Phi Dependence and Event Generator Dependence of the Acceptance for Electroproduction of Two-Body Final States

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Abstract

The acceptance function for detecting the scattered electron and $K^+$ from exclusive $K^+\Lambda$ electroproduction was studied. For a fixed bin in $\cos \theta_{K}, q^2$, and $W$, the $\phi$-dependence was fit well by the functional form, $a + b \cos \phi + c \cos 2\phi + d \cos 3\phi$. The dependence of the binned acceptance on the functional form of the event generator was also studied. The leak-through of an errant input function on the value of the extracted function was quantified. The resulting errors were found to vary inversely with the number of $\phi$ bins used in the analysis.

Introduction:

These studies were based on the ntuple, “1115”, produced by Pawel Ambrozewicz and Avto Tkabladze. For my purposes I take the quotient, evacc/evgen, as the acceptance of CLAS. I then study its kinematic dependence on $\cos \theta_{K}$, $\phi$, $q^2$, and $W$. I find that I can fit the $\phi$ dependence well as a function of four parameters only: $a + b \cos \phi + c \cos 2\phi + d \cos 3\phi$. I study the $\cos \theta_{K}$ dependence of the four parameters and find them to be smoothly varying. Note that in some places, for example very forward $\cos \theta_{K}$, that parameter $d$ is very small; i.e. the acceptance has the same shape as the extracted structure functions! This is a dangerous situation.

I then used the fitted form of the acceptance to study the effect of using different event generators to calculate the acceptance. I find that a deviation of the assumed functional form of the cross section from reality (i.e. we assume $a_1 + b_1 \cos \phi + c_1 \cos 2\phi$, but the coefficients are really $a_2$, $b_2$ and $c_2$) leads to a systematic distortion of the acceptance and subsequently to a systematic distortion of the extracted structure function parameters, $a_{ext}$, $b_{ext}$, and $c_{ext}$. These can be approximated as four partial derivatives: $\delta b_{ext}/\delta b_{in}$, $\delta b_{ext}/\delta c_{in}$, $\delta c_{ext}/\delta b_{in}$, and $\delta c_{ext}/\delta c_{in}$. In general I find that the extracted value of the constant parameter, $a_{ext}$, is unchanged. The size of the error derivatives varies strongly with $\cos \theta_{K}$; it is largest for values of $\cos \theta_{K}$ between about 0.1 and 0.7. The size of the error derivatives also varies strongly with the number of $\phi$ bins used.

Method:

I first assumed a functional form form for the cross-section to be $\text{xsn} = a + b \cos \phi + c \cos 2\phi$. I then obtained the acceptance from our ntuple; evacc/evgen. I
histogrammed and fit the acceptance as a function of \( \phi \) for each bin in \( \cos \theta_K, q^2 \) and \( W \). I then studied the effect of the functional form of the event generator on the calculated cross-section and thus on the extracted structure functions in the following way:

- I defined \( xsn1 \) to be \( a1 + b1\cos \phi + c1\cos2\phi \) and \( xsn2 \) to be \( a2 + b2\cos \phi + c2\cos2\phi \).

- I then obtained the acceptance from our ntuple; \( ev_{\text{acc}}/ev_{\text{gen}} \). I histogrammed and fit the acceptance as a function of \( \phi \) for each bin in \( \cos \theta_k, q^2 \) and \( W \). See figures 1, 2 and 3 for the shape and fit to the acceptance.

- Using the fitted acceptance function, \( \text{accep}(\phi) \) I obtained an accepted yield function, \( ay = xsn(\phi) \ast \text{accep}(\phi) \). The acceptance can now be defined as the integral of \( ay \) over a \( \phi \) bin, divided by the integral of the \( xsn \) over the \( \phi \) bin. This is referred to as the “integrated” or “binned” acceptance.

- In practice, I performed the integrations by first binning the 2 functions, \( xsn \) and \( ay \), in 160 bins in \( \phi \). I then re-binned into the final binning choice, e.g. into 8 bins, 10 bins, 16 bins, etc. This re-binning summed the original bin yields over the bounds of the new bin.

- I then produced two “observed” cross-sections. I divided the integrated accepted yield, \( ay \) calculated using \( xsn1 \) by two different values of the integrated acceptance: one calculated using \( xsn1 \) as the event generator and the second calculated using \( xsn2 \). I then fitted the observed cross-sections to our familiar form, \( a + b\cos \phi + c\cos2\phi \) and studied the difference between \( a1, b1, c1 \) and \( a2, b2, c2 \) where the number refers to which \( xsn \) was used to produce the acceptance.

Results:

Functional form of the acceptance:

See figs. 1 and 2 for a plot of the acceptance as a function of \( \phi \) for different bins in \( \cos \theta_K \). Overplotted is the fitted function. To see the values of the fit parameter, see Fig. 3. Figs. 1 - 3 were produced for the \( W = 1.8 \) GeV bin. To see a fit of the \( \theta \)-dependence of the extracted parameters, p1 - p4, refer to figs. 9 to 12 which correspond to \( W=1.7, 1.8, 1.9 \) and 2.0 GeV, respectively.

Event generator dependence of the acceptance: effect on the extracted values of \( a, b, c \):

The size of the change in \( b \) and \( c \), \( \delta b \) and \( \delta c \) was studied as a function of the size of the input change in \( b \) and \( c \), \( \delta b_{in} \) and \( \delta c_{in} \) as well as the number of \( \phi \) bins. Figures 4, 5, 6 and 7 show the change in the \( a \), \( b \) and \( c \) parameters as
Figure 1: Phi-dependence of acceptance for different bins in costhetak. The fitting function is overplotted.
Figure 2: Phi-dependence of acceptance for backward costhetak bins.
Figure 3: Costhetak dependence of the 4 fit parameters: a, b, c, d. “1” corresponds to costhetak of +1 and “10” corresponds to costhetak of -1.
a function of $\cos \theta_k$ for different bins in $W$, always for 8 bins in $\phi$ and for the single $q^2$ bin 0.5 - 0.9. Figure 8 shows the effect of increasing the number of $\phi$ bins; it is the same as fig. 4 but for 16 rather than 8 bins in $\phi$.

Conclusions:

1. The $\phi$-dependence of the acceptance can be fit with a 4-parameter function: $a + b \cos \phi + c \cos 2\phi + d \cos 3\phi$.

2. The $\cos \theta_K$ dependences of the 4 parameters is smooth. See Figs. 9 - 12.

3. There are regions of $\cos \theta_K$ where the “d” parameter is small and thus the acceptance has the same shape as the structure functions which we are extracting.

4. Our $\phi$-binning causes a systematic shift in the values of our extracted “b” and “c” coefficients, even if the input model of the cross-section is perfect. This happens because the value of the function at the bin center is not equal to the value of the function averaged over the bin. This shift is proportional to the inverse square of the number of bins and is proportional to the size of the coefficient itself. The fractional shift in $b$ is 2% for 8 bins, while the fractional shift in $c$ is 10% for 8 bins. For 16 bins the fractional shift in $b$ and $c$ are 0.5% and 2.5%, respectively.

5. Our finite-size $\phi$-binning allows mis-estimation of the cross-section in the event generator to leak through into the extracted coefficients. This effect is proportional to the inverse (not inverse square) of the number of bins and is not proportional to the size of the true coefficient (not a fractional effect). Roughly, $\delta b_{\text{ext}} = .06 \times \delta b_{\text{in}} + .17 \times \delta c_{\text{in}}$; while $\delta c_{\text{ext}} = .10 \times \delta b_{\text{in}} + .04 \times \delta c_{\text{in}}$.

6. Conclusion “5” is derived for $\cos \theta_K = 0.1$; roughly the location of the forward hole. At other values of $\cos \theta_K$, the leak-through is smaller. See figs. 4 - 8 for the values of the shifts plotted versus $\cos \theta_K$.

7. To minimize this effect, it’s most important to increase the number of $\phi$-bins in the forward region. Fortunately, this is where we have the most counts. “Take that, Murphy!”
Figure 4: Changes to the a,b,c parameters when the input parameters in the event generator are varied from 1.,-1.,-1 to 1.,-1.,-1.4 plotted versus costhetak, where “1” and “10” refer to costhetak of 1. and -1., respectively. The acceptance parameters are plotted in the lower-right sub-figure. This calculation was done for W=1.7 and for 8 ϕ bins.
Figure 5: Same as fig. 4, but for $W=1.8$. 
Figure 6: Same as fig. 4, but for W=1.9.
Figure 7: Same as fig. 4, but for W=2.0.
Figure 8: Same as fig. 4, but for 16 bins.
Figure 9: Fit to the 4 parameters (p1, p2, p3, p4) as a function of $\cos/\theta_K$ where “1” and “10” refer to $\cos/\theta_K = 1$ and -1, respectively - for $W = 1.7$
Figure 10: Same as fig. 9, but for $W = 1.8$
Figure 11: Same as fig. 9, but for $W = 1.9$. 
Figure 12: Same as fig. 9, but for W = 2.0.