NMR resonance (blue line) for deuteron.

<table>
<thead>
<tr>
<th>Target</th>
<th>21a</th>
<th>19b</th>
<th>22b</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{overall}(H)$ [%/(\mu V \cdot Gauss)]</td>
<td>14.9/240</td>
<td>29.61/230</td>
<td>46.45/230</td>
</tr>
<tr>
<td>$c_{overall}(D)$ [%/(\mu V \cdot Gauss)]</td>
<td>27.7/267</td>
<td>14.97/152</td>
<td>5.994/152</td>
</tr>
</tbody>
</table>

Table 2.2: Overall factors for different frozen spin HD target in G14 experiment, values are from Alexandrander Deur [42].
NMR single analysis

During the G14 experiment, periodic NMR measurements are conducted to monitor the target polarization. For each NMR measurement, four independent signals are collected for the NMR peaks, as in Figure 2.43, two signals from the X-channel for sweeping up and down of the magnetic field, another two from the Y-channel. The dispersion signals from Y-channel can be transformed to absorption signals using the Kramers-Kronig relations.

Figure 2.43: X-channel and Y-channel for the NMR resonance for deuteron.

We know from previous discussion that the NMR signal is proportional to the magnetization of the material $M_0$, which is proportional to the magnetic susceptibility $\chi(\omega)$, $S_{NMR} \propto M_0 \propto \chi(\omega)$. The magnetic susceptibility has both real and imaginary parts: $\chi(\omega) = \chi_1(\omega) + i\chi_2(\omega)$, which are measured by the X-channel and Y-channel respectively. $\chi_1(\omega), \chi_2(\omega)$ are related by the Kramers-Kronig relations as in Equation 2.12.

\[
\chi_1(\omega) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\chi_2(\omega')}{\omega' - \omega} d\omega', \\
\chi_2(\omega) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\chi_1(\omega')}{\omega' - \omega} d\omega',
\]

(2.12)

where $P$ is the Cauchy principal integral. These relations convert the two dispersion signals from Y-channel to absorption signals, and give two more measurement for the polarization. Before applying the Kramers-Kronig transformation, a phase rotation may be needed for some measurements. As in Figure 2.43, the X-channel signals is not flat.
around the absorption peaks. This is caused by the mix of the absorption signal and dispersion signal. A phase rotation (Eq. 2.13) can be used to solve this problem.

\[
x' = \cos(\theta)x + \sin(\theta)y, \\
y' = -\sin(\theta)x + \cos(\theta)y.
\] (2.13)

Figure 2.44: X-channel and Y-channel after phase rotation and Kramers Kronig transformation for the NMR resonance for deuteron.

Figure 2.44 gives the transformed signals for the same NMR measurement, both the X and Y channel signals have been applied the phase rotation, afterwards the Y-channel signal has been applied the Kramers Kronig transformation. Finally, if the background noise level is not at zero, a background level adjustment has to be applied.

Figure 2.43 and 2.44 have different y values, they are differed by the multiplier \(10^6 \times \frac{300}{(31 \times 128)}\). The reason for this conversion is that the x-axis does not have the unit of Gauss for magnetic field. The x values are the data points stored in the register, and the DAQ rate is 128 Hz, the sweeping rate for the magnetic field is 300 G/31 seconds. To convert the x value from data point number to Gauss, we need to multiply it by \(\frac{300}{(31 \times 128)}\). On the other hand, the y values are multiplying \(10^6\) to convert it from volts to micro-volts. Furthermore, the area under the peak is the integration of \(y\Delta x\), so the multiplier applied to x-value can be transferred to y-value without changing the result for \(y\Delta x\). This gives the final adjustment for the y-value of \(10^6 \times \frac{300}{(31 \times 128)}\) with x-value staying unchanged. After the conversion of the y values, the unit for the integrated peak area is \(\text{Gauss} \cdot \mu\text{V}\).
Since the background around the NMR peak is fluctuating, a technique referred as ‘scanning window’ is used. Figure 2.45 shows how this technique works. Firstly, a window of 300 data points is choose in order to include all the NMR peak for integration. Secondly, 36 integrations are conducted with a integration window that scans through the NMR peak with a step of 20 data points. Finally, the results for the integrations are plotted vs the integration starting point as in Figure 2.46. The plateau in Figure 2.46 is the region where the scanning window covers the whole NMR peak. The weighted mean is used to estimate the average of the peak area, and the error is the sample standard deviation of the 10 points.

\[
I = \frac{\sum_{i=1}^{10} I_i}{\sum_{i=1}^{10} \bar{I}_i},
\]

\[
\sigma = \sqrt{\frac{1}{9} \cdot \sum_{i=1}^{10} (I_i - \bar{I})^2}
\]

In [48], the error is estimated using the flat noisy background and is related to the size of the integration range. The uncertainty on one data point in the background region
can be computed as:

\[
\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}, \tag{2.15}
\]

where \( N \) is the number of sweeps for this background data point. With the assumption that the error on each points is the same, the statistical error on the peak integral can be computed by summing in quadrature: \( \epsilon = \sqrt{n} \cdot \sigma \), where \( n \) is the number of points in the integration range. It is noted that a larger integration range(larger \( \sqrt{n} \)) gives a bigger error \( \epsilon \). The error estimation used in the 'scanning window' procedure does not have this problem.

The same procedure is then applied to the other 3 peaks for the same NMR measurement. Then the 4 values are averaged using weighted mean, and their standard deviation is the error for the peak area.

To convert the integral of the peak area to the polarization value, an overall factor from Table 2.2 is used. These value are obtained from comparing the results from a NMR measurement for the HD target in thermal equilibrium state and a NMR measurement for the target in frozen spin state, in which the frozen spin target NMR measurement is referred as a reference measurement. Because the NMR measurements for the frozen spin target have been taken with different settings (RF power, RF frequency, Gain), the signals need to be normalized to the reference measurement in order to be compared to each other. The normalization is shown in Equation 2.16, since \( S_{NMR} \) is proportional to RF power, RF frequency and the circuit gain, all the signals should be normalized back to values with the reference settings.

\[
I' = \frac{f_{\text{ref}}}{f} 10^{\frac{P_{\text{ref}}-P}{20}} \frac{\text{Gain}_{\text{ref}}}{\text{Gain}} I. \tag{2.16}
\]
The polarization of target equals to the overall factor \((c_{\text{overall}})\) times the normalized peak area integral \((I')\).

\[
P_t = c_{\text{overall}} I' = c_{\text{overall}} \frac{f_{\text{ref}}}{f} \int 10^{\frac{P_{\text{ref}}-\gamma}{20}} \frac{\text{Gain}_{\text{ref}}}{\text{Gain}} I.
\]  

(2.17)

The results of the polarization for the three HD targets used in G14 experiment are shown in the following figures.

![Figure 2.47: Deuteron polarization for 21a target (silver runs).](image)

![Figure 2.48: Deuteron polarization for 19b target (gold runs).](image)

The target polarization for each run period and its statistical uncertainty are shown in Table 2.3, the weighted mean of the NMR polarization values during the period gives the average value, and the sample standard deviation is used as its statistical error. The systematic errors associated with the target polarization are summarized in Section 4.3.4, Table 4.8.
Figure 2.49: Deuteron polarization for 22b target (last target runs).

Figure 2.50: Hydrogen polarization for 21a target (silver runs).

Figure 2.51: Hydrogen polarization for 19b target (gold runs).
Figure 2.52: Hydrogen polarization for 22b target (last target runs).

Table 2.3: Polarization of the HD target from NMR signal analysis. Compared with the figures above, a 2% TC transfer loss has been applied to the values in the table.

<table>
<thead>
<tr>
<th>Run period</th>
<th>D polarization (stat. error)</th>
<th>H polarization (stat. error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver 1</td>
<td>+25.6(±0.7)%</td>
<td>+14.7(±0.2)%</td>
</tr>
<tr>
<td>Silver 2</td>
<td>+23.0(±0.6)%</td>
<td>−14.3(±1.2)%</td>
</tr>
<tr>
<td>Silver 3</td>
<td>+20.9(±0.5)%</td>
<td></td>
</tr>
<tr>
<td>Silver 4</td>
<td>−17.2(±0.5)%</td>
<td></td>
</tr>
<tr>
<td>Silver 5</td>
<td>−15.5(±0.7)%</td>
<td></td>
</tr>
<tr>
<td>Gold 1</td>
<td>+26.0(±0.8)%</td>
<td>+27.2(±0.1)%</td>
</tr>
<tr>
<td>Gold 2</td>
<td>+26.8(±0.9)%</td>
<td>+26.9(±0.4)%</td>
</tr>
<tr>
<td>Gold 3&amp;4</td>
<td>+15.8(±0.2)%</td>
<td>+18.3(±0.2)%</td>
</tr>
<tr>
<td>Last 1&amp;2a</td>
<td>+25.8(±0.8)%</td>
<td>+25.5(±0.4)%</td>
</tr>
<tr>
<td>Last 2b&amp;3</td>
<td>+24.0(±1.5)%</td>
<td>−22.6(±0.2)%</td>
</tr>
<tr>
<td>Last 4&amp;5&amp;6</td>
<td>−16.2(±1.1)%</td>
<td>+17.7(±0.3)%</td>
</tr>
</tbody>
</table>
Chapter 3

Event Selection

Three channels are studied: $\gamma n \rightarrow p\pi^-$, $\gamma p \rightarrow p\pi^+\pi^-$ and $\gamma n \rightarrow n\pi^+\pi^-$. The charged particles are detected using the CLAS drift chambers, start counters and time-of-flight counters. The drift chambers measures the particle tracking, which is used to determine the particle momentum ($p$) and flight path, while the start counters and time-of-flight counters give the particle TOF, which combining with flight-path gives particle velocity ($\beta$). The momentum and velocity are combined to give the particle mass: $m = p/\beta\gamma$.

The neutrons are detected by the forward electromagnetic calorimeter. The run conditions and procedures for selecting events for the three channels are described in the following sections.

3.1 Run Conditions

According to different targets and photon beam conditions, the G14 data is divided into different run periods. Table 3.1 shows the run periods with circularly polarized photon beam, table 3.2 shows the run periods with linearly polarized photon beam.

3.2 CLAS Banks

The reconstructed particle information is stored in the CLAS banks [49]. For some history reasons, there are two systems of banks: the 'PID' banks and 'SEB' banks. These two systems use different algorithms to reconstruct events. The 'PID' banks include HEAD, TGBI, GPID, TBID, ECHB, SCRC, STRE, TAGR, MVRT, VERT, TBER, EPIC. The 'SEB' banks include HEVT, EVNT, DCPB, TRPB, ECPB, SCPB, STPB, TGPB. The banks used in this study are the 'PID' banks. GPID is the main bank used in G14 analysis, it contains all the information for the charged particles, and the information for the tagger photons. The GPID bank is a combination of the PART and TBID banks. MVRT gives the vertex information for an event. The ECHB bank contains the information from the forward electromagnetic calorimeter, and gives the information for neutral particles. The HEAD bank contains the helicity information of the polarized beam. The TAGR bank stores the tagger information, which gives the energy, time, T counter ID, and E counter ID for the tagged photons. The TBER bank gives the
<table>
<thead>
<tr>
<th>Period</th>
<th>$E_{beam}$ (MeV)</th>
<th>Beam Polarization</th>
<th>Events</th>
<th>Torus Curr.</th>
<th>Target</th>
<th>Target Pol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver1</td>
<td>2280.96</td>
<td>-81.7%</td>
<td>830 M</td>
<td>+1920 A</td>
<td>21a</td>
<td>+D(25.6%)  +H(14.7%)</td>
</tr>
<tr>
<td>Silver2a</td>
<td>2280.96</td>
<td>-81.7%</td>
<td>300 M</td>
<td>+1920 A</td>
<td>21a</td>
<td>+D(23.0%)  -H(14.3%)</td>
</tr>
<tr>
<td>Silver2b</td>
<td>2280.96</td>
<td>-76.2%</td>
<td>870 M</td>
<td>+1920 A</td>
<td>21a</td>
<td>+D(23.0%)  -H(14.3%)</td>
</tr>
<tr>
<td>Silver3</td>
<td>2280.96</td>
<td>-76.2%</td>
<td>250 M</td>
<td>-1500 A</td>
<td>21a</td>
<td>+D(20.9%)  -H(N/A)</td>
</tr>
<tr>
<td>Silver4</td>
<td>2280.96</td>
<td>-76.2%</td>
<td>820 M</td>
<td>-1500 A</td>
<td>21a</td>
<td>-D(17.2%)  -H(N/A)</td>
</tr>
<tr>
<td>Silver5</td>
<td>2257.75</td>
<td>88.8%</td>
<td>5210 M</td>
<td>-1500 A</td>
<td>21a</td>
<td>-D(15.5%)  -H(N/A)</td>
</tr>
<tr>
<td>Gold2a</td>
<td>2541.31</td>
<td>88.2%</td>
<td>440 M</td>
<td>-1500 A</td>
<td>19b</td>
<td>+D(26.8%)  +H(26.9%)</td>
</tr>
<tr>
<td>Gold2b</td>
<td>2541.31</td>
<td>-83.4%</td>
<td>1660 M</td>
<td>-1500 A</td>
<td>19b</td>
<td>+D(26.8%)  +H(26.9%)</td>
</tr>
</tbody>
</table>

Table 3.1: Run periods for runs with circularly polarized photon beam. The first column is the name of the period, silver runs use "21a" target, gold runs use "19b" target. The second column is the energy of the electron beam. The third column is the electron beam polarization, which has an error of $1.4\%(\text{stat.}) \pm 3.0\%(\text{sys.})$. The fourth column is the number of events. The fifth column is the Torus Current, which determines the direction of the torus magnetic field. The sixth column is the target used. The seventh column is the degree of polarization for the HD target, where "±" indicate the direction of deuterons polarization along "z" direction in lab coordinate. A 2% transfer loss of polarization from TC has been applied to the numbers in the table.
<table>
<thead>
<tr>
<th>Period</th>
<th>Coherent Edge</th>
<th>Perp Events</th>
<th>Para Events</th>
<th>Amo. Events</th>
<th>Target</th>
<th>Target Pol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last tgt3</td>
<td>1800 MeV</td>
<td>506.6 M</td>
<td>483.0 M</td>
<td>86.0 M</td>
<td>22b</td>
<td>+D(24.03%), +H(22.56%)</td>
</tr>
<tr>
<td>Last tgt4</td>
<td>1800 MeV</td>
<td>307.6 M</td>
<td>304.2 M</td>
<td>52.4 M</td>
<td>22b</td>
<td>-D(16.20%), +H(17.73%)</td>
</tr>
<tr>
<td>Last tgt2a</td>
<td>2000 MeV</td>
<td>461.3 M</td>
<td>464.9 M</td>
<td>106.7 M</td>
<td>22b</td>
<td>+D(25.77%), +H(25.45%)</td>
</tr>
<tr>
<td>Last tgt2b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+D(24.03%), +H(22.56%)</td>
</tr>
<tr>
<td>Last tgt5</td>
<td>2000 MeV</td>
<td>184.7 M</td>
<td>208.8 M</td>
<td>40.8 M</td>
<td>22b</td>
<td>-D(16.20%), +H(17.73%)</td>
</tr>
<tr>
<td>gold1</td>
<td>2200 MeV</td>
<td>220.5 M</td>
<td>181.1 M</td>
<td>109.3 M</td>
<td>19b</td>
<td>+D(25.97%), +H(27.22%)</td>
</tr>
<tr>
<td>gold4</td>
<td>2200 MeV</td>
<td>20 M</td>
<td>20 M</td>
<td>0 M</td>
<td>19b</td>
<td>+D(15.75%), +H(18.27%)</td>
</tr>
<tr>
<td>last tgt1</td>
<td>2200 MeV</td>
<td>110.2 M</td>
<td>93.6 M</td>
<td>54.7 M</td>
<td>22b</td>
<td>+D(25.77%), +H(25.45%)</td>
</tr>
<tr>
<td>last tgt6</td>
<td>2200 MeV</td>
<td>93.6 M</td>
<td>148.3 M</td>
<td>10.2 M</td>
<td>22b</td>
<td>-D(16.20%), +H(17.73%)</td>
</tr>
</tbody>
</table>

Table 3.2: Run periods for runs with linearly polarized photon beam, a 30 μm diamond is used as the radiator, the electron beam energy for all these runs is 5551.77 MeV, the Torus field setting is -1500 A. The first column is the name of the period. The second column is the coherent edge energy. The third column is the number of events with E-plane of the linearly polarized photon beam perpendicular to the lab floor. The fourth column is the number of events with E-plane of the linearly polarized photon beam parallel to the lab floor. The fifth column is the number of events for runs with an amorphous radiator. The sixth column is the target name. The seventh column is the target polarization. A 2% transfer loss of polarization from TC has been applied to the numbers in the table.

covariance matrix for the CLAS detectors. SCRC bank contains information from the TOF counters.

### 3.3 Channel selection

The G14 experiments are designed to study several pseudo-scalar meson channels as listed in Table 3.3. To select the channel of interest, several particle ID filters, geometric cuts, and kinematic cuts are applied to the data.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Observables</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ + n → π^+p</td>
<td>σ_0, Σ, E, G</td>
</tr>
<tr>
<td>γ + n → π^+π^-n</td>
<td>σ_0, I^c(Σ), I^s, P_z, P^c(E), P^c_z(G), P^c_z</td>
</tr>
<tr>
<td>γ + p → π^+π^-p</td>
<td>σ_0, I^c(Σ), I^s, P_z, P^c(E), P^c_z(G), P^c_z</td>
</tr>
<tr>
<td>γ + n → K^0Λ</td>
<td>σ_0, Σ, E, G, O_x, O_y, C_x, C_y, P, T(-O_y)</td>
</tr>
<tr>
<td>γ + n → K^0Σ^0</td>
<td>σ_0, Σ, P, E, G</td>
</tr>
<tr>
<td>γ + n → K^+Σ^-</td>
<td>σ_0, Σ, E, G</td>
</tr>
</tbody>
</table>

Table 3.3: Pseudoscalar meson reactions and observables measured in G14 experiment.
3.3.1 Particle ID Filters

The first filter requires the event has the right number of final state particles. In GPID bank, each event has a list for the final state particles, which contains each particle’s PID number: proton(14), neutron(no PID in GPID bank, need ECPB bank for neutron identification), $\pi^+(8)$, $\pi^-(9)$, the numbers in the parenthesis are the PID number in the GPID bank. For the reaction $\gamma n \rightarrow p\pi^-$, the first filter requires the final state having one proton and one $\pi^-$. The second filter requires the proton and $\pi^-$ having the same 'targid', which indicates that proton and $\pi^-$ are from the event caused by the same tagged photon. The third filter is 'ngrf ==1', which requires the number of photons in the same RF bucket is one, this remove the ambiguity for events that relate to several tagged photons. The fourth filter is the "$\Delta\beta$" filter. This filter compare the $\beta$ value of the particle from two methods to remove misidentified particles. The first $\beta$ value is calculated from the timing information($\delta t$) from the start counter and the TOF counter, and the distance($\delta l$) from the start counter to the TOF counter, $\beta_1 = \frac{\delta l}{c\delta t}$. The second $\beta$ value comes from the momentum of the particle, which is calculated from the track information measured by the drift chamber. Using the definition for relativistic momentum $p = m_0v/\sqrt{1-v^2/c^2} = m_0\beta c/\sqrt{1-\beta^2}$, we get $\beta_2 = \frac{p}{\sqrt{p^2+m_0^2c^2}}$, where $m_0$ is the PDG mass for the particle. The definition for this filter is $\Delta\beta = \beta_1 - \beta_2$. A filter with $\text{abs}(\Delta\beta)<0.06$ for proton and $\text{abs}(\Delta\beta)<0.03$ for pion is used, as shown in Figure 6.4. Figure 3.2 shows the effect of this filter on the event selection.

![Graphs showing the effect of $\Delta\beta$ filter](image)

(a) Proton: $\text{abs}(\Delta\beta)<0.06$. (b) $\pi^+$: $\text{abs}(\Delta\beta)<0.03$. (c) $\pi^-$: $\text{abs}(\Delta\beta)<0.03$.

Figure 3.1: $\Delta\beta$ filter for proton, $\pi^+$ and $\pi^-$, the x-axis is $|p|$ (particle momentum).

After the four filters, the selected channel is ready for next step analysis.
Figure 3.2: The $\beta$ vs momentum plots for proton, $\pi^+$ and $\pi^-$ before (left plots) and after (right plots) $\Delta \beta$ cut. Most misidentified particles are removed with this filter.
3.3.2 Geometric Cuts

There are two geometric cuts used in this study: the fiducial cut and the vertex cut. The fiducial cut takes into account the acceptance of the CLAS detectors, and the vertex cut removes events from regions, where there is no HD.

Fiducial Cut

Fiducial cut remove the angular region where there are known obstructions to the acceptance of the detectors and near the torus coils where the torus field is not well known for good track reconstruction. The fiducial cut can be defined by a relation between the polar angle $\theta$ and azimuthal angle $\phi$ as in Equation 3.1 [52].

$$\theta > 4.0 + \frac{510.58}{(30 - \phi)^{1.5518}}$$

(3.1)

The resulting plot of $\theta$ vs. $\phi$ is shown in Figure 3.3.

Figure 3.3: Fiducial cuts for proton, $\pi^+$ and $\pi^-$. 
Vertex Cut

The HD target has a cylinder shape with the dimension 5 cm in length and 1.5 cm in diameter. The reconstruction code can determine where the event happens in the target region (the vertex information: x, y, z), this information can be used to remove the events that do not come from the HD. Two cuts are applied to the data, one is along the z direction: -10.5 cm < z < -5.5 cm (Figure 3.4), the other is along the radial direction: \( r = \sqrt{x^2 + y^2} < 1 \) cm (Figure 3.5).

Figure 3.4: Vertex z cut: -10.5 cm < z < -5.5 cm. The top plot is a schematic diagram for the HD target in the IBC (In-Beam-Cryostat). The bottom plot is the vertex plot using z coordinate from the MVRT bank, the blue line is for a HD target run, the red line is for an empty cell target run, the red curve is scaled to match the downstream Kel-F peak.

3.3.3 Kinematic Cuts

Kinematic Cuts use the conservation of 4 momentum in the reaction to make cuts on the missing mass, missing momentum, and reaction angles to select the channel of interest. For example, the reaction of a photon scattering on a neutron target resulting a recoiled proton and a recoiled \( \pi^- \) can be written in this form: \( \gamma + (n) \rightarrow p + \pi^- \). The neutron (n) is in a parenthesis, because the neutron is in the deuteron with unknown initial momentum, and we view neutron as a missing particle in this reaction. The missing mass and missing momentum for the neutron target can be calculated using the 4 momentum of the detected proton, \( \pi^- \), and incoming photon energy. Three kinematic cuts are used to remove the background events: missing mass cut, missing momentum cut, and coplanarity cut (Figure 3.6). The coplanarity cut uses the fact that there is only two final
state particles proton and $\pi^-$, in the center of momentum frame, the difference of the $\phi$ angles for these two particles should be $180^\circ$.

A problem with the cuts in Figure 3.6 is that some background events under the main peak cannot be removed.

A new procedure is developed in this study to solve this problem. This method uses the kinematic fitting, and the results for the same reaction $\gamma + (n) \rightarrow p + \pi^-$ are shown in Figure 3.7. From the plots we can see that the blue curves, which represent the events passing the cut, are restricted automatically to the kinematic cuts used in Figure 3.6, and the background events are mostly removed. In the next section, this kinematic fitting procedure will be introduced in detail.

After all the filters and cuts being applied to the data, the desired channel is selected, and are ready for the study of the cross section and polarization observables for this channel.

### 3.4 Kinematic Fitting

Kinematic Fitting is a technique that uses the least squares fit with physical constraints to improve measured quantities and to estimate unmeasured quantities.[50]. Take the reaction $\gamma + (n) \rightarrow p + \pi^-$ as an example, the kinematic fitting procedure can improve the measured photon tagged energy and the 4 momentum for the recoiled proton and $\pi^-$, and can estimate the 4 momentum for the bound neutron target. The constraints used in this procedure are the conservation of the 4 momentum before and after the reaction.

A iteration procedure is used to minimize $\chi^2 = (y - \hat{\eta})^T V_y^{-1} (y - \hat{\eta}) + 2\lambda^T f(\hat{\eta}, \hat{\xi})$, where $y$ are measured quantities, $\eta$ are fitted value for the measured quantities, $\xi$ are the unmeasured values, $V_y^{-1}$ is the covariance matrix. The minimized $\chi^2_{\text{min}}$ follows a chi-square distribution with (K-J) degree of freedom, where K is the number of constraint functions, J is the number of unmeasured variables. Based on this chi-square distribution, the confidence level and pull distributions will be calculated to measure the goodness-of-fit and to evaluate the error estimation.

Previous studies[51, 53] have shown that the correct error estimation is crucial for
kinematic fitting. In CLAS, the covariance matrix is stored in TBER bank, in which the diagonal and off-diagonal elements give the resolution errors and correlation coefficients of the tracking parameters for each track respectively. Before the kinematic fitting, the covariance matrix need to be corrected for energy loss, multiple scattering effects and different experiment settings. The confidence level and pull distributions can also help to fine tune the covariance matrix.

The results of the kinematic fitting can be used in two ways, one is to use the confidence level to remove the background events, the other is to use the fitted values to construct the new four momentum for the particles.

3.4.1 Least Squares Fitting

Suppose we have N independent experimental values $y_1, y_2, ..., y_N$ at the points $x_1, x_2, ..., x_N$. The true values of $y_i$ are unknown, but we assume there are some theoretical functions that will predicts the true value at each $x_i$, $f_i = f_i(\theta_1, \theta_2, ..., \theta_L; x_i)$, where $\theta_1, \theta_2, ..., \theta_L$ is a set of parameters, $L \leq N$.

Since each $y_i$ has measurement error $\sigma_i$, the equations $f_i = f_i(\theta_1, \theta_2, ..., \theta_L; x_i)$ cannot all be satisfied if $L < N$. For example, if we have three points not in a same line, a linear fitting function with only two parameters cannot go through all three points. But, we can require that the equation be satisfied "as closely as possible" if we define a statistic $X^2 = \sum_{i=1}^{N}((y_i - f_i)/\sigma_i)^2$, and demand that the values of $\theta_i$ be chosen so as to minimize $X^2$. Here we have assumed the measurement $y_i$ is independent, $X^2$ can be written in
Figure 3.7: Kinematic fitting procedure for selecting the channel: $\gamma + (n) \rightarrow p + \pi^-$. In each subplot, the black line is for all the events before using kinematic fitting, the blue line is for the good events that pass the cut, and the red line is for the background events that don’t pass the cut.
The Linear Least Square Model

If the functions \( f_i \) are linear functions of the parameters \( \theta_1, \theta_2, \ldots, \theta_L \); \( x_i \) = \( \sum_{i=1}^{L} a_i \theta_i \), \( i = 1, 2, \ldots, N \); \( L < N \) (3.3)

In matrix notation \( f = A\theta \), \( A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1L} \\ a_{21} & a_{22} & \cdots & a_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NL} \end{pmatrix} \), \( \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_L \end{pmatrix} \)

\( X^2 = (y - A\hat{\theta})^T V_y^{-1} (y - A\hat{\theta}) \) (4.4)

Minimizing \( X^2 \) by taking the derivatives of \( X^2 \) with respect to \( \theta \), we have,

\[ \nabla_\theta X^2 = -2(A^T V_y^{-1} y - A^T V_y^{-1} A\theta) = 0 \] (3.5)

If the matrix \( (A^T V_y^{-1} A) \) is non-singular and can be inverted, the solution for \( \theta \) is:

\[ \hat{\theta} = (A^T V_y^{-1} A)^{-1} A^T V_y^{-1} y. \] (3.6)

Substitute \( \hat{\theta} \) back to Eq. (3.4), we get the minimized \( X^2_{\text{min}} \):

\[ X^2_{\text{min}} = (y - A\hat{\theta})^T V_y^{-1} (y - A\hat{\theta}) \] (3.7)

The uncertainties for \( \theta \) can also be solved by using the formula for error propagation,

\[ V(\hat{\theta}) = \left((A^T V_y^{-1} A)^{-1} A^T V_y^{-1}\right) y \left((A^T V_y^{-1} A)^{-1} A^T V_y^{-1}\right)^T. \] (3.8)

From some algebra calculations we have:

\[ X^2 = X^2_{\text{min}} + (\theta - \hat{\theta})^T A^T V_y^{-1} A (\theta - \hat{\theta}) = X^2_{\text{min}} + (\theta - \hat{\theta})^T V^{-1} (\theta - \hat{\theta}) \] (3.9)
If the N measurements \( y_i \) are normally distributed, by definition \( X^2 \) would be a chi-square distribution with N degree of freedom. Since \( \hat{\theta} \) is linearly related to \( y_i \), it is also normally distributed. All the three terms in Eq.(3.9) will be chi-square distributed, in which \( X^2 \) has N degree of freedom, \( X^2_{\text{min}} \) has (N-L) degree of freedom, \( (\theta - \hat{\theta})^T V^{-1}(\theta - \hat{\theta}) \) has L degree of freedom.

Eq.(3.6) and (3.8) give the solution to the linear LS problem for unknown parameters \( \theta \). This result also hold when \( V_y \) is a matrix with non-zero covariance terms, in this case, the N measurements \( y_i \) are not independent.

**The Nonlinear Least Squares Model**

If \( f_i \) are nonlinear functions of the parameters \( \theta_i \), there are no exact solutions for \( \theta_i \) as in the linear case. We need to use iterative procedure to perform the minimization.

As discussed earlier, we want to minimize the quantity \( X^2 = (y - f)^T V_y^{-1} (y - f) \), where \( y \) is the vector of measurements with covariance matrix \( V_y \), and \( f = f(\theta; x) \) is the vector of predicted values, which is nonlinear function of \( \theta \). Suppose we have found of a set of approximate parameters \( \theta^\nu = \{ \theta_1^\nu, \theta_2^\nu, ..., \theta_L^\nu \} \) in the \( \nu \)-th iteration. To make the calculation simple, we assume independent measurements, \( V_{yii} = \sigma_i^2 \), the derivative of \( X^2 \) with respect to \( \theta \) at \( \theta = \theta^\nu \) is,

\[
g_l(\theta^\nu) = g_l^\nu = \frac{\partial X^2}{\partial \theta_l} = \sum_{i=1}^{N} \left( -\frac{2}{\sigma_i^2} \right) (y_i - f_i^\nu) \left[ \frac{\partial f_i}{\partial \theta_l} \right]_{\theta^\nu}, \quad l = 1, 2, ..., L \tag{3.10}
\]

We want to find an increment \( \Delta \theta^\nu \) to \( \theta^\nu \) which make \( g_l(\theta^\nu + \Delta \theta^\nu) = 0 \). To find \( \Delta \theta^\nu \), we expand \( g_l \) around \( \theta^\nu \) to first order,

\[
g_l^\nu + \frac{\partial g_l^\nu}{\partial \theta_1} \Delta \theta_1^\nu + \frac{\partial g_l^\nu}{\partial \theta_2} \Delta \theta_2^\nu + ... + \frac{\partial g_l^\nu}{\partial \theta_L} \Delta \theta_L^\nu = 0, \quad l = 1, 2, ..., L \tag{3.11}
\]

Write

\[
G_{kl}^\nu = \frac{\partial g_l^\nu}{\partial \theta_k} = \frac{\partial^2 X^2}{\partial \theta_k \partial \theta_l} = \sum_{i=1}^{N} \left( -\frac{2}{\sigma_i^2} \right) \left[ -\frac{\partial f_i}{\partial \theta_k} \frac{\partial f_i}{\partial \theta_l} + (y_i - f_i^\nu) \frac{\partial^2 f_i^\nu}{\partial \theta_k \partial \theta_l} \right] \tag{3.12}
\]

Eq.(3.11) can be written in matrix form \( g^\nu + G^\nu \Delta \theta^\nu = 0 \),

where \( g^\nu = \begin{pmatrix} g_1^\nu \\ g_2^\nu \\ \vdots \\ g_L^\nu \end{pmatrix}, \Delta \theta^\nu = \begin{pmatrix} \Delta \theta_1^\nu \\ \Delta \theta_2^\nu \\ \vdots \\ \Delta \theta_L^\nu \end{pmatrix}, G^\nu = \begin{pmatrix} \frac{\partial g_1}{\partial \theta_1} & \frac{\partial g_1}{\partial \theta_2} & \cdots & \frac{\partial g_1}{\partial \theta_L} \\ \frac{\partial g_2}{\partial \theta_1} & \frac{\partial g_2}{\partial \theta_2} & \cdots & \frac{\partial g_2}{\partial \theta_L} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial g_L}{\partial \theta_1} & \frac{\partial g_L}{\partial \theta_2} & \cdots & \frac{\partial g_L}{\partial \theta_L} \end{pmatrix} \). Then

\[
\Delta \theta^\nu = -(G^\nu)^{-1}g^\nu. \quad \text{The new parameters } \theta^{\nu+1} = \theta^\nu + \Delta \theta^\nu \text{ are used to find } (X^2)^{\nu+1}, \text{ if }(X^2)^{\nu+1} < (X^2)^{\nu}, \text{ the new parameters are better estimation, and the procedure is repeated until the improvement between two consecutive iterations is smaller than the preset value. If it is found that one iteration gives } (X^2)^{\nu+1} > (X^2)^{\nu}, \text{ one can redefine the } \nu\text{-th step by taking a smaller value, such as } \Delta \theta^\nu = \frac{1}{2} \Delta \theta^\nu \text{ to do the procedure.}
\]
Improved Measurements

In the previous sections, the least squares method is used to find the best values for the unknown parameters $\theta$, which are used in some theoretical functions $f(\theta; x)$ to predict the true observable $\eta$. However, in many situations, the unknowns are the observables $\eta$ themselves, such as in the kinematic fitting. In this case, we will use the observations $y$ and the covariance matrix $V_y$ as the initial estimates and use the principle of least squares to find the best estimates of $\eta$ which will minimize the quantity $X^2 = \epsilon^T V_y^{-1} \epsilon$, where $\epsilon = y - \eta$. The final estimates $\hat{\eta}$ of the true value $\eta$ are called the "improved measurements".

Least Squares Model with Constraints

As before, we first consider the linear LS problem with linear constraint functions:

$$X^2 = (y - A\theta)^T V_y^{-1} (y - A\theta),$$

$$B\theta - b = 0,$$

(3.13)

where the $L$ parameters $\theta$ are related through $K$ constraint functions, $B$ is a matrix with dimension $K \times L$, $b$ is a component vector. We introduce a $K$-component Lagrangian multipliers $\lambda$ and rewrite the problem in the unconstrained form with $L+K$ unknowns.

$$X^2 = (y - A\theta)^T V_y^{-1} (y - A\theta) + 2\lambda^T (B\theta - b).$$

(3.14)

Eq.(3.14) can be solved exactly if we equate to zero the derivatives of $X^2$ with respect to $\theta_l$ and $\lambda_k[50]$. In the most general situation, both the $f_i$ and the constraint functions are nonlinear of $\theta$, the iterative procedure using the method of Lagrangian multipliers will be used to minimize $X^2$. In the following, we give an example to show how to use least squares estimation with constraints to 'improve measurements' and estimate unmeasured quantities.

Suppose we have a process $\gamma + p \rightarrow p + \pi^+ + \pi^-$, where the momenta and angles of the two pions are measured, but the recoiled proton is not measured, the momentum and energy conservation give four constraint functions. For convenience, we use a vector $\eta = \{\eta_1, \eta_2, ..., \eta_N\}$ to represent the true values for the $N$ measured quantities $y = \{y_1, y_2, ..., y_N\}$, the vector $\xi = \{\xi_1, \xi_2, ..., \xi_J\}$ to represent the true values for the $J$ unmeasured quantities. The total unknowns are $N+J$ for $\eta$ and $\xi$, there are $K$ constraint equations relate the unknowns:

$$f_k(\eta_1, \eta_2, ..., \eta_N, \xi_1, \xi_2, ..., \xi_J) = 0, \quad k = 1, 2, ..., K.$$  

(3.15)

The Least Squares Principle requires that the best estimates of the unknowns $\eta$ and $\xi$ are the values making the following equations true:

$$X^2 = (y - \hat{\eta})^T V_y^{-1} (y - \hat{\eta}) = \text{minimum},$$

$$f(\hat{\eta}, \hat{\xi}) = 0,$$

(3.16)

As before, we introduce $K$ additional unknowns $\lambda = \{\lambda_1, \lambda_2, ..., \lambda_K\}$, and rewrite the problem by requiring:

$$X^2 = (y - \hat{\eta})^T V_y^{-1} (y - \hat{\eta}) + 2\lambda^T f(\hat{\eta}, \hat{\xi}) = \text{minimum},$$

(3.17)
Equating the derivatives of $X^2$ with respect to all $N+J+K$ unknowns to zero gives $N+J+K$ equations:

\[
\begin{align*}
\nabla_\eta X^2 &= -2V_y^{-1}(y - \eta) + 2F_\eta^T \lambda = 0, \\
\nabla_\xi X^2 &= 2F_\xi^T \lambda = 0, \\
\nabla_\lambda X^2 &= 2f(\eta, \xi) = 0,
\end{align*}
\]

where the matrices $F_\eta (dimension: K \times N)$ and $F_\xi (dimension: K \times J)$ are:

\[
(F_\eta)_{ki} = \frac{\partial f_k}{\partial \eta_i}, \quad (F_\xi)_{kj} = \frac{\partial f_k}{\partial \xi_j}.
\]

Since $f(\eta, \xi)$ are nonlinear functions of $\eta, \xi$, the $N+J+K$ functions cannot solve the $N+J+K$ unknowns exactly, the solution must be found by iteration.

Let us suppose that solution $\eta^\nu, \xi^\nu, \lambda^\nu$ are found in the $\nu$-th iteration, which gives $(X^2)^\nu$. We want to find the next $\eta^\nu+1, \xi^\nu+1, \lambda^\nu+1$, which gives $(X^2)^{\nu+1}$ smaller than $(X^2)^\nu$. Equations in (3.18) are used to relate the values in two iteration steps. First, by keeping the Taylor expansion of $F_\eta$ and $F_\xi$ at the starting values, we can choose $\eta^0, \xi^0, \lambda^0$. We want to find the next $\eta^\nu+1, \xi^\nu+1$. Equations in (3.18) are used to relate the values in two iteration steps. First, we make Taylor expansion of the third equation in (3.18) around $\eta^\nu, \xi^\nu$ and neglect the second and higher orders,

\[
f_k^{\nu+1} + \sum_{i=1}^N (F_\eta^\nu)_{ki}(\eta^\nu+1_i - \eta^\nu_i) + \sum_{j=1}^J (F_\xi^\nu)_{kj}(\xi^\nu+1_j - \xi^\nu_j) = 0.
\]

Then we write the other two equations in (3.18) at the $(\nu+1)$-th iteration values:

\[
\begin{align*}
\eta^\nu+1 &= y - V_y(F_\eta^T)^\nu \lambda^\nu+1, \\
(F_\xi)^\nu \lambda^\nu+1 &= 0,
\end{align*}
\]

Substitute $\eta^\nu+1$ in Eq.(3.21) to Eq.(3.20), and introduce $r = f^\nu + F_\eta^\nu(y - \eta^\nu), S = F_\eta^\nu V_y(F_\eta^T)^\nu$, we have:

\[
\begin{align*}
F^\nu + F_\eta^\nu \left[ y - V_y(F_\eta^T)^\nu \lambda^\nu+1 - \eta^\nu \right] + (F_\xi^\nu)(\xi^\nu+1 - \xi^\nu) &= 0 \\
\Rightarrow r + (F_\xi^\nu)(\xi^\nu+1 - \xi^\nu) &= S \lambda^\nu+1 \\
\Rightarrow \lambda^\nu+1 &= S^{-1} \left[ r + (F_\xi^\nu)(\xi^\nu+1 - \xi^\nu) \right].
\end{align*}
\]

Use $\lambda^\nu+1$ from Eq.(3.22) in the second equation in Eq.(3.21), we can solve $\xi^\nu+1, \lambda^\nu, \eta^\nu+1$:

\[
\begin{align*}
\xi^\nu+1 &= \xi^\nu - \left[ (F_\xi^T)^\nu S^{-1} F_\xi^\nu \right] (F_\xi^T)^\nu S^{-1} r \\
\lambda^\nu+1 &= S^{-1} \left[ r + (F_\xi^\nu)(\xi^\nu+1 - \xi^\nu) \right] \\
\eta^\nu+1 &= y - V_y(F_\eta^T)^\nu \lambda^\nu+1.
\end{align*}
\]

By keeping the Taylor expansion of $f(\eta, \xi)$ to first order, we linearize the equation set, and the completely unknown $\xi^\nu+1$ are solved at first, next the Lagrangian multipliers $\lambda^\nu+1$ and lastly the improved measurements $\eta^\nu+1$ are solved. These new values will be used to calculate the new $(X^2)^{\nu+1}$, which will be compared with $(X^2)^\nu$ to decide when to stop the iteration process. The convergence of $\Delta \eta, \Delta \xi$ may also need to be checked. For the starting values, we can choose $\eta^0 = y$, the measured value, and $\xi^0$ can be calculated form the constraint equations by inserting $\eta^0$. The covariance matrix for $\eta, \xi$ can also be calculated[50].
Confidence Level and Pull Distributions

From section 2.1, we know that the residuals \( X^2_{\text{min}} \), which is obtained from the minimization process, has a chi-square distribution. This distribution can be used to define a measure for the goodness-of-fit, which is,

\[
P_{\chi^2} = \int_{X^2_{\text{min}}}^{\infty} f(\chi^2; \nu) d\chi^2 = 1 - F(\chi^2_{\text{min}}; \nu),
\]

(3.24)

where \( f(\chi^2; \nu) \) is the PDF (probability distribution function), and \( F(\chi^2_{\text{min}}; \nu) \) is the CDF (cumulative distribution function) for the chi-square distribution with \( \nu \) degrees of freedom. In Root, \( P_{\chi^2} \) is calculated using the function TMath::Prob(chisq,ndf), which denotes the probability that an observed chi-square exceeds the value "chisq". Since CDF is uniformly distributed within \([0,1]\) (see Figure 3.8), \( P_{\chi^2} \) (often called p-value) is also uniformly distributed. If the minimization process gives \( P_{\chi^2} \) non-uniform, this means either the hypothesis is not satisfied or the measurements are bad. An example is that \( P_{\chi^2} \) is strongly peaked near zero, this indicates a contamination of the data from the background events, since background events gives bigger \( \chi^2 \), which means fatter tail in the PDF \( f(\chi^2; \nu) \), and a peak in \( P_{\chi^2} \) at low probabilities. By cutting the small \( P_{\chi^2} \), we can remove the background events, as shown in Figure 3.9.

![Figure 3.8: f(x) is PDF, F(x) is CDF for a lorentz distribution, the probability for x<x0 is P(f(x<x0))=P(F(x)<F(x0))=F(x0). The distribution which has the property P(r<r0)=r0 is the uniform distribution over the region [0,1].](image)

Quite often, a big \( X^2_{\text{min}} \) is not due to the wrong hypothesis or background events, but because either the measured values are altered by Energy loss or multi-scattering, or a wrong error estimate is used. It is helpful to define the Pull Distribution to look at each measured value,

\[
z_i = \frac{\epsilon_i}{\sigma(\epsilon_i)} = \frac{y_i - \hat{\eta}_i}{\sqrt{\sigma^2(y_i) - \sigma^2(\hat{\eta}_i)}}, \quad i = 1, 2, ..., N,
\]

(3.25)

where \( \epsilon_i = y_i - \hat{\eta}_i \) measures the deviations between the observations and the fitted values, \( \epsilon_i \) is normalized relative to its uncertainty \( \sigma(\epsilon_i) \) to allow for comparison of different
Figure 3.9: $\chi^2$ distribution with degrees of freedom $n = 4$ and its confidence level from monte carlo and real data. Right two plots are those from real data, background events gives fatter tail for $f(\chi^2; \nu)$ at large $\chi^2$ and sharp peak for $P_{\chi^2}$ around 0.
measurements. The minus sign in the calculation for $\sigma_i$ comes from the fact that $y_i, \hat{\eta}_i$ are completely correlated,

$$\sigma_i^2(y_i - \hat{\eta}_i) = V_{ii}(y) - 2\text{cov}(y, \hat{\eta})_{ii} + V_{ii}(\hat{\eta}) = V_{ii}(y) - V_{ii}(\hat{\eta}).$$  (3.26)

The pull should be normally distributed about zero with $\sigma = 1$. If $z_i$ is shifted away from zero, this indicates a systematic error in the $i$-th observation. If the observed $z_i$ is too broader/narrower than the normal distribution $N(0,1)$, the error in the $i$-th observation is consistently too small/large. Before the kinematic fitting, the effect of Energy loss, multi-scattering and other factors should be corrected for the observations and the covariance matrix. The pull distribution provide a way to test how well the corrections work.

Degrees of Freedom

The function TMath::Prob(chisq,ndf) for the calculation of the confidence level requires the degree of freedom for the distribution $\chi^2_{\text{min}}$. From Eq.(3.9) in section 2.1, we know that if we have $N$ measured values and $L$ unknown fitted values, $\chi^2_{\text{min}}$ has $(N-L)$ degree of freedom. If there are $K$ constraint functions relate the $L$ unknowns, only $(L-K)$ of them are independent, giving $(N-(L-K))$ independent terms in $\chi^2_{\text{min}}$, so $\chi^2_{\text{min}}$ is distributed as $\chi^2(N - L + K)$.

For the process $\gamma + p \rightarrow (p) + \pi^+ + \pi^-$, with recoiled proton not measured. $N = 7$, in which 1 is for incoming photon energy, 6 is for 2 sets of pion tracking variables. $L = N+J =10$, where $J=3$ is for the fitting values for the missing proton. $K = 4$ from the 4 constraint equations of the conservation of energy and momentum, this is $(N-L+K)=1C$ fit. If all the initial and final state are known, then it is a 4C fit.

3.4.2 Covariance Matrix in CLAS

From section 2.4.1, we know that an accurate covariance matrix is important for good kinematic fitting. Instead of using the 4-vector momentum to describe the status of a particle, CLAS uses the tracking parameters $q/p, \lambda, \phi$. This section will introduce how the tracking parameters are defined, the covariance matrix for these parameters, and corrections need to be done for the covariance matrix to make it more accurate.

Track Reconstruction

The track reconstruction for charged particles in the drift chambers in CLAS is done in a sector dependent system. As shown in Figure 3.10, the $x_{\text{track}}$-axis is along the $z_{\text{lab}}$ direction, the $y_{\text{track}}$-axis goes through the center of the sector, and the $z_{\text{axis}}$ is along the average magnetic field in the given sector. The relations between the tracking coordinates and lab coordinates are:

$$\begin{pmatrix}
  x_{\text{track}} \\
  y_{\text{track}} \\
  z_{\text{track}}
\end{pmatrix} = \begin{pmatrix}
  z_{\text{lab}} \\
  \cos(\alpha)x_{\text{lab}} + \sin(\alpha)y_{\text{lab}} \\
  -\sin(\alpha)x_{\text{lab}} + \cos(\alpha)y_{\text{lab}}
\end{pmatrix}$$  (3.27)

where $\alpha = \frac{\pi}{3}(N_{\text{sector}} - 1)$ is sector dependent.
The tracking parameters for the particles in TBER bank include: $q/p, \lambda, \phi, d_0, z_0$, in which $d_0$ and $z_0$ are used in vertex reconstruction, we only discuss the first three parameters here. $q$ is the charge of the particle, $p$ is its momentum in lab frame, $\lambda \in (-\pi/6, \pi/6)$ is the angle between the track and the $(x_{track}, y_{track})$ plane, $\phi$ is the angle in the $(x_{track}, y_{track})$ plane relative to $x_{track}$-axis. Using the tracking parameters, we can write the momentum of the particles in the lab frame as:

$$\begin{pmatrix} p_{x_{lab}} \\ p_{y_{lab}} \\ p_{z_{lab}} \end{pmatrix} = \begin{pmatrix} p_1 \cos(\phi) \cos(\alpha) - \sin(\lambda) \sin(\alpha) \\ p_1 \cos(\phi) \sin(\alpha) + \sin(\lambda) \cos(\alpha) \\ p \cos(\lambda) \cos(\phi) \end{pmatrix}.$$ (3.28)

The covariance matrix for the tracking parameters are also given in the TBER bank:

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{pmatrix}.$$ (3.29)

These covariance matrix elements are determined from the tracking information only without any corrections. The resolution error for $\phi$ is approximately $\sigma_{\phi_{res}} \approx \sigma_s / R$, where $\sigma_s \approx 200\mu m$ is the resolution of detector in the plane of $\phi$. $R \approx 75 cm$ is the distance from the vertex to the drift chamber, so $\sigma_{\phi_{res}}$ is about 0.27 mrad. The resolution of detector in the plane of $\lambda$ is one order of magnitude worse than that for $\phi$, $\sigma_{\lambda_{res}}$ is about 2.7 mrad. These two errors can be calculated from the TBER bank, $\sigma_{\phi_{res}} = \sqrt{c_{33}}$ and $\sigma_{\lambda_{res}} = \sqrt{c_{22}}$. Figure 3.11 gives the errors for the two tracking angle for proton, $\pi^+$ and $\pi^-$ respectively. The correlation coefficients between the tracking parameters are defined as $\rho_{ij} = c_{ij} / \sqrt{c_{ii} c_{jj}}$. From Figure 3.12, we see a strong correlation between $q/p$ and $\phi$. This is because $q/p = 1/(B r \sin(\theta))$, where $B$ is the magnetic field, $r$ is the radius of curvature of the track; on the other hand the CLAS toroidal filed B has a strong $\phi$ (which is the polar angle $\theta$ in lab coordinate) dependence, B is smaller for bigger $\phi$ as shown in Fig.5 in [29], so the two tracking parameters $q/p$ and $\phi$ are strongly correlated.
Figure 3.11: $\sigma_{\lambda_{res}}$ and $\sigma_{\phi_{res}}$ for proton, $\pi^+$ and $\pi^-$

Figure 3.12: Correlation coefficients between tracking parameters. $\rho(q/p, \phi) \approx 0.85$, $\rho(q/p, \lambda) \approx 0.15$, $\rho(\lambda, \phi) \approx -0.27$. 

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Effect of Energy Loss and Multiple Scattering on Covariance Matrix

It’s easier to study the energy loss effect using the parameter $p$ instead of $q/p$, we write the new covariance matrix in terms of $p$ as:

$$
C_{pp} = \left( \frac{\partial p}{\partial (q/p)} \right)^2 C_{qq}^2 \frac{p^4}{q^2} C_{qq},
$$

$$
C_{p\phi} = \left( \frac{\partial p}{\partial (q/p)} \right) \left( \frac{\partial \phi}{\partial \phi} \right) C_{qq}^2 = -\frac{p^2}{q} C_{qq}^2,
$$

$$
C_{p\lambda} = \left( \frac{\partial p}{\partial (q/p)} \right) \left( \frac{\partial \lambda}{\partial \lambda} \right) C_{qq}^2 = -\frac{p^2}{q} C_{qq}^2,
$$

(3.30)

It was found in previous studies that some corrections for the covariance matrix need to be done before the energy loss and multiple scattering corrections. For the two angle parameters, the values were found to be off by a factor of 1.5:

$$
\sigma_{\phi res} = 1.5 \sigma_{\phi TBER},
\sigma_{\lambda res} = 1.5 \sigma_{\lambda TBER}.
$$

(3.31)

For the momentum, the TBER bank does not account for the torus current, which makes the value off by a factor of $I_{max}/I$:

$$
\sigma_{p res} = \frac{I_{max}}{I} \sigma_p = \frac{I_{max}}{I} \frac{p^2}{q} \sigma_{(q/p) TBER},
$$

(3.32)

During the experiment, before reaching the detectors, the final state particles need to pass through some materials first, such as the target, the in-beam cryostat and the start counter. These materials will change both the value of the tracking parameters and its covariance matrix. These corrections are not included in the TBER bank, and need to be added separately.

The corrections of energy loss and multiple scattering to the tracking errors as following:

$$
\sigma_p^2 = \sigma_{p res}^2 + \sigma_{el}^2,
\sigma_\phi^2 = \sigma_{\phi res}^2 + \sigma_{ms}^2,
\sigma_\lambda^2 = \sigma_{\lambda res}^2 + \sigma_{ms}^2.
$$

(3.33)

where $\sigma_{p res}$, $\sigma_{\phi res}$ and $\sigma_{\lambda res}$ are resolution errors before the correction, $\sigma_{el}$ is the energy loss correction to the momentum error, $\sigma_{ms}$ is the multiple scattering correction to the tracking angle errors. The method to calculate $\sigma_{el}$ and $\sigma_{ms}$ can be found in section 3.1 of [53].

The Corrected Covariance Matrix

We can now write down the corrected covariance matrix for the kinematic fitting taking into account all the corrections in the previous section. Consider a reaction with L charged
particle in the final state, there are \((3L+1)\) fit parameters,

\[
C_\eta = \begin{pmatrix}
\sigma_{E_\gamma}^2 \\
p_1 \\
\lambda_1 \\
\phi_1 \\
\vdots \\
p_L \\
\lambda_L \\
\phi_L 
\end{pmatrix}, \tag{3.34}
\]

where \(E_\gamma\) is the incident photon energy and \(p_i, \lambda_i, \phi_i\) are the tracking parameters for the \(i^{th}\) charged particle.

Since the measurement of different particles are uncorrelated, the covariance between different particles are zero. The covariance matrix is,

\[
\eta = \begin{pmatrix}
\sigma_{E_\gamma}^2 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & C_{1p}^{pp} & C_{1p}^{p\lambda} & C_{1p}^{p\phi} & \ldots & 0 & 0 & 0 \\
0 & C_{1\lambda}^{1p} & C_{1\lambda}^{1\lambda} & C_{1\lambda}^{1\phi} & \ldots & 0 & 0 & 0 \\
0 & C_{1\phi}^{1p} & C_{1\phi}^{1\lambda} & C_{1\phi}^{1\phi} & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & C_{pL}^{pp} & C_{pL}^{p\lambda} & C_{pL}^{p\phi} \\
0 & 0 & 0 & 0 & \ldots & C_{\lambda L}^{\lambda p} & C_{\lambda L}^{\lambda \lambda} & C_{\lambda L}^{\lambda \phi} \\
0 & 0 & 0 & 0 & \ldots & C_{\phi L}^{\phi p} & C_{\phi L}^{\phi \lambda} & C_{\phi L}^{\phi \phi} 
\end{pmatrix}, \tag{3.35}
\]

where \(\sigma_{E_\gamma}^2\) is the error of measuring the tagged photon. \(\sigma_{E_\gamma}^2\) is calculated using the energy resolution of E-plane paddles in the tagger, which is 0.001 \(E_0\). \(E_0\) is the electron beam energy. For a photon with energy \(\bar{E}\), the energy of the photon is uniformly distributed within \([\bar{E} - 0.001E_0, \bar{E} + 0.001E_0]\) because of the resolution of the detector. The error of \(\bar{E}\) is,

\[
\sigma_{E_\gamma}^2 = \int_{E - 0.001E_0}^{E + 0.001E_0} (E - \bar{E})^2 dE = \int_{-0.001E_0}^{0.001E_0} \frac{E^2}{0.002E_0} dE' = \frac{(0.001E_0)^2}{3}. \tag{3.36}
\]

Other covariance matrix elements are calculated using the information from TBER bank:

\[
C_{i}^{pp} = \left(\frac{I_{\max p_i}^2}{I_{q_i}}\right)^2 C_{TBER}^{p\lambda} + \sigma_{poloss}^2,
\]

\[
C_{i}^{\lambda\lambda} = 2.25C_{TBER}^{\lambda\lambda} + \sigma_{ms}^2,
\]

\[
C_{i}^{\phi\phi} = 2.25C_{TBER}^{\phi\phi} + \sigma_{ms}^2,
\]

\[
C_{i}^{p\lambda} = -\left(\frac{1.5I_{\max p_i}^2}{I_{q_i}}\right) C_{TBER}^{p\lambda}, \tag{3.37}
\]

\[
C_{i}^{p\phi} = -\left(\frac{1.5I_{\max p_i}^2}{I_{q_i}}\right) C_{TBER}^{p\phi},
\]

\[
C_{i}^{\lambda\phi} = 2.25C_{TBER}^{\lambda\phi}.
\]
To check if the covariance matrix has been corrected, we can look at the pull distribution of the tracking parameters, an iteration process will help determine the best estimation for the error correction.

### 3.4.3 Preparation for Kinematic Fitting

Before the kinematic fitting, we should do the following three corrections to get the four momentum of the particles as accurate as possible: energy loss correction, momentum correction and tagger sag correction. The kinematic fitting can be used to test how good the correction is by looking at the pull distributions.

**Energy Loss Correction**

As the charged particles pass through the materials in the CLAS detector, they will lose some of their energy, thus a systematic adjustment to the particles’ energy is required. The ELOSS package [54] is used for the corrections, this package uses first principle method to calculate the energy loss by considering all the materials that the particles go through. The differences \( \Delta E = E_{after} - E_{before} \) for proton, \( \pi^+ \), \( \pi^- \) are shown in Figure 3.13. As seen in the plots, the high momentum protons have small energy loss around 15 MeV, and for low momentum protons, the correction is about 50 MeV, this is reasonable from the relation \( dE/dx \sim 1/\beta^2 \). The lighter particles \( \pi^+ \), \( \pi^- \) lost less energy than the heavier particles proton.

![Eloss correction vs. momentum for proton (top left), \( \pi^+ \) (top right), \( \pi^- \) (bottom)](image)

Figure 3.13: Eloss correction vs. momentum for proton (top left), \( \pi^+ \) (top right), \( \pi^- \) (bottom)

In order to check if the Eloss correction package works, we can plot the pull distributions before and after Eloss correction. Figure 3.14 gives this comparison for the pull distribution for \( P_{pro} \) (proton’s momentum), We can see that after the Eloss correction,
the pull distribution becomes more like a standard normal distribution, except that the mean value is still not zero, the reason will be explained in the next section.

![Pull distribution](image)

**Figure 3.14:** Pull distribution before(left plots) and after(right plots) Eloss correction for proton momentum.

### Momentum Correction and Photon Energy Correction

After the energy loss correction, the pull distributions have improved a lot, but their means are still not close to zero. The reason for this problem can be seen in Figure 3.15. We can see that the missing mass for the missing particle depends on the $\phi$ angle of the recoiled proton. This is caused by the misalignment of the drift chamber with the magnetic field. Thus we need to correct the momentum of the final state particles for each $\theta$ and $\phi$ bin, and to correct the incoming photon energy in terms of the tagger’s 767 energy bins. The way to do these two corrections is to use kinematic fitting to get the fitted value for the particles’ momenta for each $\theta$ and $\phi$ bin and photon energy for each E-counter energy bin, then take the difference between the fitted values and the measured values to get the correction equations, $\Delta p = p_{\text{fit}} - p_{\text{meas}}$, $\Delta E_{\gamma} = E_{\gamma\text{fit}} - E_{\gamma\text{meas}}$.

Momentum correction is done for $9 (\theta) \times 36 (\phi)$ bins. $\theta$ and $\phi$ are for angles in the lab frame for recoiled protons. The nine $\theta$ bins are $0^\circ < \theta \leq 20^\circ$, $20^\circ < \theta \leq 30^\circ$, $30^\circ < \theta \leq 40^\circ$, $40^\circ < \theta \leq 50^\circ$, $50^\circ < \theta \leq 60^\circ$, $60^\circ < \theta \leq 70^\circ$, $70^\circ < \theta \leq 80^\circ$, $80^\circ < \theta \leq 100^\circ$, $100^\circ < \theta \leq 180^\circ$. The 36 bins come from binning each of the six CLAS
sectors into six bins. The schematic plots for $\theta$ and $\phi$ bins can be seen in Figure 3.16. The use of both the single pion channel and double pion channel extends the dynamic range of applicability of these corrections, since pion momentum is relatively high in single pion channel, and pion momenta in double pion channel are relatively low.

Figure 3.16: $\theta$ bins (left plot) and $\phi$ bins (right plot) for momentum correction. The different sizes of the $\theta$ bins are chosen by considering the CLAS acceptance is small in the forward angle and backward angle. In the plot for $\phi$ bins, the gaps enclosed by black lines correspond to the gaps between the six CLAS sectors. The bin size near the gap is bigger because of a smaller acceptance in this region.

Momentum correction is done for proton, $\pi^+$, $\pi^-$ respectively. The correction for the bin $20^\circ < \theta \leq 30^\circ$, $-164^\circ < \phi \leq -152^\circ$ for proton is shown in Figure 3.17. For proton and $\pi^-$, the correction combines the kinematic fitted values from both the two pion channel $\gamma(p) \to p\pi^+\pi^-$ and the one pion channel $\gamma(n) \to p\pi^-$. For $\pi^+$, the correction only uses the two pion channel $\gamma(p) \to p\pi^+\pi^-$. The corrections for $\pi^+$ and $\pi^-$ for the bin $20^\circ < \theta \leq 30^\circ$, $-164^\circ < \phi \leq -152^\circ$ are shown in Figure 3.18.

Photon energy correction $\Delta E_\gamma = E_{\gamma\text{fit}} - E_{\gamma\text{meas}}$ is calculated in terms of the 767 E-counter energy bins, and the result is also a combination of the correction from both the channel $\gamma(p) \to p\pi^+\pi^-$ and $\gamma(n) \to p\pi^-$. Figure 3.19 gives the result for the photon energy correction.

After the momentum and photon energy corrections, the same pull distribution as in Figure 3.14 is plotted again in Figure 3.20. From the figure, we can see that the mean of the pull distribution moves from 0.3079 to -0.01356, and the sigma changes from 1.043 to 1.007. The momentum and photon energy corrections makes the pull distribution more like the standard gaussian distribution.

It should be noted that since the G14 experiment uses different targets and different Torus currents. The data collected is divided to different periods as in Table 3.1. The momentum and photon energy corrections are done for 4 different run periods: Silver 1 and 2, Silver 3 and 4, Silver 5, Gold 2.
Figure 3.17: Momentum correction $\Delta p$ vs $p$ for proton for the bin $20^\circ < \theta \leq 30^\circ$, $-164^\circ < \phi \leq -152^\circ$. Top left: $\Delta p$ vs $p$ for the reaction $\gamma(p) \rightarrow p\pi^+\pi^-$. Top right: $\Delta p$ vs $p$ for the reaction $\gamma(n) \rightarrow p\pi^-$. Bottom: The Gaussian fit for $\Delta p$ by binning $p$, blue triangle markers are for the two pion channel, red square markers are for the single pion channel, black circle markers are the combination of the two channel.

Figure 3.18: Momentum correction $\Delta p$ vs $p$ for $\pi^+$ and $\pi^-$ for the bin $20^\circ < \theta \leq 30^\circ$, $-164^\circ < \phi \leq -152^\circ$. Left: correction for $\pi^+$ using the channel $\gamma(p) \rightarrow p\pi^+\pi^-$. Right: correction for $\pi^-$ using both $\gamma(p) \rightarrow p\pi^+\pi^-$ and $\gamma(n) \rightarrow p\pi^-$, blue triangle markers are for the two pion channel, red square markers are for the single pion channel, black circle markers are the combination of the two channel.
Figure 3.19: Photon energy correction $\Delta E_\gamma = E_{\gamma}^{kfit} - E_{\gamma}^{meas}$ vs. Tagger E-counter bins.

Figure 3.20: Pull distribution before(left plots) and after(right plots) Momentum and Photon Energy corrections for proton momentum. Then mean value has moved from 0.31 to -0.01
3.4.4 Kinematic Fitting for $\gamma n \rightarrow p\pi^-$

In order to select the channel of interest, the kinematic fitting is used again. The four momenta of the final state particles have been applied the energy loss correction, momentum correction, and photon energy correction. For this channel, we could apply two equivalent hypothesis. One is $\gamma(n) \rightarrow p\pi^-$, which assumes an unknown moving neutron target, the other is $\gamma D \rightarrow p(p)\pi^-$, which assumes a deuteron target and a missing spectator proton in the final state. We choose the first hypothesis, since this method removes the events from the high momentum neutron in the deuteron target automatically, and the polarization observables in this study require a target with minimal initial momentum (ideally zero momentum). The confidence level cut is used to remove the background event, and the quality of the selected channel will be checked with missing mass plot, missing momentum plot, coplanarity plot, and the pull distributions for all the final state particles’ tracking parameters. These results will be shown for the gold2a period at first (Figure 3.21 and Figure 3.22). Then the confidence level plots (Figure 3.23), missing mass vs. $\phi$ plots (Figure 3.24), and the mean and sigma values of the pull distributions (Table 3.4) will be shown for all the run periods with circularly polarized beam.

Since the neutron used in this channel is a quasi-free particle, in the kinematic fitting, an effective mass for neutron is used in the procedure, and the value is estimated as following. For a free particle, the invariant mass of the particle relates to its energy and
Figure 3.22: Pull distribution for gold2a for channel $\gamma(n) \rightarrow p\pi^-$. The mean and sigma values for the gaussian fit of the pull distribution are inserted at the bottom right corner, the values are very close to a standard gaussian distribution.
Figure 3.23: 5% confidence level cut for all circular runs for the reaction $\gamma(n) \rightarrow p\pi^-$. 
Figure 3.24: Missing Mass vs. $\phi_{pro}$ for the reaction $\gamma(n) \rightarrow p\pi^-$ for all circular periods.
momentum as \( E^2 - p^2 = m^2 \). For a quasi-free particle, this relation becomes \((E - V)^2 = k^2 + m^2\), with an extra potential term \(V\). If the potential term is much smaller than the mass of the particle, we can still view the particle as a free particle but with an off-shell effective mass \(m^* : E^2 - k^2 = (m^*)^2 = m^2 + 2EV - V^2\). Since \(V \ll m\), \(m^* \approx m + V\).

For deuteron, the binding energy is 0.00223 GeV, \(m^* \approx m - C \times 0.00223\), where \(C\) is a constant, which is estimated to be 4.3 as studied in Appendix. This gives \(m^* \approx 0.930\) GeV.

<table>
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<th>(\lambda_p)</th>
<th>(\phi_p)</th>
<th>(P_{\pi^-})</th>
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<td>Gold2a</td>
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<td>-0.03</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.04</td>
<td>0.05</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>Sigma</td>
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<td>0.98</td>
<td>1.03</td>
<td>1.04</td>
<td>1.03</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>Gold2b</td>
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<td>-0.03</td>
<td>-0.01</td>
<td>-0.03</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Sigma</td>
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<td>0.98</td>
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<td>1.04</td>
<td>1.03</td>
<td>1.05</td>
<td>1.04</td>
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</table>

Table 3.4: Mean and sigma values of the pull distributions for all the circularly run periods for channel \(\gamma(n) \to p\pi^-\).

### 3.4.5 Kinematic Fitting for \(\gamma p \to p\pi^+\pi^-\)

For the reaction of two charged pion photo-production from a proton target, since the HD molecule has one free proton and one bound proton in the deuteron, we cannot separate the events in these two cases. Thus it’s not proper to assume either H or D as the target in the kinematic fitting. The only hypothesis for the kinematic fitting we can make is to assume the proton target’s momentum is unknown, and all the final state particles are detected. This makes the kinematic fitting a 1-C fit. The same set of plots as the single pion channel are shown in the following. Figure 3.25 shows the confidence cut of 5% for the gold2a period, and the effect of this cut on the missing mass, missing momentum, and coplanarity plots. Figure 3.26 shows the pull distributions for all 10 kinematic variables after energy loss correction, photon energy correction and momentum correction for gold2a period, the results are very close to the standard normal distribution as expected. Figure 3.27 shows the confidence level distribution for all circular beam runs, and figure 3.28 shows their missing mass vs phi plots.
Figure 3.25: Effect of the kinematic fitting’s confidence level cut on event selection for channel \( \gamma(p) \rightarrow p\pi^+\pi^- \) using the data from gold2a. Top left: confidence cut at 5\%. Top right: missing mass plot, bottom left: missing momentum, bottom right: coplanarity. For all the plots, the blue region is for good events that pass the cut, the red region is the background events that are removed by the cut.

Table 3.5: Mean and sigma values of the pull distributions for all the circularly run periods for channel \( \gamma(p) \rightarrow p\pi^+\pi^- \).

<table>
<thead>
<tr>
<th>Period</th>
<th>Pull Dist.</th>
<th>( P_p )</th>
<th>( \lambda_p )</th>
<th>( \phi_p )</th>
<th>( P_{\pi^+} )</th>
<th>( \lambda_{\pi^+} )</th>
<th>( \phi_{\pi^+} )</th>
<th>( P_{\pi^-} )</th>
<th>( \lambda_{\pi^-} )</th>
<th>( \phi_{\pi^-} )</th>
<th>( E_\gamma )</th>
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<td>-0.04</td>
<td>-0.00</td>
<td>-0.01</td>
<td>-0.03</td>
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<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Sigma</td>
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<td>0.98</td>
<td>1.04</td>
<td>1.04</td>
<td>0.97</td>
<td>1.04</td>
<td>1.04</td>
<td>0.99</td>
<td>1.04</td>
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</tr>
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<td>0.06</td>
<td>-0.03</td>
<td>-0.06</td>
<td>-0.01</td>
<td>-0.02</td>
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<td>-0.02</td>
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</tr>
<tr>
<td></td>
<td>Sigma</td>
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<td>0.98</td>
<td>1.02</td>
<td>1.03</td>
<td>0.96</td>
<td>1.04</td>
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<td>1.03</td>
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<tr>
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<td>-0.07</td>
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<td>-0.03</td>
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</tr>
<tr>
<td></td>
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<td>0.96</td>
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<td></td>
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<tr>
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<td>0.96</td>
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</tr>
<tr>
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<td>-0.02</td>
<td>0.03</td>
<td>0.01</td>
<td>-0.02</td>
<td>0.02</td>
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<td>0.04</td>
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<td>-0.01</td>
</tr>
<tr>
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<td>Sigma</td>
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<tr>
<td>Gold2b</td>
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<td>-0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.01</td>
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</tr>
<tr>
<td></td>
<td>Sigma</td>
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<td>0.94</td>
<td>1.04</td>
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<td>1.00</td>
<td>1.04</td>
<td>1.04</td>
<td>0.98</td>
<td>1.04</td>
<td>1.04</td>
</tr>
</tbody>
</table>
Figure 3.26: Pull distribution for gold2a for channel $\gamma(p) \rightarrow p\pi^+\pi^-$. The mean and sigma values for the gaussian fit of the pull distribution are inserted at the bottom right corner, the values are very close to a standard gaussian distribution.
Figure 3.27: 5% confidence level cut for all circular runs for the reaction $\gamma(p) \rightarrow p\pi^+\pi^-$. 
Figure 3.28: Missing Mass vs. $\phi_{pro}$ for the reaction $\gamma(p) \rightarrow p\pi^+\pi^-$ for all circular periods.
Chapter 4

E asymmetry for $\gamma n \rightarrow p\pi^-$

An accurate evaluation of the electromagnetic couplings in meson photo-production remains unknown due to the lack of experiment data on the neutron targets. Moreover, the existing data on neutron target are mainly differential cross section, only 15% are from polarization observable measurement [16]. E asymmetry for $\gamma n \rightarrow p\pi^-$ is the first measurement for this polarization observable on the neutron target.

4.1 Formalism

![Diagram of reaction](image)

Figure 4.1: Kinematics diagram for $\pi^-$ photoproduction off neutron target in the center of mass frame, $k, p_1, q, p_2$ are the momenta for the incident photon, neutron target, and final state $\pi^-$ and recoiled proton, $\theta_{CM}(\pi^-)$ is the polar angle of $\pi^-$ in center of momentum frame.

For the reaction of a photon scattering off a nucleon, the four-momenta of the incident photon, the outgoing pion, the initial nucleon and final nucleon are denoted by $k = (k, k), q = (q, \omega), p_1 = (p_1, E_1), p_2 = (p_2, E_2)$. The photo-production amplitude $A$ relates to the S matrix by the following equation:

$$S = 1 + (2\pi)^4 i\delta^4(P_f - P_i)(8\pi W N)A,$$

where $P_i = k + p_1, P_f = q + p_2, W^2 = s = (k + p_1)^2, N = (16k\omega E_1 E_2)^{1/2}$ is a normalization factor.

Since the electromagnetic interaction violates isospin conservation, the transition operator can be decomposed into an isoscalar part $\hat{S}$ and an isovector part $\hat{V}$, the pion
photo-production amplitude $A$ has three independent matrix elements with the notation $<I_f, I_3|\hat{A}|I_i, I_3>$:[17, 10]:

\[
A^{IS} = \langle \frac{1}{2}, \pm \frac{1}{2}|\hat{S}|\frac{1}{2}, \pm \frac{1}{2} > ,
\]

\[
\mp A^{IV} = \langle \frac{1}{2}, \pm \frac{1}{2}|\hat{V}|\frac{1}{2}, \pm \frac{1}{2} > ,
\]

\[
A^{V3} = \langle \frac{3}{2}, \pm \frac{1}{2}|\hat{V}|\frac{1}{2}, \pm \frac{1}{2} > .\]

The determination of the three isospin amplitudes can be obtained from the following four photoproduction reactions.

\[
A(\gamma p \rightarrow \pi^+ n) = -\sqrt{\frac{1}{3}} A^{V3} + \sqrt{\frac{2}{3}} (A^{IV} - A^{IS}) ,
\]

\[
A(\gamma p \rightarrow \pi^0 p) = +\sqrt{\frac{2}{3}} A^{V3} + \sqrt{\frac{1}{3}} (A^{IV} - A^{IS}) ,
\]

\[
A(\gamma n \rightarrow \pi^- p) = +\sqrt{\frac{1}{3}} A^{V3} - \sqrt{\frac{2}{3}} (A^{IV} + A^{IS}) ,
\]

\[
A(\gamma n \rightarrow \pi^0 n) = +\sqrt{\frac{2}{3}} A^{V3} + \sqrt{\frac{1}{3}} (A^{IV} + A^{IS}) .\]

The way to get the above expressions can be found in [10], an example for $\gamma n \rightarrow \pi^- p$ is shown in the following. The isospin part of the wave function for nucleon and pion be be written as:

\[
|p> = |\frac{1}{2}, \pm \frac{1}{2} > , |n> = |\frac{1}{2}, -\frac{1}{2} > ,
\]

\[
|\pi^+ > = -|1, +1 > , |\pi^0 > = |1, 0 > , |\pi^- > = |1, -1 > . (4.4)
\]

For the reaction $\gamma n \rightarrow \pi^- p$, the initial state isospin wave function is $|\gamma n> = |\frac{1}{2}, -\frac{1}{2} >$. The final state isospin wave function is $|\pi^- p> = |1, -1 > |\frac{1}{2}, +\frac{1}{2} > = \sqrt{\frac{1}{3}} |\frac{3}{2}, -\frac{1}{2} > -\sqrt{\frac{2}{3}} |\frac{1}{2}, -\frac{1}{2} >$. The transition amplitude $A$ for this reaction is:

\[
A(\gamma n \rightarrow \pi^- p) = \langle \pi^- p|\hat{A}|\gamma n >
\]

\[
= (\sqrt{\frac{1}{3}} < \frac{3}{2}, -\frac{1}{2}| - \sqrt{\frac{2}{3}} < \frac{1}{2}, -\frac{1}{2}|)\hat{S} + \hat{V}|\frac{1}{2}, -\frac{1}{2} >
\]

\[
= +\sqrt{\frac{1}{3}} A^{V3} - \sqrt{\frac{2}{3}} (A^{IV} + A^{IS}) .\]

Equation 4.3 shows that $A^{V3}$ can be obtained from only the reaction using proton target, the other two component can only be separated using both the proton data and the neutron data. For each reaction in Equation 4.3, the amplitude can be decomposed into helicity amplitudes or CGLN amplitudes. Helicity is the projection of spin onto the direction of momentum: $\lambda_k = \pm 1$ for incident photon, $\lambda_1 = \pm 1/2$ for initial nucleon, $\lambda_2 = \pm 1/2$ for final nucleon, $\lambda_k = 0$ for pion. The initial and final state helicities
The eight helicity amplitudes are not independent, parity conservation relates the four measurements required to determine the production amplitude $A$ without ambiguities. 

The four independent amplitudes are all complex numbers, and eight independent measurements are required to determine the production amplitude $A$ without ambiguities.

There are 16 nonzero observables for single pseudo-scalar meson photo-production as below [19]:

$$d\sigma/d\Omega(P^\gamma, P^T, P^R)$$

$$= \frac{1}{2} \{ d\sigma_0[1 - P_L^T P_y^T P_y^R \cos(2\phi_y)] + \hat{\Sigma}[ -P_L^T \cos(2\phi_y) + P_y^T P_y^R \]

$$+ \hat{\Omega}_x[ -P_L^T P_y^T P_z^R \cos(2\phi_z) - P_z^T \cos(2\phi_z)]

$$+ \hat{\Omega}_y[ -P_L^T P_y^T P_x^R \cos(2\phi_x) - P_x^T \cos(2\phi_x)]

$$+ \hat{\Omega}_z[ -P_L^T P_y^T P_y^R \cos(2\phi_y)] + \hat{\Sigma}[ -P_L^T \cos(2\phi_y) + P_y^T P_y^R \]

$$+ \hat{\Omega}_x[ -P_L^T P_y^T P_z^R \cos(2\phi_z) - P_z^T \cos(2\phi_z)]

$$+ \hat{\Omega}_y[ -P_L^T P_y^T P_x^R \cos(2\phi_x) - P_x^T \cos(2\phi_x)]

$$+ \hat{\Omega}_z[ -P_L^T P_y^T P_y^R \cos(2\phi_y)] \}$$ (4.6)

In Eq. 4.6, $P_L^T$ and $P_z^T$ are the degree of polarization for circular and linear polarized photon beam. $P_y^T$ and $P_y^R$ are the degree of polarization of target and recoil nucleon. $d\sigma_0$ is the unpolarized differential cross section. Three asymmetries which to leading order scaled by a single polarization of either beam, target or recoil ($\Sigma$, $T$, $P$) are single polarization asymmetries. Three sets of four asymmetries whose leading polarization dependence involves two polarizations of either beam–target (E, F, G, H), beam–recoil (C,$x$, C,$z$, O,$x$, O,$z$), or target–recoil ($L_x$, L,$z$, T,$x$, T,$z$) are double polarization asymmetries. Each of the double polarization asymmetry has two way to be measured in experiment, take the beam-target asymmetry $E$ as an example, $2\hat{E} = \sigma(+1, -z, 0) - \sigma(+1, +z, 0)$ (target polarization is flipped) or $2\hat{E} = \sigma(+1, -z, 0) - \sigma(-1, -z, 0)$ (beam polarization is flipped). The 16 observables are not independent, their are 37 relations between them as in appendix C of [19].

Figure 4.2 shows the list of observables that can be measured for single pseudo-scalar meson photo-production. The cross section $\sigma_0$ is un-polarization measurement, $\Sigma, T, P$ are single-polarization asymmetries, and $E, F, G, H, C_x, C_z, O_x, O_z, L_x, L_z, T_x, T_z$ are double polarization asymmetries. The complete experiment that eliminates ambiguities requires a minimum of eight observables, which can be chosen to be $\sigma_0, \Sigma, T, P$ and four double-polarization asymmetries with at least one involving recoil polarization. For the channel $\gamma p \rightarrow p\pi^-$, there are 1381 cross section points between 0.4 and 2.0 GeV, only 326 single-polarization ($\Sigma, T, P$) points, and none for the double polarization asymmetries. The G14 experiment has measured two more new asymmetries for this reaction: $E$ with circular polarized beam, and $G$ with linear polarized beam. This Chapter will show the result for the $E$ asymmetry.

This reaction needs two independent variables to determine the kinematics, which can be chosen to be the total energy $W$ and the polar angle of $\pi^- \theta_{CM}(\pi^-)$ in c.m frame. $W$
Figure 4.2: Polarization observables in the reaction of pseudoscalar meson photoproduction. Each observable can be measured in two different methods in the table. The observables in blue cells are measured in previous experiments, the ones in yellow cells are new observables that are measured in G14 experiment. Image source: [19]

is divided into 21 bins from 1500 MeV to 2300 MeV with the bin size of 40 MeV. \( \theta_{\text{CM}}(\pi^-) \) is divided into 13 bins from \( \cos(\theta_{\text{CM}}(\pi^-)) = -0.975 \) to \( \cos(\theta_{\text{CM}}(\pi^-)) = 0.975 \) with the bin size of 0.15. The E asymmetry will be calculated for each 21 \( \times \) 13 bins.

### 4.2 Extraction of E asymmetry

The E asymmetry is a beam target double polarization asymmetry as shown in Figure 4.2. The normalized E is defined as:

\[
E = \frac{\sigma(+1, -z, 0) - \sigma(-1, -z, 0)}{\sigma(+1, -z, 0) + \sigma(-1, -z, 0)},
\]

where the first value in the parenthesis represents the helicity of the circular polarized photon beam, the second value represents the direction of polarization for the longitudinal polarized target, and the third value represents the recoil nucleon polarization. Taking into account of the degree of polarization for photon beam and target, and relating the cross section to the yields of events, we can write the beam target asymmetry E and its statistical error as:

\[
E = \frac{1}{P_\gamma P_t} \frac{N_{1/2} - N_{3/2}}{N_{1/2} + N_{3/2}}
\]

\[
\sigma_E = \frac{1}{P_\gamma P_t} \frac{2(N_{1/2}N_{3/2})^{1/2}}{(N_{1/2} + N_{3/2})^{3/2}}
\]

where \( P_\gamma, P_t \) are degrees of polarization for photon beam and target. \( N_{1/2} \) is the total number of events for beam polarization and target polarization are anti parallel, \( N_{3/2} \) is the total number of events for beam polarization and target polarization are parallel.

Since the HD target has aluminum wires and KelF in the target region. We need to make corrections to the E asymmetry caused by the aluminum background.

\[
E^{\text{raw}} = \frac{1}{P_\gamma P_t} \frac{N_{1/2} - N_{3/2}}{N_{1/2} + N_{3/2}} = \frac{1}{P_\gamma P_t} \frac{N_{1/2} - N_{3/2}}{(N_{1/2} + N_{3/2})(1 + \frac{N_{\text{empty}}}{N_{1/2} + N_{3/2}})}
\]

98
\[
E = (1 + \frac{N_{\text{empty}}}{N_{1/2} + N_{3/2}})E_{\text{raw}},
\]  

(4.10)

where \(N_{\text{empty}}\) is the scaled total number of events from empty target run, \(E_{\text{raw}}\) is E asymmetry before empty target correction, \(E\) is E asymmetry after empty target correction.

For the 8 data sets in circular runs, corrections for E asymmetry from background are done for each data set separately. Figure 4.3 shows two examples from Gold2b and Silver 1 for the comparison of z vertex between full target run and empty target run. Since the empty target run and full target run have different beam time, the empty run events have to be scaled with a constant number. This number is calculated by comparing the events from the KelF peak between \(z = 0\) cm to \(z = 2\) cm as in Figure 4.3, this scale constant is 0.80 for gold2a and 0.91 for silver1. Moreover, the empty run and all the silver runs use the same target cell (21a), the gold2 runs have used a different target cell (19b). The amount of aluminum wires is different in these two cells, the 19b target only has 70% of the aluminum wires used in 21a. Within the target region: \(-10.5\) cm < \(z\) < \(-5.5\) cm, the ratio of events between the empty target run and the full target run can be calculated using the above information. For gold 2a run period, the ratio is \((10978*0.80*0.70)/251974 = 0.024\), where 0.80 is the scale factor for empty run, 0.70 is the ratio of aluminum wires between gold2a target cell and empty-a target cell. For silver 1 run period, the ratio is \((16796*0.91)/428213 = 0.036\).

The correction values of E asymmetry for these two periods are given as: \(E = 1.024E_{\text{raw}}\) for gold2a period and \(E = 1.036E_{\text{raw}}\) for silver1 period. It is found that the correction values depends on the c.m. energy \(W\), a detailed study for this dependence is shown in appendix C.

The results of E asymmetry for each run period (silver1, silver2a, silver2b, gold2a, gold2b) are shown in Figure 4.4-4.8. The comparison of the E asymmetry for different run periods is shown in Figure 4.9. The results combining all periods with the method of weighted mean are shown in Figure 4.10. In each figure, the two blue lines are from Bonn-Gatchina 2011-2 predictions for E asymmetry [13], the two lines correspond to the
upper and lower energy boundary for each energy bin. The two red lines are from SAID CM12 predictions [15].

Figure 4.4: Silver 1 run period: E asymmetry vs $\cos(\theta_{CM})$ for different center of momentum energy $W$. 
Figure 4.5: Silver 2a run period: E asymmetry vs \( \cos(\theta_{CM}) \) for different center of momentum energy \( W \).
Figure 4.6: Silver 2b run period: $E$ asymmetry vs $\cos(\theta_{CM})$ for different center of momentum energy $W$. 
Figure 4.7: Gold 2a run period: E asymmetry vs \( \cos(\theta_{CM}) \) for different center of momentum energy \( W \).
Figure 4.8: Gold 2b run period: E asymmetry vs $\cos(\theta_{CM})$ for different center of momentum energy $W$. 
Figure 4.9: Comparison of E asymmetry from 5 different run periods for 3 different center of momentum energies.
Figure 4.10: Combined result: E asymmetry vs $\cos(\theta_{CM})$ for different center of momentum energy $W$. 
4.3 Systematic Studies

Different values on the cuts used in the channel selection procedure will affect the final results for the E asymmetry. This section studies the systematic errors induced by using different 'vertex z' cuts, 'confidence level' cuts, and 'missing momentum' cuts. The systematic error is defined in the following way:

1. E asymmetry is calculated for each \( \cos(\theta_{CM}) \) and energy \( W \) bin for cut 1 and cut 2 respectively, there are 13 \( \cos(\theta_{CM}) \) bins and 21 \( W \) bins, which give a total of \( 13 \times 21 = 273 \) bins.

2. The relative difference of the E asymmetry is calculated using the two different cuts:
   \[
   \frac{\delta}{E/E_{\text{cut}2}} = \frac{E_{\text{cut}1} - E_{\text{cut}2}}{E_{\text{cut}2}} \quad \text{(a condition of \( |E_{\text{cut}2}| > 0.2 \) is imposed to avoid the case of zero denominator),}
   \]
   and the results are used to fill a 1-D histogram, which will be fitted with a Gaussian distribution. The mean \( \langle \mu \rangle \) of the Gaussian is the relative systematic error caused by using these two different cuts.

3. Calculate the mean \( \langle \mu \rangle \) and width \( \langle \sigma \rangle \) of the Gaussian distribution of \( \delta/E/E_{\text{cut}2} \) for all the circular beam run periods, and use weighted mean to obtain the overall systematic error.

\[
\sigma_{\text{systematic}} = \sqrt{\mu_{\text{Silver1}}^2 + \mu_{\text{Silver2a}}^2 + \mu_{\text{Silver2b}}^2 + \mu_{\text{Gold2a}}^2 + \mu_{\text{Gold2b}}^2 + \mu_{\text{Gold2b}}^2}
\]

where \( \sigma_{\mu} = \sigma_{\delta}/\sqrt{N} \) is the standard error of the mean value, \( N \) is the size of the sample.

4.3.1 Systematic Study for Vertex Z cuts

As shown in Chapter 3, section 3.3.2, there is a cut along the z-axis of the target: \(-10.5 \text{ cm} < z < -5.5 \text{ cm}\). This cut removes most of the events from the KelF target cell that enclose HD target. A variation of \( \pm 0.2 \text{ cm} \) of the vertex z cut is used to study the systematic effect of vertex z cut on E asymmetry.

Tightening 0.2 cm of the vertex z cut: \(-10.3 \text{ cm} < z < -5.7 \text{ cm}\), the 1-D histogram of the difference between the E asymmetry calculated from the new vertex cut and standard vertex cut (-10.5 cm < z < -5.5 cm) for circular runs silver1, silver 2, and gold2 are shown in Figure 4.11. The systematic error caused by the change of vertex z cut is calculated using Equation 4.11, and result is shown in Table 4.1. The results for loosening 0.2 cm of the vertex z cut: \(-10.7 \text{ cm} < z < -5.3 \text{ cm}\) are shown in Figure 4.12 and Table 4.2. The total systematic error caused by tightening and loosening the vertex z cut is the quadrature sum of the two errors, which is: \( \sigma_{\text{systematic}}(\text{vertex z}) = \sqrt{0.006^2 + 0.01^2} = 1.3\% \)

4.3.2 Systematic Study for Confidence level cuts

To separate the events for channel of interest from the background events, a confidence level cut of 'CL > 5%' is used. The systematic effect of this cut can be studied by varying the value for the cut. The E asymmetry will be calculated with the confidence level cuts of 2% and 10%, and the results will be compared with the E asymmetry with the 5% confidence level cut. The difference between the two E asymmetry for all bins (21 energy
Figure 4.11: Difference between the E asymmetry calculated from the tight vertex cut \((-10.3 \text{ cm} < z < -5.7 \text{ cm})\) and standard vertex cut \((-10.5 \text{ cm} < z < -5.5 \text{ cm})\).

<table>
<thead>
<tr>
<th>Period</th>
<th>(\mu)</th>
<th>(\sigma)</th>
<th>(N)</th>
<th>(\sigma_\mu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver1</td>
<td>-0.7%</td>
<td>8.6%</td>
<td>192</td>
<td>0.6%</td>
</tr>
<tr>
<td>Silver2a</td>
<td>1.9%</td>
<td>39.2%</td>
<td>185</td>
<td>2.9%</td>
</tr>
<tr>
<td>Silver2b</td>
<td>1.4%</td>
<td>18.1%</td>
<td>195</td>
<td>1.3%</td>
</tr>
<tr>
<td>Gold2a</td>
<td>0.5%</td>
<td>10.6%</td>
<td>227</td>
<td>0.7%</td>
</tr>
<tr>
<td>Gold2b</td>
<td>0.5%</td>
<td>4.3%</td>
<td>208</td>
<td>0.3%</td>
</tr>
<tr>
<td>(\sigma_{\text{systematic}})</td>
<td>(0.6%)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.12: Difference between the E asymmetry calculated from the loose vertex cut (-10.7 cm < z < -5.3 cm) and standard vertex cut (-10.5 cm < z < -5.5 cm).

<table>
<thead>
<tr>
<th>Period</th>
<th>$\mu_\delta$</th>
<th>$\sigma_\delta$</th>
<th>N</th>
<th>$\sigma_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver1</td>
<td>1.2%</td>
<td>11.9%</td>
<td>192</td>
<td>0.9%</td>
</tr>
<tr>
<td>Silver2a</td>
<td>-1.6%</td>
<td>37.8%</td>
<td>185</td>
<td>2.8%</td>
</tr>
<tr>
<td>Silver2b</td>
<td>0.2%</td>
<td>15.1%</td>
<td>195</td>
<td>1.1%</td>
</tr>
<tr>
<td>Gold2a</td>
<td>-0.4%</td>
<td>11.6%</td>
<td>228</td>
<td>0.8%</td>
</tr>
<tr>
<td>Gold2b</td>
<td>-1.3%</td>
<td>6.2%</td>
<td>209</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

$\sigma_{\text{systematic}}$ 1.1%

Table 4.2: Mean and error for the Gaussian distribution in Figure 4.12. Equation 4.11 is used to calculate the systematic error between the vertex z cut: -10.7 cm < z < -5.3 cm and -10.5 cm < z < -5.5 cm.
bins \times 13 \theta \text{ bins}) will be used to fill a 1-D histogram, and the mean and error of the fitted Gaussian distribution will be used to estimate the systematic error.

The results of comparing the 2\% and 5\% confidence level cuts are shown in Figure 4.13 and Table 4.3.

![Systematic study for confidence level cut](image)

Figure 4.13: Difference between the E asymmetry calculated from confidence level cut (CL > 2\%) and standard confidence level cut (CL > 5\%).

The results of comparing the 10\% and 5\% confidence level cuts are shown in Figure 4.14 and Table 4.4. The total systematic error caused by tightening and loosening the confidence level cut is the quadrature sum of the two errors, which is: \( \sigma_{\text{systematic}}(\text{confidence level}) = \sqrt{0.008^2 + 0.011^2} = 1.4\% \).

### 4.3.3 Systematic Study for Missing Momentum cuts

After the vertex z cut of \(-10.5 < z < -5.5 \text{ cm}\) and confidence level cut of CL > 5\%, most of the events are from the channel of interest: \( \gamma + n \rightarrow p + \pi^- \), as shown in the missing momentum plot (Figure 4.15) for gold2a data set. The missing momentum of the target neutron can go up to 0.2 GeV, which shows the quasi free feature of the neutron
<table>
<thead>
<tr>
<th>Period</th>
<th>$\mu_\delta$</th>
<th>$\sigma_\delta$</th>
<th>N</th>
<th>$\sigma_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver1</td>
<td>2.4%</td>
<td>11.2%</td>
<td>192</td>
<td>0.8%</td>
</tr>
<tr>
<td>Silver2a</td>
<td>0.8%</td>
<td>49.0%</td>
<td>186</td>
<td>3.6%</td>
</tr>
<tr>
<td>Silver2b</td>
<td>-0.3%</td>
<td>15.2%</td>
<td>196</td>
<td>1.1%</td>
</tr>
<tr>
<td>Gold2a</td>
<td>-1.6%</td>
<td>11.0%</td>
<td>227</td>
<td>0.7%</td>
</tr>
<tr>
<td>Gold2b</td>
<td>-0.1%</td>
<td>5.8%</td>
<td>209</td>
<td>0.4%</td>
</tr>
<tr>
<td>$\sigma_{\text{systematic}}$</td>
<td></td>
<td></td>
<td></td>
<td>1.1%</td>
</tr>
</tbody>
</table>

Table 4.3: Mean and error for the Gaussian distribution in Figure 4.13. Equation 4.11 is used to calculate the systematic error between the confidence level cut: $\text{CL} > 2\%$ and $\text{CL} > 5\%$.

Figure 4.14: Difference between the $E$ asymmetry calculated from confidence level cut ($\text{CL} > 10\%$) and standard confidence level cut ($\text{CL} > 5\%$).
Table 4.4: Mean and error for the Gaussian distribution in Figure 4.14. Equation 4.11 is used to calculate the systematic error between the confidence level cut: $CL > 10\%$ and $CL > 5\%$.

target. Figure 4.16 shows that the E asymmetry depends on the missing momentum of the target neutron, the magnitude of E asymmetry decrease as missing momentum increase. A cut of missing momentum: $mP < 0.1$ GeV is choose for the standard cut, since the E asymmetry is relatively flat for the region: $0 < mP < 0.1$ GeV. Another two missing momentum cuts: $mP < 0.2$ GeV and $mP < 0.05$ GeV are used for calculating the E asymmetry, which will give the systematic error of the missing momentum cut.

The results of comparing the $mP < 0.2$ GeV and $mP < 0.1$ GeV missing momentum cuts are shown in Figure 4.17 and Table 4.5. The results of comparing the $mP < 0.08$ GeV and $mP < 0.1$ GeV missing momentum cuts are shown in Figure 4.18 and Table 4.6. The total systematic error caused by tightening and loosening the missing momentum cut is the quadrature sum of the two errors: $\sigma_{\text{systematic}}(\text{missing momentum}) = \sqrt{0.07^2 + 0.022^2} = 7.3\%$.
Figure 4.16: E asymmetry vs missing momentum of target neutron for the channel: $\gamma + (n) \to p + \pi^-$. 

Table 4.5: Mean and error for the Gaussian distribution in Figure 4.17. Equation 4.11 is used to calculate the systematic error between the missing momentum cut: $m_P < 0.2$ GeV and $m_P < 0.1$ GeV.

<table>
<thead>
<tr>
<th>Period</th>
<th>$\mu_\delta$</th>
<th>$\sigma_\delta$</th>
<th>N</th>
<th>$\sigma_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver1</td>
<td>-8.2%</td>
<td>37.0%</td>
<td>192</td>
<td>2.7%</td>
</tr>
<tr>
<td>Silver2a</td>
<td>-5.1%</td>
<td>61.0%</td>
<td>186</td>
<td>4.5%</td>
</tr>
<tr>
<td>Silver2b</td>
<td>-11.6%</td>
<td>46.4%</td>
<td>196</td>
<td>3.3%</td>
</tr>
<tr>
<td>Gold2a</td>
<td>-8.9%</td>
<td>59.1%</td>
<td>226</td>
<td>3.9%</td>
</tr>
<tr>
<td>Gold2b</td>
<td>-6.4%</td>
<td>12.6%</td>
<td>208</td>
<td>0.9%</td>
</tr>
<tr>
<td>$\sigma_{systematic}$</td>
<td></td>
<td></td>
<td></td>
<td>7.0%</td>
</tr>
</tbody>
</table>

Table 4.6: Mean and error for the Gaussian distribution in Figure 4.18. Equation 4.11 is used to calculate the systematic error between the missing momentum cut: $m_P < 0.08$ GeV and $m_P < 0.1$ GeV.

<table>
<thead>
<tr>
<th>Period</th>
<th>$\mu_\delta$</th>
<th>$\sigma_\delta$</th>
<th>N</th>
<th>$\sigma_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver1</td>
<td>2.3%</td>
<td>29.5%</td>
<td>206</td>
<td>2.1%</td>
</tr>
<tr>
<td>Silver2a</td>
<td>-9.3%</td>
<td>71.3%</td>
<td>184</td>
<td>5.3%</td>
</tr>
<tr>
<td>Silver2b</td>
<td>5.0%</td>
<td>62.0%</td>
<td>197</td>
<td>4.4%</td>
</tr>
<tr>
<td>Gold2a</td>
<td>0.1%</td>
<td>38.9%</td>
<td>219</td>
<td>2.6%</td>
</tr>
<tr>
<td>Gold2b</td>
<td>1.2%</td>
<td>14.8%</td>
<td>208</td>
<td>1.0%</td>
</tr>
<tr>
<td>$\sigma_{systematic}$</td>
<td></td>
<td></td>
<td></td>
<td>2.2%</td>
</tr>
</tbody>
</table>
Figure 4.17: Difference between the E asymmetry calculated from missing momentum cut ($mP < 0.2$ GeV) and standard missing momentum cut ($mP < 0.1$ GeV).
Figure 4.18: Difference between the E asymmetry calculated from missing momentum cut (mP < 0.08 GeV) and standard missing momentum cut (mP < 0.1 GeV).
4.3.4 Other Systematic Errors

Other than the systematic errors caused by the choice of different cut values, the systematic errors of the photon beam polarization and target polarization will also introduce uncertainty in the E asymmetry. Table 4.7 gives the photon beam polarization and its error from the Moller measurement. An estimation of the systematic error for the HD target polarization is shown in table 4.8.

<table>
<thead>
<tr>
<th>Period</th>
<th>Beam polarization</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver1</td>
<td>−81.7%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Silver2a</td>
<td>−81.7%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Silver2b</td>
<td>−76.2%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Gold2a</td>
<td>88.2%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Gold2b</td>
<td>−83.4%</td>
<td>1.5%</td>
</tr>
</tbody>
</table>

Table 4.7: Photon beam polarization and its error for circular polarized beam run periods.

Uncertainties in reference Thermal Equilibrium (TE) measurement:

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD noise</td>
<td>0.3%</td>
<td>White noise in PD NMR while in HDice Lab</td>
</tr>
<tr>
<td>Temperature</td>
<td>0.2%</td>
<td>Drift, thermal gradients in HD from radiant heat load</td>
</tr>
<tr>
<td>H background</td>
<td>0.4%</td>
<td>H bkg with no target</td>
</tr>
<tr>
<td>Stoichiometry</td>
<td>0.1%</td>
<td>Deviation of H:D of 1:1, due to H2 and D2 impurities</td>
</tr>
<tr>
<td>Bkg subtraction</td>
<td>0.6%</td>
<td>Err in signal integral from imperfect separation of bkg</td>
</tr>
<tr>
<td>Incomplete relaxation</td>
<td>0.5%</td>
<td>T1 for TE measurement can be comparable to sweep time</td>
</tr>
</tbody>
</table>

Uncertainties in measurement of frozen-spin(FS) signal:

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBC noise</td>
<td>0.6%</td>
<td>Residual effect of white noise in IBC NMR and PD</td>
</tr>
<tr>
<td>Hall-B noise jumps</td>
<td>0.5%</td>
<td>Variations in signal area after correction for signal jumps</td>
</tr>
<tr>
<td>Circuit non-linearity</td>
<td>4.0%</td>
<td>From the quadratic dependence of the circuit transducer gain</td>
</tr>
<tr>
<td>RF inhomogeneity</td>
<td>1.4%</td>
<td>Field inhomogeneity</td>
</tr>
<tr>
<td>RF depolarization</td>
<td>0.1%</td>
<td>Residual uncorrected decrement from repeated rf sweeps</td>
</tr>
</tbody>
</table>

Uncertainties in relating FS signal to TE measurement:

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circuit drift</td>
<td>1.8%</td>
<td>Variation from connecting FS signal to TE reference</td>
</tr>
<tr>
<td>Lock-in gain error</td>
<td>2.9%</td>
<td>SRS 844 manufacturer’s gain error</td>
</tr>
<tr>
<td>Differential ramp-rate</td>
<td>1.0%</td>
<td>Actual ramp rate differs from nominal</td>
</tr>
<tr>
<td>TC transfer losses</td>
<td>2.0%</td>
<td>Variation in polarization loss during a TC transfer</td>
</tr>
<tr>
<td>Total systematic error</td>
<td>6.0%</td>
<td>For both H and D polarization</td>
</tr>
</tbody>
</table>

Table 4.8: Systematic errors for HD polarization measurement. [56]

From the formula of calculating E asymmetry: \( E = \frac{1}{P_y P_z} \left( \frac{N_{1/2} - N_{3/2}}{N_{1/2} + N_{3/2}} \right) \), the relative errors for photon beam polarization \( (P_y) \) and target polarization \( (P_z) \) will be propagated to the final result of E asymmetry. The systematic errors are 1.4% from photon beam polarization, and 6.30% from target polarization.
4.3.5 Total Systematic Errors

We have considered two kinds of systematic errors, one is caused by the choice of different cuts in the event selection, another is from the systematic errors in the measurement of the photon beam polarization and HD target polarization. The total error for each kind is combined using quadratic sum, a summery for the systematic errors is shown in Table 4.9.

<table>
<thead>
<tr>
<th>Sources</th>
<th>Systematic error for E asymmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex z cut (± 0.2 cm)</td>
<td>1.3%</td>
</tr>
<tr>
<td>Confidence level cut (+0.05, -0.03)</td>
<td>1.4%</td>
</tr>
<tr>
<td>Missing momentum cut (+0.1 GeV)</td>
<td>7.3%</td>
</tr>
<tr>
<td>$\sigma_{sys}(cuts)$</td>
<td>7.5%</td>
</tr>
<tr>
<td>Photon beam polarization</td>
<td>1.4%</td>
</tr>
<tr>
<td>Target polarization</td>
<td>6.0%</td>
</tr>
<tr>
<td>$\sigma_{sys}(polarization)$</td>
<td>6.2%</td>
</tr>
<tr>
<td>$\sigma_{total}$</td>
<td>9.8%</td>
</tr>
</tbody>
</table>

Table 4.9: Combine the systematic error. The main source of systematic error from using different cuts is from the missing momentum cut. The main source of systematic error from polarization measurement is from target polarization.

4.4 Conclusion

The first measurement of E asymmetry for the reaction of $\gamma + n \rightarrow p + \pi^-$ is reported. The E asymmetry is plotted vs. the polar angle of the final state $\pi^-$ in the center of momentum frame. The c.m. energy ($w_{c.m.}$) ranging from 1500 MeV to 2300 MeV has been divided into 21 bins with the bin size of 40 MeV. The results have been compared with two the predictions from two partial wave analysis group: Bonn-Gatchina and SAID. The agreement between the experiment results and the predictions from partial wave analysis is good for $w_{c.m.} < 1820$ MeV. For $w_{c.m.} > 1820$ MeV, the experiment results are different from the two predictions, and also the two partial wave analysis methods give different predictions. The result of the E asymmetry from this experiment will help the two partial wave analysis to improve their models, and give more accurate predictions for the energy $w_{c.m.} > 1820$ MeV, this energy range is also the place where most of the missing resonances are.
Chapter 5

Polarization Observables for \( \gamma p \rightarrow p\pi^+\pi^- \)

Double pion photo-production is an important channel for the study of missing resonance problem, because the cross section for this reaction is much higher than the single pion photo-production for c.m. total energy above 2 GeV, where the missing resonances are predicted to be.

5.1 Formalism

Figure 5.1: Kinemactics for the reaction of \( \gamma p \rightarrow p\pi^+\pi^- \) in c.m. frame. \( k, p_1, p_2, q_1, q_2 \) are the momenta for the incident photon, proton target, recoiled proton, and the two pions. \( \theta_{\pi\pi} \) is the polar angle of sum vector of the momenta of the two pions in center of mass frame. \( \theta_{\pi^+} \) is the angle between \( q_1 \) and \( p_{\pi\pi} \). \( \phi' \) is then angle between the plane formed by the two pion momenta and the reaction plane.

The kinematics for this process is more complicated than single pion process. The differential cross section is described by 5 kinematic variables, which can be chosen as: \( W, M_{\pi^+\pi^-} \), \( \theta_{\pi\pi}, \theta_{\pi^+} \) and \( \phi' \), in which \( W \) is the c.m. total energy, \( M_{\pi^+\pi^-} \) is invariant mass of the two final state pions, \( \theta_{\pi\pi}, \theta_{\pi^+}, \phi' \) are the angles shown in Figure 5.1. \( \phi' \) can be
obtained in the following equation:

$$\cos \phi' = \frac{(\vec{p}_1 \times \vec{q}_2) \cdot (\vec{p}_1 \times \vec{q}_2)}{|\vec{p}_1 \times \vec{q}_2||\vec{p}_1 \times \vec{q}_2|}.$$  \hspace{1cm} (5.1)

If the photon beam and target are polarized, the differential cross section can be written as:

$$\frac{d\sigma}{dx_i} = \sigma_0 \{(1 + \vec{\Lambda} \cdot \vec{P}) + \delta_\odot(I_\odot + \vec{\Lambda} \cdot \vec{P}_\odot)$$

$$+ \delta_l[\sin 2\beta(I^s + \vec{\Lambda} \cdot \vec{P}^s + \cos 2\beta(I^c + \vec{\Lambda} \cdot \vec{P}^c)]\},$$  \hspace{1cm} (5.2)

where $x_i$ are the kinematic variables, $\sigma_0$ is the unpolarized cross section, $\beta$ is the angle between the direction of the linear polarized photon beam with the x-axis in lab frame, $\delta_\odot$ and $\delta_l$ are the degree of polarization for circular and linear polarized photon beam, since there are protons in both hydrogen and deuterium in the HD target, the effective polarization of proton is $(P_H + P_D)/2$. $\Lambda$ is the target polarization. $I_\odot$ and $I^s,c$ are the beam helicity asymmetry with the use of circular and linear polarized photon beam. $\vec{P}$ is the target asymmetry. $\vec{P}_\odot$ is the double asymmetry by using circular polarized beam and polarized target. $\vec{P}^{s,c}$ are the double asymmetry by using linear polarized beam and polarized target.

For circular polarized beam and longitudinal polarized target, the above formula becomes:

$$\frac{d\sigma}{dx_i} = \sigma_0 \{(1 + \Lambda_z \cdot P_z) + \delta_\odot(I_\odot + \Lambda_z \cdot P_\odot)\}. \hspace{1cm} (5.3)$$

The three polarization observables $P_z, I_\odot, P_\odot$ can be calculated using the following equations:

$$P_z = \frac{1}{\Lambda_z} \frac{[N(\to\to) + N(\to\leftarrow)] - [N(\to\leftarrow) + N(\leftarrow\leftarrow)]}{[N(\to\to) + N(\to\leftarrow)] + [N(\to\leftarrow) + N(\leftarrow\leftarrow)]},$$

$$I_\odot = \frac{1}{\delta_\odot} \frac{[N(\to\to) + N(\to\leftarrow)] - [N(\to\leftarrow) + N(\leftarrow\leftarrow)]}{[N(\to\to) + N(\to\leftarrow)] + [N(\to\leftarrow) + N(\leftarrow\leftarrow)]},$$

$$P_\odot = \frac{1}{\Lambda_z \delta_\odot} \frac{[N(\to\to) + N(\to\leftarrow)] - [N(\to\leftarrow) + N(\leftarrow\leftarrow)]}{[N(\to\to) + N(\to\leftarrow)] + [N(\to\leftarrow) + N(\leftarrow\leftarrow)]},$$  \hspace{1cm} (5.4)

where $\to$ and $\leftarrow$ denote the circular beam polarization is parallel or antiparallel to the beam axis, $\to$ and $\leftarrow$ denote the direction of target polarization is parallel or antiparallel to the beam axis. $N(\to\to)$ is the number of events for photo beam with +1 helicity and target polarization along +z direction. $N$ is related to the differential cross section of the reaction in the following way:

$$\frac{d\sigma}{dx_i} = \frac{N}{A \cdot F \cdot \rho \cdot \Delta x_i},$$  \hspace{1cm} (5.5)

where $A$ is the acceptance, $F$ is the photon flux, $\rho$ is the target density, $\Delta x_i$ is the size of the kinematic bin. Using this relation, we can relate the experiment measured event number $N$ to the polarization observables as:

$$N = \sigma_0(A \cdot F \cdot \rho \cdot \Delta x_i)\{(1 + \Lambda_z \cdot P_z) + \delta_\odot(I_\odot + \Lambda_z \cdot P_\odot)\}. \hspace{1cm} (5.6)$$
The extraction of the three observables requires 4 different sets of measurements with different beam helicity and target polarization: \( N(\rightarrow\rightarrow), N(\rightarrow\leftarrow), N(\leftarrow\rightarrow), N(\leftarrow\leftarrow) \). Moreover, the simple form of equation 5.4 uses the assumption that the target polarization \((\Lambda_z)\), beam polarization \((\delta_\odot)\), and photon flux \((F)\) are the same for all run periods. This assumption is not valid in real experiment. In terms of target polarization, the run periods in G14 experiment can be divided into two groups. Group 1 has the target polarization along the +z direction of the lab frame, this group includes silver1, silver2a, silver2b, silver3, gold2. Group 2 has the target polarization along the -z direction, this group includes silver4 and silver5.

In Equation 5.6, we use a single letter \( C \) to represent the constant in the equation: 
\[
C = \sigma_0 \cdot A \cdot F \cdot \rho \cdot \Delta x_i.
\]
Four equations can be written for the two periods: silver5 and gold2 for different beam helicity and target polarization:

\[
\begin{align*}
N_{g2}^+ &= C_{g2}^+\{(1 + \Lambda z^2 P_z) + \delta z^2(\Lambda\circ + \Lambda z^2 P_z^\circ)\} \\
N_{g2}^- &= C_{g2}^-\{(1 + \Lambda z^2 P_z) - \delta z^2(\Lambda\circ + \Lambda z^2 P_z^\circ)\} \\
N_{s5}^+ &= C_{s5}^+\{(1 + \Lambda s5^2 P_z) + \delta s5^2(\Lambda\circ + \Lambda s5^2 P_z^\circ)\} \\
N_{s5}^- &= C_{s5}^-\{(1 + \Lambda s5^2 P_z) - \delta s5^2(\Lambda\circ + \Lambda s5^2 P_z^\circ)\}
\end{align*}
\]  

(5.7)

where \( N^\pm \) is the number of events for beam helicity equals \( \pm 1 \). And the total events for each run period can be defined as: \( N_{g2} = N_{g2}^+ + N_{g2}^- \), \( N_{s5} = N_{s5}^+ + N_{s5}^- \). The difference between the events from two beam helicity states for each run period can be defined as: \( \Delta N_{g2} = N_{g2}^+ - N_{g2}^- \), \( \Delta N_{s5} = N_{s5}^+ - N_{s5}^- \). In G14 experiment, the beam helicity flip frequency is 960.015 Hz, thus the flux for the two beam helicity states can be viewed as the same, \( C_{g2}^+ = C_{g2}^- = C_{g2}, C_{s5}^+ = C_{s5}^- = C_{s5} \). The three polarization observables can be deduced using Eq. 5.7 as following:

\[
\begin{align*}
I^\circ &= \frac{\Delta N_{g2}}{\delta z^2} - \alpha\beta \frac{\Delta N_{s5}}{\delta s5^2} \\
P_z^\circ &= \frac{\Delta N_{g2}}{\delta z^2 \Lambda s5^2} - \alpha\beta \frac{\Delta N_{s5}}{\delta s5^2 \Lambda^2} \\
P_z &= \frac{\alpha N_{s5} - N_{g2}}{\Lambda s5(N_{g2} - \alpha N_{s5})}
\end{align*}
\]  

(5.8)

where \( \alpha = \frac{C_{s5}}{C_{g2}}, \beta = \frac{\Lambda z^2}{\Lambda s5^2} \). If \( C_{g2} = C_{s5}, \Lambda s5^2 = -\Lambda z^2, \) and \( \delta z^2 = \delta s5^2 \), Eq. 5.8 goes back to Eq. 5.4. For an asymmetry \( A = (N_1 - N_2)/(N_1 + N_2) \), the error is calculated as:

\[
\sigma^2_A = \left( \frac{\partial A}{\partial N_1} \right)^2 \sigma^2_{N_1} + \left( \frac{\partial A}{\partial N_2} \right)^2 \sigma^2_{N_2} + 2 \left( \frac{\partial A}{\partial N_1} \right) \left( \frac{\partial A}{\partial N_2} \right) \text{cov}(N_1, N_2),
\]  

(5.9)

where \( \sigma^2_{N_1} = N_1, \sigma^2_{N_2} = N_2, \) and the covariance \( \text{cov}(N_1, N_2) = 0 \) for independent measurement of \( N_1, N_2 \). Using this relation, the statistical errors for these three observables
can be calculated to be:

\[
\sigma_{I^\circ} = \sqrt{\left(\frac{2N_{g2}}{\delta_{g2}^2} - \alpha \beta \left(\frac{1}{\delta_{g2}^2} - \frac{1}{\delta_{s5}^2}\right) N_{s5}^+ - \alpha \beta \left(\frac{1}{\delta_{g2}^2} + \frac{1}{\delta_{s5}^2}\right) N_{s5}^-\right)^2 N_{g2}^+}
\]

\[
\sigma_{P_x} = \sqrt{\left(\frac{2N_{g2}}{\delta_{g2}^2} + \alpha \beta \left(\frac{1}{\delta_{g2}^2} + \frac{1}{\delta_{s5}^2}\right) N_{s5}^- - \alpha \beta \left(-\frac{1}{\delta_{g2}^2} + \frac{1}{\delta_{s5}^2}\right) N_{s5}^+\right)^2 N_{g2}^-}
\]

\[
\sigma_{P_z} = \frac{\left[\alpha (\Lambda_{g2} - \Lambda_{s5})\right] \sqrt{N_{g2} N_{s5} (N_{g2} + N_{s5})}}{(\Lambda_{s5} N_{g2} - \alpha \Lambda_{s5}^2 N_{s5})^2}
\]

The ratio of the constant $C$ between gold2 and silver5 period $\alpha = \frac{C_{g2}}{C_{s5}}$ can be obtained from comparing the downstream KelF peak as in Figure 5.2, since the silver5 target lost some HD in the target region, an overall coefficient of 1.435 (silver target is 70% of the the length of a full target) is multiplied to the ratio: $\alpha = \frac{C_{g2}}{C_{s5}} = 1.435 \times \frac{K_{KelF,g2}}{K_{KelF,s5}}$. The ratio of the yields under the KelF peak at 0cm $< z < 2$cm between gold2 and silver5 periods for the six center of mass total energy bins is shown in Figure 5.3.

### 5.2 Determine the target polarization for Silver5 period

From Table 3.1, we get the polarization for the deuteron in the HD target is 15.79% along the -z direction. However, from the study of the E asymmetry in the single pion channel: $\gamma + n \rightarrow p + \pi^-$, silver5 data set gives a significant smaller value for E asymmetry if the deuteron polarization is $P_D = -15.79\%$, as shown in Figure 5.4, this result implies that the actual deuteron polarization is much smaller than the value measured by the NMR.
Figure 5.2: Vertex $z$ distribution for channel $\gamma p \rightarrow p\pi^+\pi^-$ for Gold2 (left) and Silver5 (right) periods.

Figure 5.3: The ratio of the yields under the KelF peak at $0 \text{cm} < z < 2 \text{cm}$ between gold2 and silver5 periods for the six center of mass energy bins.
This is the reason why in the previous chapter, the data set of silver 5 is not included in the calculation for the E asymmetry. However, in the calculation of $P_z, I^0, P_z^0$ for the two pion photo-production, silver5 data set is required to be used in the calculation, since this data set has deuteron polarization along -z direction. Thus we need to get the right value for deuteron polarization for silver5 data set. The method we are using is to vary the value of deuteron polarization for silver5 data set, until we get the closest value for E asymmetry in the single pion channel between the silver5 and the combined results from previous chapter. A $\chi^2$ value is defined in the following equation:

$$\sigma \chi^2 = \sum_i \frac{(E_{s5} - E_{\text{combine}})^2}{\sigma^2(E_{s5} - E_{\text{combine}})}$$

(5.13)

Figure 5.5 plots the $\chi^2$ vs. the deuteron polarization for silver5 target. $\chi^2$ is minimized at 5.1%. The error for the polarization can be determined by varying the $\chi^2$ by 1 near
its minimal, which is 0.2%. The result for the deuteron polarization: $D_{\text{pol}} = (5.1 \pm 0.2)\%$ is significantly smaller than the polarization measured by the NMR, which is $(15.47 \pm 0.70)\%$.

![Chi2 vs. Deuterons polarization (Silver5) graph](image)

Figure 5.5: $\chi^2$ vs. the deuteron polarization for silver5 target.

From Eq 5.8, we can see that the small value of deuteron polarization for silver5 target won’t affect the calculation for $I^\odot$, but will affect the other two observables $P_z, P_z^\odot$.

### 5.3 Beam Helicity Asymmetry $I^\odot$

The results for beam helicity asymmetry are shown in Figure 5.6. The results are compared with previous CLAS experiment g1c [20] and g9a [21], and they are in overall agreement. The prediction from the theory by A.Fix [23] is also shown for the first two energy bins.

We notice that for lower energy bins the amplitudes of the curves from G14 experiment is bigger that the results from G1c and G9a, for higher energy bins the amplitudes of the curves from G14 experiment is smaller that the results from G1c and G9a. These differences are caused by the use of different ways to integrate the polar angle $\theta_{\pi\pi}$. In this study, we integrate $I^\odot$ over $\theta_{\pi\pi}$ by dividing $\cos(\theta_{\pi\pi})$ into 5 bins from -1 to 1, calculate $I_c$ for each $\cos\theta_{\pi\pi}$ bin, and then add them together as:

$$I^\odot(\phi') = \frac{\int_{-1}^{1} I^\odot(\theta_{\pi\pi}, \phi') d(\cos\theta_{\pi\pi})}{\int_{-1}^{1} d(\cos\theta_{\pi\pi})} = \frac{1}{2} \sum I^\odot(\theta_{\pi\pi}, \phi') \Delta(\cos\theta_{\pi\pi})$$  \hspace{1cm} (5.14)

1/2 in the equation is the normalization number from the inverse of the $\cos(\theta_{\pi\pi})$ integration range of 2 from -1 to 1.
Figure 5.6: Beam helicity asymmetry vs. $\phi'$ for different center of mass energy $w$. The results from previous CLAS measurements $g_{1c}$ [20] and $g_{9a}$ [21], and the prediction from [23] are also shown in the plots.
On the other hand, the way of the integration the other way experiments used is to view \( \cos \theta_{\pi\pi} \) as one bin, and calculate \( I^\odot(\phi') \) by summing all the events from all \( \cos \theta_{\pi\pi} \) bins for each \( \phi' \) bin. We also used the second way of \( \cos \theta_{\pi\pi} \) integration, Figure 5.7 shows that the results from three different experiments are very close to each other by using the same method for \( \cos \theta_{\pi\pi} \) integration.

Figure 5.7: Beam helicity asymmetry vs. \( \phi' \) for different center of mass energy \( W \), calculated under the assumption that \( I^\odot(\phi') \) is independent of \( \theta_{\pi\pi} \). The results from previous CLAS measurements \( g1c \) [20] and \( g9a \) [21], and the prediction from [23] are also shown in the plots.

However, the second way of integration makes an assumption that the observable \( I^\odot(\phi') \) is uniformly distributed along \( \cos \theta_{\pi\pi} \) for a fixed \( \phi' \) bin. Figure 5.8 shows that this assumption is not true, since \( I^\odot(\phi') \) fluctuates along \( \cos \theta_{\pi\pi} \) direction for fixed \( \phi' \) bins. Base on this observation, we believe that the first method introduced earlier is the right way for \( \cos \theta_{\pi\pi} \) integration. Since this two methods gives different values for \( I^\odot(\phi') \), we suggest the data in \( g1c \) and \( g9b \) should be replotted using the right way for \( \cos \theta_{\pi\pi} \)
integration. While the simple differences in the approach to $\theta_{\pi\pi}$ integration account for most of the variations between these results and those of g1c and g9a, we note that at the highest energies the present experiment sees significantly less beam helicity asymmetry near the extreme $\phi'$ angles of $-\pi$ and $+\pi$. 

Figure 5.8: Beam helicity asymmetry $I^\circ(\phi')$ vs. $\phi'$ and $\cos \theta_{\pi\pi}$ for center of mass energy $W = 1750$ MeV.

5.4 Target Asymmetry $P_z$

The results for target asymmetry are shown in Figure 5.9, the results using the $\theta_{\pi\pi}$ integration with the assumption that $P_z$ is independent of $\theta_{\pi\pi}$ are shown in Figure 5.10.

5.5 Beam Target Double Asymmetry $P_{z\circ}$

The results for beam target double asymmetry are shown in Figure 5.11, the results using the $\theta_{\pi\pi}$ integration with the assumption that $P_{z\circ}$ is independent of $\theta_{\pi\pi}$ are shown in Figure 5.12.

5.6 Conclusion

The three polarization observables for $\gamma p \rightarrow p\pi^+\pi^-$ using circular polarized photon beam and longitudinal polarized target are compared with previous experiment [20] [21]. $I^\circ$ are in overall agreement between the G14 measurement and the G9a measurement [21], but $P_z$ and $P_{z\circ}$ has big discrepancy between these two experiments.
Figure 5.9: Target asymmetry vs. $\phi'$ for different center of mass energy $\omega$. 
Figure 5.10: Target asymmetry vs. $\phi'$ for different center of mass energy $w$, calculated under the assumption that $P_z(\phi')$ is independent of $\theta_{\pi\pi}$.
Figure 5.11: Beam target double asymmetry vs. $\phi'$ for different center of mass energy $w$. 
Figure 5.12: Beam target double asymmetry vs. $\phi'$ for different center of mass energy $w$, calculated under the assumption that $P_{Z}^{\phi'}(\phi')$ is independent of $\theta_{\pi\pi}$. 
Chapter 6

Polarization Observables for

\[ \gamma n \rightarrow n\pi^+\pi^- \]

The reaction of double pion photo-production with the neutron target has the same Kinematics and polarization observables as the reaction using the proton target. The diagram for the reaction kinematics is shown in Figure 6.1.

Figure 6.1: Kinematics for the reaction of \( \gamma n \rightarrow n\pi^+\pi^- \) in c.m. frame. \( k, p_1, p_2, q_1, \) \( q_2 \) are the momenta for the incident photon, neutron target, recoiled neutron, and the two pions. \( \theta_{\pi\pi} \) is the polar angle of sum vector of the momenta of the two pions in center of mass frame. \( \theta_{\pi+} \) is the angle between \( q_1 \) and \( p_{\pi\pi} \). \( \phi' \) is then angle between the plane formed by the two pion momenta and the reaction plane.

6.1 Event selection

Similar filters are used in the neutron target channel as in the proton target channel.

6.1.1 Particle ID filters

The first set of filters are particle ID filters: PID = 8 for \( \pi^- \), PID = 9 for \( \pi^+ \). For the identification of neutrons, CLAS uses the Electromagnetic Calorimeter to detect neutral.
particles, in the G14 experiment, there are two neutral particles involved: photons and neutrons. These two neutral particles are separated by a $\beta$ cut at 0.9. The neutrons are identified with $\beta < 0.9$. (Figure 6.2).

![Figure 6.2: $\beta$ value for the neutral particles detected by EC, a cut of $\beta < 0.9$ is used to separate neutrons from photons.](image)

Moreover, 'targid' and 'ngrf == 1' are used to better define the reaction channels. The 'targid' filters require all the final state particles come from the same tagged photon; and the 'ngrf == 1' filter require the number of photons in the same RF bucket is one.

### 6.1.2 $\Delta \beta$ cuts

Since the EC module of CLAS detectors does not have high efficiency detecting neutrons (Figure 6.3), we will loose the the $\Delta \beta$ cuts to increase statistics: $\Delta \beta$ cut for $\pi^+$ and $\pi^-$ will be chosen to be $|\Delta \beta| < 0.5$ instead of $<0.3$. Figure 6.5 gives the $\beta$ vs momentum plot for $\pi^+$ and $\pi^-$ particles after the $|\Delta \beta|$ cut.

### 6.1.3 Fiducial cuts

The same fiducial cuts are used for $\pi^+$ and $\pi^-$ using the Equation 6.1, the angular distribution before and after the fiducial cuts for $\pi^+$ and $\pi^-$ are shown in Figure 6.6.

$$\theta > 4.0 + \frac{510.58}{(30 - \phi)^{1.5518}}$$ (6.1)

For neutrons, extra cuts need to be used in order to remove some of the events that misidentify protons as neutrons [57]. As shown in Figure 6.7, there are two peaks in the top right plot (b) for the $\theta$ collinear distribution, the peak on the right side indicates that there are some protons misidentified as neutrons. In the center of momentum frame, the sum of the polar angles for the sum vector of $\pi^+, \pi^-$ and the neutron should be
Figure 6.3: Measured neutron detection efficiency vs. neutron momentum in EC detectors. Image Source: [39]

(a) $\pi^+ : \text{abs}(\Delta \beta)<0.05.$

(b) $\pi^- : \text{abs}(\Delta \beta)<0.05.$

Figure 6.4: $\Delta \beta$ filter for $\pi^+$ and $\pi^-$

Figure 6.5: The $\beta$ vs momentum plots for $\pi^+$ and $\pi^-$ after $\Delta \beta$ cut.
Figure 6.6: Fiducial cuts for $\pi^+$ and $\pi^-$. The top two plots are the angular distributions of $\pi^+, \pi^-$ before fiducial cuts, the bottom two plots are the distributions after fiducial cuts.

$180^\circ$ in the reaction: $\gamma + n \rightarrow n + \pi^+ + \pi^-$. However, since the DC detectors have some bad regions, which cannot identify protons, these protons will hit the EC detectors and induce electromagnetic showers as the same as neutrons. The method for CLAS to separate protons from neutrons detected by EC is to use the particle charge information from DC. If part of the DC does not work, the protons go through this part will be misidentified as neutrons. Moreover, since protons will be bent in the torus magnetic field, the sum of $\theta$ for proton and $\pi^+ \pi^-$ will not be $180^\circ$, which gives the right peak in Figure 6.7(b). The bottom two plots in Figure 6.7 show the regions of EC where the misidentified protons are. Plot (c) uses a cut of $\theta_{\pi^+ \pi^-} + \theta_{\text{neu}} - \pi > 0.1$, two regions of the angular distribution plot have more events than other regions, on the other hand, plot (d) with a cut of $\theta_{\pi^+ \pi^-} + \theta_{\text{neu}} - \pi < 0.1$ (good neutron events) does not show the two enhanced regions. In order to remove the events with the misidentified protons, we will make an extra cut in the fiducial cut to remove the three regions with misidentified protons in Figure 6.6(c). As shown in Figure 6.8, after the extra fiducial cuts for the misidentified protons, the right peak in the $\theta$ collinear plot disappears. Since the protons bend in the torus magnetic field, the regions for misidentified protons in EC should change position if the torus field changes direction, Figure 6.9 gives the change of the regions for positive and negative torus field. For the other magnetic field setting, the collinear $\theta$ plot before and after the extra fiducial cut for the misidentified protons are shown in Figure 6.10.
(a) Neutron angular distribution after normal fiducial cuts.

(b) \( \theta_{\pi^+ \pi^-} + \theta_{\text{neut}} - \pi \) in center of momentum frame

(c) Neutron angular distribution with cut of \( \theta_{\pi^+ \pi^-} + \theta_{\text{neut}} - \pi > 0.1 \)

(d) Neutron angular distribution with cut of \( \theta_{\pi^+ \pi^-} + \theta_{\text{neut}} - \pi < 0.1 \)

Figure 6.7: The problem of misidentification of protons as neutrons.

---

(a) The three regions where the misidentified protons are are removed with extra cuts.

(b) \( \theta_{\pi^+ \pi^-} + \theta_{\text{neut}} - \pi \) in center of momentum frame after the extra fiducial cuts

Figure 6.8: Effect of the extra fiducial cuts on the removal of misidentified protons.
(a) Neutron angular distribution for positive torus magnetic field.

(b) Neutron angular distribution for negative torus magnetic field.

Figure 6.9: The regions for misidentified protons change positions for different torus field.

(a) $\theta_{\pi^-}\pi^+ + \theta_{\text{neu}} - \pi$ in center of momentum frame before misidentified protons region cut for negative torus field runs.

(b) $\theta_{\pi^-}\pi^+ + \theta_{\text{neu}} - \pi$ in center of momentum frame after misidentified protons region cut for negative torus field runs.

Figure 6.10: Effect of the extra fiducial cuts for negative torus field runs.
6.2 Kinematic Fitting for $\gamma n \rightarrow n\pi^+\pi^-$

For the kinematic fitting of this channel, since there is no information of the covariance matrix for the final state neutrons, we cannot apply the same kinematic fitting procedures as we did for $\gamma n \rightarrow p\pi^-$, $\gamma p \rightarrow p\pi^+\pi^-$, in which all the final state particles are detected with the information of covariance matrix. We will use an approximate kinematic fitting for this channel, in which we assume the neutron target is a free neutron. The fitting is a '1C fit' as: $\gamma n \rightarrow (n)\pi^+\pi^-$, with the final state neutron missing (its 3 momentum are free parameters in the fitting). Even though the final state neutron is viewed as 'missing', we have required that a neutron is detected in EC for the reaction in the event selection process. The pull distribution and confidence level are shown in Figure 6.11 and Figure 6.12. The cut of confidence level $CL > 2\%$, its effect on the missing mass, missing momentum and coplanarity plots are shown in Figure 6.13. The subplots in Figure 6.13 indicate that some background events are not removed by the confidence level cut, thus extra cuts for missing mass ($0.8\text{GeV} < m_m < 1.1 \text{ GeV}$), coplanarity ($2.7 < |\delta(\phi)| < 3.5$) and vertex-z ($-10.5 \text{ cm} < z < -5.5 \text{ cm}$) are used after the confidence level cut.

![Pull distribution for $\gamma n \rightarrow (n)\pi^+\pi^-$](pull_distribution.png)

Figure 6.11: Pull distribution for $\gamma n \rightarrow (n)\pi^+\pi^-$. 

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Figure 6.12: Confidence level for $\gamma n \rightarrow (n)\pi^+\pi^-$, a cut of CL > 2% are used to remove the background events.

(a) Missing mass: $\gamma n \rightarrow (n)\pi^+\pi^-$.  
(b) Missing momentum: $\gamma n \rightarrow (n)\pi^+\pi^-$.  
(c) Coplanarity: $|\phi(n) - \phi(\pi^+\pi^-)|$.  
(d) Vertex z cut.

Figure 6.13: Effect of the kinematic fitting’s confidence level cut on event selection for channel $\gamma n \rightarrow (n)\pi^+\pi^-$. For all the plots, the blue region is for good events that pass the cut, the red region is the background events that are removed by the confidence level cut. Extra cuts for missing mass ($0.8\text{GeV} < mm < 1.1\text{GeV}$), coplanarity ($2.7 < |\delta(\phi)| < 3.5$) and vertex-z(-10.5 cm < z < -5.5 cm) are used after the confidence level cut.
6.3 Results and discussions

The polarization observables for the channel of $\gamma n \to n\pi^+\pi^-$ that can be extracted from the G14 data set are the same as the channel of $\gamma p \to p\pi^+\pi^-$ in Chapter 5. The observables are the beam helicity asymmetry $I^\odot$, target polarization asymmetry $P_z$, and beam-target double polarization asymmetry $P^\odot_z$. The data from two run periods of G14 experiment are used in this study: gold2 and silver5. The gold2 data set has a deuteron polarization $P_D = 26.9\%$, the silver5 data set has a deuteron polarization of $P_D = -5.1\%$.

6.3.1 Beam Helicity Asymmetry $I^\odot$

The results for beam helicity asymmetry are shown in Figure 6.14. The results are compared with $I^\odot$ with the proton target. From the plots, it is notice that the beam helicity asymmetry is very close for higher c.m. energy of 1850 MeV, 1950 MeV, 2050 MeV, and 2150 MeV, but for lower c.m. energy of 1650 MeV, 1750 MeV, the discrepancy is significant. This observation indicates that at lower c.m. energy, the resonances are different between the double pion reaction using the proton target and the neutron target. From Figure 1.7 in the first Chapter, we notice that $F_{15}(1680)$ is missing for the single pion photon-production with neutron target. This situation could happen in the double pion photo-production, and our observation of the difference of $I^\odot$ between the proton target and neutron target provide useful information for groups using PWA to check the model for double pion photo-production.

6.3.2 $P_z$ and $P^\odot_z$

The results for beam helicity asymmetry are shown in Figure 6.15, and the results for beam target double asymmetry are shown in Figure 6.16. These two sets of plots shows the comparison for $P_z$ and $P^\odot_z$ for the reaction of double pion photo-production using polarized proton target and polarized neutron target respectively. For both targets, since the run period (silver5) for target polarization along -z direction has a very small target polarization: $-5.1\%$ for neutron, and $-2.55\%$ for proton, there are big error bars on the measurement of $P_z$ and $P^\odot_z$, which make it hard for us to extract useful information from the measurement of these two observables.
Figure 6.14: Beam helicity asymmetry $I^\odot$ vs. $\phi'$ for different center of mass energy $w$. 
Figure 6.15: Target asymmetry $P_z$ vs. $\phi'$ for different center of mass energy $w$. 
Figure 6.16: Beam target double asymmetry $P^\circ_z$ vs. $\phi'$ for different center of mass energy $w$. 

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Chapter 7

Summery and Outlook

In this dissertation, we have introduced the first experiment using polarized HD target at Jefferson Lab: the G14 experiment. In the experiment, a highly polarized photon beam and longitudinal polarized HD target have been used. Both circularly polarized and linearly polarized photon beam have been used in the experiment. Only the data set with circularly polarized beam has been analyzed in this study. The polarization of the photon beam was in a range from 76.2% to 88.8% depending on different run periods, the energy of the photon beam covers a range from 0.5 GeV to 2.2 GeV. During the circularly polarized photon beam runs, two polarized targets were used. The first target (target 21a) was operated in both "+z" direction and "-z" direction, the deuteron was polarized as high as 25.60% for '+z' mode. However, in the '-z' mode, the polarization is only −5.1%. The second target (target gold2) was operated only in the '+z' direction, and it had a higher deuteron polarization of 26.78%. The final state particles of the reaction is detected by the nearly 3π CLAS detectors hosted in Hall B.

Polarization observables from three reaction channels have been analyzed. They are: \( \gamma + n \rightarrow p + \pi^- \), \( \gamma + p \rightarrow p + \pi^+ + \pi^- \), \( \gamma + n \rightarrow n + \pi^+ + \pi^- \). In the analysis procedure, a new method to remove the background events has been developed based on kinematic fitting. And the confidence level cut has been proved to be a powerful tool to select separate events of interest from background events.

In the single pion channel \( \gamma + n \rightarrow p + \pi^- \), the beam target double asymmetry \( E \) has been analyzed, this is the first measurement for this observable on the polarized neutron target, and will be used to solve the discrepancy between different PWA models in the high c.o.m. energy region.

For the double pion channels, three polarizations are measured: \( I^\odot, P_z, P^\odot_z \). The beam helicity asymmetry \( I^\odot \) using proton target has been compared with two earlier measurements (G1c and G9a), and the results are agreed. However, it was found that \( I^\odot \) is not an uniform function along \( \cos \theta_{\pi\pi} \), thus the integration along \( \cos \theta_{\pi\pi} \) should be done in a different way other than the previous two experiments have done. Using the new way of integration, some discrepancies have been found between the new results from G14 and the results from G1c and G9a, thus we suggest that the results in G1c and G9a should be checked again using the new way of integration of \( I^\odot \) along \( \cos \theta_{\pi\pi} \).

The beam helicity asymmetry \( I^\odot \) using neutron target has also been measured, this is the first measurement for this observable. The results from proton and neutron target for \( I^\odot \) have been compared, and big difference has been found for the c.o.m energy.
bins to be 1650 MeV and 1750 MeV. It is possible that the difference comes from the missing of $F_{15}(1680)$ resonance in the reaction with neutron target. Confirmation will await a detailed coupled channel partial wave analysis. For the other two observables $P_z, P^z$, since an unexpected small target polarization was found for the data set of target polarization along '-z' direction, the results have big error bars and we could not extract useful information from these two observables.

For the study of hadron spectroscopy, a significant progress in experiment has been made for the last decade, especially for the reaction using photon beam. The new experiment results have helped us get a better understanding of the hadron system in the medium energy region. However, many more experiment measurements need to be done in order to get the complete picture for hadron spectroscopy. Especially for reactions with more than one final state pions, there are very few data available. The reaction channels with more than one final state pions have bigger cross sections than the single pion reactions in the energy region where most of the 'missing resonances' are (above 2 GeV), this fact puts this reaction in higher priority in the study of the 'missing resonance problem'. The double pion channel is very promising for us to find those missing resonances, thus more experiments aim this reaction are necessary.
Appendices
Appendix A

NMR signal analysis

A.1 Steps to prepare the four signals

Step1: Phase rotation. Figure A.1 and A.2 shows that X and Y signals mix the absorption signal and dispersion signal, a phase rotation is needed to get pure absorption and dispersion signal.
Step2: Background subtraction. Figure A.4 shows that if the background signal of Y channel is not zero, the Kramer Kronig transformed signal will have a slope.
Step3: Kramer Kronig transformation. After the phase rotation and background subtraction(Figure A.4), 4 clean absorption signals (Figure ??) are ready to deduce the polarization value.

A.2 Background subtraction

Which most of the NMR measurements have very clean and flat background signals, there are some measurements have irregular background signals due to electrical or mechanical noises. There are four types of background that need to be corrected. The first type is as shown in previous section, in which the X and Y signals are shifted away from zero with a constant value.

The second type of background is that the background line is a linear line with a slope (Figure A.6), then the Kramer Kronig transformed signal will have a exponential tail (Figure A.7). The corrected signals are shown in Figure A.8, A.9.

The third type of background is that the background is a step function (Figure A.10), the background subtracted signal is shown in Figure A.11.

The fourth type of background is that the background has sharp peaks (Figure A.12), the background subtracted signal is shown in Figure A.13.
Figure A.1: Original signal from x- and y- channels of the NMR system.
Figure A.2: Signal from x- and y- channels after phase rotation.
Figure A.3: Signal from x- and y- channels after phase rotation. Y-channel signal has been applied the Kramer Kronig transformation, there is a slope in the background, which is due to the nonzero background of the y-channel signal before Kramer Kronig transformation.
Figure A.4: Signal from x- and y- channels after phase rotation, the two signals have been applied background subtraction.
Figure A.5: Signal from x- and y- channels after phase rotation and background subtraction, the background of the Kramer Kronig transformed y-channel signal becomes flat.
Figure A.6: Signal from x- and y- channels after phase rotation.
Figure A.7: Signal from x- and y- channels after phase rotation. Y-channel signal has been applied the Kramer Kronig transformation, there is a exponential tail in the background, which is due to the linear background of the y-channel signal before Kramer Kronig transformation.
Figure A.8: Signal from x- and y- channels after phase rotation, the two signals have been applied background subtraction.
Figure A.9: Signal from x- and y-channels after phase rotation and background subtraction, the background of the Kramer Kronig transformed y-channel signal becomes flat.
Figure A.10: Signal from x- and y- channels after phase rotation.
Figure A.11: Signal from x- and y- channels after phase rotation and background subtraction.
Figure A.12: Signal from x- and y- channels after phase rotation.
Figure A.13: Signal from x- and y- channels after phase rotation and background subtraction.
Appendix B

The effective mass of nucleon inside deuteron

The technique of reaction channel selection used in this study is the 1-C kinematic fitting. One of the most important parameters used in this procedure is the effective mass nucleon inside deuteron. Such as in the channel $\gamma(n) \rightarrow p\pi^-$, there are four constrain equations from the conservation of energy and momentum, and three unknown values for the target neutron’s initial 3 momentum. A key component for the kinematic fitting to work is an accurate estimate of the effective mass for the quasi free neutron.

An straightforward solution for this problem is to use Gaussian fit to find the central of the missing mass plot for this reaction, as in Figure B.1. However, as shown in the figure, the central values for the peaks from two different run periods are different. This difference is caused by an offset of the photon beam energy.

Figure B.1: Missing mass plot for the reaction of $\gamma(n) \rightarrow p\pi^-$ before photon beam energy correction. The left plot is for Silver1 period, the right plot is for Gold2a period.

In order to correct the photon beam energy, the missing mass is plotted for the same reaction, but in a different way. Using the spectator model, the same reaction can be written as $\gamma D \rightarrow p(p)\pi^-$, with $p$ in parenthesis representing the spectator proton, which does not involved in the reaction. The spectator proton should have a on-shell mass of 0.9383 GeV. The missing mass plot for this reaction is shown in Figure B.2. Both of the central values for the missing mass peaks are not at 0.9383 GeV. This indicate the photon beam energy is not very accurate. For Silver1 period, the photon beam energy should increase by 0.0030 GeV, and for Gold2a period, the photon beam energy should increase by 0.0082 GeV. The new missing mass plots for the reaction $\gamma D \rightarrow p(p)\pi^-$ after photon beam correction are shown in Figure B.3.
Figure B.2: Missing mass plot for the reaction of $\gamma D \rightarrow p(p)\pi^-$ before photon beam energy correction. The left plot is for Silver1 period, the right plot is for Gold2a period.

Figure B.3: Missing mass plot for the reaction of $\gamma D \rightarrow p(p)\pi^-$ after photon beam energy correction. The left plot is for Silver1 period, the right plot is for Gold2a period.

The missing mass plots for the reaction after the photon beam energy correction is:

Figure B.4: Missing mass plot for the reaction of $\gamma(n) \rightarrow p\pi^-$ after photon beam energy correction. The left plot is for Silver1 period, the right plot is for Gold2a period.

The energy correction for photon beam and the calculated effective mass for quasi-free neutron for all circular run periods are shown in Table B.1. The average and statistical error for the effective mass is: $m^* = 0.9303 \pm 0.0003$ GeV.
<table>
<thead>
<tr>
<th>Period</th>
<th>$\Delta E$ (GeV)</th>
<th>$m^*$ (neutron) (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver1</td>
<td>0.003</td>
<td>0.9307</td>
</tr>
<tr>
<td>Silver2a</td>
<td>0.0015</td>
<td>0.9305</td>
</tr>
<tr>
<td>Silver2b</td>
<td>0.0015</td>
<td>0.9303</td>
</tr>
<tr>
<td>Silver3</td>
<td>0.0015</td>
<td>0.9303</td>
</tr>
<tr>
<td>Silver4</td>
<td>0.0015</td>
<td>0.9300</td>
</tr>
<tr>
<td>Silver5</td>
<td>-0.0015</td>
<td>0.9305</td>
</tr>
<tr>
<td>Gold2a</td>
<td>0.0082</td>
<td>0.9299</td>
</tr>
<tr>
<td>Gold2b</td>
<td>0.0082</td>
<td>0.9302</td>
</tr>
</tbody>
</table>

Table B.1: The photon beam energy offset and the effective mass of quasi free neutron for all circular run periods.
Appendix C

Empty target correction

As shown in Figure 2.31 in Chapter 2, there are aluminum wires inside the HD target cell. The channel selection procedures, especially the Kinematic Fitting will remove most of the events from the target cell, but there are still some background events that will pass the cuts. During the experiment, an empty target run has been conducted for the study of empty target subtraction. Figure C.1 gives the z vertex distribution for gold2a period and emptya period, the blue line in the target region on right plot indicates a small amount of events from aluminum wires have passed the kinematic fitting cut.

![Figure C.1: Z vertex distribution for gold2a(left) and emptya(right). Black line is for all the events before the kinematic fitting cut, blue line is for events that pass the cut, and red line is for events fail the cut.](image)

These background events will dilute the events from HD target, and makes the polarization observables smaller. And this effect can be corrected using the following equation:

\[
E = (1 + \frac{N_{empty}}{N_{HD}})E_{raw},
\]

where \(N_{empty}\) is for the events normalized background events from empty target run, and \(N_{HD}\) is for the events from HD target.

A comparison of z vertex distribution between full target run and empty target run for different c.m. energy is shown in Figure C.2, it is noticed that the the ratios of events for the downstream KeF peak are different for different c.m. energy. Thus, an energy dependent background correction for E asymmetry is needed.
Figure C.2: Comparison of z vertex between Gold2a (blue line) and Empty-a (red line) periods for different c.m. energies.

Figure C.3 shows the ratios of events between the full target run periods and empty target run period for all circular runs.

The ratio of events between empty target runs and full target runs in the target region (-10.5 cm < z < -5.5 cm) can be obtained by using the scale constant from Figure C.3. For gold2a and gold2b periods, the fact that only 70% aluminum wires in use should be included in the calculation. The results are shown in Figure C.4.
Figure C.3: Ratio vs. W(c.m.). Ratio = KelF(full target runs)/KelF(empty target runs)
Figure C.4: Ratio vs. W(c.m.). Ratio = N_cell_scaled/(N_HD+N_cell_scaled), where N-cell-scaled is the scaled number of events from empty target, N_HD is the number of events from HD.
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