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Measurement of Pion-Pion Final State Interactions in $\eta \rightarrow \pi + \pi - \gamma$ with CLAS at Jefferson Lab

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MEASUREMENT OF PION-PION FINAL STATE
INTERACTIONS IN $\eta \rightarrow \pi^+\pi^-\gamma$ WITH CLAS AT JEFFERSON
LAB

by

Torri C. Jeske

A Dissertation Submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
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ABSTRACT

MEASUREMENT OF PION-PION FINAL STATE INTERACTIONS IN $\eta \rightarrow \pi^+\pi^-\gamma$ WITH CLAS AT JEFFERSON LAB

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Old Dominion University, 2020
Director: Dr. Moskov Amaryan

Decays of pseudoscalar mesons proceed from the chiral anomaly, which arises from spontaneous chiral symmetry breaking. In the limit of massless quarks (chiral limit), the $\eta \rightarrow \pi^+\pi^-\gamma$ decay width is determined solely by the box anomaly term in the Wess Zumino Witten Lagrangian. Since the physical quarks are not massless, the decay region of the η meson is far from the chiral limit and thus proper inclusion of the momentum dependence is essential to reproduce the measured decay width. Several theoretical frameworks have been proposed to describe these interactions. We report a new measurement of the α parameter which measures the contribution of pion-pion final state interactions to the differential decay width.

The data was collected during the g11a run period using the CEBAF Large Acceptance Spectrometer located in Hall B at the Thomas Jefferson National Accelerator Facility in 2004. The data was collected using a tagged photon beam with energy range of 1.5-3.5 GeV incident on a liquid Hydrogen target. Our results are in agreement with the latest experimental measurement and theoretical calculations.

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DEDICATION

This work is dedicated to my son, Adam. You are the light of my life.

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CHAPTER 1

INTRODUCTION

The structure of this thesis is organized as follows. Chapter 1 gives an introduction to the Standard Model and introduces the theoretical background of our experimental measurement.

Chapter 2 describes the CLAS spectrometer during the CLAS6 era when the data for this thesis was taken.

Chapter 3 summarizes the event selection and corrections applied to the data and describes the analysis.

Chapter 4 summarizes the results of our work.

1.1 THE STANDARD MODEL

The Standard Model describes fundamental particles and interactions that comprise and govern the visible matter of our Universe. The fundamental particles are called leptons, quarks, and bosons. The leptons include electrons, muons, and neutrinos and their corresponding antiparticles. The bosons include the photon, W and Z bosons, gluons, and the Higgs boson. The bosons (excluding the Higgs) are known as force mediators: the photon mediates the Electromagnetic force; the gluon mediates the Strong Nuclear Force; and the W and Z bosons mediate the Weak Nuclear Force. There are six quarks (q) and antiquarks (\bar{q}): up (u), down (d), strange (s), top (t), bottom (b) and charm (c). Quarks and leptons combine to form the visible matter in our Universe. The defining feature of quarks is that they possess color charge. Inside a hadron, quarks are very strongly bound to each other resulting (this is known as confinement) in a colorless composite states known as hadrons. There are two ways to form a colorless hadron: a quark-antiquark pair or a colorless combination of three quarks. A quark-antiquark pair is known as a meson while a combination of three quarks is known as a baryon. There is recent evidence suggesting the existence of exotic quark combinations, mainly tetraquarks and pentaquarks. Fig. 1 summarizes the fundamental particles and their properties. The Standard Model is written in the language of Quantum Field Theory, which describes the mechanics of very light particles traveling near the speed of light.

30 Even with the predictive success of the Standard Model, there are limitations. For
 31 instance, it does not yet incorporate gravity, predict the quark masses, describe neutrino
 32 oscillations, or incorporate any dark matter particle. In Fig. 1, the fundamental particles
 33 and their properties are summarized in visual form.

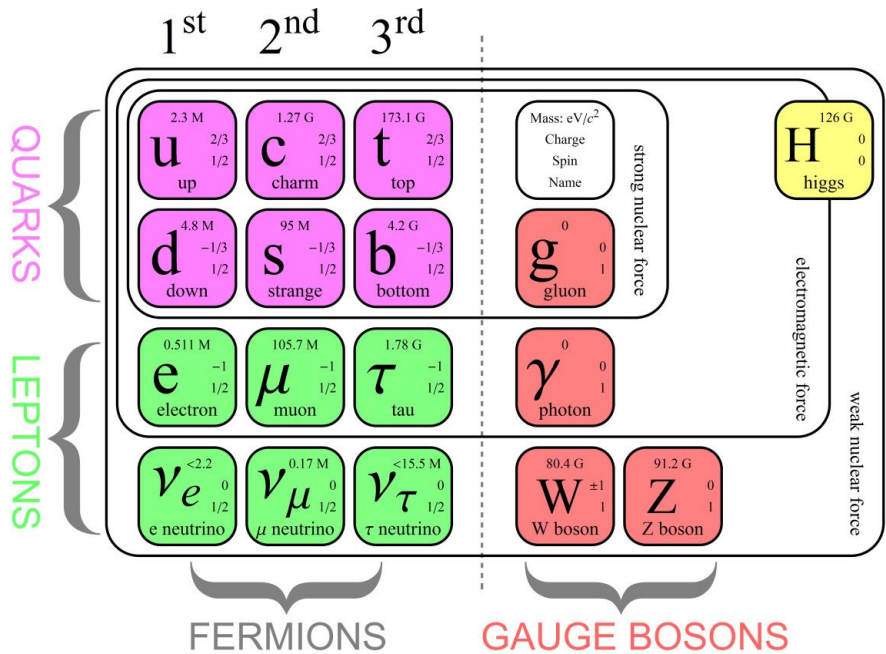


FIG. 1: Visual summary of the fundamental particles and their properties described in the Standard Model. Each square denotes the particle mass, charge, spin, and name from top to bottom. The first, second, and third notations correspond to the particle’s generation. Image from [1].

34 To construct hadrons from quarks, the Quark Model was proposed by Gell-Man, Zweig,
 35 and Ne’eman. Assuming three types of quarks (u, d, and s), the colorless combinations
 36 yielded baryons and mesons. The baryons are summarized in Table 1. The meson combina-
 37 tions yield the meson nonet seen in 2 and tabulated in Table 2.

TABLE 1: Baryon Decuplet

qqq	Q	S	Baryon
uuu	2	0	Δ^{++}
uud	1	0	Δ^+
udd	0	0	Δ^0
ddd	-1	0	Δ^-
uus	1	-1	Σ^{*+}
uds	0	-1	Σ^{*0}
dds	-1	-1	Σ^{*-}
uss	0	-2	Ξ^{*0}
dss	-1	-2	Ξ^{*-}
sss	-1	-3	Ω^-

TABLE 2: Meson Nonet

$q\bar{q}$	Q	S	Meson
$u\bar{u}$	0	0	π^0
$u\bar{d}$	1	0	π^+
$d\bar{u}$	-1	0	π^-
$d\bar{d}$	0	0	η
$u\bar{s}$	1	1	K^+
$d\bar{s}$	0	1	K^0
$s\bar{u}$	-1	-1	K^-
$s\bar{d}$	0	-1	\bar{K}^0
$s\bar{s}$	0	0	η'

38 This approach led to the prediction of the Ω^- baryon and its discovery in 1964, for which
 39 Gell-Mann won the Nobel Prize.

40 Since baryons have three valence quarks, each with spin $\frac{1}{2}$, they have half integer spin and
 41 are thus fermions. Fermions obey Fermi-Dirac statistics and the Pauli exclusion principle.
 42 The Pauli exclusion principle states that two or more identical fermions cannot occupy the
 43 same state simultaneously. Mesons have two valence quarks and have integer spin and are
 44 bosons. Bosons obey Bose-Einstein statistics. Now, according to Table 1, there are baryons
 45 with the same quarks, which are also fermions. This appeared to violate the Pauli exclusion

46 principle until the introduction of the color degrees of freedom in Quantum Chromodynamics.
 47 The two up quarks in the proton (and the down quark) must have different color, thus obeying
 48 the Pauli exclusion principle.

49 Much like classifying every day objects by their shape, size, and color, we can further
 50 classify subatomic particles by their intrinsic properties such as angular momentum and how
 51 they transform under certain operations. These properties are encoded in J^{PC} notation,
 52 where J is the *total* angular momentum, L is the angular momentum, and S is the spin
 53 angular momentum. The values J can take are:

$$|L - S| \leq J \leq |L + S|. \quad (1.1.1)$$

54 A naive example of the difference between L and S can be explained using the rotation
 55 of the Earth around the Sun (L) and the rotation of the Earth about its own axis (S). P
 56 represents parity conjugation, and inverts the sign of the spatial coordinates, $P = (-1)^{L+1}$.
 57 C represents charge conjugation and turns particles into antiparticles, $C = (-1)^{L+S}$. The
 58 classification of mesons using the quantum numbers above is presented in Table 3.

TABLE 3: Types of Mesons

Type	L	S	J	P	J^{PC}
Pseudoscalar	0	0	0	-	0^{-+}
Scalar	1	1	0	+	0^{++}
Vector	0	1	1	-	1^{--}
Axial Vector	1	0	1	+	1^{+-}
Tensor	1	1	2	+	2^{++}

59 1.2 THE η MESON AND ALLOWED DECAY MODES

60 The η meson is a pseudoscalar with quantum numbers $J^{PC} = 0^{-+}$. It is a light meson
 61 with mass of 547.862 ± 0.017 MeV and a lifetime of $(5.0 \pm 0.3) \cdot 10^{-19}$ s. It is part of the
 62 pseudoscalar meson nonet along with the charged and neutral pions and kaons.

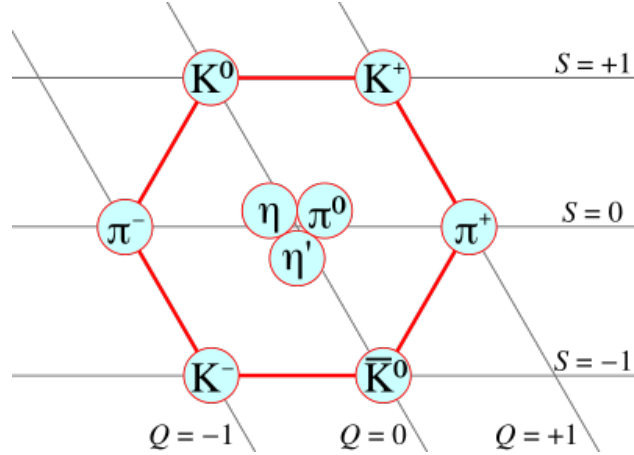


FIG. 2: Nonet of pseudoscalar mesons. The vertical axis is *strangeness*. It is equal to the difference in antistrange quarks and strange quarks. The diagonal axis corresponds to electric charge.

63 The physically observed η is a linear combination of the octet and singlet states η_8 and
 64 η_0 , respectively. The quark content of the octet and singlet states are

$$\begin{aligned}\eta_0 &= \sqrt{\frac{2}{3}} (u\bar{u} + d\bar{d} + s\bar{s}) \\ \eta_8 &= \sqrt{\frac{1}{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}).\end{aligned}\tag{1.2.2}$$

65 The octet state is a pseudo-Goldstone boson whose mass vanishes in the chiral limit, whereas
 66 the singlet state is not because of the anomalous breaking of the U(3) to SU(3) symmetry
 67 [24]. The combination is described using an experimentally determined mixing angle θ_{mix}
 68 and can be written as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} -\sin \theta_{\text{mix}} & \cos \theta_{\text{mix}} \\ \cos \theta_{\text{mix}} & \sin \theta_{\text{mix}} \end{pmatrix} \cdot \begin{pmatrix} \eta_0 \\ \eta_8 \end{pmatrix}\tag{1.2.3}$$

69 where $\theta_{\text{mix}} \approx 20 \text{ deg}$ [24]. The mixing angle is an important quantity to test theoretical
 70 predictions with experimental results. The $\eta \rightarrow \pi^+\pi^-\gamma$ decay is C-conserving, with the $C(\eta)$
 71 $= 1$, $C(\gamma) = -1$, and $C(\pi^+\pi^-) = (-1)^l$. Therefore, C invariance is only given for odd values
 72 of l . The most common decay modes of η meson are described in Table 4.

Decay	Branching Ratio
$\eta \rightarrow 2\gamma$	$39.3 \pm 0.20\%$
$\eta \rightarrow 3\pi^0$	$32.56 \pm 0.23 \%$
$\eta \rightarrow \pi^+\pi^-\pi^0$	$22.73 \pm 0.28 \%$
$\eta \rightarrow \pi^+\pi^-\gamma$	$4.22 \pm 0.08 \%$
$\eta \rightarrow e^+e^-\gamma$	$(7.0 \pm 0.7) \cdot 10^{-3} \%$

TABLE 4: Most common decay modes of the η meson.

73 1.3 SYMMETRIES AND THE CHIRAL ANOMALY

74 All of the dynamics of a classical system can be determined from a function called the
75 Lagrangian. The Lagrangian is equal to the difference in kinetic and potential energies of
76 the system. From this, the equations of motion can be derived using the Euler-Lagrange
77 equations: In one dimension, this is written as:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0, \quad (1.3.4)$$

78 where the single dot notation represents the first time derivative. Another important quantity
79 is called the action, defined as

$$S = \int L dt. \quad (1.3.5)$$

80 A very important principle in physics is that of least action: given an infinite number of paths
81 between two points, a particle will take the one which minimizes the action. Examples of
82 classical actions and their symmetries are summarized in Table 5. In quantum field theory,
83 the Lagrangian becomes a function of spacetime, and for a field ϕ , the Euler-Lagrange
84 equations become

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial [\partial_\mu \phi]} \right) = 0 \quad (1.3.6)$$

85 Symmetries arise from transformations that do not change the physics of a system.
86 Noether's theorem expresses the relationship between symmetries and conservation laws:
87 for every continuous transformation that leaves the action invariant there exists a time de-
88 pendent classical charge Q and corresponding conserved current, $\partial_\mu J^\mu = 0$. The quantity
89 inside the parenthesis in Eq.1.3.6 is defined as the current J^μ . The conserved charge is found
90 by integrating the time component of the current,

$$Q = \int d^3x J^0. \quad (1.3.7)$$

TABLE 5: Symmetries and Conservation Laws

Symmetry	Conservation Law
Translation in time	Energy
Translation in space	Momentum
Rotation	Angular Momentum
Gauge transformation	Charge

91 In general, there are three different types of symmetries: exact, broken, and approxi-
 92 mate. Exact symmetries are valid under any condition; approximate symmetries are valid
 93 under certain conditions; and broken symmetries arise under various circumstances where
 94 the Lagrangian is still invariant under the given transformation but the ground state is not.
 95 When a classical symmetry is broken in the realm of quantum physics it is an anomalous
 96 symmetry.

97 The chiral symmetry, $SU_L(3) \times SU_R(3)$, of QCD occurs when the three light quarks: u,
 98 d, and s are massless. This symmetry is spontaneously broken which gives rise to the octet
 99 of massless Goldstone bosons, which are identified as the pseudoscalar pions, kaons, and eta
 100 mesons. A chiral transformation is one of the form:

$$\Psi \rightarrow \Psi' = e^{-i\theta\gamma_5}\Psi, \quad (1.3.8)$$

101 where θ measures the rotation and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is the product of the four gamma matrices.
 102 As a straightforward example, one can apply the chiral transformation to demonstrate the
 103 conservation of the axial vector current, $j^{\mu 5} = \bar{\Psi}\gamma_\mu\gamma_5\Psi$, using the Dirac Lagrangian

$$L_{Dirac} = \bar{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi. \quad (1.3.9)$$

104 The wavefunctions under a chiral transformation become

$$\begin{aligned} \Psi &\rightarrow \Psi' = e^{-i\theta\gamma^5}\Psi \\ \bar{\Psi} &\rightarrow \bar{\Psi}' = \bar{\Psi}e^{i\theta\gamma^5}. \end{aligned} \quad (1.3.10)$$

105 Using the axial vector current and the Dirac equation, the divergence can be calculated as

$$\partial_\mu j^{\mu 5} = (\partial_\mu\bar{\Psi})\gamma^\mu\gamma^5\Psi - \bar{\Psi}\gamma^5\gamma^\mu\partial_\mu\Psi = 2im\bar{\Psi}\gamma_5\Psi. \quad (1.3.11)$$

106 Clearly, if the mass of the particle is zero the axial vector current is conserved. When a
 107 gauge field (A_μ) is present, as in,

$$\mathcal{L} = \bar{\Psi}[i\gamma^\mu(\partial_\mu - ieA_\mu) - m]\Psi, \quad (1.3.12)$$

108 one can find that the axial vector current is no longer conserved even when the particle mass
109 is zero:

$$\partial^\mu j_{5\mu} = -\frac{e^2}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}, \quad (1.3.13)$$

110 where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-
111 Civita tensor, and e is the electric charge. This result was first discovered by Adler, Bell,
112 and Jackiw and is known as the ABJ or chiral anomaly.

113 1.4 QCD AND THE WZW LAGRANGIAN

114 Quantum Chromodynamics (QCD) is the theory that describes the strong interaction
115 that exists between the quarks and gluons in hadrons. Two main features of QCD include:

- 116 1. Color confinement: This is why we do not observe individual quarks themselves. In
117 order to separate quarks inside a hadron, increasing amounts of energy are required.
118 This would become so great that a quark-antiquark pair would be produced, resulting
119 in a pair of hadrons.
- 120 2. Asymptotic freedom: The asymptotic freedom of QCD was discovered by David Gross
121 and Frank Wilczek and independently by David Politzer. Asymptotic freedom de-
122 scribes a reduction in the strength of interactions between the quarks and gluons as
123 the energy scale increases.

124 The QCD Lagrangian is given by:

$$\mathcal{L}_{QCD} = -\frac{1}{2} \text{Tr}[G_{\mu\nu} G^{\mu\nu}] + \bar{q}(i\gamma_\mu D^\mu - m)q \quad (1.4.14)$$

125 where

$$\begin{aligned} G_{\mu\nu} &= \partial_\mu G_\nu - \partial_\nu G_\mu - ig[G_\mu, G_\nu] \\ D_\mu q &= (\partial_\mu - igG_\mu)q \end{aligned} \quad (1.4.15)$$

126 and where $G_\mu = G_\mu^a \lambda^a / 2$ is the vector field of the gluons and $G_{\mu\nu}$ is the field strength
127 tensor. In low energy QCD, Chiral Perturbation Theory exploits the global $SU(3)_L \times SU(3)_R$
128 symmetry in the limit of vanishing quark masses. Following Witten's approach in [5], the
129 lowest order effective chiral action is:

$$S = \frac{f_\pi^2}{4} \int d^4x \text{Tr}[(D_\mu U)(D^\mu U^\dagger)] \quad (1.4.16)$$

130 with U being the chiral unitary matrix

$$U = \exp\left(\frac{i\sqrt{2}}{f_\pi}P\right) \quad (1.4.17)$$

131 and with P being the pseudoscalar field matrix

$$\left(\begin{array}{ccc} P = \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{array}\right). \quad (1.4.18)$$

132 The Wess-Zumino-Witten Lagrangian is of the form

$$\begin{aligned} \Gamma_{WZW} &= \frac{iN_c}{240\pi^2} \int_M d\Sigma^{ijklm} Tr((U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U)(U^\dagger \partial_l U)(U^\dagger \partial_m U)) \\ &= -\frac{N_c}{240\pi^2 F_\pi^5} \int dx^4 Tr(\phi^a \partial_\mu \phi^a \partial_\nu \phi^a \partial_\alpha \phi^a \partial_\beta \phi^a) \epsilon^{\mu\nu\alpha\beta} + \text{higher orders} \end{aligned} \quad (1.4.19)$$

133 where N_c is the number of colors and F_π is the pion decay constant. When the Lagrangian
134 is coupled to a photon field A_μ , it is of the form [24]

$$\begin{aligned} \mathcal{L}_{WZW}(UA_\mu) &= \mathcal{L}_{WZW}(U) \\ &+ \frac{eN_c}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \int d^4x A_\mu Tr(Q(R_\nu R_\alpha R_\beta + L_\nu L_\alpha L_\beta)) \\ &- \frac{iN_c e^2}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} \int d^4x F_{\mu\nu} A_\alpha Tr\left(Q^2(R_\beta + L_\beta) + \frac{1}{2}(QU^\dagger QU R_\beta + QUQU^\dagger L_\beta)\right) \end{aligned} \quad (1.4.20)$$

135 where Q is the quark charge matrix, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength
136 tensor, and R_μ and L_μ are defined as $(\partial_\mu U^\dagger)U$ and $U\partial_\mu U^\dagger$, respectively. The first term
137 corresponds to the quintangle anomaly $K^+K^- \rightarrow \pi^+\pi^-\pi^0$. The second term represents the
138 QCD box anomaly and the third term corresponds to the triangle anomaly. The triangle
139 and box diagrams are shown in Fig.3.

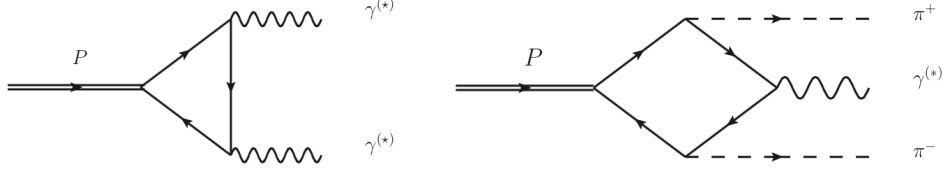


FIG. 3: The decay of a pseudoscalar meson P to two photons and to $\pi^+\pi^-\gamma$. The AVV anomaly (left) describes the coupling of two vector mesons and a pseudoscalar, while the VAAA anomaly (right) describes the coupling of three pseudoscalar mesons and a vector meson.

140 At the chiral limit ($s_{\pi^+\pi^-} = 0$), the decay amplitude can be written [24] from the box
 141 anomaly term of the \mathcal{L}_{WZW} coupled to a photon field:

$$\mathcal{A}_{\eta \rightarrow \pi^+\pi^-\gamma}(0, 0, 0) = \frac{eN_c}{12\sqrt{3}\pi^2 F_\pi^3} \left(\frac{F_\pi}{F_8} \cos \theta - \sqrt{2} \frac{F_\pi}{F_0} \sin \theta \right) \epsilon^{\mu\nu\alpha\beta} \epsilon_\mu^* p_{+\nu} p_{-\alpha} k_{\gamma\beta} \quad (1.4.21)$$

142 where F_0 and F_8 are the decay constants for the singlet and octet η states, θ is the mixing
 143 angle, p_\pm and k are the momenta of the pions and photon, and ϵ^* is the polarization of
 144 the photon. The resulting decay rate $\Gamma_{\eta \rightarrow \pi^+\pi^-\gamma} = 35.7$ eV [24] is nearly a factor of two
 145 smaller than the experimental value of $\Gamma_{\eta \rightarrow \pi^+\pi^-\gamma} = 60 \pm 4$ eV. The physical decay region,
 146 $4m_{\pi^\pm}^2 \leq s_{\pi^+\pi^-} \leq m_\eta^2$, is far from this region, thus proper inclusion of the momenta of the
 147 pions and final state interactions is essential.

148 1.5 DECAY RATE AND AMPLITUDE CALCULATIONS

149 In this section we will derive the amplitude and the decay rate for the decay $\eta \rightarrow \pi^+\pi^-\gamma$
 150 following the procedure in [6]. For the reaction $P(P) \rightarrow \pi^+(p_+)\pi^-(p_-)\gamma(k)$, momentum
 151 conservation gives

$$P = p_+ + p_- + k. \quad (1.5.22)$$

152 The invariant decay amplitude can be written as given in [6]

$$\mathcal{A}(\eta \rightarrow \pi^+\pi^-\gamma) = \frac{i}{m_\eta^3} \left(M_G \epsilon_{\mu\nu\alpha\beta} \epsilon^\mu k^\nu p_+^\alpha p_-^\beta + E_g [(\epsilon \cdot p_+)(k \cdot p_-) - (\epsilon \cdot p_-)(k \cdot p_+)] \right), \quad (1.5.23)$$

153 where M and E are the magnetic and electric form factors. To calculate the squared ampli-
 154 tude, and eventually the decay rate, we can insert $p_- = P - p_+ - k$ to take advantage of the

155 antisymmetric tensor:

$$\mathcal{A}(\eta \rightarrow \pi^+ \pi^- \gamma) = \frac{i}{m_\eta^3} \left(M_G \epsilon_{\mu\nu\alpha\beta} \epsilon^\mu k^\nu p_+^\alpha P^\beta + E_G [(\epsilon \cdot p_+)(k \cdot P) - (\epsilon \cdot P)(k \cdot p_+)] \right). \quad (1.5.24)$$

In the rest frame of the η , $P^\mu = m_\eta \delta^{\mu 0}$, the amplitude then reads

$$\mathcal{A}(\eta \rightarrow \pi^+ \pi^- \gamma) = \frac{i}{m_\eta^2} \left(-M_G \epsilon^{ijk} \epsilon^i \tilde{k}^j \tilde{p}_+^k + E_G [(-\epsilon \cdot \tilde{\mathbf{p}}_+)(\tilde{E}_\gamma)] \right)$$

156

$$= \frac{i \tilde{E}_\gamma}{m_\eta^2} \left(M_G \hat{\mathbf{k}} \cdot (\boldsymbol{\epsilon} \times \tilde{\mathbf{p}}_+) - E_G \boldsymbol{\epsilon} \cdot \tilde{\mathbf{p}}_+ \right). \quad (1.5.25)$$

157 Squaring the amplitude yields

$$\begin{aligned} |\mathcal{A}|^2(\eta \rightarrow \pi^+ \pi^- \gamma) &= \frac{\tilde{E}_\gamma^2}{m_\eta^4} \left(|M_G|^2 |\hat{\mathbf{k}} \cdot (\boldsymbol{\epsilon} \times \mathbf{p}_{+\perp})|^2 + |E_G|^2 |\boldsymbol{\epsilon} \cdot \mathbf{p}_{+\perp}|^2 \right. \\ &\quad \left. + E_G^* M_G \left[\hat{\mathbf{k}} \cdot (\mathbf{p}_{+\perp} \times \boldsymbol{\epsilon}) \right] (\boldsymbol{\epsilon} \cdot \mathbf{p}_{+\perp})^* + M_G^* E_G \left[\hat{\mathbf{k}} \cdot (\mathbf{p}_{+\perp} \times \boldsymbol{\epsilon}) \right]^* (\boldsymbol{\epsilon} \cdot \mathbf{p}_{+\perp})^* \right). \end{aligned} \quad (1.5.26)$$

158 Ref. [6] defines the polarization vectors as:

$$\boldsymbol{\epsilon}_1 = \frac{(\mathbf{p}_+ \times \mathbf{k}) \times \mathbf{k}}{|(\mathbf{p}_+ \times \mathbf{k}) \times \mathbf{k}|} = \frac{\hat{\mathbf{k}}(\mathbf{p}_+ \cdot \hat{\mathbf{k}}) - \mathbf{p}_+}{|\hat{\mathbf{k}}(\mathbf{p}_+ \cdot \hat{\mathbf{k}}) - \mathbf{p}_+|} = -\hat{\mathbf{p}}_{+\perp} \quad (1.5.27)$$

159 and

$$\boldsymbol{\epsilon}_2 = \frac{\mathbf{p}_+ \times \mathbf{k}}{|\mathbf{p}_+ \times \mathbf{k}|} = \frac{\mathbf{p}_{+\perp} \times \hat{\mathbf{k}}}{|\mathbf{p}_{+\perp} \times \hat{\mathbf{k}}|} = \hat{\mathbf{k}} \times (-\hat{\mathbf{p}}_{+\perp}). \quad (1.5.28)$$

160 This results in an unpolarized square decay amplitude:

$$\sum_{\text{pol}=1}^2 |\mathcal{A}|^2(\eta \rightarrow \pi^+ \pi^- \gamma) = \frac{\tilde{E}_\gamma^2 |\mathbf{p}_+^*|^2 \sin^2 \theta_\pi}{m_\eta^4} (|M_G(s_{\pi\pi})|^2 + |E_G|^2). \quad (1.5.29)$$

161 Rewriting Eq. 1.5.29 in terms of standard variables $s_{\pi\pi}$ and θ_π is

$$\sum_{\text{pol}=1}^2 |\mathcal{A}|^2(\eta \rightarrow \pi^+ \pi^- \gamma) = \frac{\lambda(m_\eta^2, s_{\pi\pi}, 0) s_{\pi\pi} \beta_\pi^2 \sin^2 \theta_\pi}{16 m_\eta^2} (|M_G|^2 + |E_G|^2), \quad (1.5.30)$$

162 where

$$\beta_\pi = \sqrt{1 - \frac{4m_\pi^2}{s_{\pi\pi}(\tilde{E}_\gamma)}}. \quad (1.5.31)$$

163 The partial decay rate for $\eta \rightarrow \pi^+\pi^-\gamma$ is given by eq.(46.19) in [2].

$$\begin{aligned}
 d\Gamma &= \frac{1}{(2\pi)^5} \frac{1}{16m_\eta^2} |\mathcal{A}|^2 |\mathbf{P}_+^*| |\tilde{E}_\gamma| dm_{\pi\pi} d\cos\theta_\pi d\phi_\pi^* d\cos\tilde{\theta}_\gamma d\tilde{\phi}_\gamma \\
 &= \frac{1}{2^{12}\pi^3} \left(1 - \frac{s_{\pi\pi}}{m_\eta^2}\right)^3 \frac{s_{\pi\pi}^{3/2}}{m_\eta^3} \beta_\pi^3 \sin^2\theta_\pi (|M_G(s_{\pi\pi})|^2 + |E_G|^2) d\sqrt{s_{\pi\pi}} d\cos\theta_\pi,
 \end{aligned}
 \tag{1.5.32}$$

164 where

$$M_G(s_{\pi\pi}) = m_\eta^3 \mathcal{M}(s_{\pi\pi}, k^2 = 0), \tag{1.5.33}$$

165 and where

$$M(s_{\pi\pi}, k^2 = 0) = \mathcal{A}_{\eta \rightarrow \pi\pi\gamma}(0, 0, 0) \times F. \tag{1.5.34}$$

166 Eq.1.5.34 is a product of the chiral limit amplitude with a form factor F. For the specific
 167 decay of interest, \mathcal{M} is equal to Eq.1.4.21. To leading order, the electric form factor, $|E_G|$ is
 168 set to zero. In the next section, we describe different approaches to describe the form factor
 169 F in Eq.1.5.34.

170 1.6 MODEL PREDICTIONS FOR $\eta \rightarrow \pi^+\pi^-\gamma$

171 Proper inclusion of the final state interactions has been attempted using various theoret-
 172 ical models. Each of these models are briefly summarized below.

173 a) VMD: In this model, the decay amplitude is modified either using the simplest possible

174

$$F_V(s_{\pi\pi}) = -\frac{m_\rho^2}{s_{\pi\pi} - m_{\rho^2} i m_\rho \Gamma_\rho} \tag{1.6.35}$$

175 The resulting total decay width is $\Gamma^{VMD}(\eta \rightarrow \pi^+\pi^-\gamma) = 62.3$ eV.

176 b) N/D: The $\pi\pi$ final state interactions are taken into account using contributions from
 177 both vector meson dominance as well as one-loop corrections. The modification to the
 178 decay amplitude is

$$\left(1 - c + c \cdot \frac{1 + a s_{\pi\pi}}{D_1(s_{\pi\pi})}\right) \tag{1.6.36}$$

179 The Omnes function, $D_1(s_{\pi\pi})$ incorporates $\pi\pi$ scattering phase space. The parameter c
 180 is set to 1, while $a = \frac{1}{2m_{\rho^2}}$. The modified decay width using this approach is $\Gamma^{N/D}(\eta \rightarrow$
 181 $\pi^+\pi^-\gamma) = 65.7$ eV [24], and the predicted value for the α parameter is 0.64 ± 0.02 .

182 c) Inclusion of pion loop corrections and higher order momenta on the order $\mathcal{O}(p^6)$. The
 183 form factor was modified using the one-loop formulation

$$F_V(s_{\pi\pi}) = 1 + \frac{1}{6f_\pi^2}(s_{\pi\pi} - 4m_\pi^2)\bar{J}(s_{\pi\pi}) + \frac{s_{\pi\pi}}{6} \left(\langle r^2 \rangle + \frac{1}{24\pi^2 f_\pi^2} \right). \quad (1.6.37)$$

184 This leads to the predicted decay width of $\Gamma(\eta \rightarrow \pi^+\pi^-\gamma) = 47 \text{ eV}$ [19]. The prediction
 185 for the α parameter is -0.7 ± 0.1 .

186 d) Uses the Hidden Local Symmetries (HLS) model, which describes γ -V transitions. The
 187 vector mesons are treated as degrees of freedom with pseudoscalar mesons and contact
 188 terms. The Lagrangian describing the $\eta \rightarrow \pi^+\pi^-\gamma$ decay consists of both a resonant
 189 term and a contact term, which contains the box anomaly term. The decay rate
 190 calculated from this model is $\Gamma(\eta \rightarrow \pi^+\pi^-\gamma)^{HLS} = 56.3 \pm 1.7 \text{ eV}$ [21]. The prediction
 191 for the α parameter is 0.23 ± 0.01 .

192 e) Kubis and Plenter [40] consider the effects of the a_2 tensor meson, which is the lowest
 193 lying resonance that can contribute in the $\pi\eta$ system. This analysis did not report a
 194 predicted decay width and instead provided a new prediction of the α parameter to be
 195 1.52 ± 0.06 .

196 Models a-d essentially modify the amplitude in the chiral limit, Eq. 5, by a form factor
 197 specific to each model. The predicted decay widths are in better agreement with the exper-
 198 imental value compared to the decay width from the chiral limit. The shape of the dipion
 199 invariant mass spectra is more sensitive to the description of the final state interactions
 200 which will give more insight into the agreements between the experimental decay rates and
 201 the theoretical predictions mentioned previously.

202 1.7 MODEL-INDEPENDENT ANALYSIS OF $\eta \rightarrow \pi^+\pi^-\gamma$

203 The approach to obtain the α parameter is based on [7] and will be described briefly
 204 below. It combines chiral perturbation theory (ChPT), extended from SU(3) to U(3), and
 205 a dispersive analysis. It is general and can be applied to all decays of mesons with a $\pi\pi\gamma$
 206 final state, where soft bremsstrahlung does not occur and where the pion pair is of invariant
 207 mass square below the first significant $\pi\pi$ threshold. The need for this approach arises from
 208 the significant deviation between the theory predictions and data, which mostly stems from
 209 the non-perturbative $\pi\pi$ final state interaction.

210 The method involves two main steps. The first step is to fit the spectral decay data with
 211 a function of the form

$$\frac{d\Gamma}{ds_{\pi\pi}} = |AP(s_{\pi\pi})F_V(s_{\pi\pi})|^2\Gamma_0(s_{\pi\pi}), \quad (1.7.38)$$

212 where the normalization parameter A has the dimensions of mass⁻³ and where

$$\Gamma_0 = \frac{1}{3 \cdot 2^{11} \cdot \pi^3 m_\eta^3} (m_\eta^2 - s_{\pi\pi})^3 s_{\pi\pi} \sigma(s_{\pi\pi})^3, \quad (1.7.39)$$

213 and

$$\sigma(s_{\pi\pi}) = \sqrt{1 - \frac{4m_\pi^2}{s_{\pi\pi}}}. \quad (1.7.40)$$

214 represents the phase-space terms and the kinematics of the absolute square of the simplest
 215 gauge invariant matrix element (for point particles). The decay rate is equivalent to the
 216 decay rate derived in Section 1.4, provided the electric form factor is set to zero in the
 217 leading order, and the form factor is replaced with the pion vector form factor and the
 218 reaction specific polynomial $P(s_{\pi\pi})$.

219 The pion vector form factor $F_V(s_{\pi\pi})$ is well known from both theory ([8]-[14]) and mea-
 220 surements of $e^+e^- \rightarrow \pi^+\pi^-$ ([15]-[18]). It collects all non-perturbative $\pi\pi$ interactions and
 221 is universal. It is defined in terms of the vector-isovector current:

$$\langle \pi^+(p')\pi^-(p) | V_\mu^3 | 0 \rangle = (p - p')_\mu F_V(s_{\pi\pi}). \quad (1.7.41)$$

222 In the elastic regime, the form factor is defined as

$$F_V = M_V + T_{\pi\pi} G_{\pi\pi} M_V, \quad (1.7.42)$$

223 where M_V , $G_{\pi\pi}$, and $T_{\pi\pi}$ stand for the production vertex, the two-pion propagator and the
 224 $\pi\pi$ scattering amplitude, respectively. In this approach, M_V is assumed to be real. From
 225 Eq. 1.6.35, we can write

$$\text{Im}(F_V(s_{\pi\pi})) = \sigma(s_{\pi\pi}) T_{\pi\pi}(s_{\pi\pi})^* F_V(s_{\pi\pi}). \quad (1.7.43)$$

226 It is at this point where the variance between different models can be seen. In terms of Vector
 227 Meson Dominance, the relative strength of the first and second term is a free parameter.
 228 With the model independent approach used for our analysis, this is not a possibility. Instead,
 229 the authors use a twice subtracted dispersion integral:

$$F_V(s_{\pi\pi}) = \exp\left(\frac{1}{6} s_{\pi\pi} \langle r^2 \rangle + \frac{s_{\pi\pi}^2}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_{11}(s)}{s^2(s - s_{\pi\pi} - i\epsilon)}\right), \quad (1.7.44)$$

230 where $\langle r^2 \rangle$ is the mean pion radius and δ_{11} is the phase, in order to guarantee that the
 231 integral over the phase space converges in the elastic regime. For our kinematic region, the
 232 pion vector form factor is approximated as

$$|F_V(s_{\pi\pi})| \approx 1 + (2.12 \pm 0.01)s_{\pi\pi} + (2.13 \pm 0.01)s_{\pi\pi}^2 + (13.80 \pm 0.14)s_{\pi\pi}^3. \quad (1.7.45)$$

233 The form factor contains both the Born term as well as the final state interactions. The
 234 right plot in Fig. 4 shows the form factor in the appropriate kinematic range for the η decay.
 235 From the plot, visible differences between the one loop calculation and the result derived
 236 from the dispersion integral are already apparent.

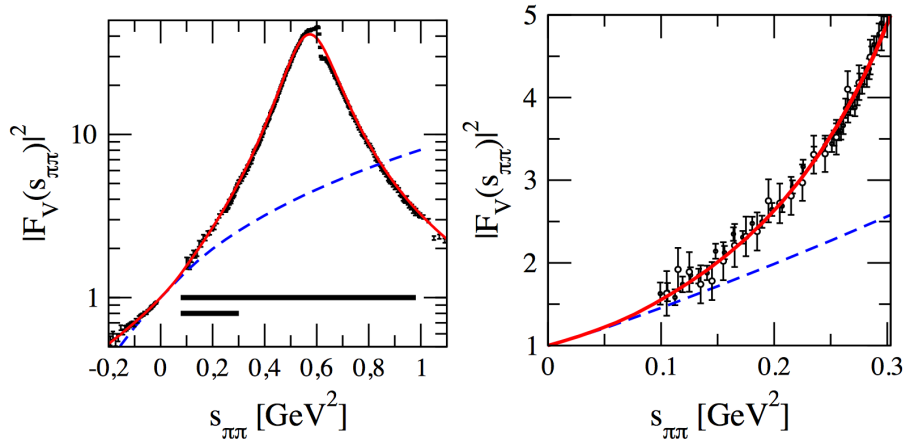


FIG. 4: The (red) solid band shows the form factor derived from Eq. 1.6.39, the (blue) dashed line is the result from one-loop ChPT with identical values of the pion radius. The time-like data is shown as solid and open circles, respectively. The space-like data are from. The short (long) thick, horizontal bar in the left panel denotes the kinematic range covered in the decay of the η (η') meson. The right panel shows a zoom into the $s_{\pi\pi}$ range relevant for the η decay.

237 The normalization parameter A and the function $P(s_{\pi\pi})$ are reaction specific. The poly-
 238 nomial $P(s_{\pi\pi})$ is expanded in a Taylor series around $s_{\pi\pi} = 0$,

$$P(s_{\pi\pi}) = 1 + \alpha s_{\pi\pi} + \mathcal{O}(s_{\pi\pi}^2). \quad (1.7.46)$$

239 The parameters A and α allow insights into the physics underlying the decay process. This
 240 is described in the second step of the method by developing a matching scheme to relate A
 241 and α to the parameters of the underlying effective field theory.

CHAPTER 2

CEBAF, CLAS, AND THE G11 EXPERIMENT AT

JEFFERSON LAB

To study fundamental and/or composite particles and their interactions, physicists can gain insight using three different methods. The first involves scattering events. This essentially corresponds to firing one particle at another and recording what happens. The second involves decays, in which one particle decays into many particles and we observe the decay products. The third method involves studying bound states, or the combination of two or more particles. At Jefferson Lab, these types of interactions are studied using an electron accelerator (CEBAF) that can send electrons (or for some Halls, photons) at different experimental targets.

This chapter describes the Continuous Electron Beam Accelerator Facility (CEBAF), the CEBAF Large Acceptance Spectrometer, and the G11 Experiment that took place at the Thomas Jefferson National Accelerator Facility located in Newport News, Virginia before the 12 GeV upgrade. Since the data for this thesis was taken in 2004, the detector and the lab facilities will be described as they were during data taking. The main components of CEBAF and CLAS are described and a brief overview of the G11 experiment is provided.

2.1 THOMAS JEFFERSON NATIONAL ACCELERATOR FACILITY

The data for this thesis was collected in Hall B at the Thomas Jefferson National Acceleration Facility (JLAB) during the Summer of 2004. At the time, JLAB was home to the Continuous Electron Beam Accelerator Facility (CEBAF), three Experimental Halls: A, B, and C, and numerous other research facilities. Hall B is home to the CEBAF Large Acceptance Spectrometer (CLAS) detector. The CLAS detector is composed of many detector systems in order to provide charged particle resolution and a large angular acceptance.



FIG. 5: Aerial view of the Thomas Jefferson National Accelerator Facility in Newport News, VA. The three experimental Halls can be seen in the lower right corner of the image.

266 2.2 CONTINUOUS ELECTRON BEAM ACCELERATOR FACILITY

267 Construction on the Continuous Electron Beam Accelerator Facility started on Febru-
 268 ary 13, 1987. The machine has a racetrack configuration with two superconducting linear
 269 accelerator (LINACs) segments connected by beam lines to allow recirculation passes. The
 270 superconducting radio frequency (SRF) cavities are used to accelerate electrons and provide
 271 beam to all experimental halls.

272 The electron beam is produced at the injector using an electron gun. Three diode lasers
 273 produce pulses which illuminate a GaAs photocathode. The pulses are timed so that each
 274 experimental hall receives electron bunches every 2 ns. Initially, the electrons are accelerated
 275 to 100 keV and then an optical chopper improves the separation of the bunches. Then,
 276 CEBAF uses superconducting radio frequency (SRF) cavities in order to accelerate the
 277 electrons. Superconducting cavities are non-resistive, allowing CEBAF to obtain a 100%
 278 duty factor. The electrons are then sent into the recirculating linear accelerators (LINACs).
 279 Fig. 6 displays a pair of Niobium SRF cavities.



FIG. 6: Pair of superconducting Niobium cavities.

280 Each LINAC consists of 168 superconducting RF Niobium cavities. Each cavity is im-
 281 mersed in liquid Helium within a cryomodule and is cooled to -271deg C . Each LINAC is
 282 capable of providing 600 MeV of acceleration. The LINACs are connected by 9 recirculating
 283 arcs. The arcs allow the beam to make up to five passes through each LINAC, resulting in
 284 a maximum energy of ≈ 6 GeV. Each hall can extract the beam after any number of passes,
 285 although no two halls can run with the same lower energy.

286 The operating conditions and parameters for CEBAF are summarized in Table ??.

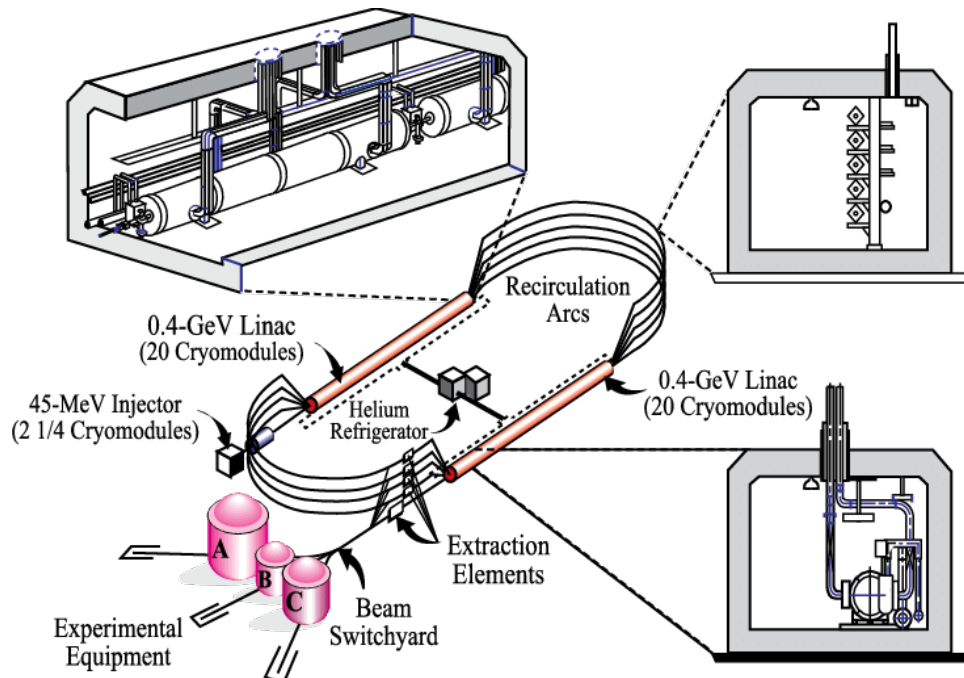


FIG. 7: Schematic of the Continuous Electron Beam Accelerator Facility (CEBAF).

287 2.3 PHOTON TAGGER

288 For our experiment, the electron beam must be converted into a photon beam. This is
 289 accomplished with the photon tagging system, shown in Fig. 8. The electron beam interacts
 290 with a gold foil radiator, producing photons via Bremsstrahlung radiation. Bremsstrahlung,
 291 from *bremesen* "to brake" and *strahlung* "radiation", refers to the process of producing ra-
 292 diation from decelerated charged particles deflected from nuclei. Since the nucleus of an
 293 atom is much heavier than the electron, the energy transferred to the nucleus is negligible.
 294 To satisfy conservation of energy, the energy of the Bremsstrahlung photon is equal to the
 295 difference of the incident electron energy and the deflected electron energy. For the setup
 296 in CLAS, the Bremsstrahlung photons and the decelerated electrons are traveling along the
 297 same direction as the initial incident electron.

298 A radiator with 10^{-4} radiation lengths was used during the production runs, while a
 299 much thinner radiator was used during normalization runs. The radiation length refers to
 300 the thickness of a material required to reduce the mean energy of an electron by a factor of
 301 e . It is defined as

$$\frac{1}{L_0} = \frac{4Z(Z+1)r_e^2 N_0}{137 \cdot A} \cdot \ln \frac{183}{Z^{1/3}}, \quad (2.3.47)$$

302 where $r_e = \frac{e^2}{mc^2}$, N_0 is Avogadro's number, Z (A) is the atomic (mass) number of the
 303 nucleus, and where e (m) is the charge (mass) of the electron.

304 After interacting with the Gold foil target, the beam now consists of a mix of electrons
 305 and photons. A dipole magnetic field is used to separate electrons from the photon beam
 306 and allows the photons to proceed towards the CLAS target. The magnetic field directed
 307 the recoil electrons towards two hodoscope planes, each made of overlapping arrays of scin-
 308 tillators. The first hodoscope plane, called the E plane, measures the momentum of the
 309 recoil electrons and the energy of the emitted photon. The second hodoscope, called the T
 310 plane, measures the timing information needed for the coincidence with events triggered by
 311 the interaction of corresponding photons in the target. Together, the E and T planes can
 312 tag photon energies in the range of 20-95% of the initial electron energy. Fig. 9 shows a
 313 schematic of the E and T planes.

314 The remaining components of the tagging system are collimators. These are used to
 315 trim the beam halos. There are sweeping magnets between the collimators to remove any
 316 unwanted particles from the interaction of the photon beam with the first collimator.

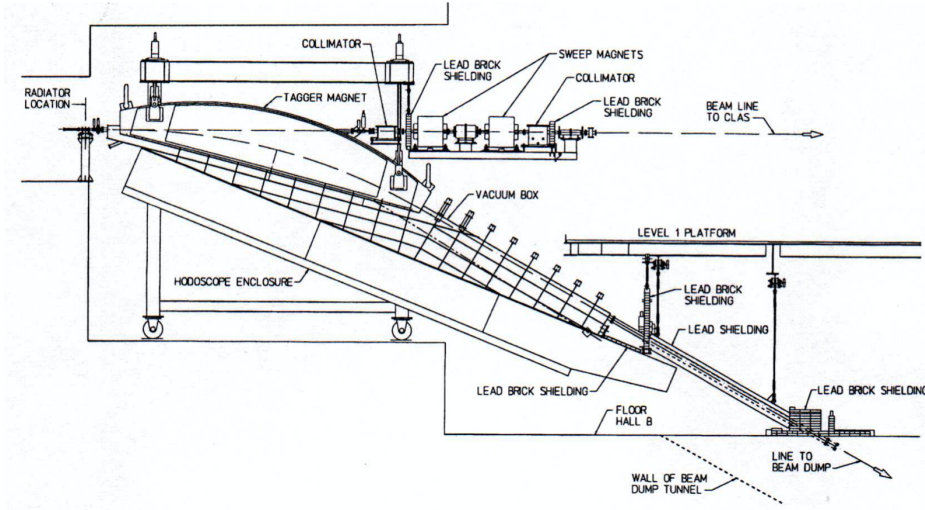


FIG. 8: Photon Tagging system in Hall B.

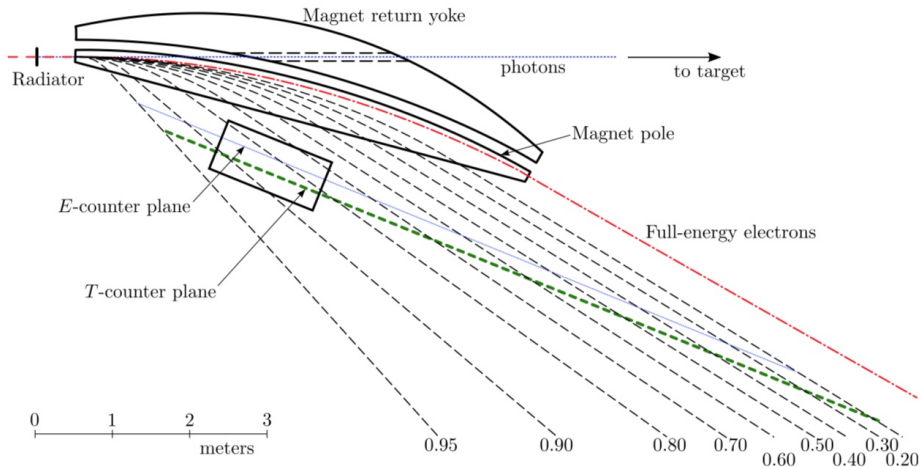


FIG. 9: Diagram of tagger E and T planes.

317 The tagging system is also used to calculate the event start time. The timing system can
 318 identify which RF beam bucket the photon is associated with. This is useful to calculate the
 319 event vertex time, which is the time when the final state particles were located in the same
 320 physical location, i.e. the event vertex. Complete details of the photon tagging system can
 321 be found in [25].

322 2.4 CEBAF LARGE ACCEPTANCE SPECTROMETER

323 The CEBAF Large Acceptance Spectrometer (CLAS) detector is located in Hall B. The

324 detector consists of many layers of sub-components. Fig.11 shows a schematic of the detector.
 325 The detector was designed based on a toroidal magnetic field, and optimized to measure
 326 charged particles with good momentum resolution, geometrical coverage of charged particles
 327 to large angles in the laboratory, and keep the region around the target free of a magnetic
 328 field magnetic so that dynamically polarized targets can be used. The CLAS Collaboration
 329 consists of many institutions throughout the world whose members designed, assembled,
 330 commissioned and continue to operate the detector to this day.

331 The detector is used to measure momenta and angles of outgoing charged particles pro-
 332 duced from the interaction of the photon beam with target. The components of the detector
 333 are laid out in an onion like pattern surrounding the target, with six visible sectors.

334 Each component will be briefly described below.

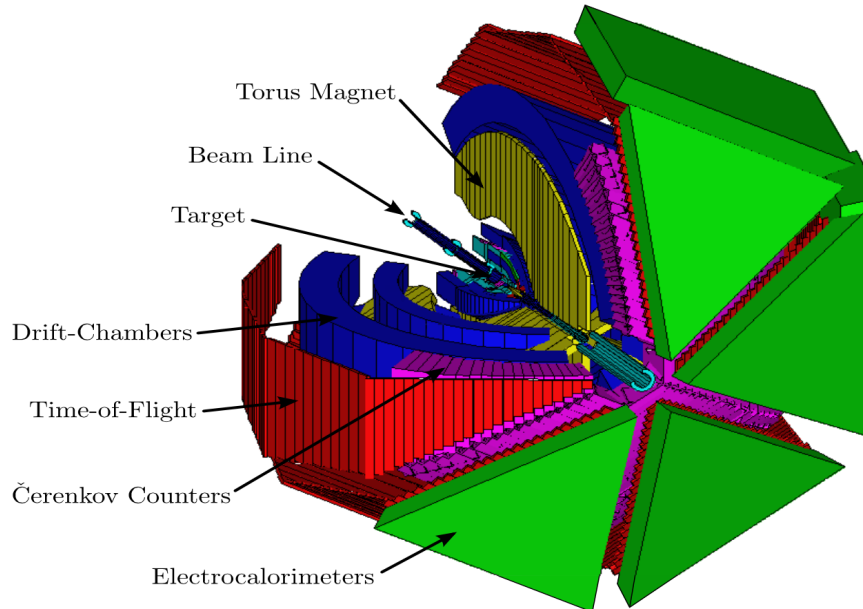


FIG. 10: A schematic of the CLAS detector housed in Hall B at Jefferson Lab.

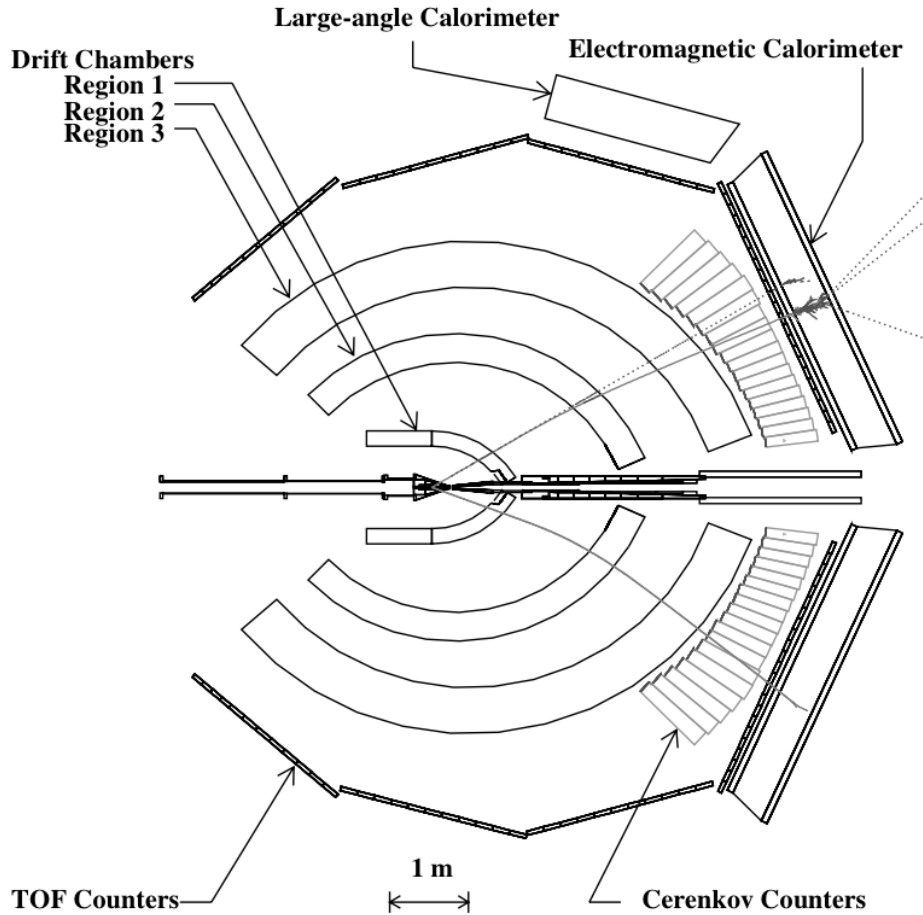


FIG. 11: A top view of the CLAS detector cut along the beam line. Typical photon, electron, and proton tracks are superimposed on the figure.

335 2.4.1 TARGET

336 The g11 target cell was cylindrical in shape and made from Kapton by CLAS technician
 337 Steve Christo. The target cell is 40 cm long with a radius of 2 cm. The target material was
 338 liquid Hydrogen. A schematic of the target cell is shown below.

TABLE 6: Specifications of the cryogenic target

Length	40.0 cm
Radius	2.0 cm
Temperature	19.3 K
Pressure	1122 mBar
Density	0.07177 g/cm ³



FIG. 12: A schematic of the target inside the CLAS detector.

339 **2.4.2 START COUNTER**

340 A new start counter was installed for the g11 run period. The start counter is divided to
341 six sectors each with four scintillator paddles. Each section was made of four 502 mm long
342 straight scintillator paddles with tapered ends that form the 30 mm long nose of the counter.
343 Each paddle was 29 mm wide and 2.15 mm thick and connected to an acrylic light guide
344 which was connected to a photomultiplier tube. The start counter measures the interaction
345 time of incident photons in the target by detecting any charged particle produced in the
346 reaction. The timing resolution is ≈ 400 ps. The timing information was included in the
347 Level 1 trigger during the experiment. More details on this detector element can be found
348 in [29].

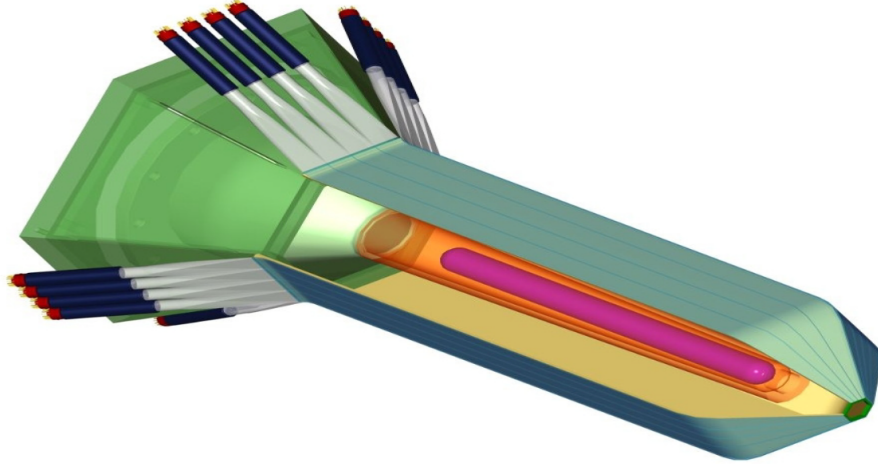


FIG. 13: A schematic of the new start counter installed before the g11 run.

349 2.4.3 SUPERCONDUCTING TOROIDAL MAGNET

350 The CLAS detector contained six kidney shaped superconducting toroidal magnets. Each
 351 coil is located in its own cryostat. Each coil is about 5 m long and 2.5 m wide with 60°
 352 separation in the azimuthal direction. Together, the 6 coils produced an approximately
 353 toroidal magnetic field, with the maximum field strength of 3.5 T.

354 When charged particles encounter magnetic fields, their trajectory is bent. During the
 355 g11 experiment, negatively charged particles were bent towards the beam pipe and positively
 356 charged particles were bent away from the beam pipe. Running at higher currents provides
 357 better momentum resolution but decreases the detector's acceptance for negative particles, it
 358 was decided that running at a lower current was the optimal choice. The charged particles in
 359 CLAS travel through the drift chambers. When the strength and direction of the magnetic
 360 field is known, the trajectory of the particle can be used to determine its momentum.

361 2.4.4 DRIFT CHAMBERS

362 In order to determine the momentum of charged particles, the particles must be tracked
 363 as they travel through the field generated by the magnet. The particles were tracked using
 364 three separate drift chamber regions. Region 1 is located inside the torus coils. Region 2
 365 was mounted directly to the magnet's cryostats, where the magnetic field was the strongest.
 366 Region 3 was positioned outside the torus coils. Region 1 and Region 3 are both located
 367 where the magnetic field is weak.

368 Each region of the drift chamber spans the same polar angle range and consists of two
369 superlayers of wires. Each superlayer contains six layers of hexagonal Gold-plated Aluminum
370 alloy field wire cells of $140\ \mu\text{m}$ diameter surrounding $20\ \mu\text{m}$ Gold-plated Tungsten sense wires.
371 The first superlayer had wires along the axial direction (perpendicular to the beamline).
372 The second superlayer has wires tilted at 6° with respect to the axial wires. Together, the
373 information from both superlayers allows us to reconstruct the polar and azimuthal angles.
374 There are 35,148 hexagonal cells in total that can detect charged particles with momenta
375 greater than $0.2\ \text{GeV}$ and cover a polar angle range from 8° to 143° . Each drift chamber is
376 filled with a 90% Argon and 10% Carbon Dioxide gas mixture. The gas mixture supports
377 drift velocities of $4\ \text{cm}/\mu\text{sec}$ and very high operational voltage.

378 Charged particles passing through the drift chambers ionize gas molecules. After the gas
379 is ionized, the electrons and ions move towards the sense and field wires, respectively. The
380 hit positions of the initial charged particles is found using the drift time information from the
381 detected signals and hit positions of the initial charged particles. More information about
382 the CLAS drift chamber system can be obtained from [30] and [31].

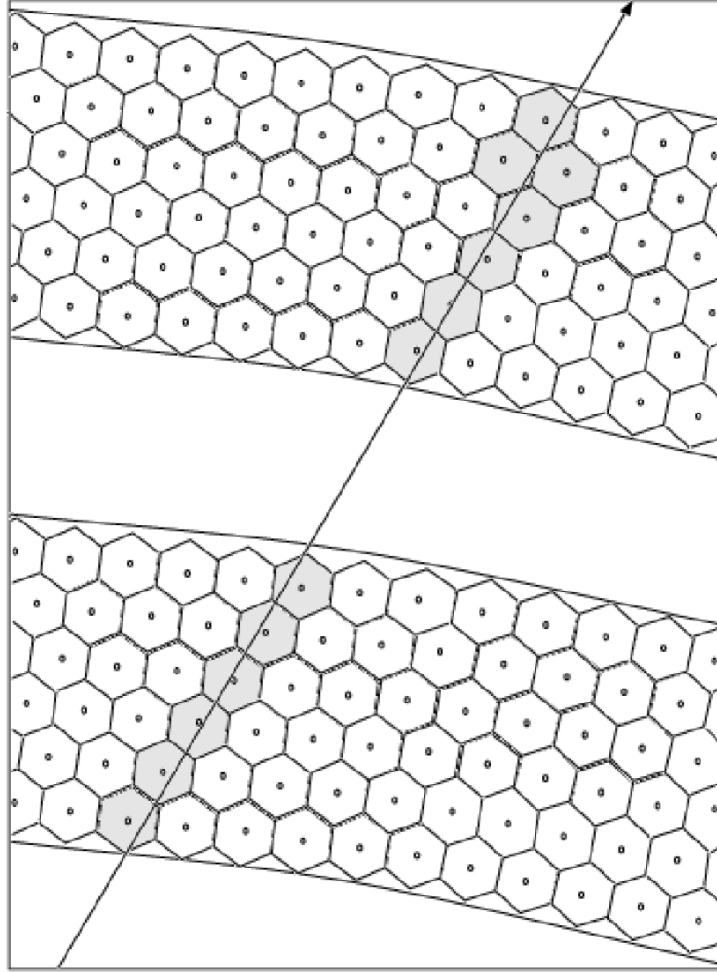


FIG. 14: A schematic of a section of the drift chambers showing two super layers. The wires are arranged in hexagonal cells. The sense wires are located in the center and the field wires are located at each corner of each cell. The track of a charged particle is depicted by the arrow passing through the drift chambers. The shaded hexagons represent hit cells.

383 2.4.5 TIME-OF-FLIGHT DETECTORS

384 The next detector element used during G11 consists of six segmented scintillator walls.
 385 Each wall has four panels with 57 scintillator paddles with variable lengths of 30 to 450 cm
 386 and widths of either 15 or 22 cm. The thickness of the paddles is 5.08 cm. The primary
 387 purpose of this TOF system is to measure the arrival time of charged tracks. The timing
 388 resolution was 80 to 160 ps, from shortest to longest paddle length, respectively. The timing
 389 resolution allows the separation of pions and protons up to a momentum of 2.5 GeV/c.

390 With the event vertex time measured from the photon tagger, the arrival time gives the
 391 total time of flight of the particle through the detector. Once this is known, the speed of
 392 the particle is obtained by dividing the path length by the arrival time. Then, knowing the
 393 momentum and speed of the particle allows us to compute its mass

$$m = \frac{p\sqrt{1 - \beta^2}}{\beta}, \quad (2.4.48)$$

394 thus identifying the particle. The details of the TOF system can be found in [30].

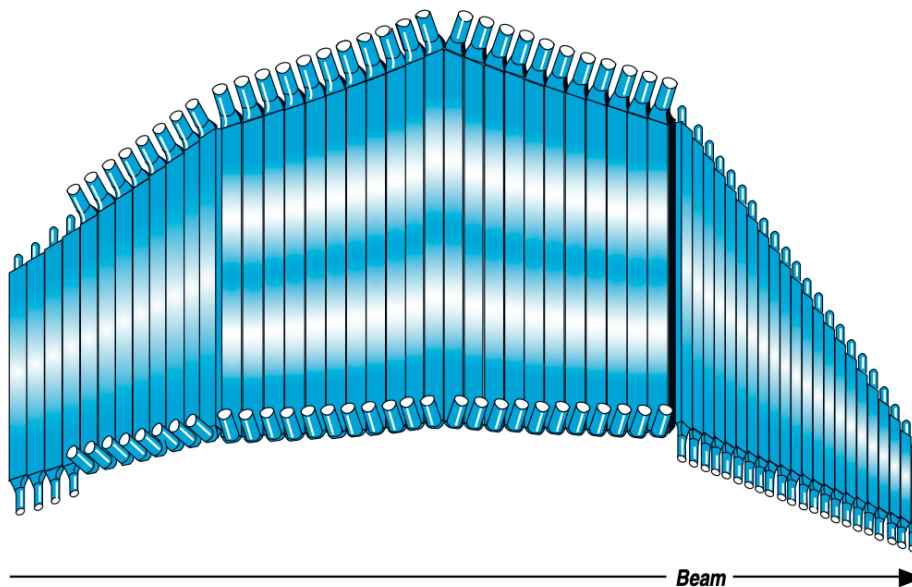


FIG. 15: Section of Time-of-Flight detector system used in each sector of CLAS. Each of the four panels consists of different sized scintillator paddles.

395 2.4.6 CHERENKOV DETECTORS

396 When a charged particle moves through a medium faster than the speed of light in that
 397 medium, Cherenkov radiation is produced. This is analogous to the shock wave produced
 398 by a jet traveling faster than the speed of sound. The gas Cherenkov detectors are used to
 399 discriminate between negative pions and electrons for momenta lower than 2.5 GeV *after*
 400 passing through the drift chambers. The critical angle, θ_c is defined as

$$\cos \theta_c = \frac{1}{n\beta}, \quad (2.4.49)$$

401 where $\beta = v/c$, and n is the index of refraction is unique to each particle.

Optical Mirror System

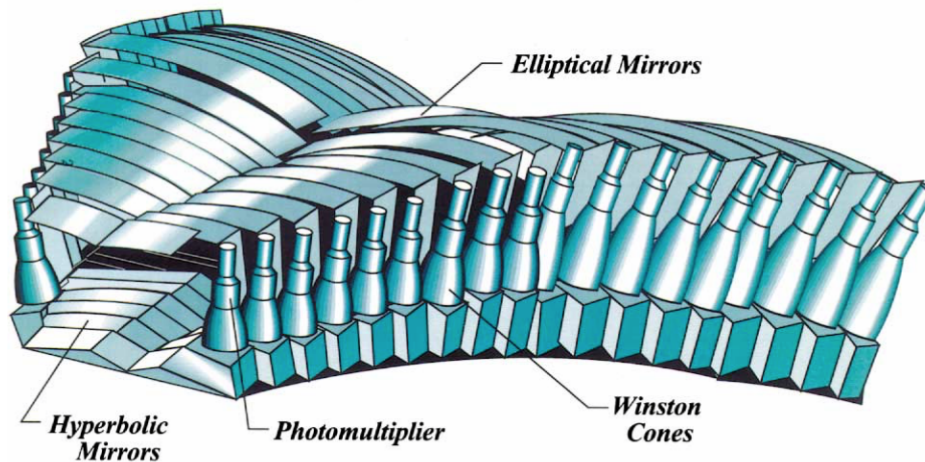


FIG. 16: A schematic of a single Cherenkov Counter showing mirrors and Winston tubes.

402 The gas used was perfluorobutane (C_4F_{10}). This was chosen due to its high index of
 403 refraction (1.00153) which produces a higher yield of photons. The minimum energy for a
 404 pion to produce Cherenkov radiation in this material is 2.7 GeV, while for electrons it is 9.2
 405 GeV. Complete details of the CLAS Cherenkov detector can be found in [26].

406 2.4.7 ELECTROMAGNETIC CALORIMETERS

407 The electromagnetic calorimeters are used to identify electrons above 0.5 GeV and pho-
 408 tons above 0.2 GeV. It is primarily used for reconstructing the radiative decays of the π^0 and
 409 η mesons, but is also used for leptons and neutral particles. The calorimeter is constructed
 410 of 39 layers consisting of alternating lead sheets and scintillator bars. The scintillators are
 411 arranged in three views, called U, V, and W, that allow for electromagnetic showers to
 412 be spatially located by the pixels created with the overlapping layers. The design of the
 413 calorimeter was such that the following conditions could be achieved:

- 414 1. e/γ energy resolution $\sigma/E \leq 0.1/\sqrt{E(\text{GeV})}$;
- 415 2. position resolution $\delta r \approx 2$ cm at 1 GeV;
- 416 3. π/e rejection greater than 99% at $E \leq 1$ GeV;
- 417 4. fast (< 100 ns) total energy sum for the event trigger;
- 418 5. mass resolution for two photon decays $\delta m/m \leq 0.15$;

419 6. neutron detection efficiency $> 50\%$ for $E_n > 0.5$ GeV;

420 7. time of flight resolution ≈ 1 ns.

421 More information on the electromagnetic calorimeters can be found in [32].

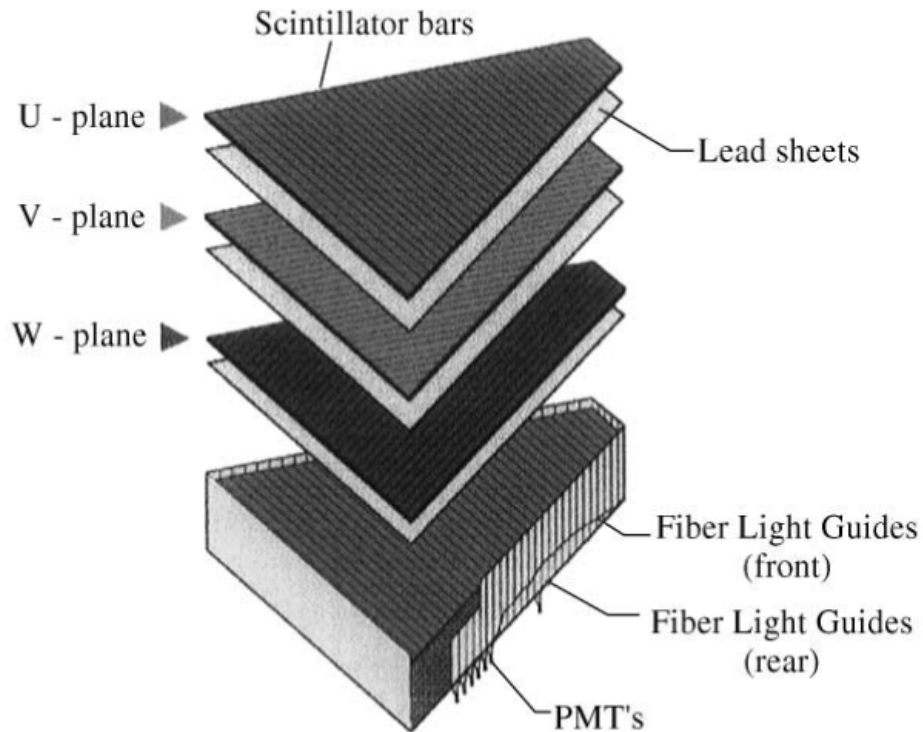


FIG. 17: View of the electromagnetic calorimeters U, V, and W planes.

422 2.5 BEAM LINE DEVICES

423 Aside from the CLAS detector, Hall B houses more instruments for various purposes.
 424 Upstream from the detector these instruments include: beam position monitors, harps, and
 425 current measuring devices to monitor the quality of the beam. Downstream, the total absorp-
 426 tion shower counter, pair spectrometer, and pair converter are used to measure the photon
 427 flux incident on the target. A brief description of these instruments is given below.

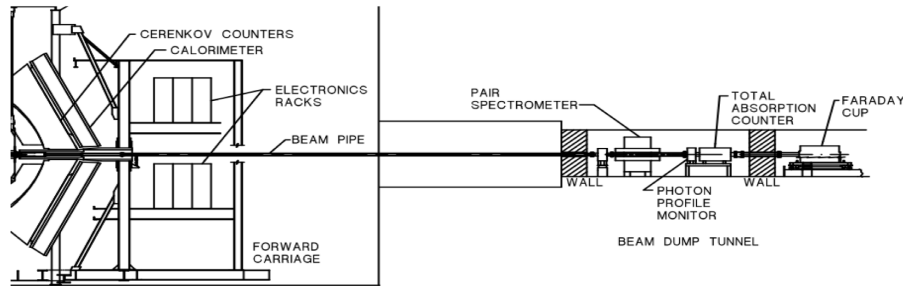


FIG. 18: Beamline components downstream of the CLAS detector.

428 2.5.1 BEAM POSITION MONITORS

429 Beam Position Monitors are used to monitor the x and y position with a resolution of
 430 less than 0.1 mm and a current measurement. The BPM's reported the beam position at
 431 a rate of 1 Hz. The BPMs were located at 36.0, 24.6, and 8.2 m upstream from the target
 432 [33].

433 2.5.2 HARPS

434 Harps allow the beam profile to be studied. They consist of a pair of wires oriented to
 435 scan the beam profile in orthogonal directions. These wires are made from both Tungsten
 436 and Iron. The position of the harps are located at 36.7m, 22.1m, and 15.5m upstream from
 437 the target. Electrons scattered by the harps were detected by an array of PMT detectors
 438 downstream from the location of the harps. More information on the harps can be found in
 439 [33].

440 2.5.3 TOTAL ABSORPTION SHOWER COUNTER

441 The TASC was used to obtain the photon flux at beam currents lower than 100 pA.
 442 During production data taking, the TASC was removed from the beamline and reinserted
 443 during the lower-intensity normalization runs.

444 It consisted of four lead glass blocks made of 55% PbO and 45% SiO_2 with a radiation
 445 length of 2.36 cm. The blocks were (10x10x40) cm and were mounted in a 2x2 array. Each
 446 block contained a Philips XP4312B phototube. More information about the TASC can be
 447 found in [33].

448 2.5.4 PAIR SPECTROMETER

449 The pair spectrometer was used to measure the photon flux at higher beam intensities
450 than that of the TASC. The pair spectrometer consisted of a large aperture dipole magnet,
451 an aluminum converter, and an array of eight scintillator paddles. As photons struck the
452 converter, they produced e^+e^- pairs which were then swept out of the beam line and directed
453 towards the scintillator paddles. The converters had a radiation length of 1% or 2%. The
454 scintillators were positioned so that the spectrometer would detect e^+e^- pairs over the full
455 energy range of the tagger spectrometer.

456 The pair spectrometer remained in the beam line during higher intensity runs of up to
457 several nano amps, in contrast with the TASC.

458 Fig. 19 displays a top view of the upstream beam line components and their location
459 relative to the CLAS detector.

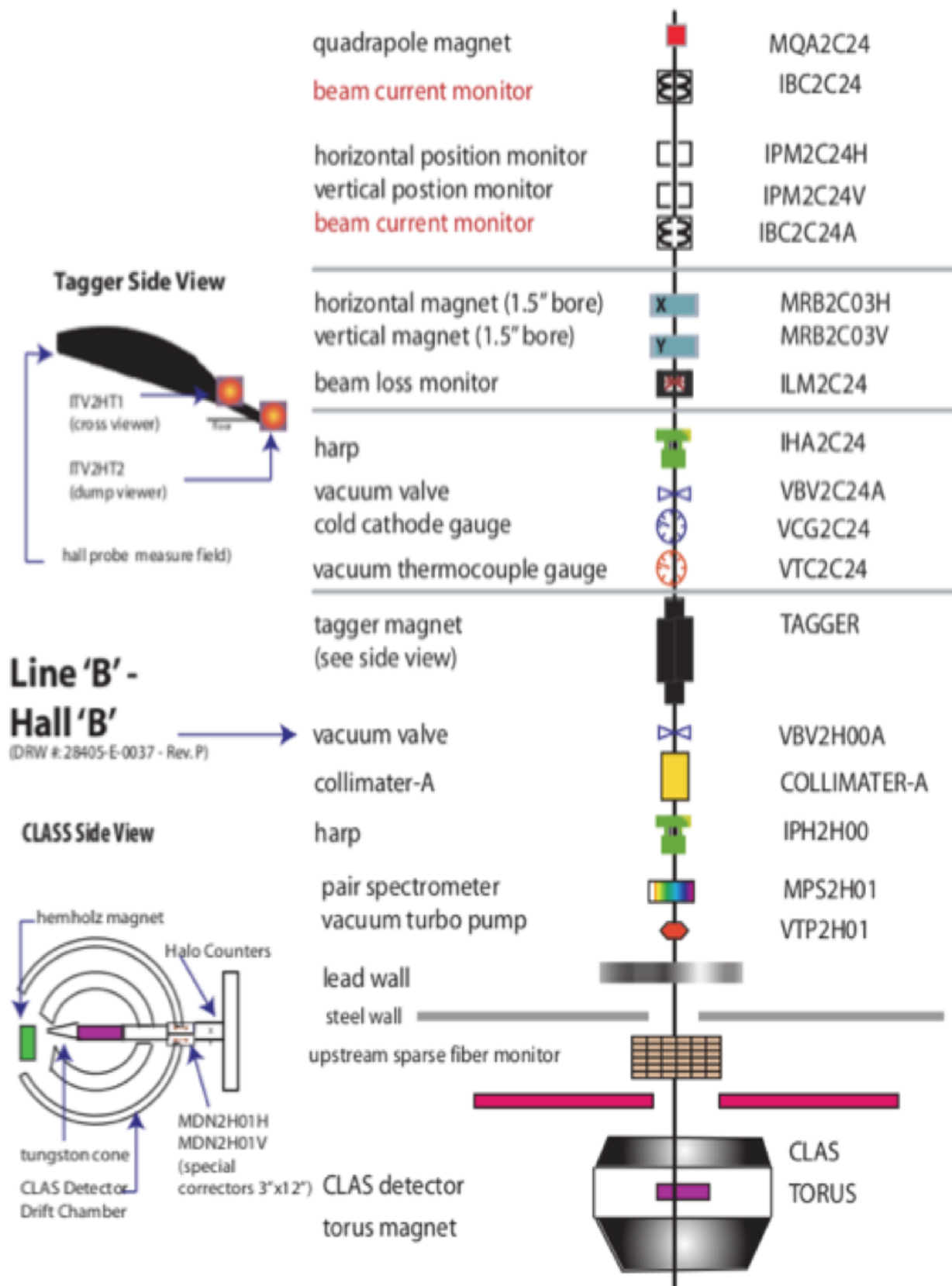


FIG. 19: Beamline components in Hall B.

2.6 DATA ACQUISITION

Signals from each detector system are read out by electronic crates that convert the analog signals into digital signals. Analog to Digital Converters (ADCs) are used for signal amplitude while Time to Digital Converters (TDCs) are more sensitive to the signal rise or fall time. Triggers are used to separate real physics events from electronic signals from other sources, such as cosmic radiation or electronic noise. Once there is a trigger, the DAQ collected the signals and wrote them to magnetic tape for analysis.

Once an event satisfies the level 1 trigger, the DAQ is gated and begins to read the event. Signals from the detector elements in CLAS are digitized and then read out by 24 VME readout controllers, known as ROC's. Then, data tables from the various systems were sent to the CLAS online acquisition computer in the control room. The Event Builder associates the tables into banks, and then packaged the event into its final form before storing it in a shared memory bank. For the g11 run period, events were selected using information from the tagger, start counter, and the time-of-flight scintillators. Events were recorded when both the tagger Master OR (MOR) and the CLAS Level 1 hardware trigger fell within a timing window of 15 ns. The Level 1 trigger required a signal from any of the 4 start counter paddles and any of the 48 TOF paddles from two separate sectors of CLAS within 150 ns.

2.7 THE G11 EXPERIMENT

The g11 run period was part of experiment E04021, *Spectroscopy of Excited Baryons with CLAS: Search for Ground and First Excited States* was conducted in Hall B during the summer of 2004. The goal for the experiment was to search for the Θ^+ pentaquark state. As a result, data for a number of different final states was obtained. The process of calibrating the subsystems of CLAS and converting the raw information into events containing tracks with momentum and timing information is called cooking. Cooking was done by Maurizio Ungaro. The process is described in [34].

2.7.1 BEAM ENERGY

The electron beam energy during the g11 experiment was 4.017 GeV. Towards the end of the run period, an additional 26 runs were collected with a beam energy of 5.021 GeV. The data analyzed in this work does not include the runs at 5.021 GeV. The photon beam energy for the g11 experiment ranges from 1.5 to 3.5 GeV.

490 2.7.2 G11A TRIGGER

491 The trigger was chosen specifically to maximize multi-track event detection in CLAS.
 492 The trigger required at least 2 charged tracks in different sectors of the TOF system within
 493 a 100 ns timing coincidence window. In addition, two hits in the Start Counter matching
 494 the sectors of the TOF hits within a 15 ns timing window were required. The MOR from
 495 the photon tagger was required as well. The photon tagger was only triggered on the first
 496 40 of the 61 T-counters, which preferentially selected events originating from higher energy
 497 photons.

498 2.8 EXCLUDED RUNS

499 Data taken during the g11a run period was grouped into runs, which consist of 10M
 500 events per run. The g11a run period includes runs 43490 to 44133. Runs 43490 to 44107 were
 501 taken with the electron beam energy equal to 4.019 GeV, while runs 44108 to 44133 were
 502 taken with an electron beam energy of 5.021 GeV. Only runs 43490 to 44133 are included in
 503 this analysis. In addition, runs 43490 to 43525 were commissioning runs and are not used for
 504 physics analysis. The table below lists additional runs that are not included in our analysis:

Run	Description
43490-43525	Commissioning Runs
43586-43596	TOF Problem in Sector 3
43675-43778, 44013	Different Trigger Configuration
43871	Data Processing Error
43981-43982	Logbook Lists DC Problems
43989-43991	Logbook Lists DAQ Problems
44000-44002, 44007-44008, 44010-44012	TOF Problem in Sector 2
44108-44133	5.021 Beam Energy

TABLE 7: g11a cooked runs that are not included in physics analysis

505 2.9 EVENT SELECTION

506 Events were required to have three charged tracks in the final state identified as a proton,
 507 π^+ , and π^- and a photon tagged by an electron in the tagger. Charged particles were
 508 identified using the CLAS Simple Event Builder (SEB) package. The SEB package calculates

509 the velocity of the detected particle and compares it with an expected velocity corresponding
 510 to the measured momentum and the masses of the different particle candidates. The particle
 511 is chosen based on the minimum difference between the measured and candidate velocities.
 512 The final state photon is identified by having an energy deposition in the ECAL without a
 513 corresponding charged track and simultaneously having $\beta > 0.9c$. To suppress accidental
 514 coincidences from different beam bunches, there is an additional requirement to have only
 515 one photon present in the tagger within the ± 2 ns time interval between the tagger and the
 516 start counter, i.e. $|ST_{time} - TAG_{time}| < 2$ ns.

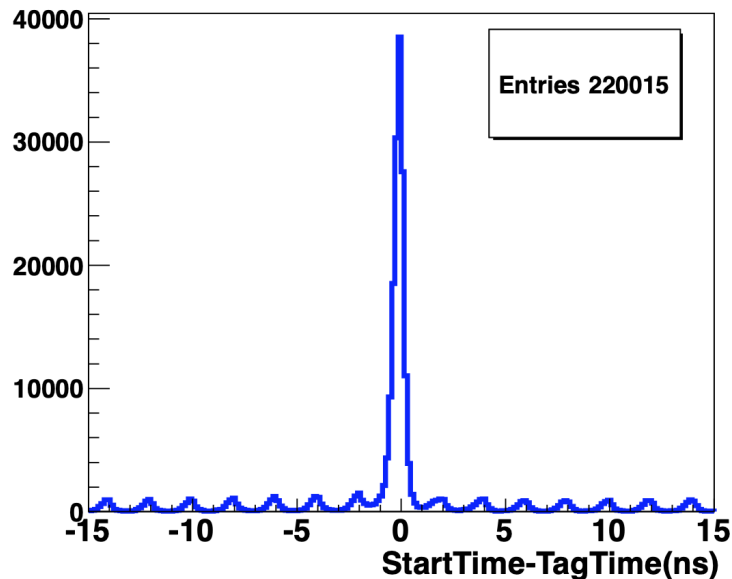


FIG. 20: Tagger time subtracted from the event start time. Image from [41].

517 This cut ensures we choose the correct photon in the case that multiple photons were
 518 detected close to the event start time. An additional cut was required that each event only
 519 have one photon detected in the tagger during the 2 ns interval.

CHAPTER 3

DATA ANALYSIS

This chapter describes the corrections applied to the raw data, including the standard CLAS corrections involving Tagger Energy, Energy Loss, and Momentum corrections. The CLAS detector performance cuts are discussed. After applying the standard CLAS cuts, there are kinematic cuts that are applied which are specific to our analysis. We describe the method used to extract our signal events and then describe the standard CLAS software packages used for simulations.

3.1 CORRECTIONS

This section summarizes the corrections applied to the g11 data set. These were discovered either during or after data was take and subsequently applied to our skimmed data set. These corrections were not applied to the Monte Carlo data.

3.1.1 TAGGER ENERGY CORRECTIONS

Alignment issues in the photon tagger's focal plane lead to an inaccurate photon energy constructed from the raw trigger information. The alignment issues arise because of the sagging of the tagger under gravity. As a consequence, there is a deviation between the proton mass and the missing mass of the $\gamma p \rightarrow K^+ K^- X$ reaction, $\Delta M = M_X - M_P$. This shift is visualized in Fig. 21. The deviation was dependent on the run number and was corrected on a run-by-run basis. The correction factor on average was $R = E_{corrected}/E_{initial} = 1.005$, which indicates the reported beam energy was about 0.5% less than the actual value.

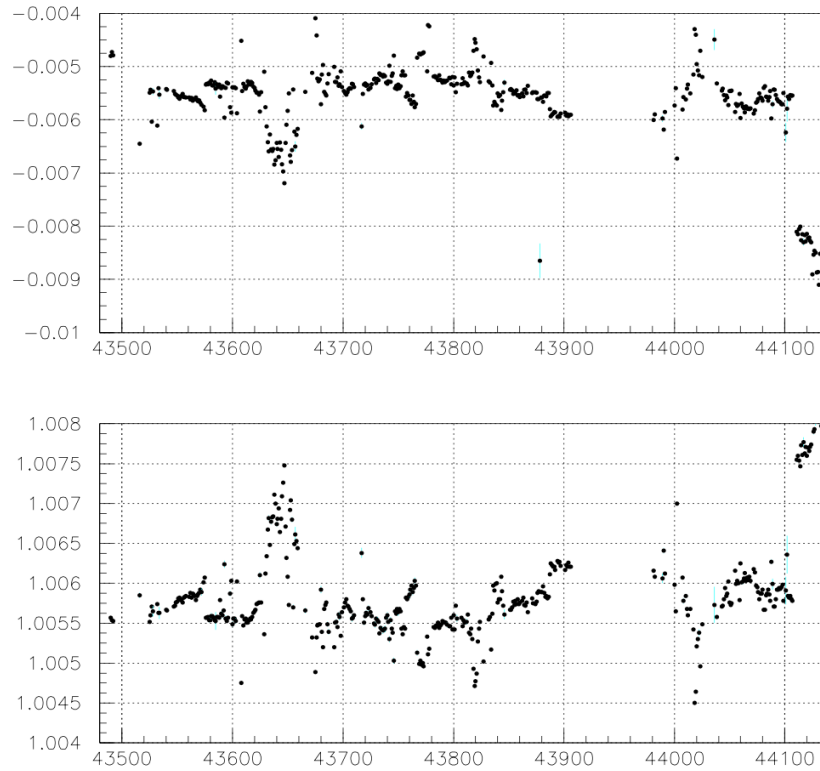


FIG. 21: Upper Panel: Missing mass shift as function of run number. Lower panel: photon energy corrections as a function of run number. Image from

540 The photon energy corrections were checked using $\gamma p \rightarrow \pi^+ \pi^- X$. After the corrections
 541 were applied, the proton mass was shifted to the correct value. The tagger energy corrections
 542 as a function of the Tagger ID are shown in 22. The red line corresponds to the corrections
 543 before electron energy corrections. After the electron energy corrections the photon energy
 544 is shifted by 0.5%. The black curve confirms that after the electron beam energy corrections
 545 and tagger corrections are applied, the missing proton mass has the correct value.

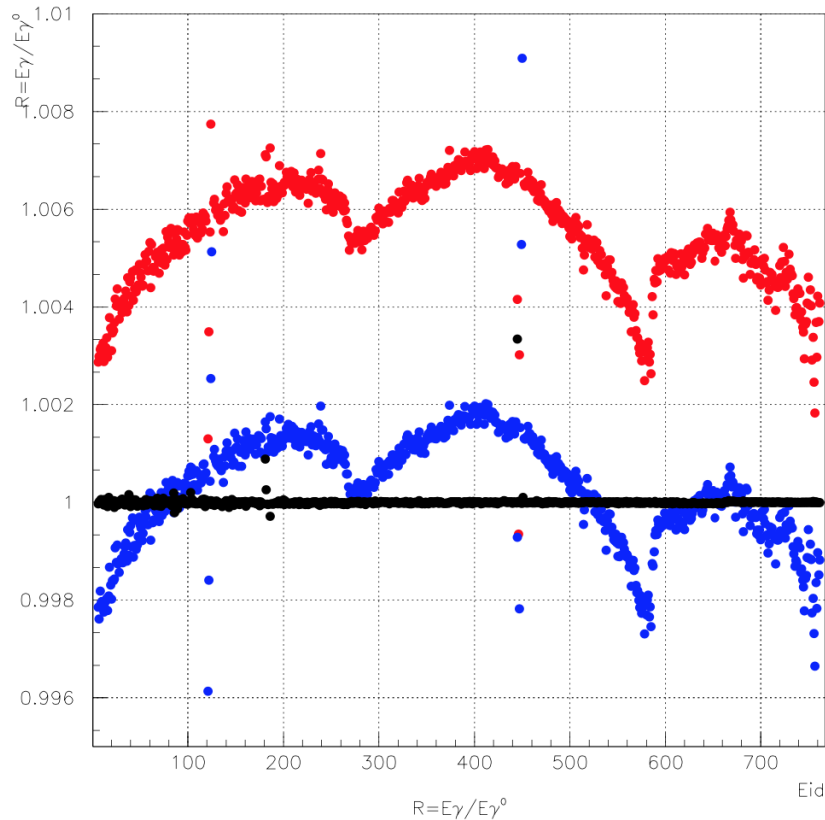


FIG. 22: Tagger energy correction as function of Tagger ID. Red (blue) line corresponds to the corrections before (after) electron energy corrections. Image from

546 The inconsistent points correspond to cable swaps. This was discovered during the photon
 547 energy calibration after the data taking was completed. They were accounted for via a
 548 correction procedure from [35].

549 3.1.2 ENERGY LOSS CORRECTIONS

550 Particles traveling through materials lose energy primarily due to excitation and ioniza-
 551 tion. Corrections were derived to account for energy loss in the target, the beam pipe, the
 552 start counter, and the air gap between the start counter and the Region 1 drift chambers.
 553 The corrections were applied with the CLAS Eloss package written by Eugene Pasyuk [27].
 554 For the g11 experiment, this was modified to include the geometry of the target and new
 555 start counters.

556 3.1.3 MOMENTUM CORRECTIONS

557 Discrepancies in the toroidal magnetic field map and/or in the drift chamber survey infor-
 558 mation can lead to inaccuracies in the reconstructed momenta. The momentum corrections
 559 used in this analysis were derived by Valery Kubarovksy [35]. The corrections were deter-
 560 mined using the missing mass technique. The reactions $\gamma p \rightarrow \pi^+ \pi^- p$ and $\gamma p \rightarrow K^+ K^- p$
 561 were used to extract the corrections as a function of the ϕ angle, after the energy loss and
 562 tagger corrections were applied. They are individually calculated for π^+ , π^- , K^+ , K^- , and
 563 protons.

564 The corrections for the π^+ and π^- are obtained from $\gamma p \rightarrow \pi^+ \pi^- p$ inclusive reaction,
 565 requiring only one of each final state particle, and for the K^+ and K^- from the $\gamma p \rightarrow K^+ K^- p$
 566 reaction again with only one of each particle in the final state. The correction factors are
 567 on the order of $\pm 1\%$ for ϕ and are much smaller for θ and P and are thus ignored. Fig.23
 568 shows the momentum corrections for positive particles as a function of ϕ . Fig. 24 shows the
 569 corrections for negative particles.

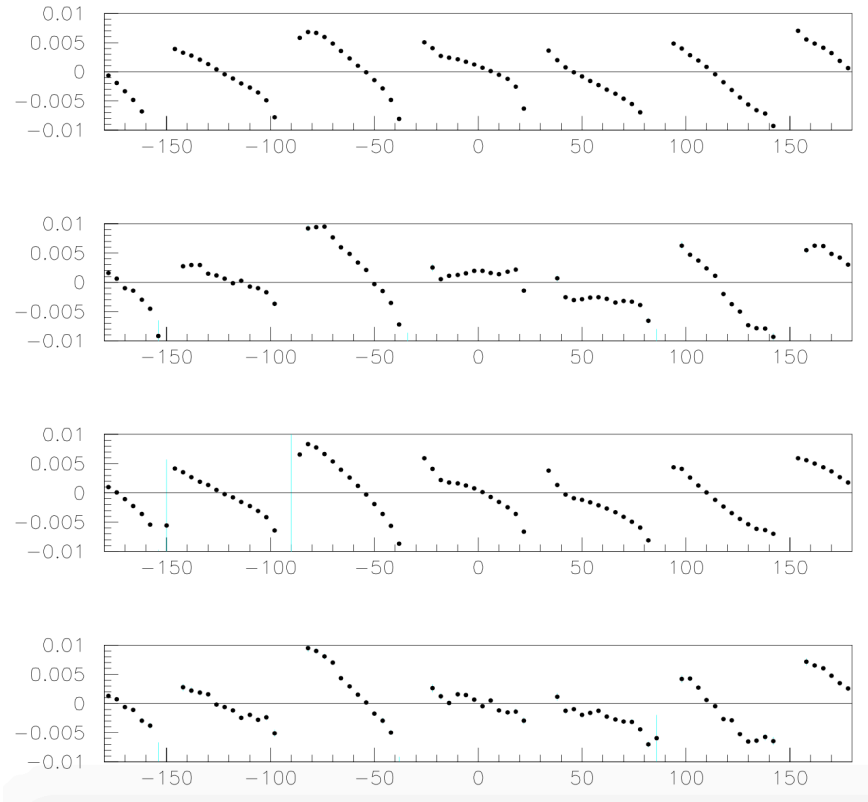


FIG. 23: Momentum corrections, $R = P_{corrected}/P_{measured} - 1$, as a function of ϕ for positive particles estimated from the missing mass distributions in $\gamma p \rightarrow \pi^+ \pi^- p$ reaction for π^+ , protons, and for K^+ and protons from $\gamma p \rightarrow K^+ K^- p$. Figure taken from [35].

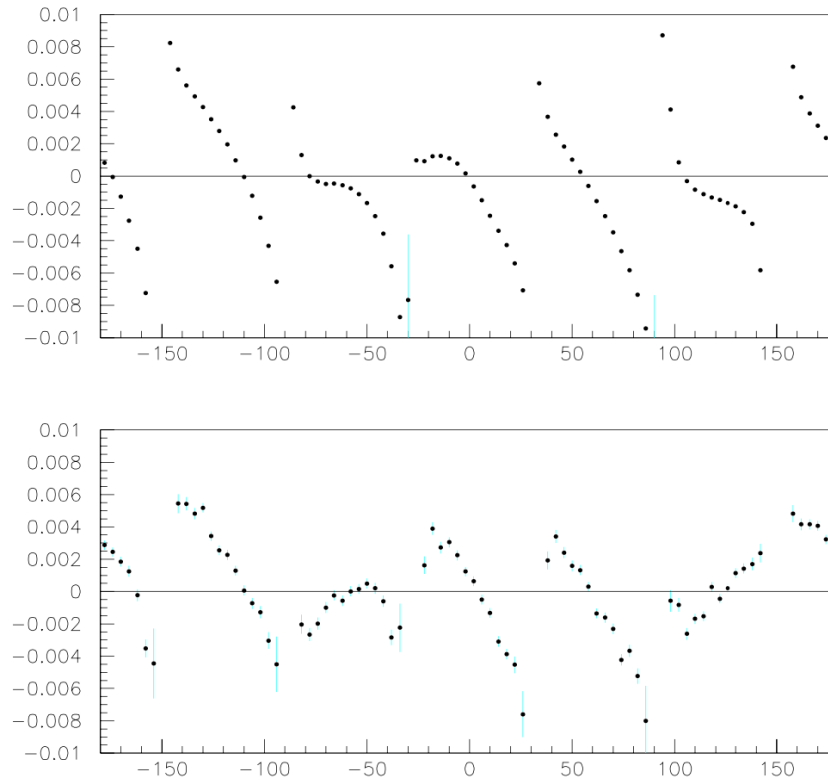


FIG. 24: Momentum corrections, $R = P_{corrected}/P_{measured} - 1$, as a function of ϕ for negative particles K^- and π^- . Figure taken from [35].

570 Since the corrections involved reactions and final states different from the one under
 571 study, it is important to determine the dependence of η as a function of θ_P , E_{beam} , and P_P .
 572 With no dependence on these quantities, the η signal should appear as a flat band. This is
 573 shown in Figs. 25-27.

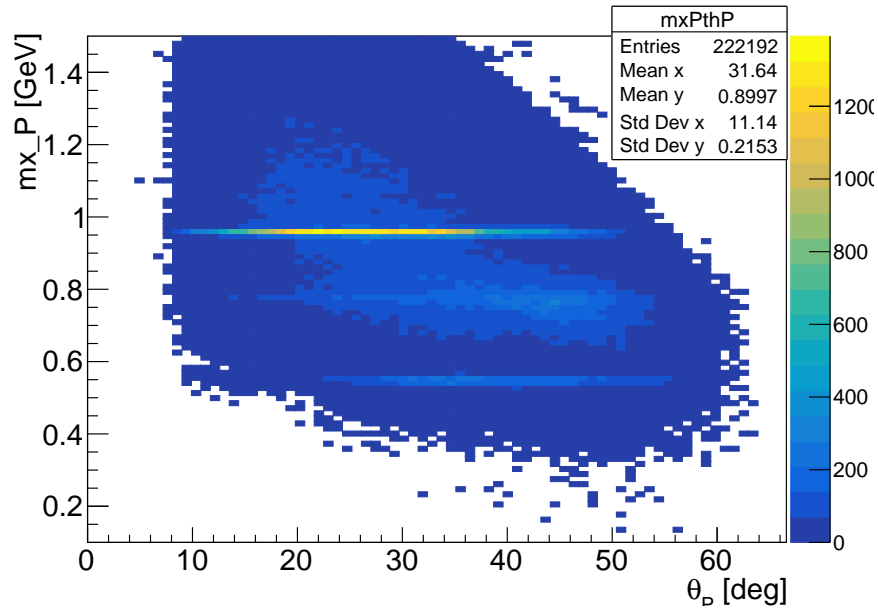


FIG. 25: $m_x(P)$ as function of θ_P . The signal of η appears as a flat band around 0.547 GeV.

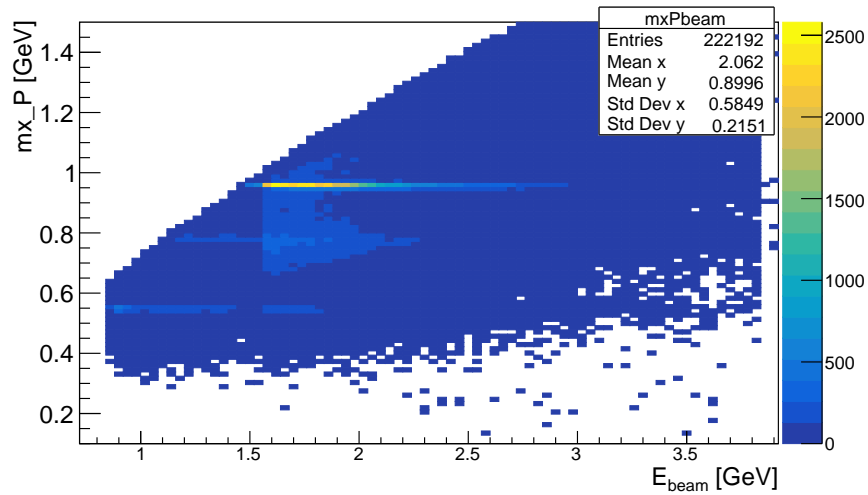


FIG. 26: $m_x(P)$ as function of E_{beam} . Note the minimum beam energy for g11 was 1.51 GeV. The η signal appears as a flat band around 0.547 GeV.

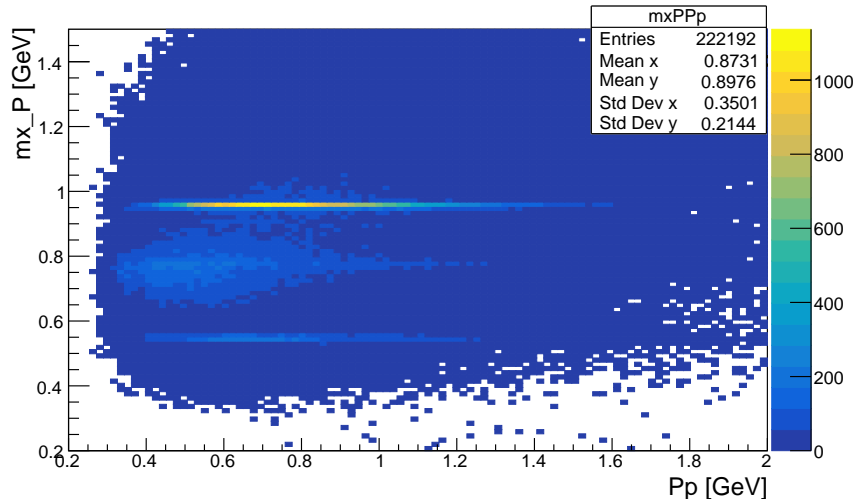


FIG. 27: $m_x(P)$ as function of θ_P . The η signal appears as a flat band around 0.547 GeV.

574 Since the η signal appears as a flat band, it is evident our signal peak does not depend
 575 on the beam energy, θ_P or momentum of the proton, as expected.

576 3.2 DETECTOR PERFORMANCE CUTS

577 This section summarizes the cuts on the data related to the CLAS detector itself.

578 3.2.1 MINIMUM MOMENTUM CUTS

579 Protons with low momentum are difficult to model in CLAS because they lose a significant
 580 amount of energy as they pass through material in the detector. Matt Bellis, a post-doctoral
 581 researcher at Carnegie Mellon University, performed a study that compared the acceptance
 582 of the data and Monte Carlo using the reaction $\gamma p \rightarrow p\pi^+\pi^-$. An acceptance asymmetry,
 583 defined as:

$$A = \frac{|A_{data} - A_{mc}|}{A_{data} + A_{mc}} \quad (3.2.50)$$

584 was calculated. This quantity should be zero. Fig. 28 shows this quantity for protons. In
 585 most areas, the data and MC acceptances are equal except in the very forward region and
 586 for low momentum protons, thus these regions are cut.

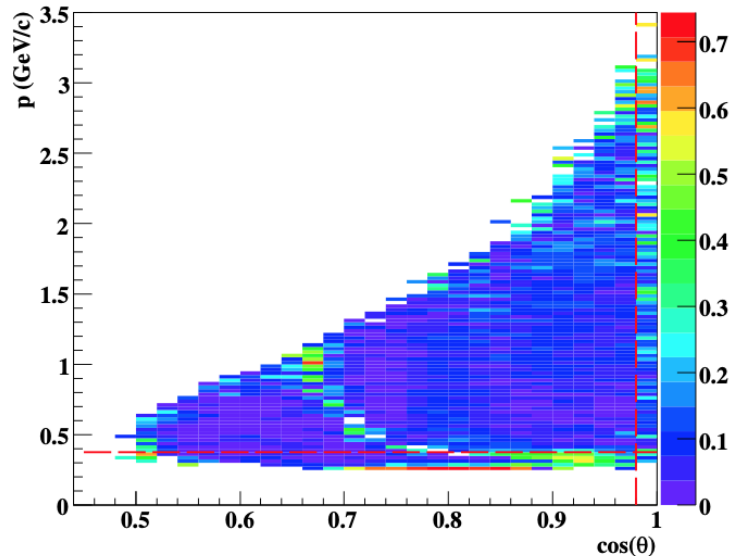


FIG. 28: Acceptance asymmetry vs p (GeV/ c) and $\cos\theta$ for protons. The vertical dashed line indicates a cut $\cos\theta = 0.985$ and the horizontal dashed line indicates a cut at $p = 375\text{MeV}/c$. The curved segment corresponds to a bad TOF paddle.

587 For our specific analysis, the low momentum proton cut was placed at $P_p = 350\text{MeV}$.
 588 For the charged pions, a minimum momentum cut is placed at P_{π^+} and $P_{\pi^-} > 0.1\text{ GeV}$.

589 3.2.2 TOF PADDLE KNOCK OUTS

590 Dead time-of-flight particles were removed if they had noticeable discrepancies for pions
 591 and/or protons. The paddles were identified by examining occupancy plots of both the data
 592 and Monte Carlo. Table 3.2.2 summarizes the knocked out paddles for each sector.

TABLE 8: Bad time-of-flight paddles that were not used for analysis.

Sector	Knocked Out Paddles
1	18,26,27,33
2	none
3	11,24,25
4	26
5	20,23
6	25,30,34

593 **3.3 KINEMATIC CUTS**

594 After the corrections, we can plot the missing mass of the proton from the reaction
 595 $\gamma p \rightarrow pX$ from our data. In Fig. we see three peaks corresponding to the η , ρ/ω , and the
 596 η' mesons as identified by their mass, demonstrating that particle X in the reaction above
 597 could have been the mesons shown.

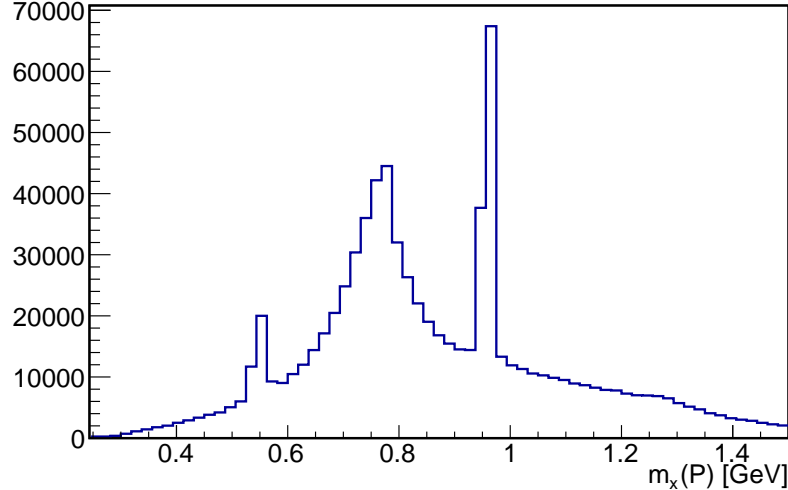


FIG. 29: Missing mass of the proton, $m_x(P)$, showing the η , ρ , and η' peaks, respectively. We required $|m_x^2(P\pi^+\pi^-\gamma)| < 0.0005$ and $|m_x^2(P\pi^+\pi^-)| < 0.005$.

598 To identify the $\pi^+\pi^-\gamma$ final state from the decay of η meson we require the squared
 599 missing mass of all final state particles to be around zero, $|m_x^2(P\pi^+\pi^-\gamma)| \leq 0.0005 \text{ GeV}^2$.
 600 This exclusivity cut ensures there are no other particles in our desired final state. This is
 601 shown in Fig. 30. To select the mother particle of interest, in this case eta, we require
 602 the missing mass of the proton to be $|m_x(P) - M_\eta| \leq 0.03 \text{ GeV}$. This cut corresponds to
 603 approximately 2.3σ obtained from the fit of $m_x(P)$. Furthermore, we require the missing
 604 mass of all charged particles to be $|m_x^2(P\pi^+\pi^-)| < 0.005 \text{ GeV}^2$, shown in Fig. 31. This
 605 ensures we are looking at reaction $\eta \rightarrow \pi^+\pi^-\gamma$ and not $\eta \rightarrow \pi^+\pi^-\pi^0$. Fig. 32 shows the
 606 signal of η with the vertical bars showing our defined signal region.

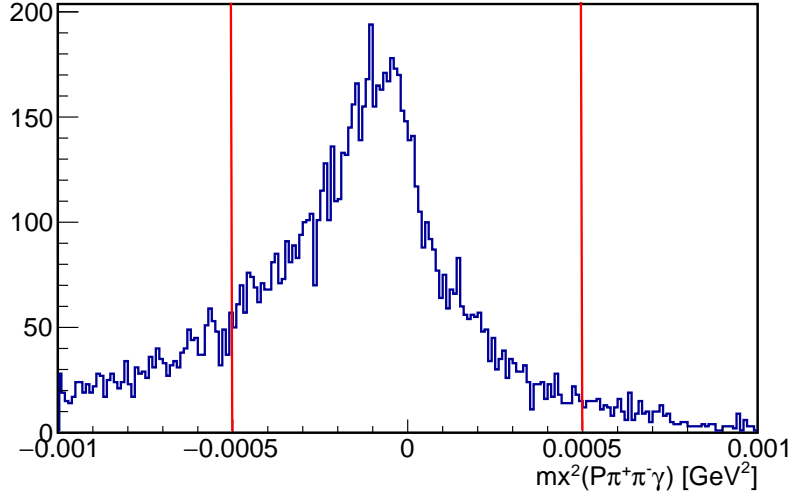


FIG. 30: Missing mass squared of all final state particles, $m_x^2(P\pi^+\pi^-\gamma)$. The cut value used in this analysis is $|m_x^2(P\pi^+\pi^-\gamma)| < 0.0005 \text{ GeV}^2$.

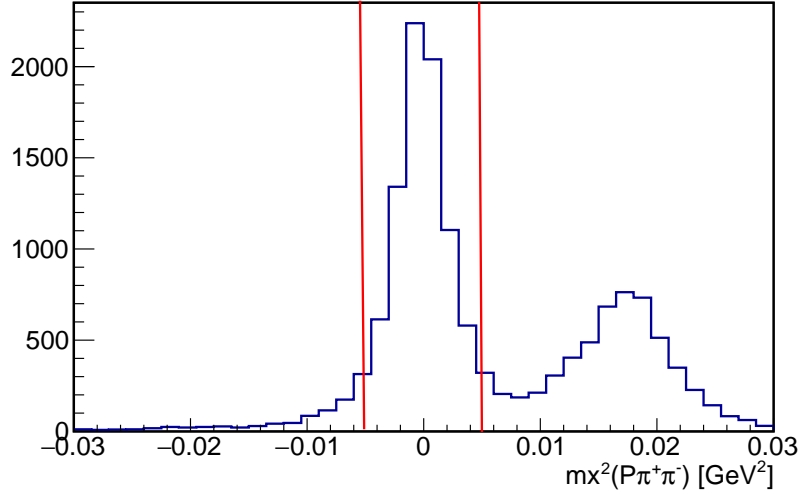


FIG. 31: Missing mass squared of proton, π^+ , and π^- , $m_x^2(P\pi^+\pi^-)$. In order to separate the photons from π^0 , we select the region $|m_x^2(P\pi^+\pi^-)| < 0.005 \text{ GeV}^2$.

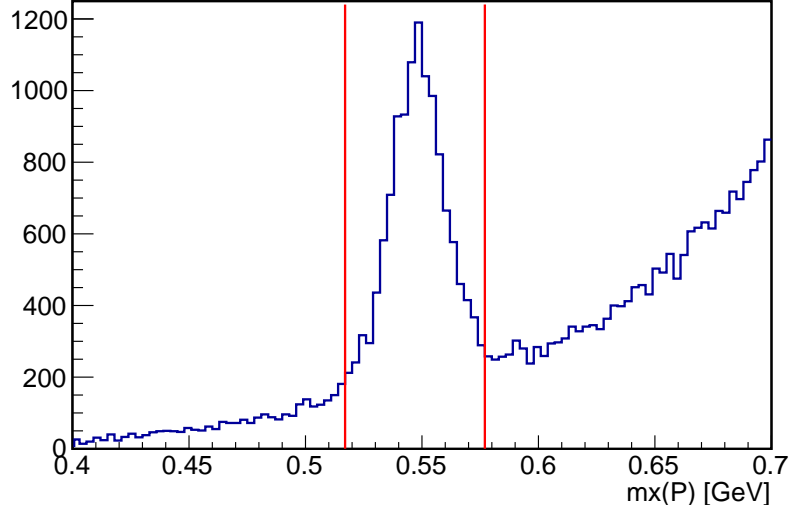


FIG. 32: Missing mass of the proton, $m_x(P)$ showing the η peak. Cut value used to select η region is $|m_x(P) - M_\eta| < 0.03$ GeV.

607 In addition to the cuts mentioned previously, we require the missing energy of the proton,
 608 π^+ , and π^- and the momentum of the outgoing photon to be greater than 0.1 GeV. This cut
 609 is based on the threshold of the ECAL photon detection threshold of 75 MeV. We require
 610 the difference between these two cuts to be small, ensuring that the missing energy of the
 611 three particles match the momentum of the outgoing photon.

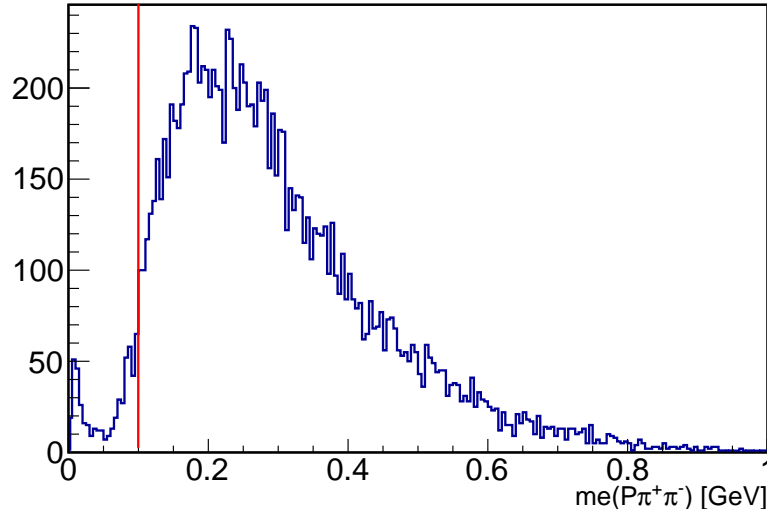


FIG. 33: Missing energy of proton, π^+ , and π^- . Cut value is $me(P\pi^+\pi^-) > 0.1$ GeV.

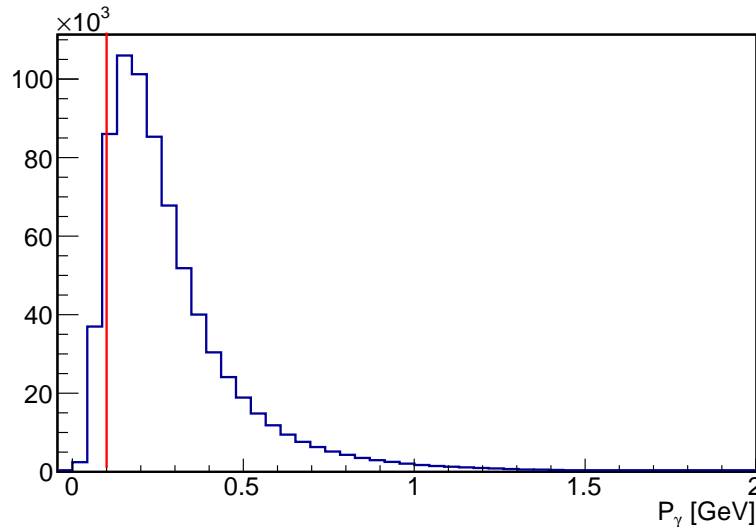


FIG. 34: Momentum of outgoing photon. Cut value is $P_\gamma > 0.1$ GeV

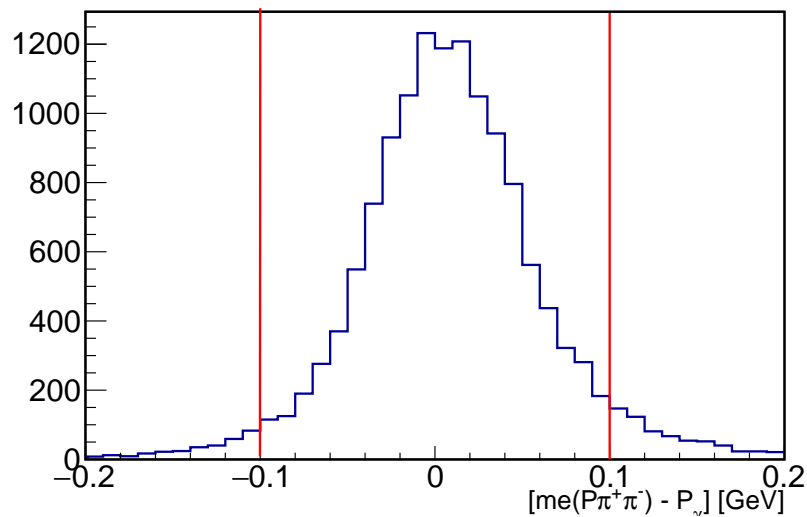


FIG. 35: Difference of missing energy of the proton, π^+ , π^- and P_γ . The cut value used in this analysis is $|me(P\pi^+\pi^-) - P_\gamma| < 0.1$ GeV.

612 An additional cut has been placed on θ_γ to be in the range of 15-45 deg. This is shown
 613 in Fig. 53. For this analysis, all relative cuts were applied to each variable excluding the cut
 614 on the variable itself.

615 3.4 RECONSTRUCTING η

616 We first check to see if we can resolve the peak of η from the missing mass of the proton
 617 in each bin of $IM(\pi^+\pi^-)$ since the two pion invariant mass spans the range from $[2m_\pi, m_\eta]$.
 618 The invariant mass is binned in 21, 10 MeV wide bins. This is shown in Fig. 39 - 42. There
 619 is a clear signal of η in all bins except those larger than $|IM(\pi^+\pi^-) - 0.505| < 0.05$, therefore
 620 we exclude those in our analysis.

621 3.4.1 SIDEBAND SUBTRACTION

622 Sideband subtraction is a method used to subtract the background under a signal. In
 623 principle, it requires at least two variables: the variable of interest and the separation vari-
 624 able. In our case, our variable of interest is the squared missing mass of the proton, π^+ , and
 625 π^- , labeled as $m_x^2(P\pi^+\pi^-)$ while our separation variable is the missing mass of the proton,
 626 m_xP . The separation variable is used to separate the background from the signal region us-
 627 ing the regions away from the Gaussian peak, called sidebands. The sideband regions must
 628 be chosen such that they are sufficiently far from the η peak, not in the region of another
 629 resonance, and with the sum of the two sideband regions equal in width to the signal region.

630 The procedure is as follows. First, a plot of $m_x^2(P\pi^+\pi^-)$ is produced with a cut on the
 631 m_xP in the sideband regions. Then, the $m_x^2(P\pi^+\pi^-)$ is plotted with a cut on m_xP from the
 632 signal region, which includes signal (S) and background (B) events. Then, we integrate the
 633 sideband and signal regions and calculate the ratio $R = A_{signal}/A_{sidebands}$. Once this ratio
 634 is obtained, the events in $m_x^2(P\pi^+\pi^-)$ are scaled by the ratio R. This gives an estimate of
 635 the number of background events under the signal peak. Finally, this weighted histogram is
 636 subtracted from the $m_x^2(P\pi^+\pi^-)$ obtained from the signal region of $m_x(P)$. An example
 637 construction for one bin is provided below.

638 For each bin of $IM(\pi^+\pi^-) \in [0.305, 0.505]$, if the η peak is present, we fit the distribution
 639 with a Gaussian and linear background polynomial and estimate the number of background
 640 events in the signal and sideband regions. We show the η signal as postage stamp plots in Fig.
 641 39 - 42. An example fit is shown in Fig. 37 for the bin corresponding to $|IM(\pi^+\pi^-) - 0.345| <$
 642 0.005 . Fits for all bins are shown in the Appendix.

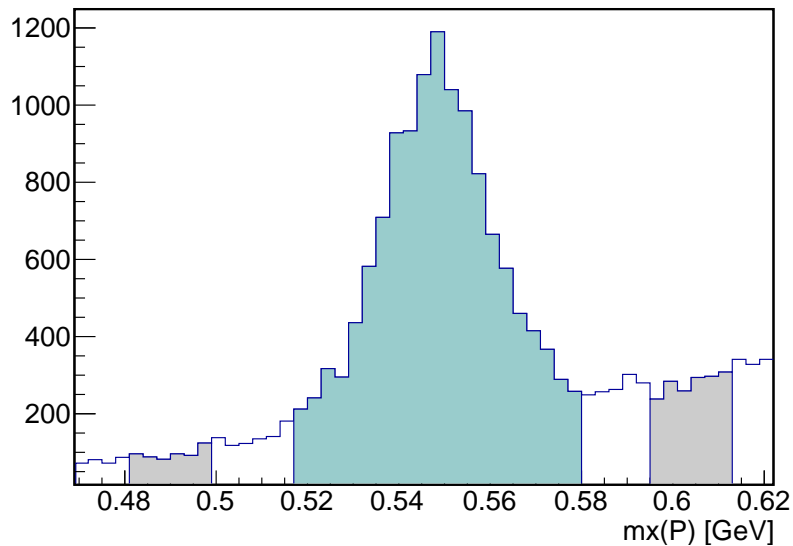


FIG. 36: Missing mass of the proton with signal (green) and sideband regions (grey) labeled.

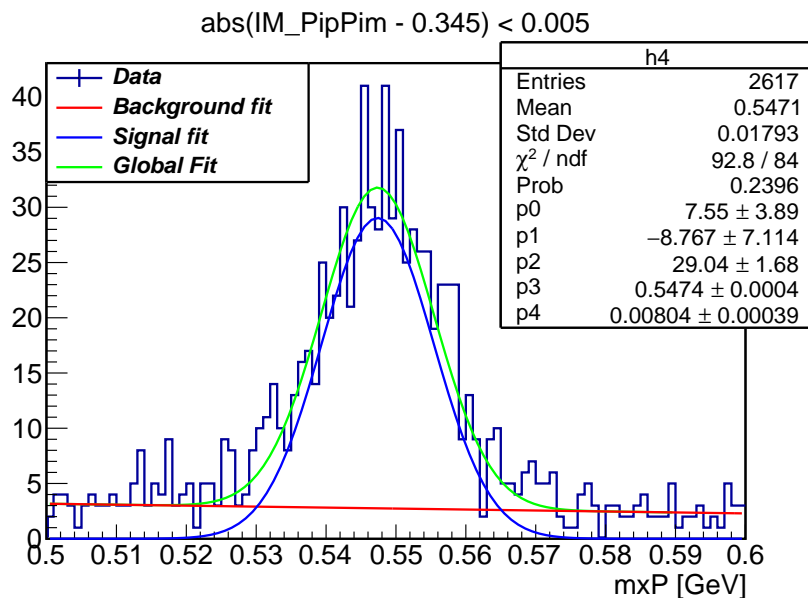


FIG. 37: Missing mass of the proton for $|\text{IM}(\pi^+\pi^-) - 0.355| < 0.005$ GeV. The total fit function is in green, the background polynomial is red, and the Gaussian signal is blue. The number of events in the signal and sideband regions was obtained via integrating the total function in the three respective regions.

643 The signal region is defined as $|m_x(P) - M_\eta| < 0.03$ GeV. The sideband regions are
 644 chosen to be from $|m_x(P) - 0.490| < 0.0075$ GeV and $|m_x(P) - 0.604| < 0.0075$ GeV. The

645 number of background events in the signal region, containing both signal and background
 646 events, and the number of events in the sideband regions is obtained. They are denoted as
 647 A_{peak} , A_{left} , and A_{right} .

$$A_{region} = \frac{1}{BW} \int_{M_l}^{M_h} (a_0 + b_0 m) dm \quad (3.4.51)$$

648 where M_h and M_l denote the upper and lower limits of the defined mass region. We
 649 then plot $m_x^2(P\pi^+\pi^-)$ from the signal region of η , labeled Φ_{signal} and from the left and right
 650 sideband regions, labeled $\Phi_{sidebands}$. An example bin is shown in Fig. 38. The left most
 651 peak corresponds to the photon peak, while the other peak visible in lower $IM(\pi^+\pi^-)$ bins
 652 corresponds to neutral pions. The sideband subtraction is then performed in each bin using
 653 the following:

$$\Phi_{sub} = \Phi_{signal} - \frac{A_{peak}}{A_{left} + A_{right}} \Phi_{sideband}, \quad (3.4.52)$$

654 and is shown in for all bins are shown in Fig. 43-47.

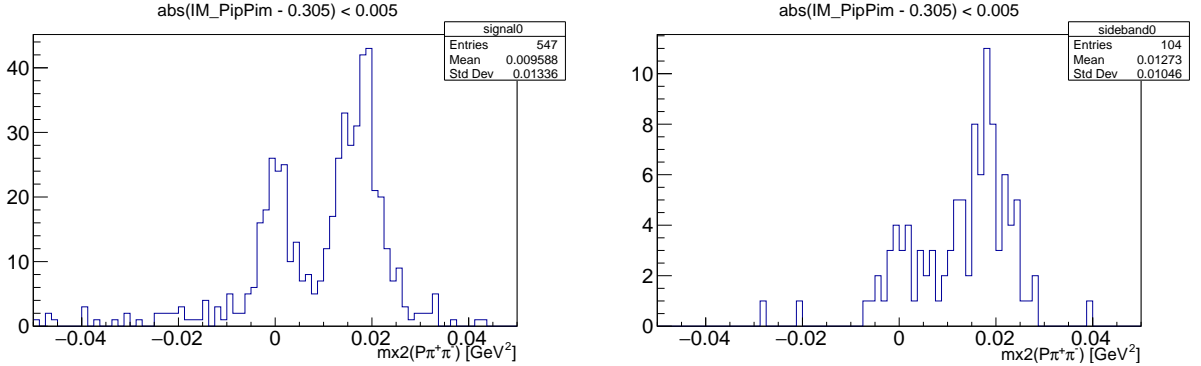


FIG. 38: Left plot: $m_x^2(P\pi^+\pi^-)$ from the signal region of η , $|m_x(P) - 0.547| < 0.03$ GeV. Right plot: same as left but from the sideband regions of η , $|m_x(P) - 0.490| < 0.0075$ GeV and $|m_x(P) - 0.604| < 0.0075$ GeV. The invariant mass bin selected is $|IM(\pi^+\pi^-) - 0.345| < 0.005$ GeV.

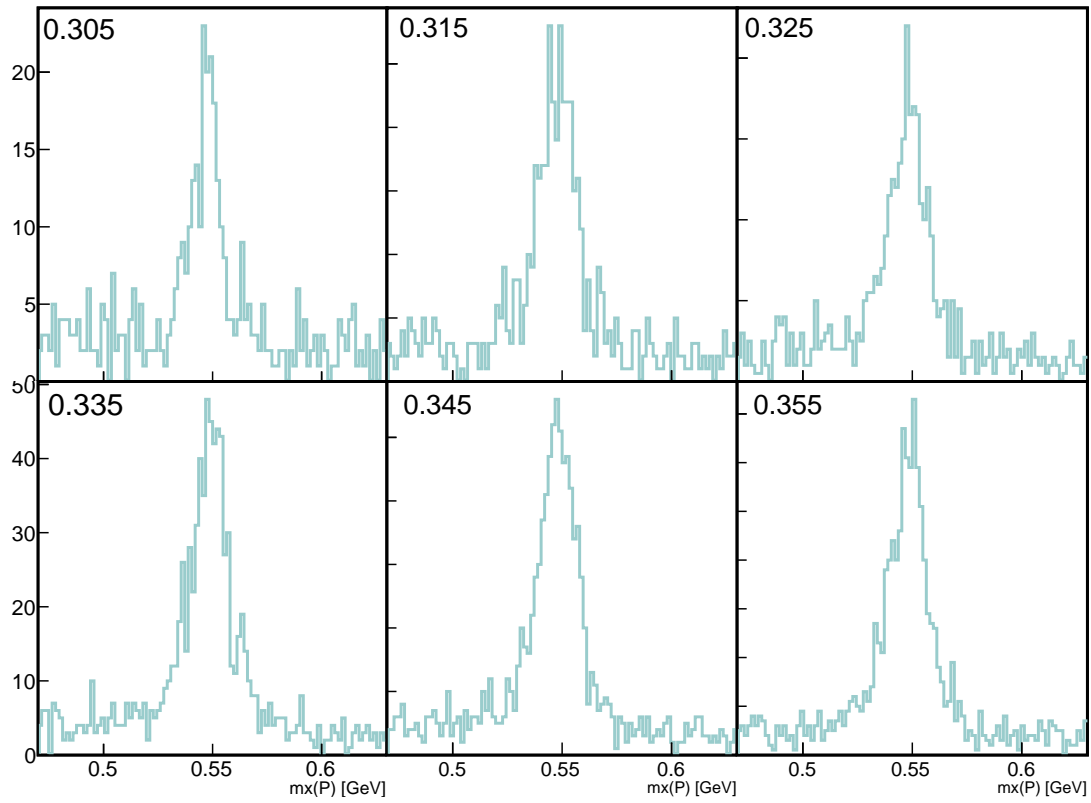


FIG. 39: $m_x(P)$ for bins $|IM(\pi^+\pi^-)| \in [0.305, 0.355]$. The centroid of the invariant mass bin is printed in the upper left corner of each plot.

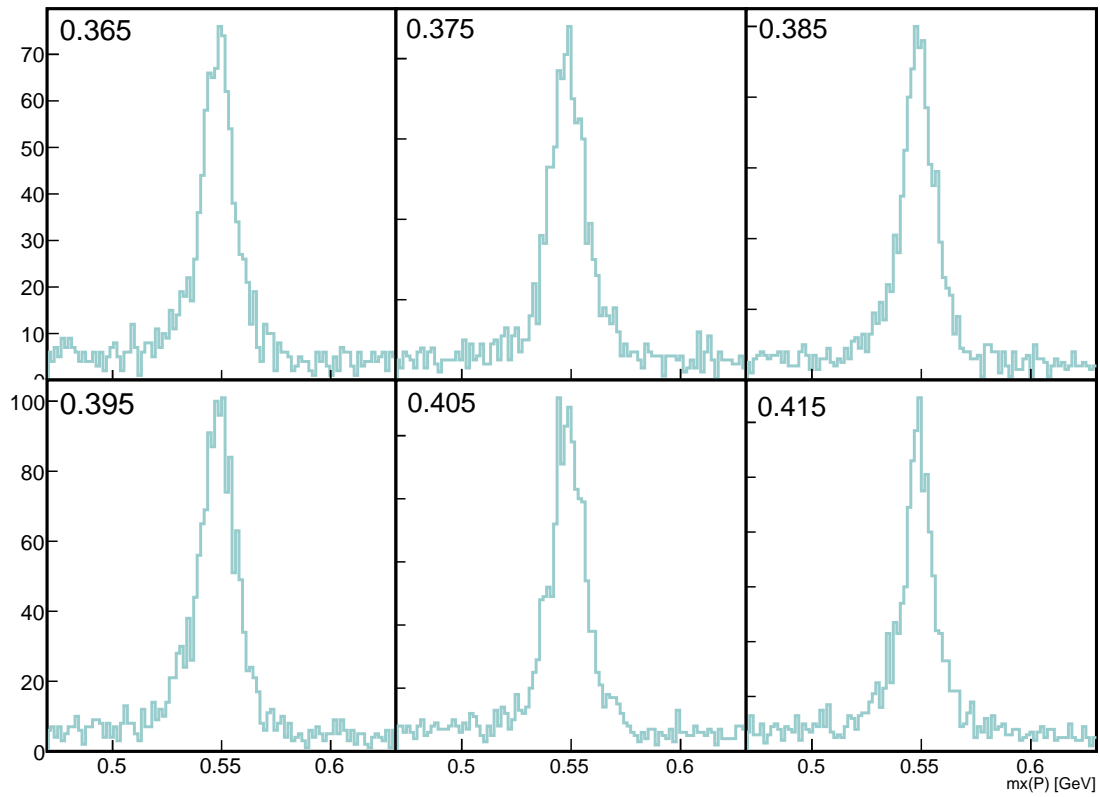


FIG. 40: $m_x(P)$ for $|IM(\pi^+\pi^-)| \in [0.365, 0.415]$. The centroid of the invariant mass bin is printed in the upper left corner of each plot.

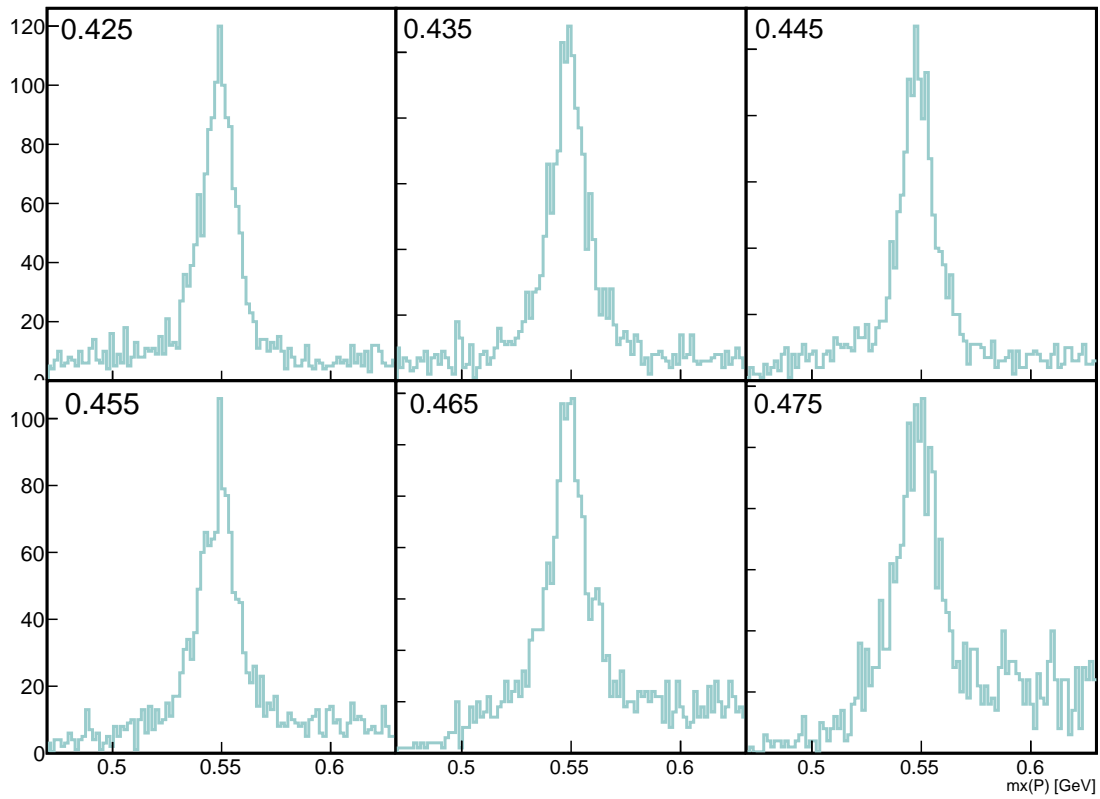


FIG. 41: $m_x(P)$ for $|IM(\pi^+\pi^-)| \in [0.425, 0.475]$. The centroid of the invariant mass bin is printed in the upper left corner of each plot.

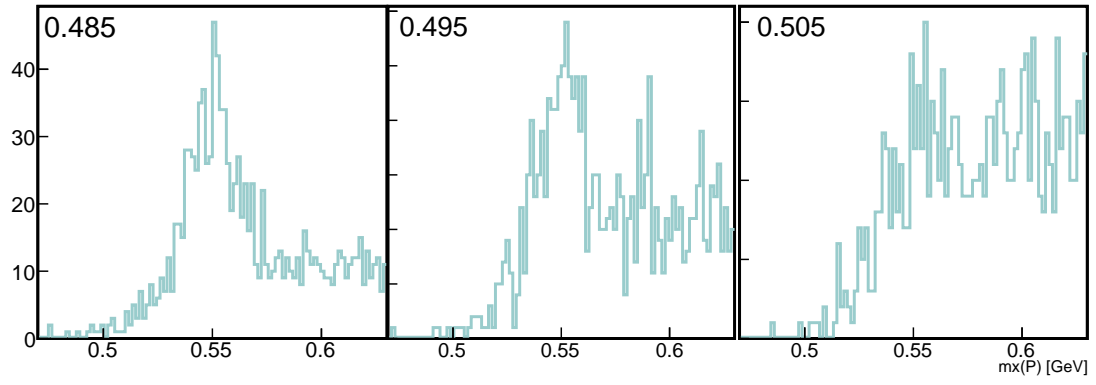


FIG. 42: $m_x(P)$ for $|IM(\pi^+\pi^-)| \in [0.485, 0.505]$. Bins above $|IM(\pi^+\pi^-) - 0.495|$ are excluded from the analysis.

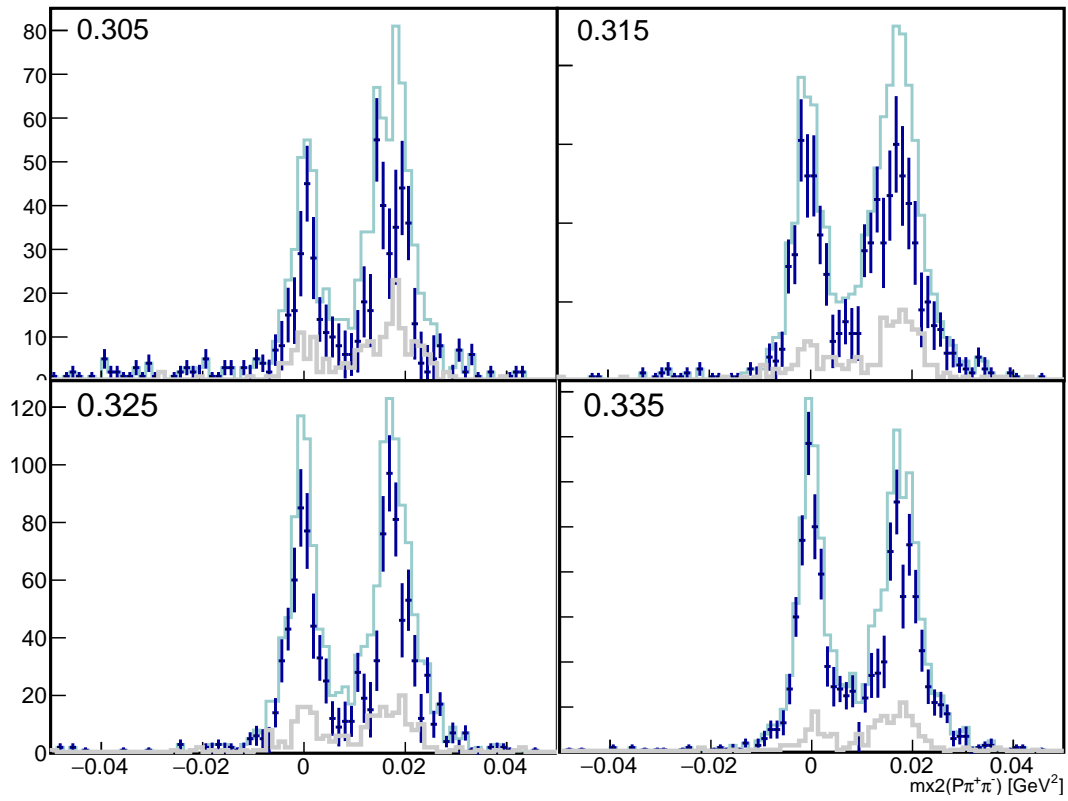


FIG. 43: $m_x^2(P\pi^+\pi^-)$, from signal region (green) and sidebands (grey). The dark blue histogram is the result of the weighted sideband events subtracted from the signal events. The selected bin of $IM(\pi^+\pi^-)$ is printed at the top of each plot.

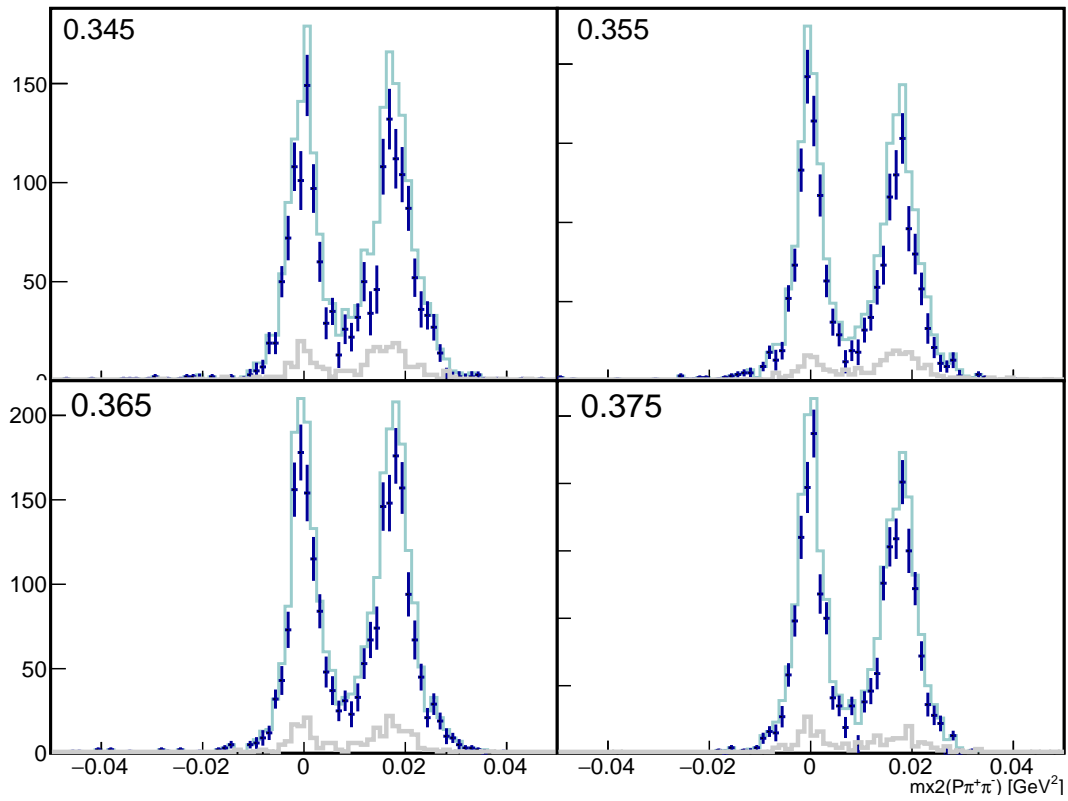


FIG. 44: $m_x^2(P\pi^+\pi^-)$, from signal region (green) and sidebands (grey). The dark blue histogram is the result of the weighted sideband events subtracted from the signal events. The selected bin of $IM(\pi^+\pi^-)$ is printed at the top of each plot.

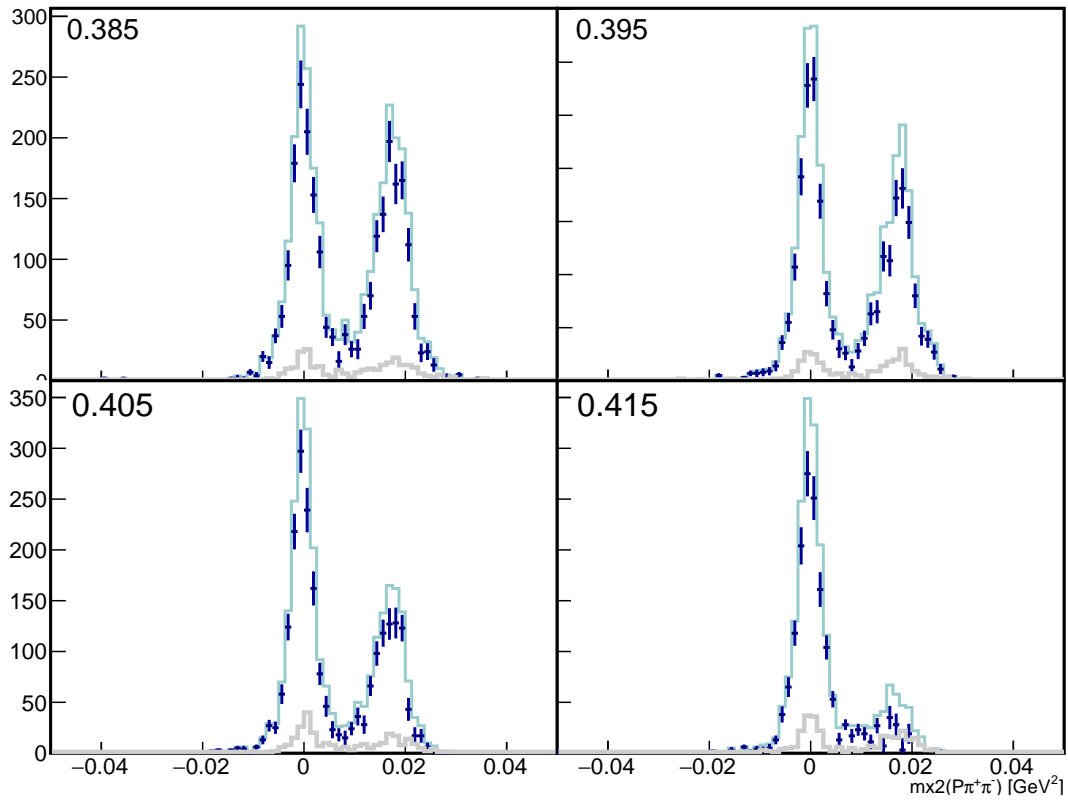


FIG. 45: $m_x^2(P\pi^+\pi^-)$, from signal region (green) and sidebands (grey). The dark blue histogram is the result of the weighted sideband events subtracted from the signal events. The selected bin of $IM(\pi^+\pi^-)$ is printed at the top of each plot.

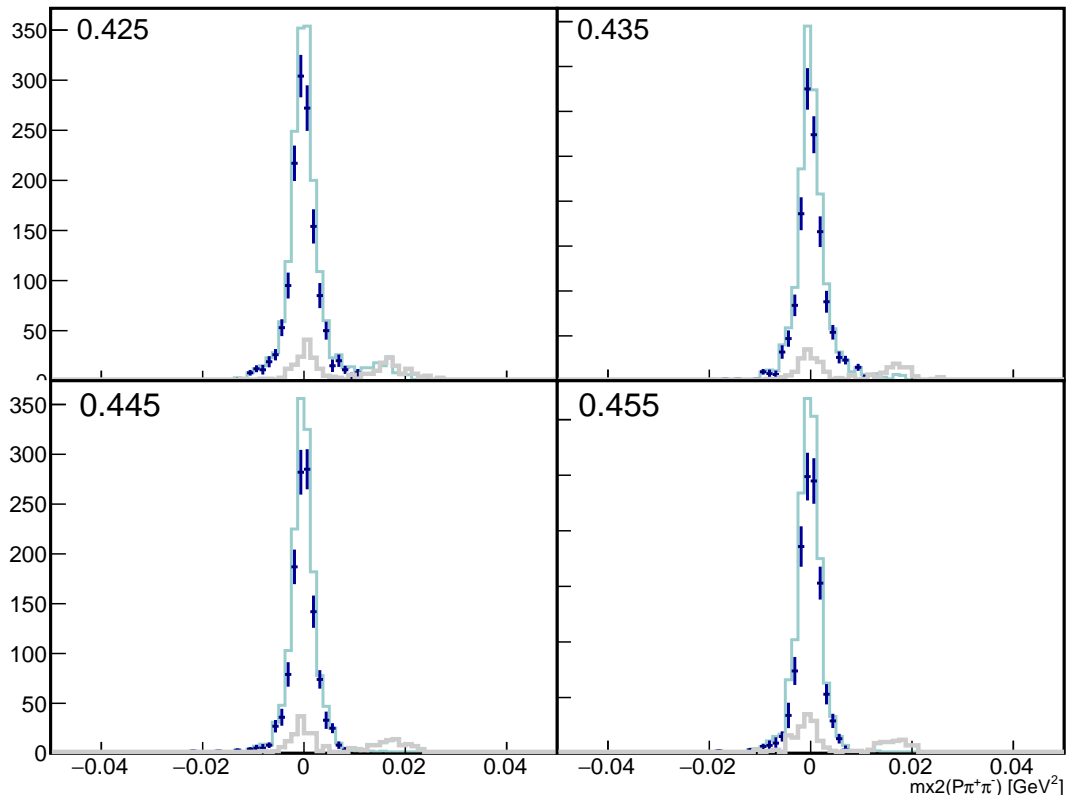


FIG. 46: $m_x^2(P\pi^+\pi^-)$, from signal region (green) and sidebands (grey). The dark blue histogram is the result of the weighted sideband events subtracted from the signal events. The selected bin of $IM(\pi^+\pi^-)$ is printed at the upper left of each plot.

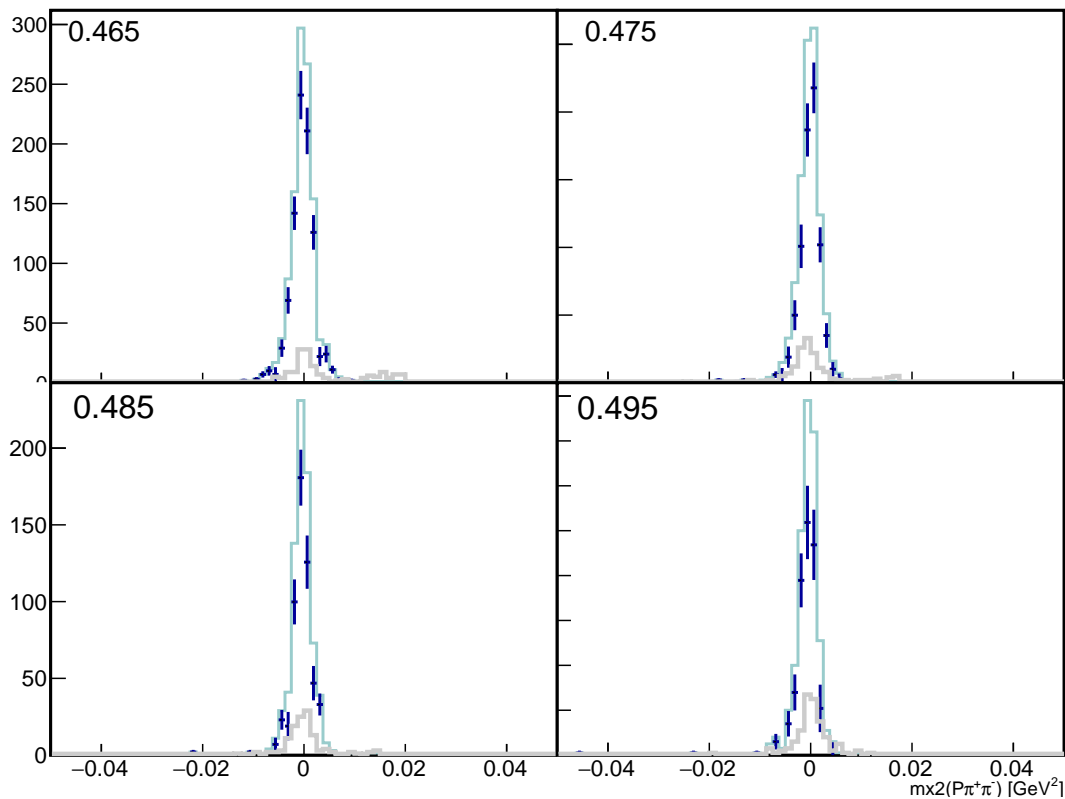


FIG. 47: $m_x^2(P\pi^+\pi^-)$, from signal region (green) and sidebands (grey). The dark blue histogram is the result of the weighted sideband events subtracted from the signal events. The selected bin of $IM(\pi^+\pi^-)$ is printed at the top of each plot.

655 3.5 EXTRACTING N_γ

656 Each Φ_{sub} histogram is fitted with either a double Gaussian and polynomial or single
 657 Gaussian and polynomial as appropriate, see Fig. 48. The number of events from the
 658 photon peak is then plotted as a function of $s(\pi^+\pi^-) = IM^2(\pi^+\pi^-)$ in Fig. 49. The total
 659 number of signal events, N_s , is 17302 ± 131 .

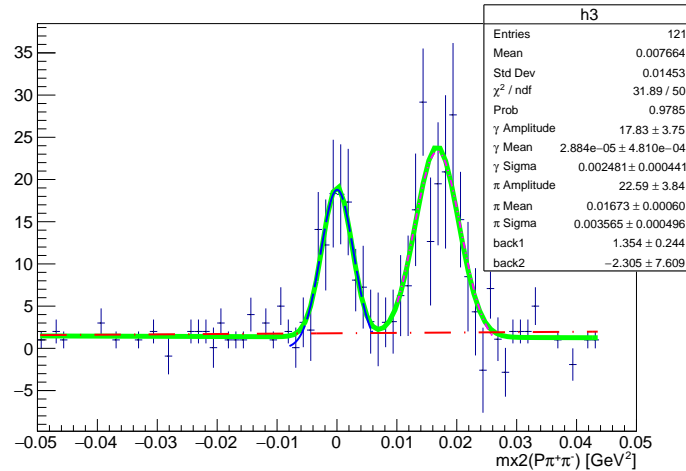


FIG. 48: Fit to $m_x^2(P\pi^+\pi^-)$ after sideband subtraction. The green solid curve is the total fit, the red dashed curve is the linear background function, and the blue and pink dashed curves correspond to the Gaussian fits for the photon and pion peaks, respectively.

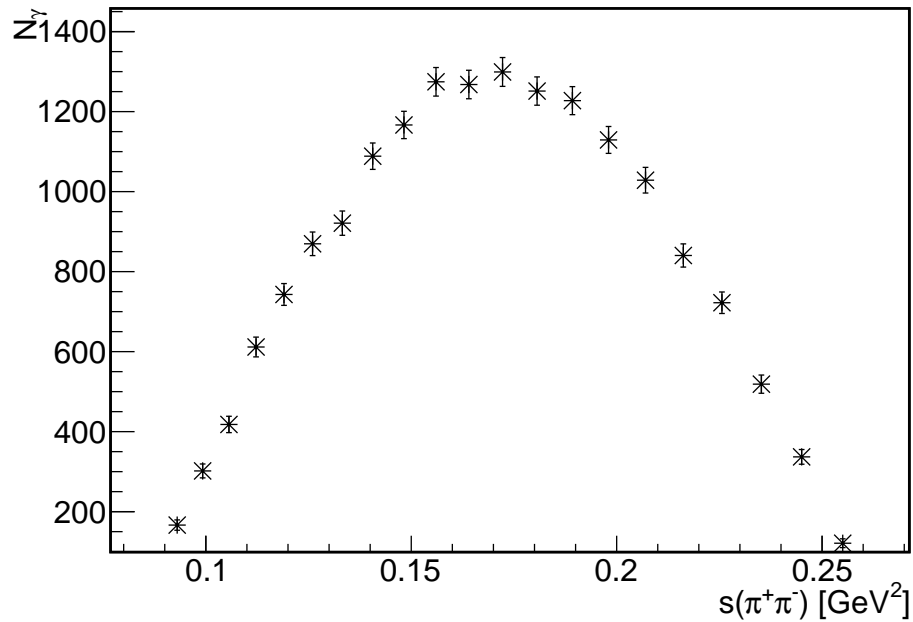


FIG. 49: Number of photons as function of $s(\pi^+\pi^-)$. This does not include any acceptance corrections. The total number of signal events, N_s , is 17302 ± 131 .

661 In the CLAS detector, there are certain regions in which physics events cannot be
 662 recorded, such as in the area dividing each sector. In addition, each sector in CLAS has
 663 inefficiencies in the subsystems: inoperable DC wires, PMT inefficiencies, dead scintillator
 664 strips, and bad paddles. Simulations are performed to understand the effects of these inef-
 665 ficiencies and dead regions on the experimental data. The simulation chain is described in
 666 this section. More information can be found in the Appendices.

667 The software used in this analysis was run inside a Docker container. A container is a
 668 standard unit of software that packages up code and all its dependencies so the application
 669 runs quickly and reliably independently of the computing environment. A Docker container
 670 image is a lightweight, standalone, executable package of software that includes everything
 671 needed to run an application: code, runtime, system tools, system libraries and settings [36].
 672 Nick Tyler, a Graduate Student at the University of South Carolina, compiled most of the
 673 CLAS6 software inside a Docker container, thus allowing the usage of old software when
 674 analysing old data without having to recompile on the new Jefferson Lab computers.

675 3.6.1 SGEN

676 The event generator is used to generate Monte Carlo files for the reaction $\eta \rightarrow \pi^+\pi^-\gamma$.
 677 In each Monte Carlo file, 100,000 decays were generated containing the initial ($\gamma + p$) and
 678 final state particles $p, \pi^+\pi^-\gamma$. The total number of events generated is approximately 1.3
 679 billion. The beam energy range was 1.5 to 3.5 GeV. The decay was generated according to
 680 the following:

$$|M|^2 \approx |F(s_{\pi\pi})|^2 E_\gamma^2 q^2 \sin^2 \theta \quad (3.6.53)$$

681 where

$$E_\gamma = \frac{1}{2} \left(m_\eta - \frac{s_{\pi\pi}}{m_\eta} \right) \quad (3.6.54)$$

682 and where q is the pion momentum in the pion-pion rest frame. An input value of 1.80
 683 for the α parameter was also used. The pion vector form factor parameterization was the
 684 same used in Fig. 4. The η production angle was taken according to the differential cross
 685 section published from Williams, et al. The following table tabulates the number of events
 686 generated for each IM($\pi^+\pi^-$) bin.

$IM(\pi^+\pi^-)$ (GeV)	N_{gen} (M)	$IM(\pi^+\pi^-)$ (GeV)	N_{gen} (M)
0.305	10.2	0.415	94.8
0.315	18.0	0.425	95.2
0.325	26.8	0.435	93.2
0.335	36.3	0.445	88.9
0.345	46.0	0.455	82.3
0.355	55.8	0.465	73.5
0.365	65.2	0.475	62.9
0.375	73.8	0.485	51.1
0.385	81.4	0.495	38.7
0.395	87.6	0.505	26.7
0.405	92.2	total:	1.3007E9

TABLE 9: Number of events generated for each $IM(\pi^+\pi^-)$ bin. The value of the bin displayed is the center.

687 SGEN produces a text file containing the particle id, momenta, and energy information
688 for every particle in each event.

689 3.6.2 GAMP2MC

690 After event generation, the output text files from SGEN are converted to input bos files
691 for GSIM using a program called gamp2MC. The original script was written by Michael
692 Kunkel and was recompiled by Raffaella De Vita. The software flags used are summarized
693 in the table below.

Name	Value	Meaning
-m		make MC banks instead of PART bank
-r	43582	run number
-o		output file
-T		put beam particle in TAGR bank
-S	0.321, -0.254, 0.378, 0.407	distribute xy vertices according to $\mu_x, \mu_y, \sigma_x, \sigma_y$
-z	-30.0,10.0	distribute z-vertex in given range

TABLE 10: gamp2MC flag parameters and definitions

694 **3.6.3 GSIM**

695 In order to calculate the acceptance, the CLAS GSIM package was used. GSIM
 696 is a GEANT based simulation of the CLAS detector and was the standard simulation
 697 package used when data was taken. After event generation, GSIM propagates each of
 698 the particles through the CLAS detector, resulting in a simulated set of detector signals
 699 for each track. GSIM takes into account the inefficiencies described previously in the
 700 CLAS_CALDB_RUNINDEX. This contains information about the inefficiency of each sub-
 701 system in CLAS. The run index used is calib_user.RunIndexg11a. The parameters used for
 702 GSIM are summarized in Table 11.

Name	Value	Name	Value
AUTO	1	LIST	
KINE	1	BEAM	
MAGTYPE	2	MAGSCALE	0.4974
FIELD	2	GEOM	ALL
NOSEC	OTHE	TARGET	g11a
TGPOS	0.0 0.0 0.0	STZOFF	-10.0
STTYPE	1.0	RUNG	43582
CUTS	5e-3	DCCUTS	1e-4
ECCUTS	5e-4	SCCUTS	1e-4
STCUTS	5e-5	NOGEOM	EC1 CC MINI PTG
FASTCODE		TRIG	500000

TABLE 11: GSIM flag parameters contained in ffreed card and definitions

703 The flag KINE specifies the kinematics generator. There are default generator options
 704 or a separate kinematics generator can be used. For a separate generator, the kinematics
 705 information can be written into the MCVX and MCTK banks in the input bos files. This was
 706 the case for our simulations. The remaining flag parameters are described in the Appendix.
 707 All output bank information is stored in BOS files for further processing.

708 **3.6.4 GPP**

709 The output of GSIM is then processed by software called GSIM Post Processor (GPP).
 710 This program smears the particle momentum, timing information, and removes dead wires
 711 to more accurately reflect the actual resolution of the detector. The parameters used for

712 GPP are listed in Table 12.

Name	Value	Meaning
P	0x1f	remove dead wires
R	43582	run number for the wire map
Y		drops the DC hits according to the efficiency in the GPP map and the DC wire map in the database
f	1.0	time smearing
a	1.0	DOCA smearing region 1
b	1.0	DOCA smearing region 2
c	1.0	DOCA smearing region 3

TABLE 12: GPP flag parameters and definitions

713 The DOCA smearing values of 1.0 indicate default values where minimal smearing is
714 used. After GPP, the events are passed through RECSIS.

715 3.6.5 RECSIS

716 After post-processing, the simulated data is processed through the user_ana routine,
717 which is an implementation of RECSIS. RECSIS, REConstruction and analySIS, is the same
718 framework that was used to cook the g11a data. The reconstructed Monte Carlo events are
719 then analyzed using the same cuts as the experimental data.

720 3.7 SIMULATION VALIDATION

721 In order to check the accuracy of our event generator, we compare momenta, θ , and ϕ
722 of each final state particle, in addition to the photon beam energy. The comparisons are
723 shown in Figures 50-, 53. The experimental data is shown in blue and the simulated data
724 is in red. Each histogram is normalized to unity using its integral. For θ_γ , angles less than
725 15 deg and greater than 45 deg were removed from the analysis. For most quantities, the
726 agreement between the Monte Carlo data and experimental data is good.

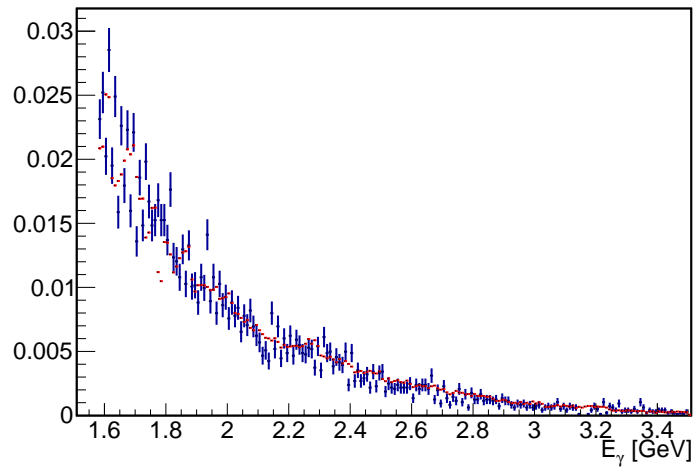


FIG. 50: Comparison of photon beam energy. Experimental data is in blue and simulated data is in red.

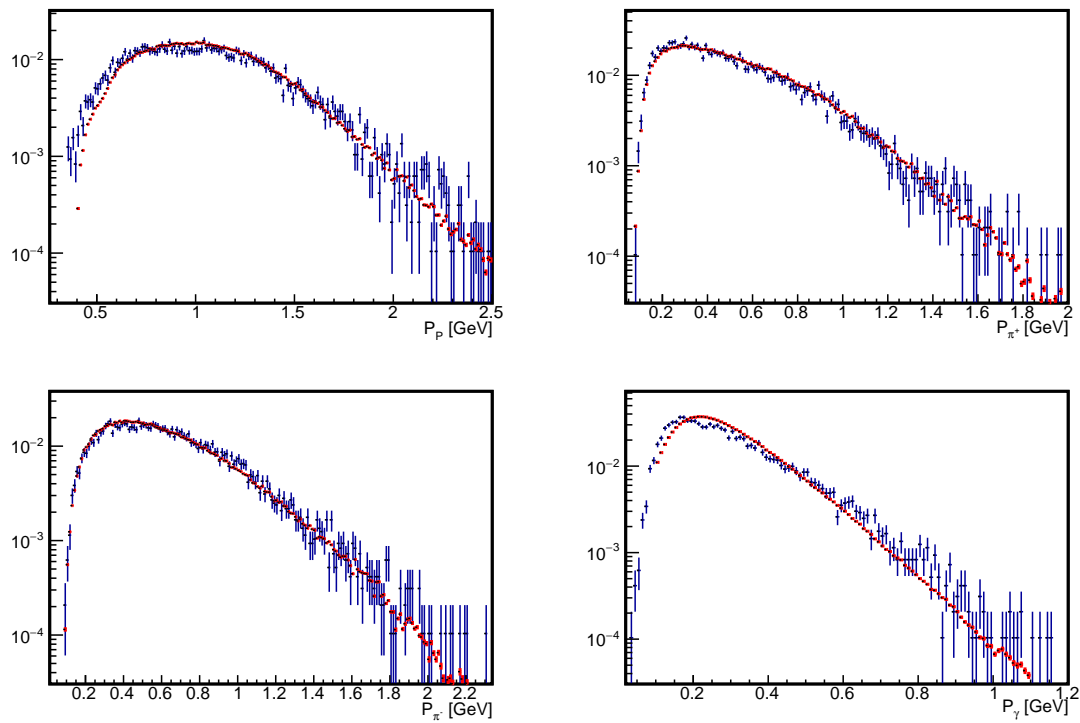


FIG. 51: Comparison of momenta of each final state particle from Monte Carlo and experimental data. Experimental data is in blue and simulated data is in red.

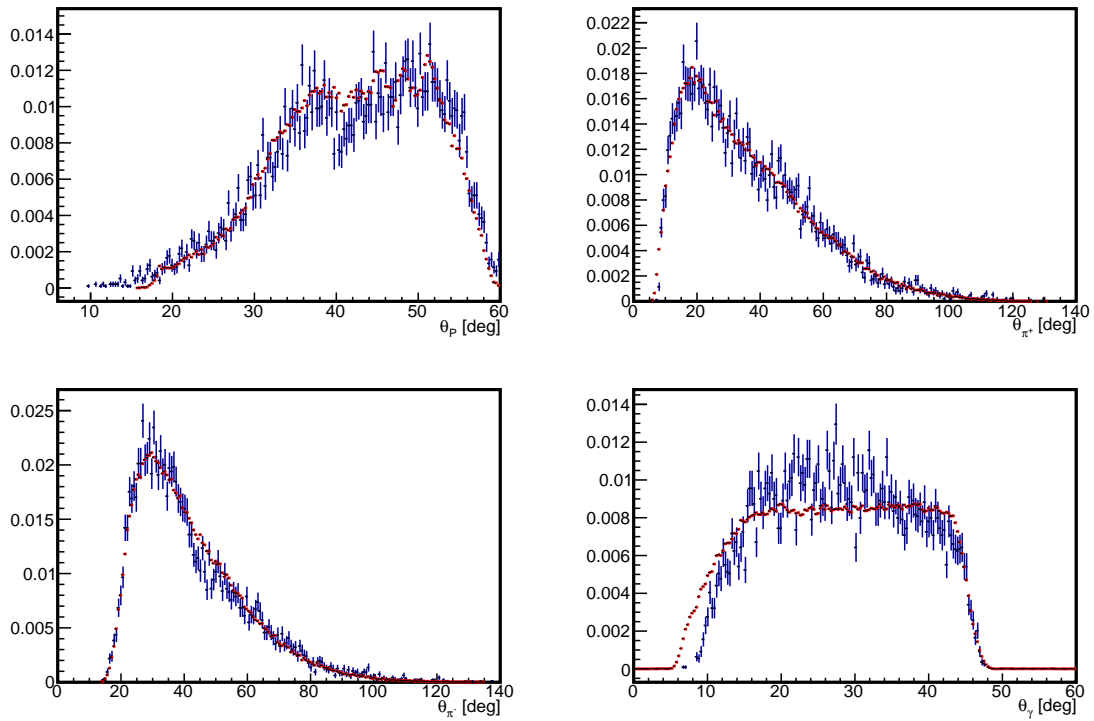


FIG. 52: Comparison of θ of each final state particle from Monte Carlo and experimental data. Experimental data is in blue and simulated data is in red. An additional cut has been placed on θ_γ to be in the range of 15-45 deg.

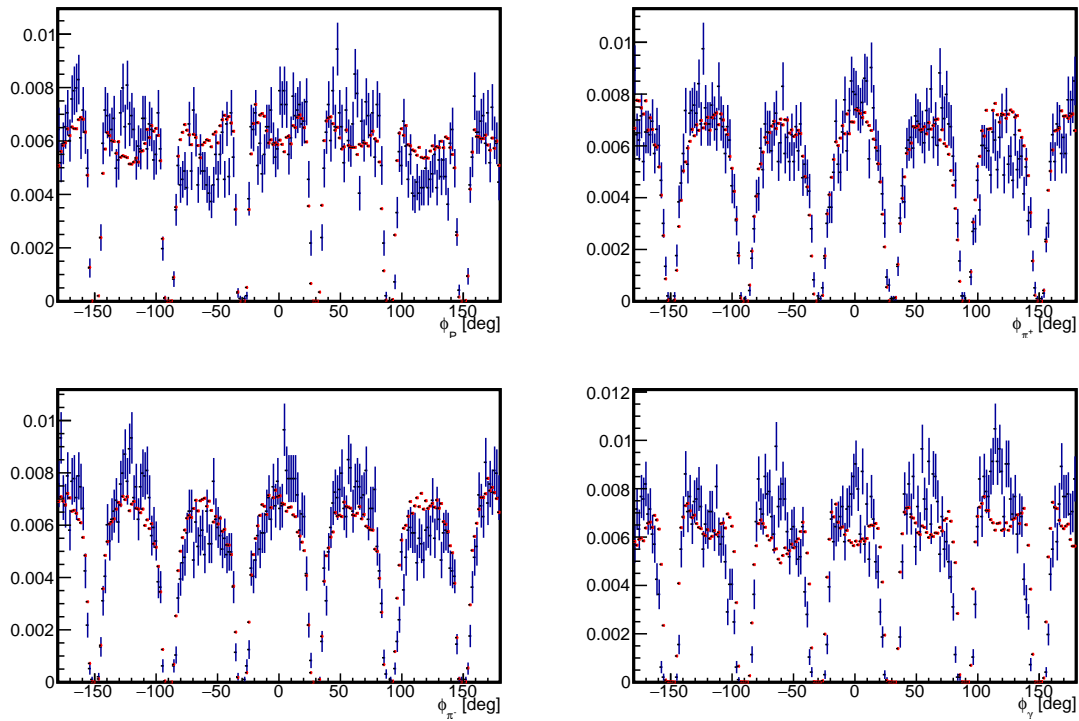


FIG. 53: Comparison of ϕ of each final state particle. Experimental data is in blue and simulated data is in red.

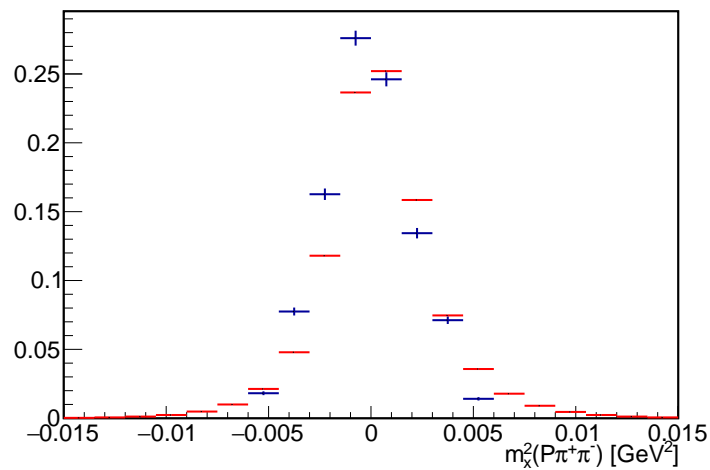


FIG. 54: Comparison of $m_x^2(P\pi^+\pi^-)$. Experimental data is in blue and simulated data is in red.

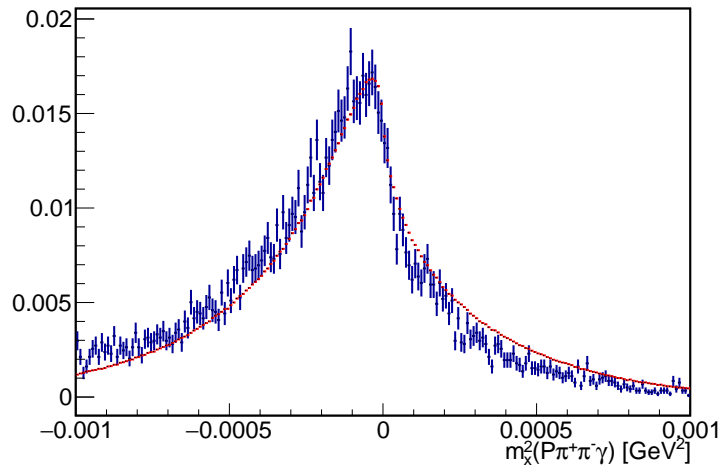


FIG. 55: Comparison of $m_x^2(P\pi^+\pi^-\gamma)$. Experimental data is in blue and simulated data is in red.

727 The comparison between the data and reconstructed Monte Carlo events is also shown
 728 for $m_x^2(P\pi^+\pi^-)$ and $m_x^2(P\pi^+\pi^-\gamma)$ in Fig. 54 and Fig. 55. The overall agreement between
 729 the g11 data and the reconstructed Monte Carlo is good.

CHAPTER 4

730

RESULTS

731

4.1 CLAS DETECTOR ACCEPTANCE

732

733 The acceptance is the probability that an event will be kept after it has been passed
 734 through the simulation chain (described in Chapter 3) and subject to the same cuts as in
 735 the data analysis. Events that do pass are called reconstructed events. The acceptance for
 736 each $s(\pi^+\pi^-)$ bin is calculated as:

$$A(s_{\pi^+\pi^-}) = \frac{N_{rec}}{N_{gen}} \quad (4.1.55)$$

737 where N_{rec} is the number of successfully reconstructed events and N_{gen} is the number of
 738 generated events in each bin. The error on each data point is calculated using the standard
 739 error propagation for Poisson statistics,

$$\begin{aligned} \sigma(N_{rec}) &= \sqrt{N_{rec}} \\ \sigma(N_{gen}) &= \sqrt{N_{gen}} \end{aligned} \quad (4.1.56)$$

740 From Eq. 4.1.55, the error for each data point is derived as

$$\left(\frac{\sigma_A}{A}\right) = \left[\left(\frac{\sigma(N_{rec})}{N_{rec}}\right)^2 + \left(\frac{\sigma(N_{gen})}{N_{gen}}\right)^2 \right]^{\frac{1}{2}}. \quad (4.1.57)$$

741 The acceptance as a function of $s(\pi^+\pi^-)$ is shown in Fig. 56. The acceptance is fit with
 742 a fourth-order polynomial. The data is then corrected by the value of the polynomial fit at
 743 each $s(\pi^+\pi^-)$ bin.

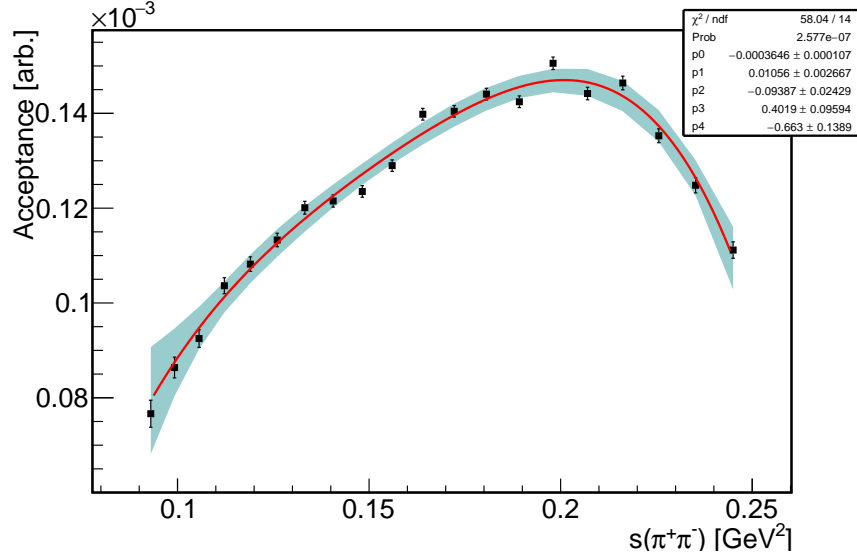


FIG. 56: Acceptance as function of $s(\pi^+\pi^-)$. Red line is a fit with 4th order polynomial. Green band represents 95% confidence interval.

744 4.2 α PARAMETER

745 After obtaining the acceptance, the data in Fig. 49 is then corrected by the value of
 746 the fitted function at the corresponding bin. To account for the errors on the parameters,
 747 the covariance matrix was recorded and errors were calculated using the general formula
 748 presented in Eq. 15.

$$\sigma_{polynomial}^2 = \begin{pmatrix} \frac{\partial f}{\partial p_0} & \cdot & \cdot & \cdot & \frac{\partial f}{\partial p_n} \end{pmatrix} \begin{pmatrix} \sigma_{a_0}^2 & \cdot & \cdot & \rho\sigma_{a_0}\sigma_{a_n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \rho\sigma_{a_n}\sigma_{a_0} & \cdot & \cdot & \sigma_{a_n}^2 \end{pmatrix} \begin{pmatrix} \frac{\partial f}{\partial p_0} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial f}{\partial p_n} \end{pmatrix} \quad (4.2.58)$$

749 .

750 The resulting data is then fit with Eq. 2 in order to extract α . Fig. 57 displays the fit to
 751 the acceptance corrected data. The error bars contain the statistical error of the number of
 752 reconstructed photons and the error from the acceptance added in quadrature. Our result
 753 is $\alpha = 1.51 \pm 0.24_{stat}$.

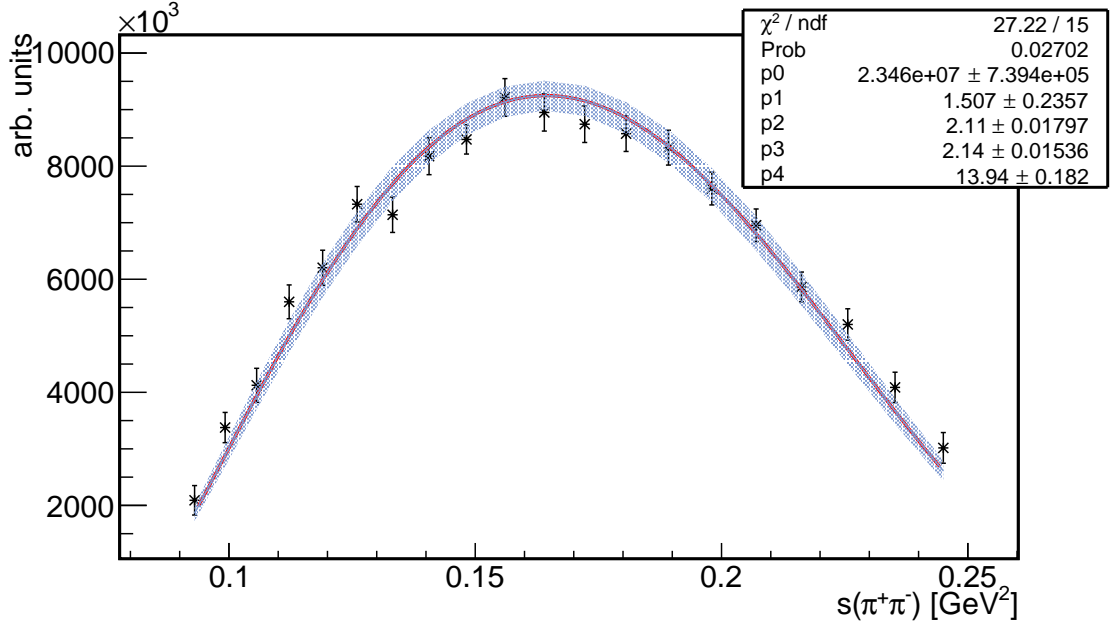


FIG. 57: Acceptance corrected N_γ in arbitrary units as function of $s(\pi^+\pi^-)$. Error bars contain statistical errors and the error from the acceptance added in quadrature. The red line is the fit function presented in Eq. 2. The purple band is 95% confidence interval.

754 4.3 SYSTEMATIC ERRORS

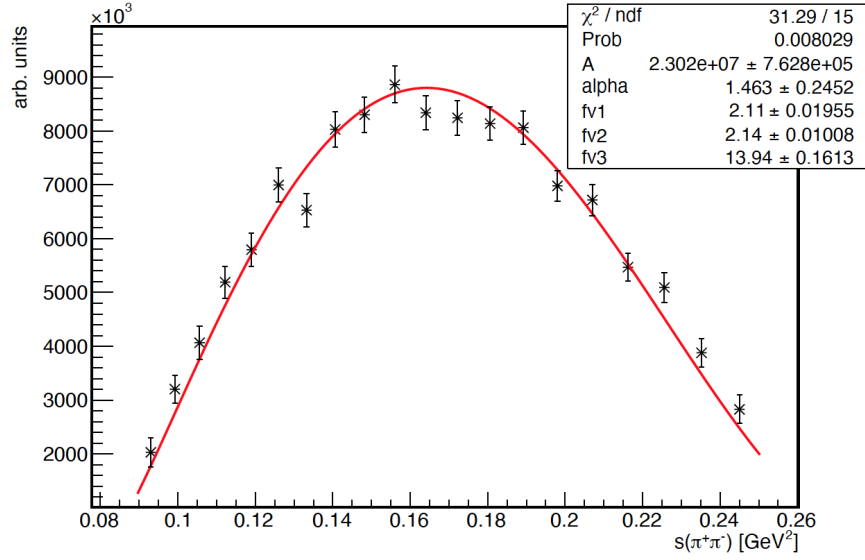
755 In order to calculate the systematic error of our measurement, the following cuts were
756 varied from their original values:

- 757 • $|m_x(P) - M_\eta| < 0.03$
- 758 • $|m_x(P) - 0.605| < 0.0075$
- 759 • $|m_x(P) - 0.490| < 0.0075$
- 760 • $|m_x^2(P\pi^+\pi^-\gamma)| < 0.0005$

761 The width of the signal and sideband regions defined when performing the background
762 subtraction can contribute a systematic error to the measurement. We also vary the exclu-
763 sivity cut. The changes in the acceptance due to the new cuts is assumed to be negligible.
764 To account for the systematic errors, the cuts are adjusted and then the analysis procedure
765 described in Chapter 3 is repeated for each cut individually. The fit results for the systematic
766 variations can be seen in Fig. 59 -60.

Original Cut	Cut Variation	Description
$ m_{x_P} - M_\eta < 0.3$	$ m_{x_P} - M_\eta < 0.34$	adjusting signal region
$ m_{x_P} - 0.605 < 0.0075$	$ m_{x_P} - 0.605 < 0.0085$	adjusting right sideband
$ m_{x_P} - 0.490 < 0.0075$	$ m_{x_P} - 0.490 < 0.0085$	adjusting left sideband
$ m_{x2_P\pi^+\pi^-\gamma} < 0.0005$	$ m_{x2_P\pi^+\pi^-\gamma} < 0.00055$	less exclusive cut
$ m_{x2_P\pi^+\pi^-\gamma} < 0.0005$	$ m_{x2_P\pi^+\pi^-\gamma} < 0.00045$	more exclusive cut

TABLE 13: Cuts used in calculating systematic errors.

FIG. 58: Fit results for $m_x^2(P\pi^+\pi^-\gamma) < 0.00045$.

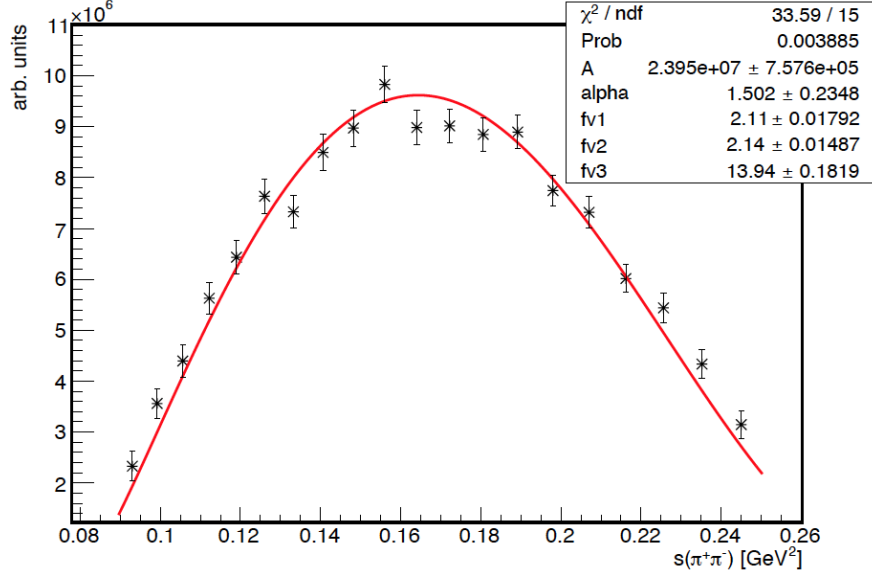


FIG. 59: Fit results for $m_x^2(P\pi^+\pi^-\gamma) < 0.00055$.

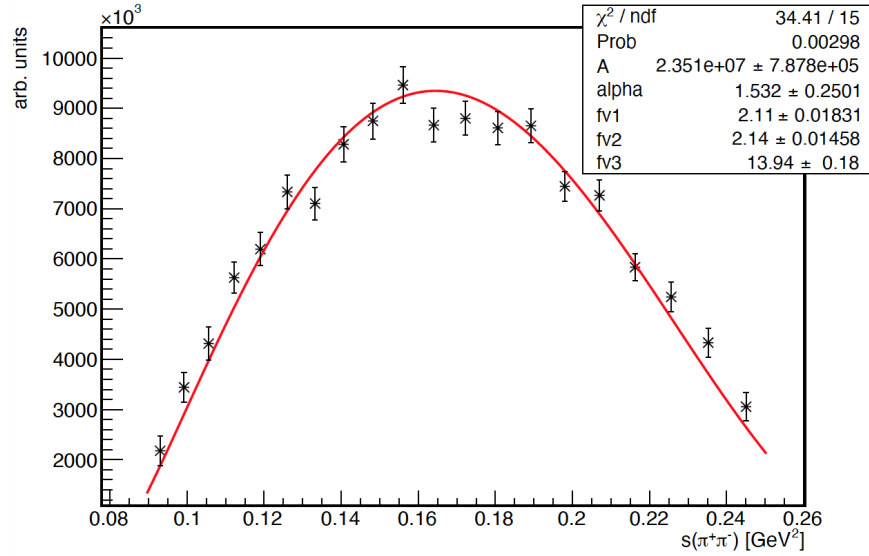


FIG. 60: Fit results for adjusting signal region to $|m_x(P) - M_\eta|$ and sideband regions to $|m_x(P) - 0.605| < 0.0085$ and $|m_x(P) - 0.490| < 0.0085$.

767 The relative difference between the reference value, α_r , and the systematic variation,
 768 α_{sys} , is calculated as

$$\Delta\alpha_i = \frac{|\alpha_r - \alpha_{sys}|}{\alpha_r}. \quad (4.3.59)$$

769 The relative difference is calculated for each systematic error and the results are added
 770 in quadrature. The results are tabulated in Table 14. The largest contribution to the total
 771 systematic error is due to the exclusivity cut.

Systematic	Relative Error
$ m_x(P) - M_\eta < 0.034$	
$ m_x(P) - 0.605 < 0.0085$	
$ m_x(P) - 0.490 < 0.0085$	0.006
$ m_x^2(P\pi^+\pi^-\gamma) < 0.00055$	0.016
$ m_x^2(P\pi^+\pi^-\gamma) < 0.00045$	0.039
Total Systematic Error	0.043

TABLE 14: Tabulated systematic errors and their total.

772 4.4 COMPARISON TO OTHER MEASUREMENTS

773 The most recent measurements of the α parameter, produced by WASA-at-COSY and
 774 KLOE collaborations, are summarized in the table below and plotted together with various
 775 theoretical calculations in Fig. 61. The average of the experimental measurements (KLOE,
 776 CLAS, WASA) is 1.57 ± 0.34 and is represented by the purple band. Only the recent
 777 theoretical work from Kubis and Plenter [40] is in agreement with the latest experimental
 778 measurements.

779 4.4.1 COMPARISON WITH PREVIOUS EXPERIMENTAL MEASURE- 780 MENTS

781 The first reported measurement of the α parameter was obtained from Gormley, et al.
 782 in 1970. The analysis of 7250 $\eta \rightarrow \pi^+\pi^-\gamma$ events yielded an $\alpha = 1.8 \pm 0.4$. In 1973, Layter,
 783 et al. analysed 18150 $\eta \rightarrow \pi^+\pi^-\gamma$ events yielding an $\alpha = -0.9 \pm 0.1$. The most recent
 784 measurements were performed by the WASA-at-COSY collaboration, analysing 13960 ± 140
 785 events, yielding an $\alpha = 1.89 \pm 0.25_{stat} \pm 0.59_{sys} \pm 0.02_{theo}$. The KLOE collaboration measured
 786 an $\alpha = 1.32 \pm 0.2_{total}$. It is important to note this measurement takes in to account the effects
 787 of the a2 tensor meson. Our measurement is in agreement with the result from the KLOE
 788 collaboration and is more precise than that of the WASA collaboration. Our measurement
 789 differs significantly from that of Layter, et al.

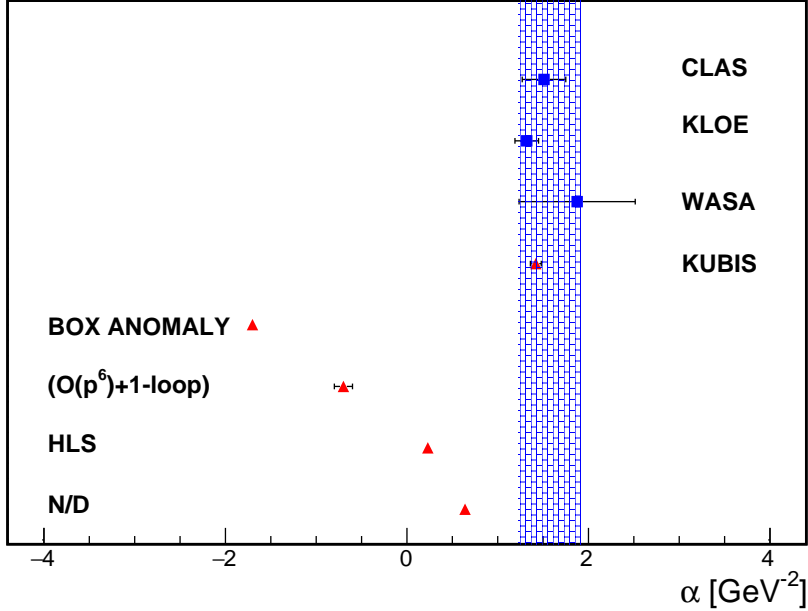


FIG. 61: Graphical representation of experimental measurements (squares) and theoretical calculations (triangles) of α from the decay $\eta \rightarrow \pi^+\pi^-\gamma$. The purple shaded region represents the average of the three experimental measurements. Error bars for the experimental measurements contain statistical and systematic errors added in quadrature. References for data points (top to bottom) KLOE [38], WASA [39], Kubis [40], Box Anomaly [24], $(O(p^6)+1\text{-loop})$ [19], HLS [21], and N/D [23].

790 4.4.2 COMPARISON WITH THEORY PREDICTIONS

791 From Fig. 61 it is easily seen that the experimental measurements (within the average
 792 band) agree with the most recent theoretical result from Kubis [40]. The remaining theo-
 793 retical predictions, corresponding to the different models described in Sec. 1.5, are in stark
 794 contrast with our measurement.

795 4.5 CONCLUSION

796 We extract the α parameter from a model independent fit to the dipion invariant mass
 797 distribution using CLAS g11 data. This parameter measures the contribution of the dipion
 798 momentum dependence to the partial decay width of $\eta \rightarrow \pi^+\pi^-\gamma$, which proceeds from the
 799 box anomaly. Our value, $\alpha = 1.51 \pm 0.24_{\text{stat}} + 0.04_{\text{sys}}$ is in agreement with the most recent

800 experimental measurement from the KLOE collaboration and the theoretical calculation
801 from Kubis et al. Our result strongly excludes the previous theoretical calculations.

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APPENDIX A

876

SIMULATION SCRIPTS

877

A.1 GAMP2MC

878

879 The executable to run the gamp2MC script is located here:

```
880 /work/clas/clasg11/devita/gamp2MC/build/bin/gamp2MC
```

881 . The command used is

```
882 /work/clas/clasg11/devita/gamp2MC/build/bin/gamp2MC -m -r43582
```

```
883 -o<output file>-T -S-0.321,-0.254,0.378,0.407
```

```
884 -z-30,10 <input file>
```

A.2 GSIM

885

886 The command used to run gsim:

```
887 gsim_bat -ffread gsiman.input -kine 1 -mcin
```

```
888 <input bos file> -bosout
```

889 FFREAD card used for simulations:

```
890 CUTS 5.e-3 5.e-3 5.e-3 5.e-3 5.e-3
```

```
891 DCCUTS 1.e-4 1.e-4 1.e-4 1.e-4 1.e-4
```

```
892 ECCUTS 5.e-4 5.e-4 5.e-4 5.e-4 5.e-4
```

```
893 SCCUTS 1.e-4 1.e-4 1.e-4 1.e-4 1.e-4
```

```
894 STCUTS 5.e-5 5.e-5 5.e-5 5.e-5 5.e-5
```

```
895 AUTO 1
```

```
896 KINE 1
```

```
897 MAGTYPE 2
```

```
898 MAGSCALE 0.4974
```

```
899 FIELD 2
```

```
900 GEOM 'ALL'
```

```
901 NOGEOM 'EC1' 'CC' 'MINI' 'PTG'
```

```

902 NOSEC 'OTHE'
903 TARGET 'g11a'
904 TGPOS 0.0 0.0 0.0
905 STZOFF -10.0
906 STTYPE 1
907 RUNG 43582
908 TRIG 500000
909 STOP

```

910 A.3 GPP

911 Command line to run GPP:

```

912 gpp -P0x1f -oclas.centos6.gpp -R43582
913 -a1.0 -b1.0 -c1.0 -f1.0 -Y clas.centos6.evt

```

914 A.4 RECSIS

915 Command line for cooking:

```

916 user_ana -t new_recsis.tcl
917 RECSIS file used for cooking g11 data
918 source /group/clas/builds/test3/
919 src/clas6-trunk/reconstruction/recsis/recsis_proc.tcl;
920 turnoff ALL;
921 global_section off;
922 turnon seb trk tof egn user pid;
923 inputfile          clas.evt;
924 setc chist_filename ntuple.hbook;
925 setc log_file_name logfile;
926 setc outbanknames(1) "all";
927 outputfile clas.out PROC 2047;
928 setc prlink_file_name "prlink_g11_1920.bos";
929 setc bfield_file_name "bgrid_T67to33.fpk";
930 set torus_current 1920;
931 set mini_torus_current 0;

```

```
932 set poltarget_current 0;
933 set TargetPos(3) -10.;
934 set trk_maxiter 8;
935 set trk_minhits(1) 2;
936 set trk_lrambfit_chi2 50.;
937 set trk_tbtfit_chi2 70.;
938 set trk_prfit_chi2 70.;
939 set trk_statistics 3 ;
940 set dc_xvst_choice 0;
941 set def_adc -1;
942 set def_tdc -1;
943 set def_atten -1;
944 set def_geom -1;
945 set st_tagger_match 15.;
946 set lst_do -1;
947 set lpid_make_trks 0;
948 set photon_trig_type 4;
949 set lseb_nt_do -1;
950 set lall_nt_do -1;
951 set lscr_nt_do -1;
952 set lpart_nt_do -1;
953 set lst_nt_do -1;
954 set ltbt_nt_do -1;
955 set lmvrt_nt_do -1;
956 set lmctk_nt_do -1;
957 set lgpid_do -1;
958 fpack "timestop_ -999999999"
959 setc rec_prompt "CLASCHEF_recsis>_";
960 go 2000000;
961 exit_pend;
```

APPENDIX B

962

DOCKER CONTAINER

963

964 To run software interactively inside the Docker container:

```
965 module load singularity
```

966 then attach the following command ahead of the program you want to run:

```
967 singularity exec /work/clas/clase1/tylern/clas6.img
```

968 for example, to run GSim:

```
969 singularity exec /work/clas/clase1/tylern/clas6.img gsim_bat -ffread <ffr
```

APPENDIX C

USEFUL DEFINITIONS

C.1 4-VECTORS

A 4-vector is a mathematical object with four components. It differs from a Euclidean 3 vector in how its magnitude is determined. The energy and 3-momentum of a particle form a 4-vector $P^\mu = (\vec{p}, E)$, when squared gives the squared mass of the particle, $P \cdot P = E^2 - |\vec{p}|^2 = m^2$. For this work, we use the following four vectors in calculations:

$$\begin{aligned}
 P_{beam} &= (0, 0, P_{beam}, E_{beam}) \\
 P_{target} &= (0, 0, 0, M_P) \\
 P_P &= (Px_P, Py_P, Pz_P, E_P) \\
 P_{\pi^+} &= (Px_{\pi^+}, Py_{\pi^+}, Pz_{\pi^+}, E_{\pi^+}) \\
 P_{\pi^-} &= (Px_{\pi^-}, Py_{\pi^-}, Pz_{\pi^-}, E_{\pi^-}) \\
 P_\gamma &= (Px_\gamma, Py_\gamma, Pz_\gamma, E_\gamma),
 \end{aligned} \tag{3.1.60}$$

where the four vector represents the target at rest.

C.2 KINEMATIC DEFINITIONS

We define the missing mass of the proton, $m_x(P)$ from the reaction $\gamma p \rightarrow pX$ as

$$(P_{beam} + P_{target} - P_P)^2 = P_X^2 = m_X^2. \tag{3.2.61}$$

The missing mass of the proton, π^+ , and π^- , $m_x^2(P\pi^+\pi^-)$ is calculated as

$$(P_{beam} + P_{target} - P_P - P_{\pi^+} - P_{\pi^-})^2 = P_X^2 = m_X^2, \tag{3.2.62}$$

where for our reaction, the missing particle could have been a photon or neutral pion. The missing energy, $m_e(P\pi^+\pi^-)$, is defined as

$$E_{beam} + E_{target} - E_P - E_{\pi^+} - E_{\pi^-} = E_X. \tag{3.2.63}$$

The invariant mass of π^+ and π^- , is calculated as the square root of the Mandelstan s variable

$$s = IM^2(\pi^+\pi^-) = (P_{\pi^+} + P_{\pi^-})^2 = M_{\pi^+}^2 + M_{\pi^-}^2 + 2(E_{\pi^+}E_{\pi^-} - \vec{p}_{\pi^+} \cdot \vec{p}_{\pi^-}). \tag{3.2.64}$$

In this work, the invariant mass ranges from $4m_\pi^2 < s(\pi^+\pi^-) < m_\eta^2$.

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APPENDIX D

987

FITS TO DATA

988 This Appendix shows the fits to the $m_x(P)$ and $m_x^2(P\pi^+\pi^-)$ distributions in order to per-
 989 form the sideband subtraction and extraction of the number of photons per bin of $IM(\pi^+\pi^-)$.

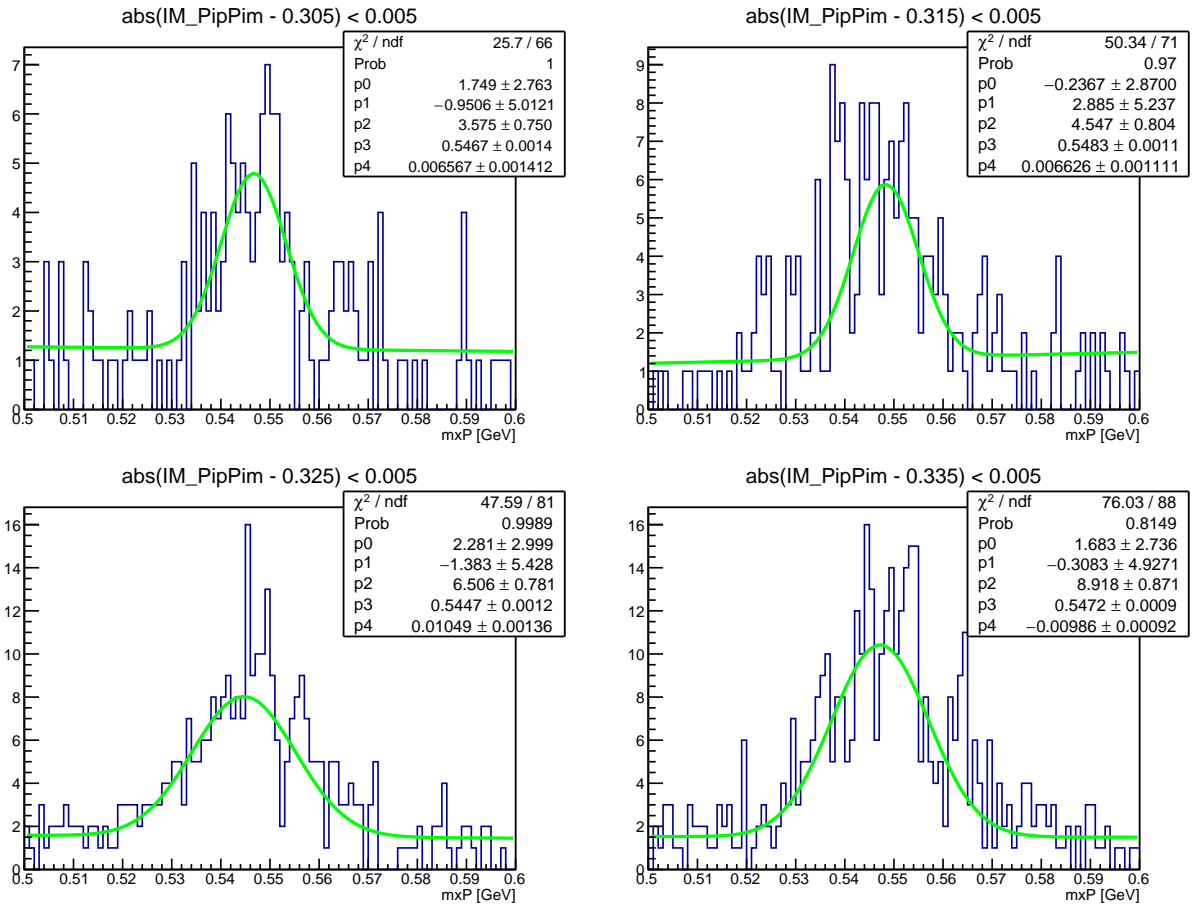


FIG. 62: Fit to η peak in the missing mass of the proton, $m_x(P)$. The invariant mass bin is specified at the top of each figure.

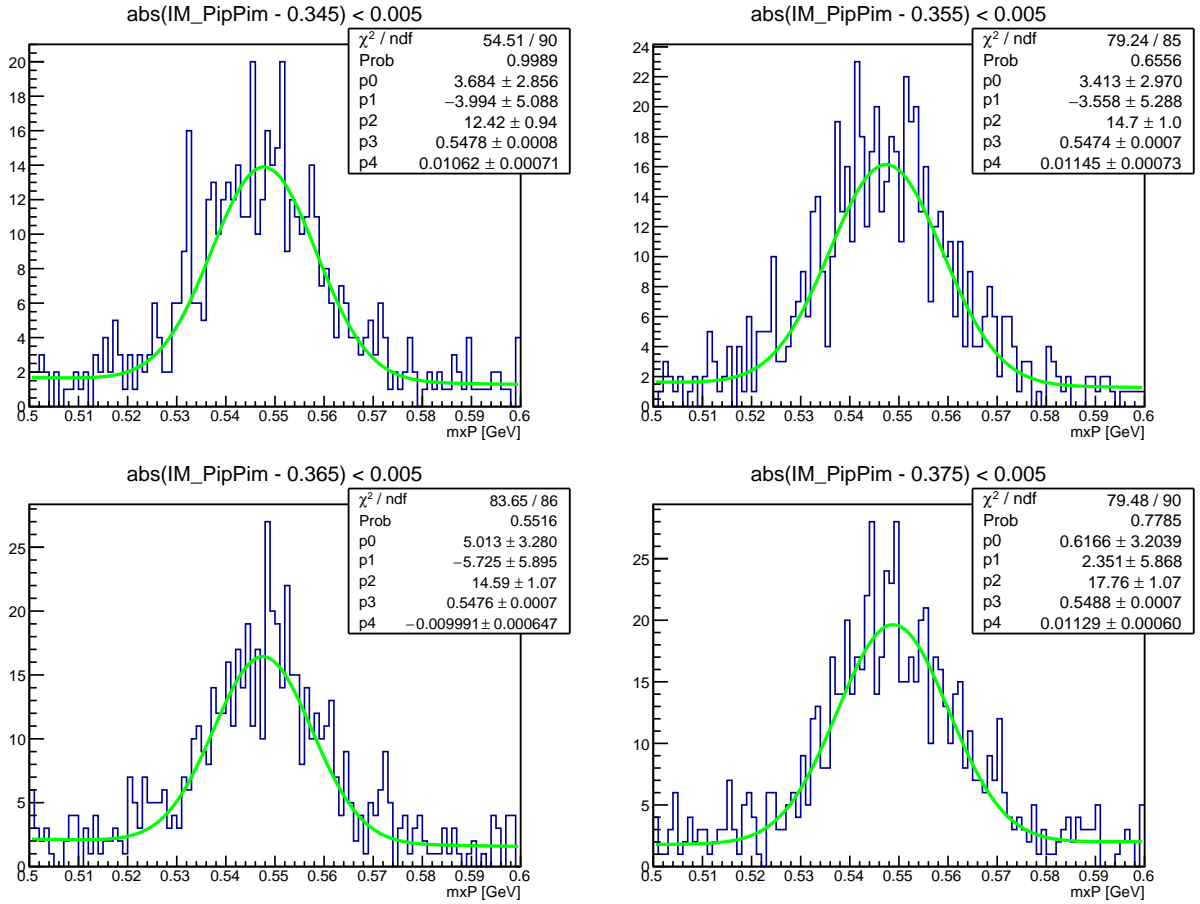


FIG. 63: Fit to η peak in the missing mass of the proton, $m_x(P)$. The invariant mass bin is specified at the top of each figure.

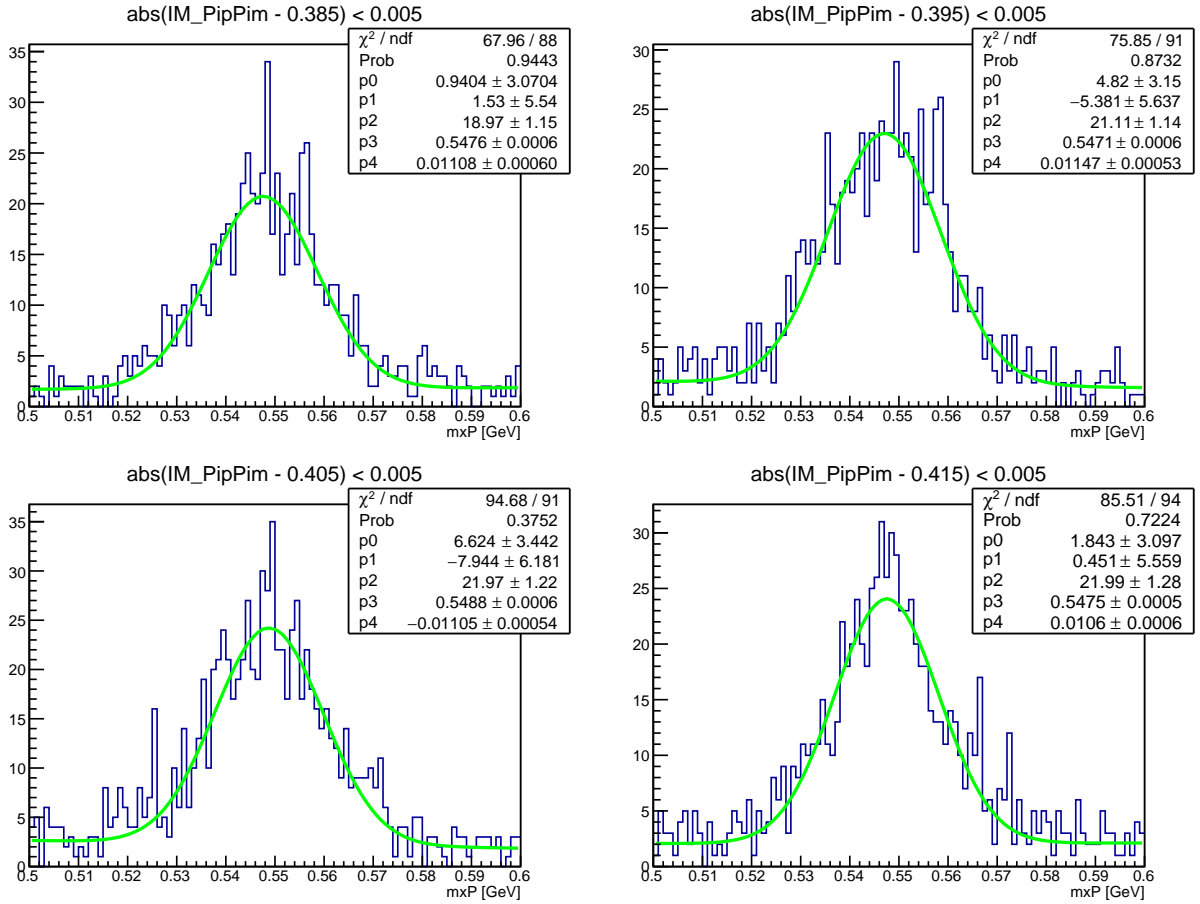


FIG. 64: Fit to η peak in the missing mass of the proton, $m_x(P)$. The invariant mass bin is specified at the top of each figure.

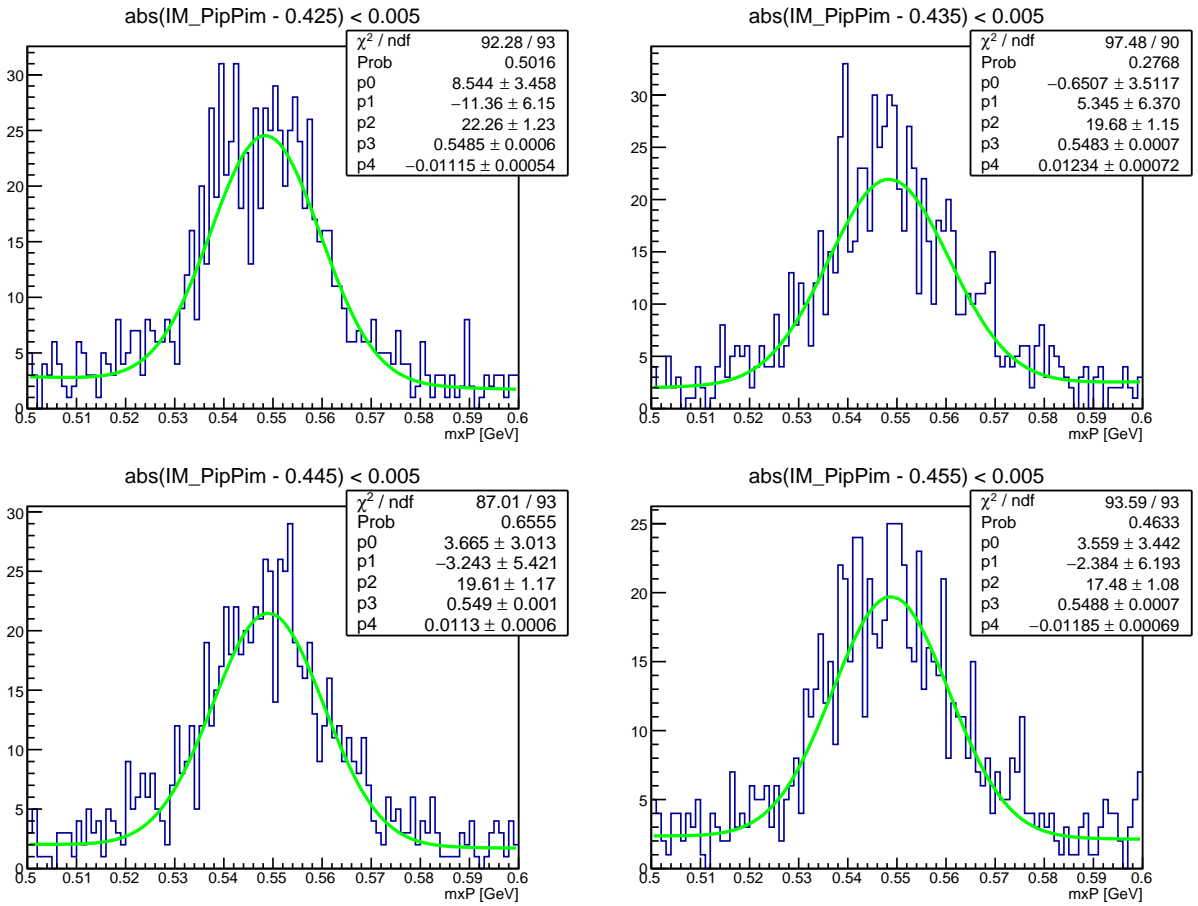


FIG. 65: Fit to η peak in the missing mass of the proton, $m_x(P)$. The invariant mass bin is specified at the top of each figure.

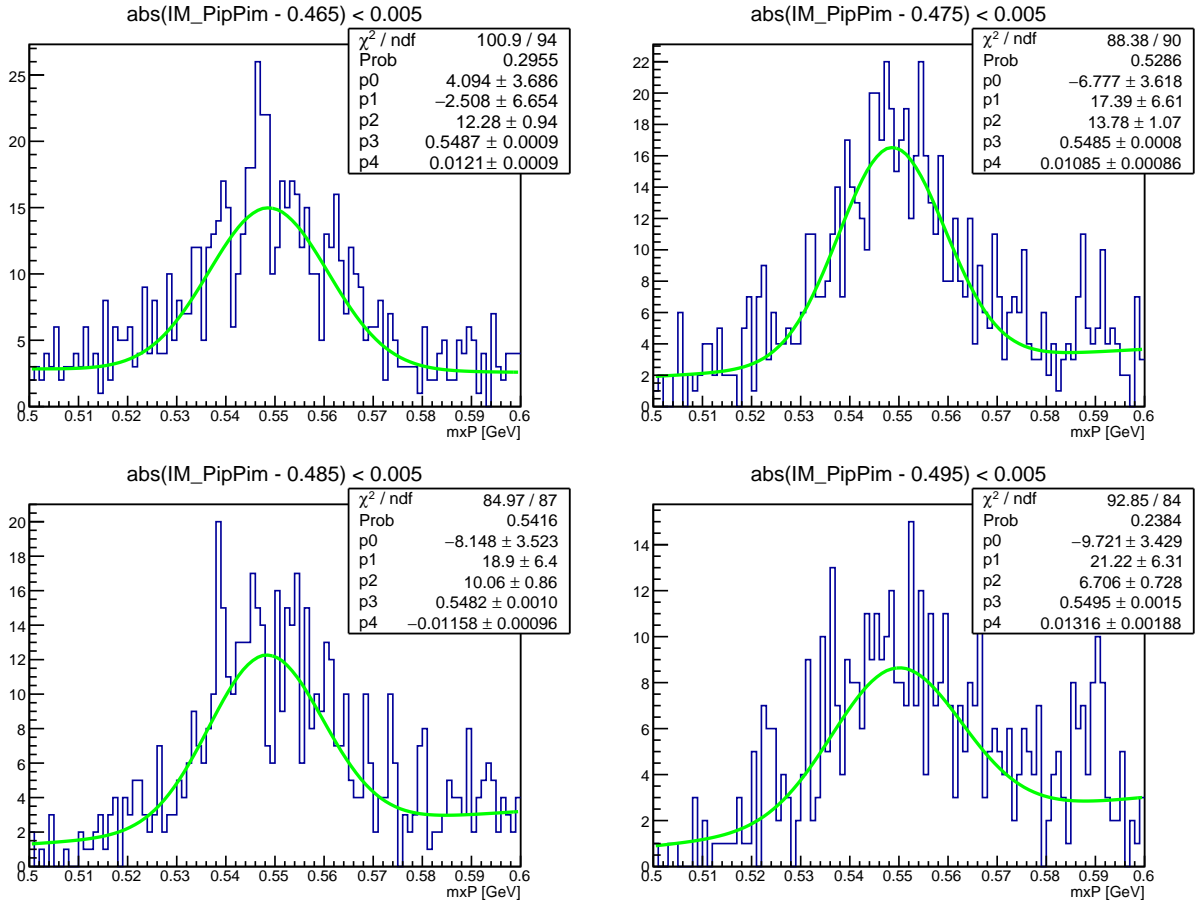


FIG. 66: Fit to η peak in the missing mass of the proton, $m_x(P)$. The invariant mass bin is specified at the top of each figure.

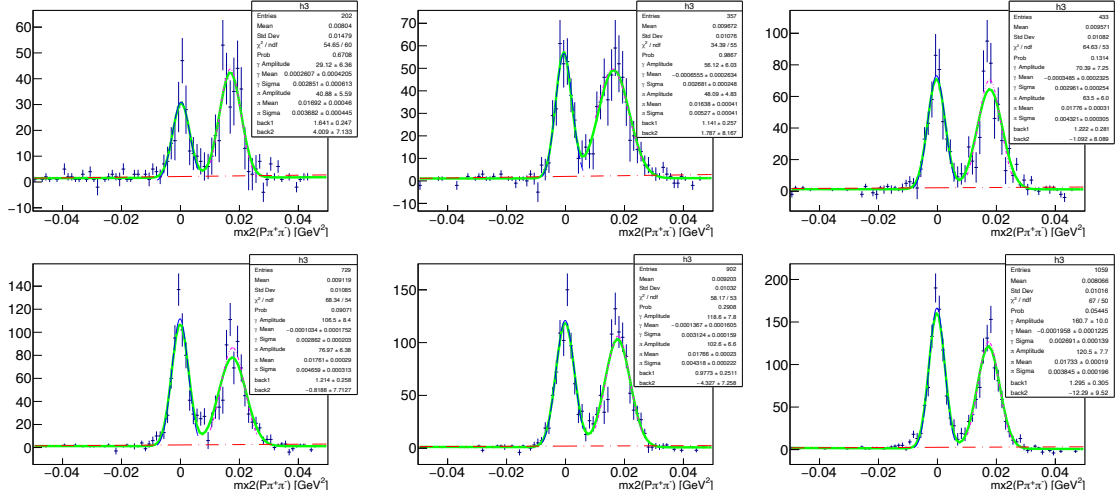


FIG. 67: Fit to $m_x^2(P\pi^+\pi^-)$ for $IM(\pi^+\pi^-) \in [0.305, 0.355]$. Red dash curve is linear background function. Green is total fit function: sum of 2 Gaussians and linear polynomial.

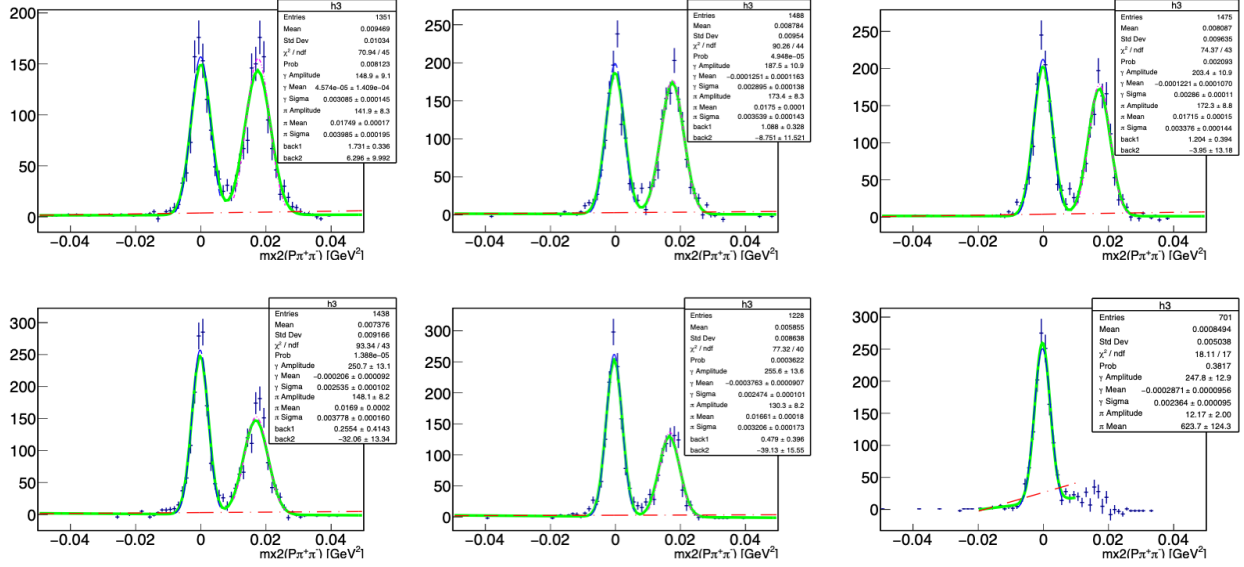


FIG. 68: Fit to $m_x^2(P\pi^+\pi^-)$ for $IM(\pi^+\pi^-) \in [0.365, 0.415]$. Red dash curve is linear background function. Green is total fit function: sum of 2 Gaussians and linear polynomial.

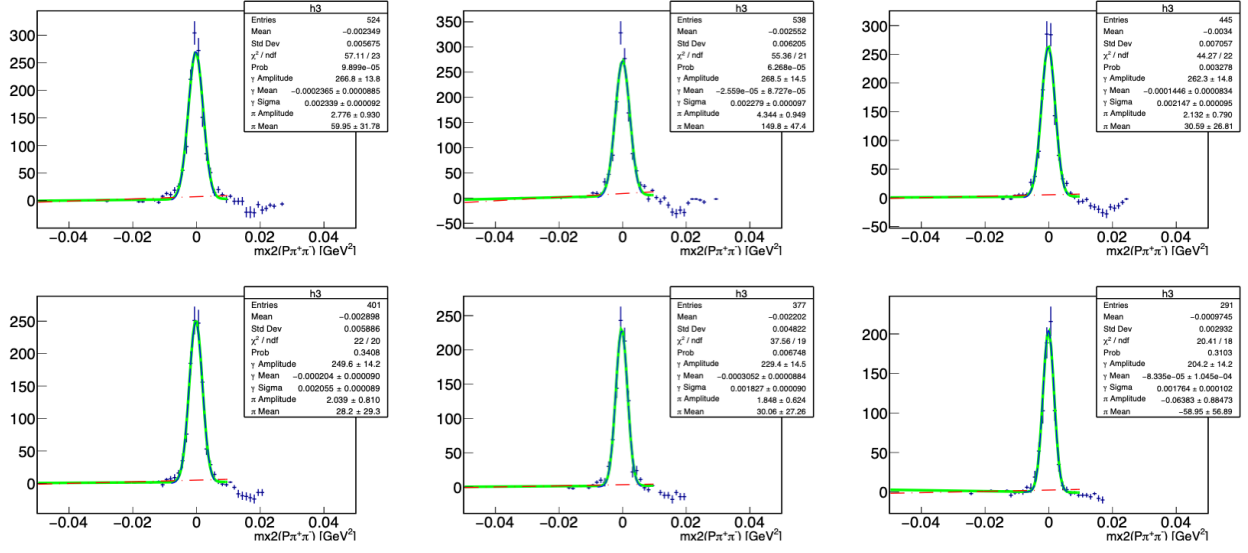


FIG. 69: Fit to $m_x^2(P\pi^+\pi^-)$ for $IM(\pi^+\pi^-) \in [0.425, 0.475]$. Red dash curve is linear background function. Green is total fit function: sum of 2 Gaussians and linear polynomial.

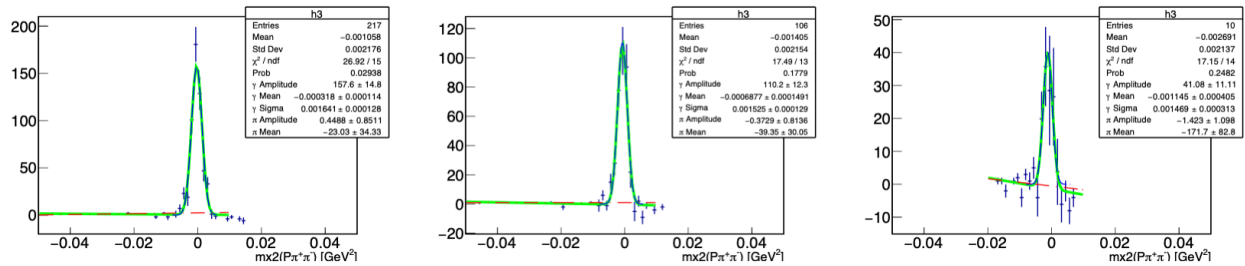


FIG. 70: Fit to $m_x^2(P\pi^+\pi^-)$ for $IM(\pi^+\pi^-) \in [0.485, 0.505]$. Red dash curve is linear background function. Green is total fit function: sum of Gaussian and linear polynomial.

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