Semi-inclusive pion production with CLAS

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Abstract

We propose the measurement of the semi-inclusive charged pion cross section off the proton ($ep \to e'\pi^+X$ and $ep \to e'\pi - +X$) using the CLAS detector. In deep inelastic scattering this process can be factorized in the abpsoriton of the virtual photon by a single quark and the subsequent fragmentation of the quark into hadrons. Existing measurements indicates that this picture may be applicable at rather low values of the four momentum transfer Q^2 that can be rached at the JLab energies. The large acceptance of the CLAS detector would allow a detailed study of the semi-inclusive 5-dimenaional cross section, providing a tool to test the factorization picture and investigate the hadronization process. This analysis will be based on the data recorded at energy of 4 and 6 GeV during the E1c, E1d, and E1-6 run periods, providing kinematical coverage for $x = 0.2 - 0.6$ and $Q^2 = 1 - 6$ GeV^2.

1 Motivation

Deep Inelastic Scattering is a fundamental tool to study the nucleon structure. The measurement of the inclusive lepton-nucleon cross section at large momentum transfer Q^2 gives access to information on the parton distributions inside the nucleon. A natural extension of this measurement is represented by semi-inclusive scattering in which an hadron is detected in coincidence with the scattered electron in the final state. In the deep inelastic regime, this process can be described by a simple and intuitive picture shown in figure 1. The semi-inclusive process is factorized into three parts: the emission of a parton from the target nucleon, the elementary interaction between the parton and the virtual photon, and the fragmentation of the struck parton into hadrons. The virtual photon-parton interaction can be calculated in pQCD. On the contrary the other two parts are soft terms and need to be parameterized. The first term is sensitive to the nucleon inner structure and is described by the nucleon structure function $f(x, Q^2)$ ($x = Q^2/2M\omega$). The second term is related to the probability of the struck parton fragmenting into an hadron h. This is usually parameterized in term of the fragmentation function $D_{h/q}(z, p_\perp^h, Q^2)$, where $z = E_h^{lab}/\omega$ is the energy fraction carried by the outgoing hadron, and $p_\perp^h$ is the hadron trasverse momentum in respect to the quark.
Both the parton distribution functions and the fragmentation functions are universal quantities that do not depend on the particular process that was used for their extraction. They are directly related to the inner structure of hadrons and are therefore fundamental objects for the understanding of strong interactions.
The possibility of factorizing the semi-inclusive process in the virtual photon-quark interaction and the subsequent quark hadronization is the key element to access these objects from the cross section measurement. While this factorization has been tested successfully at high energy, its applicability at momentum transfers and invariant masses of few GeV is a controversial point and need to be proved. As Q^2 and W decrease, non perturbative processes as higher twist, final state interaction, etc., become sizeable and could spoil the simple picture of semi-inclusive deep inelastic scattering. In addition at the JLab energies, the separation between the products of the quark fragmentations (current fragmentation) and the target remainants (target fragmentation) is not clear, and therefore an accurate test of whether the factorization picture holds is necesary to pursue these study.
The energy region that can be study at JLab is of particularly interesting to understand the mechanism that drives the transition from the perturbative region to the region of strong interactions. In such intermediate range, the test of factorizaton can provide a tool to understand how the transition occurs. In the case factorization holds, then semi-inclusive electron scattering can become an important source of information about the nucleon structure function and hadron fragmentation functions. The flavor structure of the parton distribution function can be accessed by tagging different hadrons in the final states which in fact would give different flavor combinations for the measured structure functions. In addition it would allows the extraction of the quark fragmentation functions for which the data are more sparse.

2 Formalism

The general structure of the semi-inclusive hadron production on the nucleon $eN \to e'hX$ can be written as

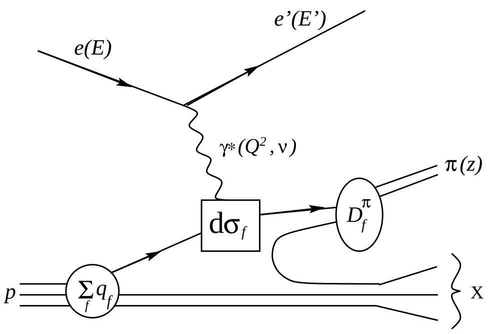

Figure 1: Semi-inclusive pion electroproduction diagram.

$$\frac{2E_h d\sigma}{dE' d\Omega d^3 p_h} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{Q^2}{\mathbf{q^2}} \left[W_T + \epsilon W_L + \sqrt{\frac{\epsilon(\epsilon+1)}{2}} W_{LT} \cos\phi_h + \epsilon W_{TT} \cos 2\phi_h \right],$$

where Q^2 is the squared four-momentum transfer, $\mathbf{q^2}$ is the squared three-mpmentum transfer, ϵ is the virtual photon polarization, and W_i are the structure function which parameterize the process dynamics. This expression is very general and can be used to describe both semi-inclusive and exclusive cross sections in unpolarized lepton-nucleon scattering. In the deep inelastic case is convenient to adopt the structure functions H_i introduced by Levelt and Mulders (see Ref. [1]) defined as

$$\begin{aligned} 2zH_1 &= MW_T \\ 2zH_2 &= \frac{\omega Q^2}{\mathbf{q^2}}(W_T + W_L) \\ 2zH_3 &= \frac{Q^3}{|\mathbf{q}||\mathbf{p_h}|} W_{LT} \\ 2zH_4 &= \frac{Q^4}{\mathbf{q^2 p_h^2}} W_{TT} \end{aligned}$$

The advantage of these new functions is that the cross section expressed in terms of the H_i becomes an expansion of powers of $1/Q$, i.e. a twist expansion. BY introducing the invariants $x = Q^2/2m\omega$ and $y = P \cdot q / P \cdot k = \omega/E$, the cross section of eq. 1 becomes

$$\begin{aligned} \frac{d\sigma}{dx dy dz d^2\mathbf{p}_{h\perp} d\phi_h} &= \frac{4\pi\alpha^2 MEQ^2}{Q^4} \frac{E_h}{\mathbf{p}_{h||}} \left[xy^2 H_1 + \left(1 - y - \frac{Mxy}{2E}\right) H_2 + \right. \\ &+ \frac{|\mathbf{p}_{h\perp}|}{Q}(2-y)\sqrt{\frac{1-y-\frac{Mxy}{2E}}{1+\frac{2Mxy}{2E}}} \cos\phi_h H_3 \\ &+ \left. \frac{|\mathbf{p}_{h\perp}|^2}{Q^2} \frac{1-y-\frac{Mxy}{2E}}{1+\frac{2Mxy}{2E}} \cos 2\phi_h H_4 \right] \end{aligned} \quad (1)$$

where the $\cos\phi_h$ and $\cos 2\phi_h$ appears naturally has twist-3 and twist-4 terms, while the ϕ-independent terms represent the leading order (twist-2) term.
At the order $\mathcal{O}(Q^{-1})$, the authors of ref. [1] showed that the structure functions H_i can be expressed as the product of the parton distribution function and of the hadron fragmentation function. In particular the twist-2 terms are connected by a generalized Callan-Gross relation, being

$$2xH_1 = H_2,$$

and H_1 (or H_2) can be written as the product of the parton ditribution function $f(x,Q^2)$ and of the hadron fragmentation function $D(z,p_\perp,Q2)$, implying the validity of factorization. Precisely, at the order $\mathcal{O}(Q^{-1})$, the cross section for semiinclusive hadron production can be written as

$$\begin{aligned}
\frac{d\sigma(eN\to e'hX)}{dxdydzd^2\mathbf{p}_{h\perp}d\phi_h} &= \frac{8\pi\alpha^2MEQ^2}{Q^4}\sum_{i=q\bar{q}}\int d^2\mathbf{k}_\perp\left\{\left(\frac{y^2}{2}-y+1\right)xf_{i/N}D_{h/i}\right.\\
&+ 2(2-y)\sqrt{1-y}\cos\phi_h\frac{\mathbf{p}_{h\perp}}{Q}\frac{x}{z}\left[f_{i/N}\left(\frac{1}{z}D^\perp_{h/i}-D_{h/i}\right)\right] \qquad (2)\\
&+ \left.2(2-y)\sqrt{1-y}\cos\phi_i\frac{\mathbf{k}_\perp}{Q}\frac{x}{z}\left[-f_{i/N}D^\perp_{h/i}+z\left(f_{i/N}-xf^\perp_{i/N}\right)D_{h/i}\right]\right\}
\end{aligned}$$

where $k_\perp$ is the quark transverse momentum with respect to the virtual photon direction, $p^h_\perp = k_\perp + k^h_\perp$ is the hadron transverse momentum with respect to the virtual photon, and $D_{h/i} = D_{h/i}(z,k^h_\perp,Q^2) = D_{h/i}(z,p^h_\perp - zk_\perp,Q^2)$. For generality, the dependence of the structure function f on the quark transverse momentum has been included in the above formula and therefore $f_{i/N} = f_{i/N}(x,k_\perp,Q^2)$. However it was shown that the $k_\perp$ dependence of the parton distibution functions can be written in factorized form as $f(x,Q^2)e^{-\alpha k^2_\perp}$. Based on this assumption, the integral over $k_\perp$ becomes calculable and the cross section can be finally written as a product of a function of x and a function of the pion energy fraction z.

3 Experimental Status of Semiinclusive Lepton Scattering

Measurements of semi-inclusive lepton-nucleon scattering have been performed in several facilities as CERN (EMC) [6], DESY (H1, ZEUS, HERMES) [7], and less recentely at Cornell [8] and SLAC [9]. In addition an experiment to study duality in meson electroproduction was approved in Hall C [10]. In the following we will focus on the measurements performed at HERMES, Cornell, and SLAC, since their kinematics is closer to what can be reached with CLAS.
The HERMES experiment measured semi-inclusive pion production on the proton and the neutron in the DIS regime, for W between 4.5 and 6 GeV, and Q^2 between 1 and 10 GeV^2 ($\langle Q^2\rangle = 2.3$ GeV^2). Measuring the charged pion multiplicity on proton and neutron and directly assuming factorization, they were able to extract the sea flavour asymmetry $(\bar{d}-\bar{u})/(u-d)$ for $0.02 < x < 0.3$. The results of the HERMES measurement were found

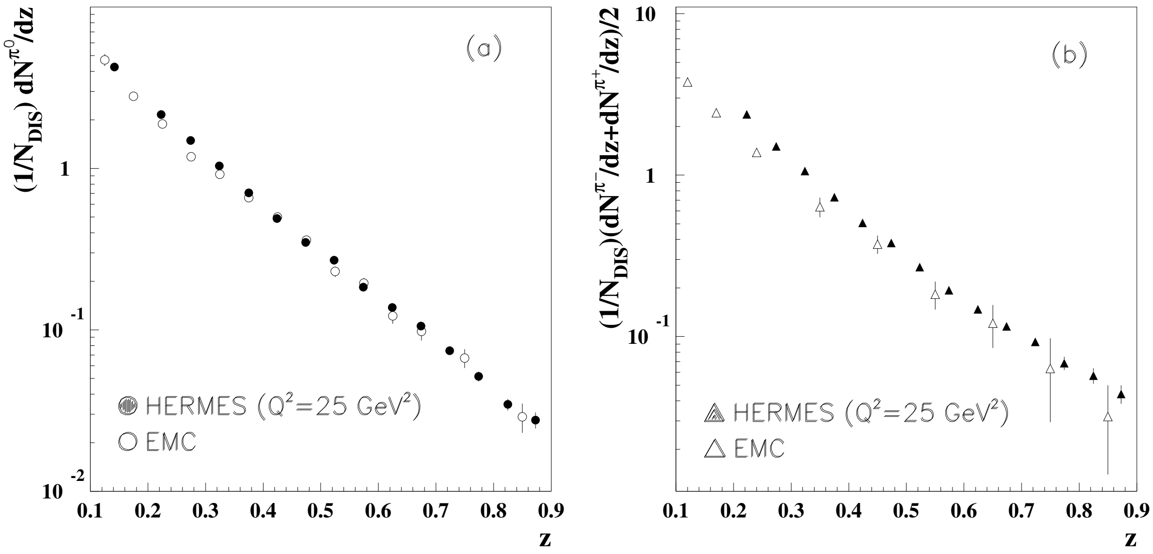

Figure 2: Multiplicities for neutral (a) and charged (b) pions. The HERMES results have been evolved to Q2=25 GeV2 using a NLO QCD model. Only statistical uncertainties are shown. The systematic uncertainties for neutral (charged) pions are 9% (7%) for HERMES and less than or equal 13% (10%) for EMC.

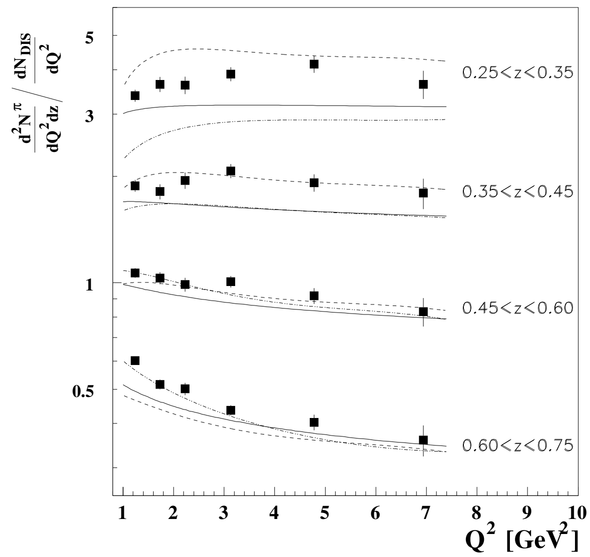

Figure 3: Total(neutral plus charged) pion multiplicity as a function of Q^2 for various z-bins measured by HERMES. The three curves shown are NLO QCD calculations of fragmentation functions (see ref. [7]

to be in good agreement with the Drell-Yan data at high energies [11]. Similarly, the HERMES experiments also measured the π^+, π^- e π^0 multiplicities, which in the assumption of factorization are directly related to the fragmentation fucntion. The extracted multiplicities were compared with the fragmentation function data of EMC at Q^2=25 GeV^2, and after accounting for pQCD evolution effects, they were found to be compatible. Both results seems to indicate that factorizaation may hold also for rather low values of Q^2.
Semi-inclusive hadron production on hydrogen and deuteron was measured at SLAC [9] for $0.25 < Q^2 < 0.3$ GeV^2 and $12 < W^2 < 30$. Positively and negatively charged hadrons were detected in coincidence with the scattered electrons. Because of the low statistics, a direct extraction of the fragmentation functions were not attempted, but a likelihood fit of the data was performed using the function

$$\frac{1}{\sigma_{tot}(Q^2,W)}\frac{d\sigma(Q^2,W)}{dx_F d\mathbf{p}^2_{h\perp} d\phi} = \frac{dN}{dz} b e^{-b\mathbf{p}^2_{h\perp}} \frac{1 + A\cos\phi + B\cos 2\phi}{2\pi} \tag{3}$$

where the Feynman variable $x_F = 2p_{||}/W$ is directly related to z. This analysis showed that the data were compatible with the distribution of equation 3, which implicitely assumes factorization, and that at 90% confidence level no ϕ dependence was observed. Based on what discussed in section 2, this would imply a negligible contribution of the twist-3 and twist-4 terms to the cross section.
Measurements of π^+ and π^- production on the proton and neutron were performed at Cornell for invariant mass W between 2 and 3 GeV and four-momentum transfer Q^2 between 1 and 4 GeV^2 [8]. An equivalent of the structure function H_1 (or H_2) introduced in section 2 was extacted from the measured cross section, without the separation of the parton distribution and fragmentation functions. This quantities was found to be only weakly dependent on W and Q^2.
The study of duality in meson electroproduction is the subject of an approved experiment in Hall C. This experiments would measure charged pion semi-inclusive production for fixed Q^2 between 1.8 and 6.0 GeV^2 and x between 0.2 and 0.7. The pions emitted in the directions of the virtual photon ($p^h_\perp$) will be detected. A test run was performed in Hall C in 1999 and the results of the data analysis already gave indications that factorization may hold at this low energy.

4 Semi-inclusive meson production with CLAS

The large acceptance of the CLAS detector allows a detailed study of the 5-dimensional semiinclusive cross section, spanning a large range of four momentum transfer Q^2 and x. Figure 4 show the Q^2 and x range covered respectively with a beam energy of 4.5 and 5.6 GeV at the maximum torus field (I_{torus}=3375 A). The solid line shows the $W = 2$ GeV limit. For any bin in this range, CLAS allows the measurement of the z, $p_\perp$, and ϕ_h distributions of the outgoing meson. In particular the capability of measuring the ϕ_h distribution is very important for the study of the semi-inclusive process since it would permit the separation of the different terms in eq. 1.
To study in detail the feasibility of this measurement with the CLAS detector, we have performed a preliminary analysis of the data recorded during the E1C run period at 4.46 GeV with inbending Torus field at $I_{torus} = 3375$ A. Typical z and $p_\perp$ distributions for $ep \to e'\pi^+X$

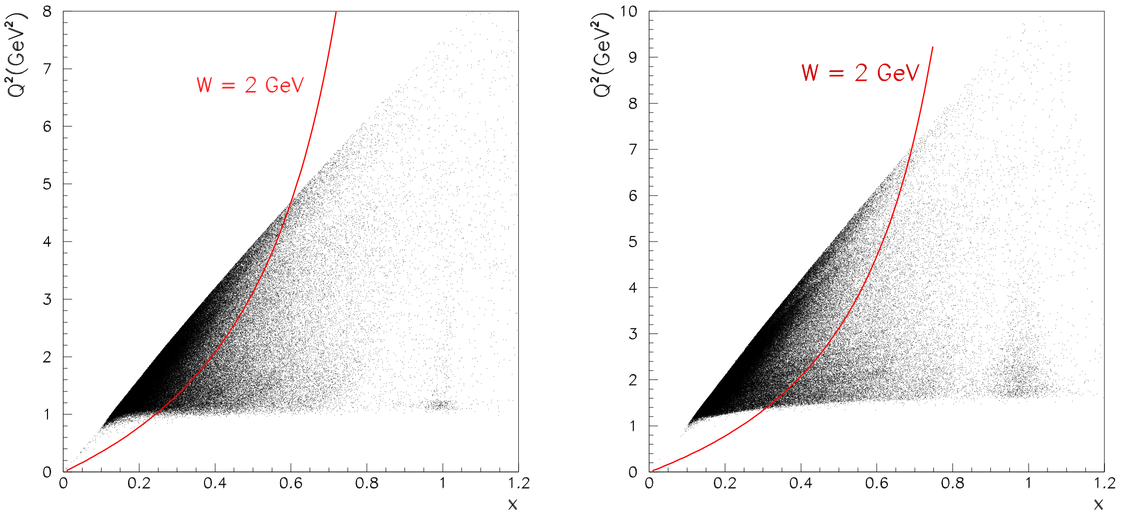

Figure 4: Q^2 versus x distributions measured by CLAS at a beam energy of 4.5 GeV (left) and 5.7 GeV (right)

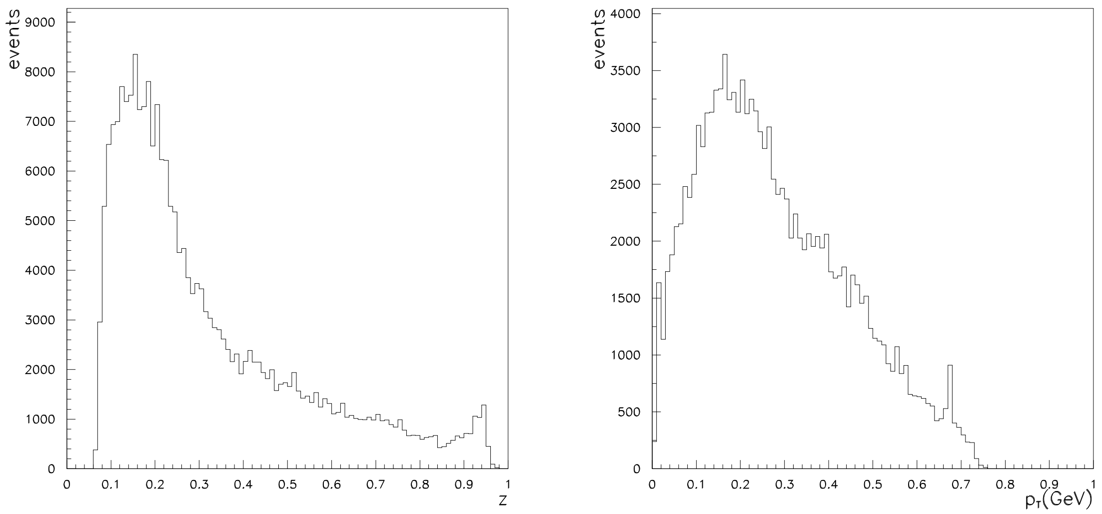

Figure 5: z and $p_\perp$ distribution for $ep \to e'\pi^+X$ events extracted from the CLAS data at 4.46 GeV (E1C run period). The four momentum transfer Q^2 is between 2.0 and 2.5 GeV^2.

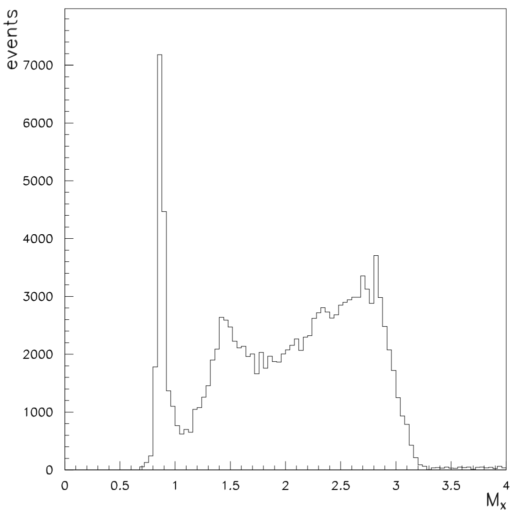

Figure 6: Missing mass distribution for $ep \to e'\pi^+X$ events extracted from the CLAS data at 4.46 GeV (E1C run period). The four momentum transfer Q^2 is between 2.0 and 2.5 GeV^2.

events extracted from these data are shown in figure 5. This very preliminary distributions were obtained using events in which both the scattered electron and the outgoping pion were inside the fiducial regions of the detector and correcting for the geometrical acceptance of this cuts. As shown by these plots, CLAS allows the measurement of the z dependence of the cross section from values of the order of 0.1 up to the maximum value of 1, for transverse pion momenta up to 0.7-0.8 GeV.
Examples of the $p_\perp^{\pi^+}$ versus z of pions emitted in the forward emisphere ($\theta_\pi^{CM} < 90°$) are shown in figure 7. These plots show the typical correlation between z and $p_\perp$ of the outgoing pion: the kinematical limit is represented by the external circle which corresponds to the exclusive reaction $ep \to e'\pi^+n$.
The ϕ versus $p_\perp$ distributions are shown in figure 8. The ϕ range is almost completely covered by the data, allowing the separetion of the different terms of the semi-inclusive cross section as already discussed. Such distributions corrected for the geometrical acceptance of the fiduccial cuts were integrated over the meson transverse momentum to obtain a very preliminary estimate of the semi-inclusive π^+ cross section at 4.5 GeV. The results are shown in figure 9 for different bins of W and for a Q^2 interval between 2.0 and 2.5 GeV^2.

4.1 Test of the validity of factorization

The formalism introduced in section 2 is intrinsically based on the hypothesis of factorization of the cross section into the absorption of the virtual photon by a quark and the subsequent fragmentation on the struck quark into hadrons. Without this hypothesis it is not possible to connect the semi-inclusive cross section with the fragmentaion functions as was done in eq. 2. For this reason one the key issue in the proposed analysis is the test of factorization. As already mentioned, the factorization of the reaction mechanism results in the cross section being written as the product of a function of the quark momentum fraction x and of a

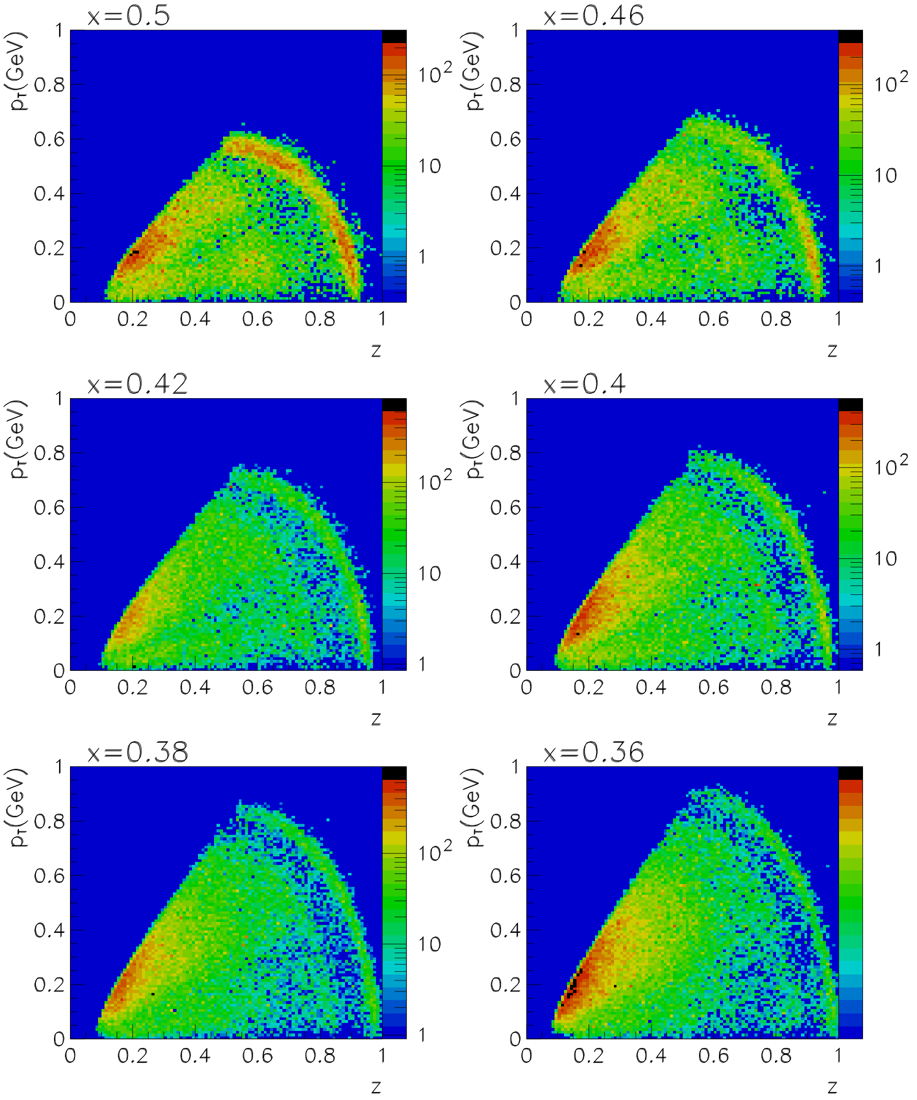

Figure 7: $p_{\perp}$ versus z distribution for $ep \to e'\pi^+X$ events extracted from the CLAS data at 4.46 GeV (E1C run period). The four momentum transfer Q^2 is between 2.0 and 2.5 GeV^2. Each panel corresponds to a different x interval.

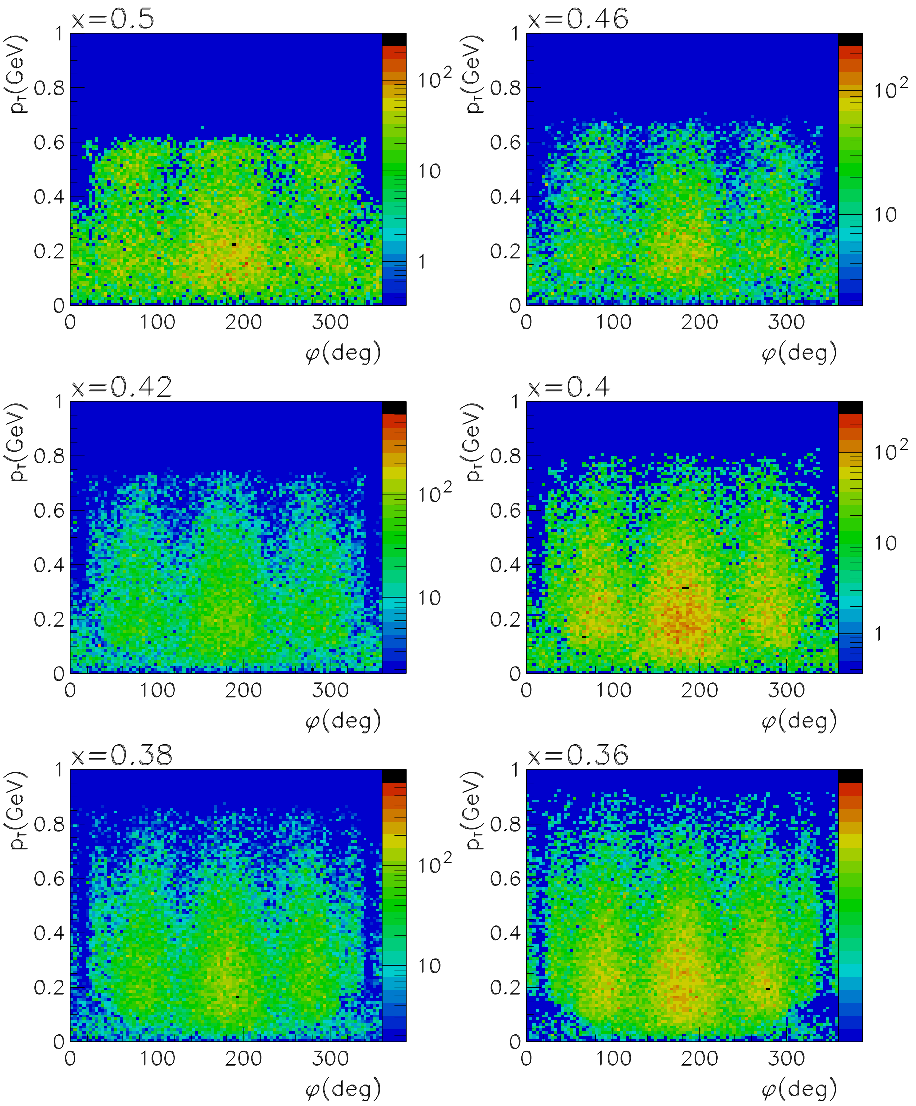

Figure 8: $p_\perp$ versus ϕ distribution for $ep \to e'\pi^+X$ events extracted from the CLAS data at 4.46 GeV (E1C run period). The four momentum transfer Q^2 is between 2.0 and 2.5 GeV^2. Each panel corresponds to a different x interval.

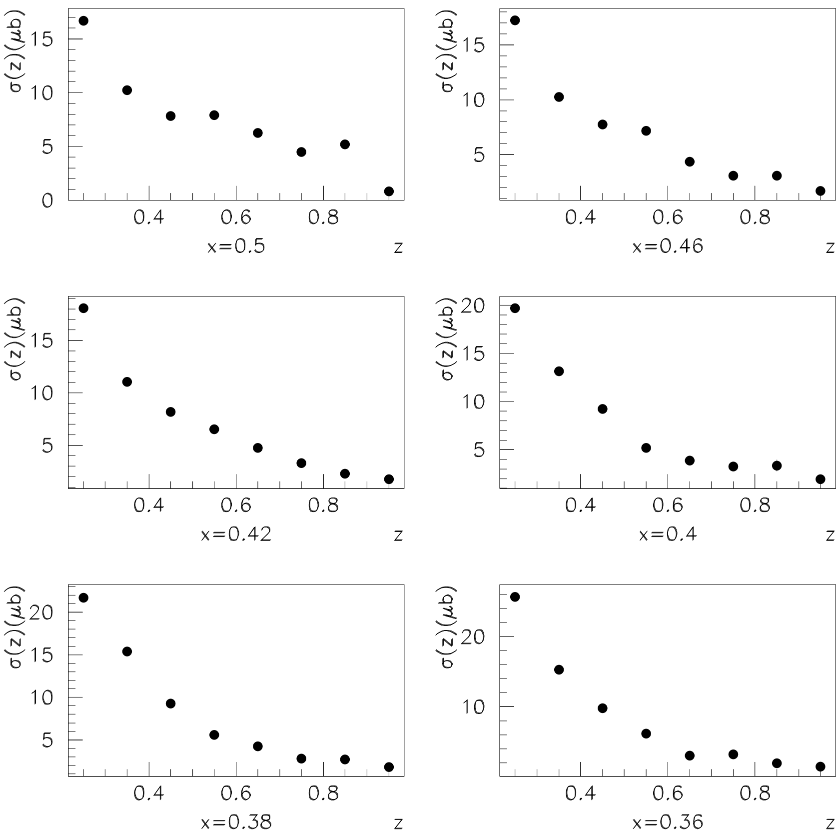

Figure 9: z dependence of ϕ-independent term of the cross section for $ep \to e'\pi^+X$ events extracted from the CLAS data at 4.46 GeV (E1C run period). Events with both electron and pion within the detector fiducial regions were selected and corrections for the geometrical acceptance of these cuts were applied. The cross section was integrated oevr the pion transverse momentum. The four momentum transfer Q^2 is between 2.0 and 2.5 GeV^2. The statistical error is smaller than the dot size

function of the hadron energy fraction z. The test of factorization is therefore possible by comparing the z dependence of the semi-inclusive cross section for different values of x. In particular if factorization holds, the twist-2 part of the cross section can be schematically written as

$$\sigma(z, Q^2, z, p_\perp) \to f(x, Q^2) D(z, p_\perp, Q^2). \tag{4}$$

Consequentely, the ratio of the cross section for two different values of x (x_1 and x_2) at the same value of Q^2 is expected to be independent of z and $p_\perp$. More precisely to be able to apply this simple expression without additional assumptions, we propose to measure the ϕ-independent term of the cross section for charged pion production ($\sigma^{\pi^+ + \pi^-}$). This quantity can be written as

$$\frac{d\sigma(\pi^+ + \pi^-)}{dxdydzd^2\mathbf{p}_{h\perp}d\phi_h} = \frac{8\pi\alpha^2 MEQ^2}{Q^4} \sum_{i=q\bar{q}} \left(\frac{y^2}{2} - y + 1\right) x \left[f_{i/N} D_{\pi^+/i} + f_{i/N} D_{\pi^-/i}\right] \tag{5}$$

If we neglect s-quark and gluon contributions, and assume isospin symmetry, this becomes

$$\begin{aligned} \frac{d\sigma(\pi^+ + \pi^-)}{dxdydzd^2\mathbf{p}_{h\perp}d\phi_h} &= \frac{8\pi\alpha^2 MEQ^2}{Q^4} \left(\frac{y^2}{2} - y + 1\right) x \left[f_{u/N} D_{\pi^+/u} + f_{d/N} D_{\pi^+/d} + \right. \\ &+ \left. f_{u/N} D_{\pi^-/u} + f_{d/N} D_{\pi^-/d}\right] \\ &= \frac{8\pi\alpha^2 MEQ^2}{Q^4} \left(\frac{y^2}{2} - y + 1\right) x \left[f_{u/N}(D_{\pi^+/u} + D_{\pi^-/u}) + \right. \\ &+ \left. f_{d/N}(D_{\pi^+/d} + D_{\pi^-/d})\right] \\ &= \frac{8\pi\alpha^2 MEQ^2}{Q^4} \left(\frac{y^2}{2} - y + 1\right) x (f_{u/N} + f_{d/N})(D_{\pi^+/u} + D_{\pi^-/u}) \end{aligned} \tag{6}$$

which is explicitely factorized.
This feature can be exploited to test the validity of factorization. An indication of the sensitivity of this method to violation of the factorization can be obtained considering the variation of $xf(x)$ in the x range accessible with CLAS: existing measurements of the parton distribution functions shows that $xf(x)$ has a variation of a factor 3 going from $x \sim 0.3$ to 0.1 for $x \sim 0.6$.

4.2 Duality in the fragmentation functions

If factorization is proved, the fragmentation functions $D(z, p_\perp, Q^2)$ could be extracted from the measured cross section by using existing parameterization of the parton distribution functions. As a first step it would be possible to evaluate the ratio

$$\frac{\sigma(z, Q^2, z, p_\perp)}{\int dz dp_\perp \sigma(z, Q^2, z, p_\perp)} \tag{7}$$

which is directly proportional to the fragmentation functions. These are not well know in the region of intermediate Q^2 (1-6 GeV^2) which is accessible at Jefferson Lab. For this reason this measurement would be fundamental to complete the existing information. The study of the Q^2 dependence of the fragmentation functions down to Q^2 of the order of 0.5 - 1 GeV^2 would also allow the study of duality as was performed for the structure function F_2 [12, 13].

In addition, thanks to the large acceptance of the CLAS detector, it would be possible to perform a detailed study of the dependence of the fragmentation functions on the meson transverse momentum $p_\perp$ for which only qualitative information exists.

4.3 Higher twist analysis of the semi-inclusive cross section

As discussed in section 2, the semi-inclusive cross section appears as a twist expansion, where the different twists are distinguishable from their different ϕ dependence. Such dependence can be measured with the CLAS detector thanks to its large acceptance. The preliminary analysis that was performed on the 4 GeV data has shown that close to full ϕ coverage can be achieved for most of the covered Q^2 and x range. Fitting the ϕ dependence of the cross section allow therefore the separation of the ϕ-indenpendent, $\cos\phi$, and $\cos 2\phi$ terms. The relative weight of these terms will provide direct information on the higher twist contributions to this process.

5 Conclusion

We propose the measurement of the semiinclusive cross section for charged pion production for electron beam energy from 4 to 6 GeV using the data recorded in the E1C, E1D, and E1-6 run periods. By detecting the pion in coincidence with the scattered electron we will extract the five-dimensional differential cross section $d\sigma(\pi^+ + \pi^-)/dxdydzd^2\mathbf{p}_{h\perp}d\phi_h$. The analysis of the azimuthal dependence of the cross section will allow the extraction of the twist-2 term of the cross section. This quantity is fundamental to test the validity of factorization in the kinematical region accessible at Jefferson Lab. Sparse data from previous measurements suggest that we may be in the right kinematical region of study the onset of such phenomenon in pion electroproduction. If factorization is proved, it would then be possible to study in detail the meson fragmentation functions and flavour dependence of the distribution function in the region of costituent quarks.

References

[1] L. Levelt and P. Mulders, Phys. Rev. **D49**, 96 (1994).

[2] H. Georgi and H. D. Politzer, Phys. Rev. Lett. **40**, 3 (1978).

[3] R. N. Cahn, Phis. Rev. **D40**, 3107 (1989).

[4] E. Berger, Z. Phys. **C4**, 289 (1980).

[5] A. Konig and P. Kroll, Z. Phys. **C16**, 89 (1982).

[6] M. Arneodo *et al.*, Z. Phys. **C34**, 277 (1988).

[7] A. Airapetian *et al.*, hep-ex 0104004; Phys. Rev. Lett. **81**, 5519 (1998).

[8] C. J. Bebek *et al.*, Phys. Rev. Lett. **34**, 759 (1975); Phys. Rev. Lett. **37**, 1320 (1976); Phys. Rev. **D15**, 3085 (1977).

[9] J. T. Dakin *et al.*, Phys. Rev. Lett. **31**, 786 (1973)

[10] R.Ent *et al.*, *Duality in Meson Electroproduction* **PR-00-108**, (2000).

[11] H. L. Lai *et a.*, Phys. Rev., **D55**, 1280 (1997).

[12] M. Osipenko *et al.* (CLAS Collaboration), to be published.

[13] I. Niculescu *et a.*, Phys. Rev. Lett., **85**, 1182 (2000), Phys. Rev. Lett., **85**, 1186 (2000)