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Track Fitting in an Inhomogeneous Magnetic Field

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ABSTRACT

We describe an efficient method of track fitting for charged particles moving in an arbitrary inhomogeneous magnetic field. The method is a least-squares fit in 3-dimensional space which assumes local field uniformity to approximate the orbit as a set of linked helical segments. We propagate the orbit and error matrix in order to calculate the residuals and their derivatives analytically at the position of the hit wires. The accuracy of the method depends only on the step size of the linked helical segments. The fit converges rapidly to the point of minimum χ^2; one needs only a few iterations even for poor starting values of track parameters.

1. Introduction

There exist a great number of different algorithms for fitting momenta of charged particles in a magnetic field. In most cases they make use of detector particularities and are therefore of little general interest. One may, however, group them in the following way:

(*i*) A fit to an analytical correct track model. It may be illustrated by a fit to a straight line in a field-free region or to a helix in a homogeneous magnetic field.

(*ii*) A fit to a numerical correct track model. Here one solves the equation of motion numerically for the track parameters. This procedure can be made as precise as requested and always works but is the slowest one.

(*iii*) A fit to an analytical incorrect track model. A model of the track is chosen which is a sufficient approximation to the real track, but which in itself does not fulfill the equation of motion. In this category we find polynomial approximations (parametrization), and in particular spline curves[1]. The parametrization method is sensitive to detector inefficiencies, major chamber displacements and the phase-space region. Therefore, many different parametrizations have to be performed. The method may be used to give a first estimate of the track parameters for a subsequent fit.

Usually, the experimental layout is chosen such that the first approach can be taken. This means, in practice, placing all chambers either outside a magnetic field or in a homogeneous one. Even if an analytical model is employed (i), it is advisable to implement the numerical method (ii) first. In particular, there always remains a tricky phase-space region or magnetic field region where the numerical model must be utilized because the other methods do not work, or it is not worth the effort as long as the percentage of tracks concerned is small.

The numerical method of fitting, when all drift chambers are placed in a solenoidal magnetic field, is discussed elsewhere[2,3]. The derivatives needed for least-squares fitting are calculated assuming a homogeneous field along the drift chamber wires. The analytical formulae, used for calculation of derivatives, are good approximations even for a slightly non-uniform magnetic field.

In this paper we describe in detail the numerical method of track fitting, developed for the CLAS detector[4] at CEBAF. The CLAS detector is a magnetic multi-gap spectrometer based on a large iron-free toroid with six superconducting coils. In each of the six sectors of the toroidal magnet, the CLAS tracking detector is organized in three separate packages (Regions 1,2 and 3) each consisting of an axial and a stereo superlayer. A superlayer consists of 6 layers of sense wires; each layer consists of 192 sense wires (in Region 1 of 128 sense wires) which are parallel to the local z axis and perpendicular to the middle (x, y) plane in a given sector. The wires in stereo superlayers are canted at 6° with respect to the z axis. The incident beam is parallel to the x axis. The cells are oriented such that subsequent layers are shifted by one-half of a cell in order to resolve the left-right ambiguity. Region 1 is a low magnetic field region ($\sim$ 0.05 kG) inside the inner coil of the toroid and located at a distance of $\sim$ 60 cm from a target position. Region 2 is a high field region ($\sim$ 10 kG) situated within the toroid, while Region 3 is located beyond the outer coil (magnetic field $\sim$ 1 kG). The magnetic field vector is along the wires ($H_x = H_y = 0$) only in the middle plane between the coils; near the coils the field components (H_x, H_y) become substantial and may even change sign along the particle trajectory.

The method is a least-squares fit in 3-dimensional space which assumes local field uniformity to approximate the orbit as a set of linked helical segments. We propagate the orbit and error matrix[5] in order to calculate the residuals and their derivatives analytically at the position of the hit wires. The accuracy of the method depends only on the step size of the linked helical segments, and on the representation of the inhomogeneous magnetic field.

2. Measured tracks

We assume that a charged particle traverses the drift chamber and fires N sense wires close to its trajectory. By means of the space time relation the drift time measured at wire i can be converted to a distance of closest approach $d_{meas,i}$. The measured distances form an N-dimensional vector $\vec{d}_{meas}$. The resolution function gives the uncertainties σ_i of the measurements.

In the following it will be assumed that the calibration is completed, i.e. the proper space time relation and resolution function are known. We shall also assume that pattern recognition has succeeded in

(*i*) grouping the measurements according to track candidates,

(*ii*) ordering the measurements for one track according to increasing path length from some starting point, and

(*iii*) resolving the left-right ambiguities at all wires.

3. Reference frame and parameters

The dip angle λ and azimuthal angle φ are defined by the following expressions for the momentum components in a $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$ lab reference frame

$$\vec{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} p\cos\lambda\cos\varphi \\ p\cos\lambda\ \sin\varphi \\ p\sin\lambda \end{pmatrix} = p\cdot\vec{t} \tag{1}$$

The particle traveling with constant velocity in a magnetic field is described by specifying five parameters at a uniquely defined reference point. The reference point is usually chosen to be the track point which is closest to the origin (target position) or to the point of closest approach to the innermost hit wire. A convenient set of parameters is:

$$\vec{q} = \begin{pmatrix} 1/p \\ \lambda \\ \varphi \\ d \\ z \end{pmatrix} \tag{2}$$

The theoretical track will be denoted by a N-dimensional vector $\vec{d}(\vec{q})$ which is a function of the 5-dimensional parameter vector $\vec{q}$. The parameters $\vec{q}$ are determined by fitting the theoretical track points $\vec{d}(\vec{q})$ to the measured points $\vec{d}_{meas}$.

4. Least-squares fit

The purpose of the track fit is to find the set of track parameters $\vec{q}$ which minimizes the quadratic form

$$\chi^2 = \sum_i^N \frac{[d_{meas,i} - d_i(\vec{q})]^2}{\sigma_i^2} \tag{3}$$

Minimizing χ^2 as a function of $\vec{q}$ is equivalent to finding a solution of the system of equations

$$\frac{\partial \chi^2}{\partial q_\mu} = 0 \quad for \quad \mu = 1, ..., 5 \tag{4}$$

The system of eqs. (4) is nonlinear because the function $\vec{d}(\vec{q})$ is nonlinear, and an iterative procedure has to be used to solve for $\vec{q}$. In order to linearize $\vec{d}(\vec{q})$ and to obtain the true solution $\vec{q}$, the Taylor expansion around an approximate solution $\vec{q}_n$ is used. Neglecting higher order terms, this yields the linear system of equations:

$$D^T W D(\vec{q} - \vec{q}_n) = D^T W[\vec{d}_{meas} - \vec{d}(\vec{q}_n)] \tag{5}$$

where D is the Jacobian matrix $(D_{i\mu}) = (\partial d_i / \partial q_\mu)$ and W is a weight matrix (the inverse of the covariance matrix of the measured drift distances d_i). Equation (5) can be solved for the parameter correction $\Delta\vec{q} \equiv \vec{q} - \vec{q}_n$. With this correction one gets a new approximation

$$\vec{q}_{n+1} = \vec{q}_n + \Delta\vec{q} \tag{6}$$

After a few iterations n, one usually obtains a solution close to $\vec{q}$, provided one starts with a good first approximation $\vec{q}_0$, given by the pattern recognition procedure.

5. The particle trajectory

For the calculation of the distances of closest approach $\vec{d}(\vec{q})$, one needs an analytical expression for the trajectory of a charged particle moving in a magnetic field. First, we consider the case of a homogeneous magnetic field in the z-direction $\vec{H} = (0, 0, H)$ and assume that the track starts at

$$\vec{x}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

with a momentum

$$\vec{p}_0 = p \cdot \vec{t}_0 = p \begin{pmatrix} 0 \\ \cos\lambda \\ \sin\lambda \end{pmatrix} \tag{8}$$

where $\vec{t}_0$ denotes the initial tangent unit vector and λ the dip angle with respect to the (x,y) plane. The particle moves on a helix with its axis parallel to $\vec{H}$. The helix is conveniently parametrized by the turning angle Θ in the (x,y) plane:

$$\vec{x}(\Theta) = \rho \begin{pmatrix} 1 - \cos\Theta \\ \sin\Theta \\ \Theta\tan\lambda \end{pmatrix} \tag{9}$$

The angle Θ is related to the track length l, which would be an equally good choice for the independent trace parameter:

$$\Theta = \frac{QH}{p} l \tag{10}$$

where Q is the charge of the particle. The projection of the particle trajectory into the plane perpendicular to the field is a circle with radius

$$\rho = \frac{p\cos\lambda}{QH} \tag{11}$$

Differentiation of eq. (9) together with eqs. (10 and 11) yields an expression for the track tangent as a function of l:

$$\frac{\partial\vec{x}(l)}{\partial l} = \vec{t}(l) = \cos\lambda \begin{pmatrix} \sin\Theta \\ \cos\Theta \\ \tan\lambda \end{pmatrix} \tag{12}$$

For an arbitrary magnetic field $\vec{H}(\vec{x})$, the particle trajectory can be described by linked helical segments. The particle is traced from a point $\vec{x}_i$ with tangent vector $\vec{t}_i$ and magnetic field $\vec{H}(\vec{x}_i)$ to point $\vec{x}_{i+1}$ with tangent vector $\vec{t}_{i+1}$. If the distance between $\vec{x}_i$ and $\vec{x}_{i+1}$ is sufficiently small, the magnetic field in between can be assumed to be constant.

For the tracing procedure it is useful to define a local coordinate system which varies along the particle's trajectory. At the point $\vec{x}_i$ it is given by the following unit vectors:

$$\vec{h}_i = \frac{\vec{H}(\vec{x}_i)}{|\vec{H}(\vec{x}_i)|}$$

$$\vec{n}_i = \frac{\vec{t}_i \times \vec{h}_i}{|\vec{t}_i \times \vec{h}_i|} \tag{13}$$

$$\vec{b}_i = \vec{h}_i \times \vec{n}_i$$

The ordered vector triplet $(\vec{n}_i, \vec{b}_i, \vec{h}_i)$ defines an orthonormal right-handed coordinate system. The orientations of the initial momentum and the magnetic field in this system satisfy the initial conditions (8). Hence one can immediately adopt the results (9) for the orbit and (12) for the track tangent. For a tracing step from $\vec{x}_i$ to $\vec{x}_{i+1}$, the new point is obtained using eq. (9):

$$\vec{x}_{i+1} = \vec{x}_i + \rho[(1 - \cos\Theta_i)\cdot\vec{n}_i + \sin\Theta\cdot\vec{b}_i + \Theta_i \tan\lambda_i \cdot \vec{h}_i] \tag{14}$$

The new tangent direction is given by eq. (12):

$$\vec{t}_{i+1} = \cos\lambda_i(\sin\Theta_i\cdot\vec{n}_i + \cos\Theta_i\cdot\vec{b}_i + \tan\lambda_i\cdot\vec{h}_i) \tag{15}$$

Thus, starting from a track point $\vec{x}_0$ with tangent $\vec{t}_0$, these formulae allow for the calculation of the particle trajectory in an arbitrary magnetic field. The accuracy of the method of linked helical segments depends on the step sizes $|\vec{x}_{i+1} - \vec{x}_i|$ and on the homogeneity of the field. Deviations from the true particle trajectory can be easily kept small compared to the spatial resolution of the drift chamber.

However, one can save the calculation of $\vec{b}_i$ and the normalization of $\vec{n}_i$ by replacing

$$\vec{b}_i = \frac{\vec{t}_i - \sin\lambda_i\cdot\vec{h}_i}{\cos\lambda_i}$$

and

$$\vec{n}_i = \frac{\vec{t}_i \times \vec{h}_i}{\cos\lambda_i} \tag{16}$$

This yields the following forms for equations (14) and (15):

$$\vec{x}_{i+1} = \vec{x}_i + \frac{\rho}{\cos\lambda_i}[(1-\cos\Theta_i)\cdot(\vec{t}_i \times \vec{h}_i) + \sin\Theta_i\cdot\vec{t}_i + (\Theta_i - \sin\Theta_i)\sin\lambda_i\cdot\vec{h}_i] \tag{17}$$

and

$$\vec{t}_{i+1} = \sin\Theta_i \cdot (\vec{t}_i \times \vec{h}_i) + \cos\Theta_i \cdot \vec{t}_i + (1 - \cos\Theta_i)\sin\lambda_i \cdot \vec{h}_i \tag{18}$$

These expressions are used in our program.

6. Distance of closest approach

The ultimate purpose of tracing is the determination of the distances of closest approach $\vec{d}(\vec{q})$ of a track to the hit wires. As the track can pass on either side of the wire, the distance of closest approach d_i must be a signed quantity. The direction $\vec{a}_i$ along which d_i is measured is perpendicular to $\vec{w}_i$ and $\vec{t}_i$ and given by

$$\vec{a}_i = \frac{\vec{w}_i \times \vec{t}_i}{|\vec{w}_i \times \vec{t}_i|} \tag{19}$$

The distance of closest approach is given by the projection of $\vec{x}_i - \vec{W}_{0i}$ onto this axis

$$d_i = (\vec{x}_i - \vec{W}_{0i}) \cdot \vec{a}_i, \tag{20}$$

where $\vec{W}_{0i}$ is the wire position in the plane $z = 0$. For axial wires ($\vec{w}_i = \vec{e}_z$) the distance d_i is measured in the (x,y) plane.

The sign convention for d_i becomes clear in the view against $\vec{w}_i$. Following a particle along $\vec{t}_i$, a track passing on the left (right) side of a wire gives $d_i > 0$ ($d_i < 0$). The same signs must be applied to the measured drift distances.

7. Transport matrix and derivatives

As was shown in section 4, an iterative track fit requires the calculation of the derivatives $\partial d_i/\partial q_\mu$ at any point $\vec{x}_i$ along the particle trajectory.

Before discussing a general case it is useful to consider the case of a homogeneous magnetic field in the z-direction (as in a solenoidal detector). In this case, to calculate the derivatives at any track point $\vec{x}_i$, one has to insert eq.(17) into eq.(20) and to differentiate with respect to all track parameters $q_{0\mu} = (1/p, \lambda_0, \varphi_0, d_0, z_0)$ for a track $(\vec{x}_0, \vec{t}_0)$ at the origin. This yields the following results:

$$\frac{\partial d_i}{\partial q_{0\mu}} = \begin{pmatrix} p(\vec{x}_0 - \vec{x}_i) \cdot \vec{a}_i \\ L_i(\vec{e}_z \cdot \vec{a}_i)/\sin\lambda_0 \\ [(\vec{x}_i - \vec{x}_0) \times \vec{a}_i]_z \\ \vec{u}_0 \cdot \vec{a}_i \\ \vec{e}_z \cdot \vec{a}_i \end{pmatrix} \tag{21}$$

where L_i is the total track length between the space points $\vec{x}_0$ and $\vec{x}_i$, and

$$\vec{u}_0 = \frac{\vec{e}_z \times \vec{t}_0}{|\vec{e}_z \times \vec{t}_0|} \tag{22}$$

None of the above derivatives refers to the sign or magnitude of the magnetic field explicitly, i.e. they are purely geometrical quantities calculated in the fixed (lab) coordinate system. One can see immediately that the derivatives $\partial/\partial\lambda_0$ and $\partial/\partial z_0$ are zero for the axial wires, since vector product $(\vec{e}_z \cdot \vec{a}_i)$ is equal to zero. The three parameters $(1/p, \varphi_0, d_0)$ describe a circle which is the projection of the helix onto the (x, y) plane. Information about these parameters comes from axial as well as stereo wires, whereas information regarding λ_0 and z_0 can be obtained from stereo wires only.

In the following sections we will discuss the calculation of the derivatives for an arbitrary magnetic field.

7.1 Local coordinate system

We define a local coordinate system (transverse system), in which $x_\perp$ coordinate is measured along the particle momentum $\vec{t}$, and $(y_\perp, z_\perp)$ coordinates are measured along directions $\vec{u}$ and $\vec{v}$ perpendicular to $\vec{t}$:

$$\vec{u} = \frac{\vec{e}_z \times \vec{t}}{|\vec{e}_z \times \vec{t}|} \tag{23}$$

and

$$\vec{v} = \vec{t} \times \vec{u} \tag{24}$$

The transformation matrix between coordinates (x, y, z) in the lab reference frame $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$ and coordinates $(x_\perp, y_\perp, z_\perp)$ in the local coordinate system $(\vec{t}, \vec{u}, \vec{v})$ is:

$$\begin{pmatrix} x_\perp \\ y_\perp \\ z_\perp \end{pmatrix} = \begin{pmatrix} \cos\lambda\cos\varphi & \cos\lambda\sin\varphi & \sin\lambda \\ -\sin\varphi & \cos\varphi & 0 \\ -\sin\lambda\cos\varphi & -\sin\lambda\sin\varphi & \cos\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{25}$$

7.2 Calculation of the transport matrix

In this section we will discuss the error propagation, from a starting point of the track propagated to the end of the track, in the presence of a magnetic field. Given an error matrix R_0 at $l = 0$ for the variables $(1/p, \lambda, \varphi, y_\perp, z_\perp)$, which refers to the plane perpendicular to the track at $l = 0$, we ask for corresponding error matrix R_l

after a path length l, refering now to the plane perpendicular to the track at point l. If T_l is the transformation matrix between the variables at $l = 0$ and l

$$(\delta\vec{q})_l = \begin{pmatrix} \delta 1/p \\ \delta\lambda \\ \delta\varphi \\ \delta y_\perp \\ \delta z_\perp \end{pmatrix}_l = T_l \cdot \begin{pmatrix} \delta 1/p \\ \delta\lambda \\ \delta\varphi \\ \delta y_\perp \\ \delta z_\perp \end{pmatrix}_{l=0} = T_l \cdot (\delta\vec{q})_0 \tag{26}$$

then R_l is obtained from R_0 by

$$R_l = T_l \cdot R_0 \cdot T_l^T \tag{27}$$

We will now compute the transformation matrix T_l in the presence of a magnetic field, which varies both in strength and direction along the track.

Before calculating the errors $(\delta\vec{q})_l$ at the end of the track we consider first the error propagation along a short length dl from l to $(l + dl)$. The error at $(l + dl)$ will have the following contributions:

$$(\delta\vec{q})_{l+dl} = (\mathbf{I} + dl \cdot \mathbf{A}_{l+dl} + dl \cdot \mathbf{B}_{l+dl}) \cdot (\delta q)_l = \mathbf{M} \cdot (\delta q)_l \tag{28}$$

The first two contributions to the matrix M would be present even without a magnetic field. This part takes into account the fact that an error in the direction of the track at l causes an error in the position of the track at $(l + dl)$. The term $dl \cdot B_{l+dl} \cdot (\delta q)_l$ is due to the deflections caused by the magnetic field. In eq. (28) I is the unit matrix.

By dividing the total track length l into small steps of size $\Delta l = l/N$ and applying the above relation for each step n, one obtains:

$$(\delta\vec{q})_l = (\prod_{n=1}^{N} \mathbf{M}_n) \cdot (\delta\vec{q})_0 = \mathbf{T}_l \cdot (\delta\vec{q})_0 \tag{29}$$

The matrices M_n may be calculated during the track tracing, and transport matrix T_l gives the full error propagation from point $l = 0$ to l.

Calculation of the matrix A. Only the elements connected with $(\delta y_\perp)_{l+dl}$ and $(\delta z_\perp)_{l+dl}$ are different from zero since $\delta\lambda/dl$ and $\delta\varphi/dl$ vanish if no magnetic field is present.

The error $\delta z_\perp$ at $(l + dl)$ due to $\delta\lambda$ at l is

$$\delta z_\perp = dl \cdot \delta\lambda \tag{30}$$

Similarly the error $\delta y_\perp$ at $(l + dl)$ due to an error $\delta\varphi)$ at l is

$$\delta y_\perp = dl \cdot \cos\lambda \cdot \delta\lambda \tag{31}$$

With eqs. (30) and (31) we obtain matrix A:

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\lambda & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{32}$$

Calculation of the matrix B. To compute the matrix B we use the relation (10)

$$d\Theta = -\vec{H} \cdot \frac{dl}{p} = \begin{pmatrix} \sin\lambda d\varphi \\ -d\lambda \\ \cos\lambda d\varphi \end{pmatrix} \tag{33}$$

where $\vec{H}$ stands for $0.3 \times 10^{-3} \times charge \times field \times [GeV \cdot cm^{-1} \cdot kG^{-1}]$.
Equation (33) tells that the direction of a particle with momentum p is rotated by the field $\vec{H}$ on the path length dl by an angle $d\Theta$, with the axis of rotation parallel (or antiparallel) to $\vec{H}$.
More explicitly eq. (33) reads

$$\begin{pmatrix} d\lambda \\ d\varphi \end{pmatrix} = -\frac{dl}{p} \begin{pmatrix} -H_2 \\ \frac{H_3}{\cos\lambda} \end{pmatrix} \tag{34}$$

where H_2 is the field component perpendicular to $(z_\perp, x_\perp)$ plane and H_3 is the field component perpendicular to $(x_\perp, y_\perp)$ plane. We also define H_0 and H_1 which will be used below:

$$\begin{aligned} H_1 = (\vec{H} \cdot \vec{t}) &= \quad H_0 \cos\lambda + H_z \sin\lambda = -\frac{\partial H_3}{\partial\lambda} \\ H_2 = (\vec{H} \cdot \vec{u}) &= -H_x \sin\varphi + H_y \cos\varphi = \frac{\partial H_0}{\partial\varphi} \\ H_3 = (\vec{H} \cdot \vec{v}) &= -H_0 \sin\lambda + H_z \cos\lambda = \frac{\partial H_1}{\partial\lambda} \end{aligned} \tag{35}$$

where $H_0 = H_x \cos\varphi + H_y \sin\varphi = -\partial H_2/\partial\varphi$. Thus, (H_1, H_2, H_3) is the field vector represented in the transverse $(\vec{t}, \vec{u}, \vec{v})$ reference frame.

From eq. (34) we obtain:

$$\delta\begin{pmatrix} d\lambda \\ d\varphi \end{pmatrix} = -\begin{pmatrix} -H_2 \\ \frac{H_3}{\cos\lambda} \end{pmatrix} dl\cdot\delta(1/p)_l - \frac{1}{p}\begin{pmatrix} -H_2 \\ \frac{H_3}{\cos\lambda} \end{pmatrix}\delta(dl) - \frac{dl}{p}\cdot\delta\begin{pmatrix} -H_2 \\ \frac{H_3}{\cos\lambda} \end{pmatrix} \quad (36)$$

Now we derive the term $\delta(dl)$. To define completely an error matrix it is also necessary to specify a few conventions. We will adopt the condition $\delta x_\perp = 0$, which means that the error matrix refers to a plane perpendicular to the track. The condition $\delta x_\perp = 0$ implies that in general $\delta(dl) \neq 0$. Because of the influence of the magnetic field in the interval $(l+dl)$, the coordinate system $(x_\perp, y_\perp, z_\perp)$ is rotated by the angle $d\Theta$, so that we get at $(l+dl)$:

$$\begin{pmatrix} \delta x_\perp \\ \delta y_\perp \\ \delta z_\perp \end{pmatrix}_{l+dl} = \begin{pmatrix} 1 & \cos\lambda d\varphi & d\lambda \\ -\cos\lambda d\varphi & 1 & \sin\lambda d\varphi \\ -d\lambda & -\sin\lambda d\varphi & 1 \end{pmatrix}\begin{pmatrix} 0 \\ \delta y_\perp \\ \delta z_\perp \end{pmatrix}_l + \begin{pmatrix} \delta(dl) \\ 0 \\ 0 \end{pmatrix} \quad (37)$$

In the last column of this equation, terms of second order in dl have been suppressed. The condition $(\delta x_\perp)_{l+dl} = 0$ implies

$$\delta(dl) = -\cos\lambda d\varphi\cdot(\delta y_\perp)_l - \delta\lambda\cdot(\delta z_\perp)_l \quad (38)$$

By differentiating the last term of eq. (36) with respect to λ and φ we obtain

$$\delta\begin{pmatrix} -H_2 \\ \frac{H_3}{\cos\lambda} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{H_0}{\cos^2\lambda} \end{pmatrix}(\delta\lambda)_l + \begin{pmatrix} H_0 \\ -\tan\lambda\cdot H_2 \end{pmatrix}(\delta\varphi)_l \quad (39)$$

With eqs. (34), (36), (38) and (39) we find for matrix B

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ H_2 & 0 & -\frac{H_0}{p} & \frac{H_2 H_3}{p^2} & -\frac{H_2 H_2}{p^2} \\ -\frac{H_3}{\cos\lambda} & \frac{H_0}{p\cos^2\lambda} & \frac{\tan\lambda H_2}{p} & -\frac{H_3 H_3}{p^2\cos\lambda} & \frac{H_3 H_2}{p^2\cos\lambda} \\ 0 & 0 & 0 & 0 & -\frac{H_3\tan\lambda}{p} \\ 0 & 0 & 0 & \frac{H_3\tan\lambda}{p} & 0 \end{pmatrix} \quad (40)$$

Thus, for an infinitesimal path length dl, transport matrix M is given by

$$\mathbf{M} = \mathbf{I} + (\mathbf{A}+\mathbf{B})\cdot dl \quad (41)$$

where I is the unit matrix.

Derivatives $D_{i\mu}$. The derivatives with respect to the track parameters at the origin $q_{0\mu} = (1/p, \lambda_0, \varphi_0, d_0, z_0)$ are given by

$$D_{i\mu} = \frac{\partial d_i(\vec{q}_0)}{\partial q_{0\mu}} = \begin{pmatrix} T_i(4,1) \\ T_i(5,2) \cdot a(3)_i \\ T_i(4,3) \\ T_i(4,4) \\ T_i(5,5) \cdot a(3)_i \end{pmatrix} \tag{42}$$

The parameters $\vec{q}_0$ are determined by fitting the track points $\vec{d}_i(\vec{q}_0)$ along the trial trajectory to the measured points $\vec{d}_{meas,i}$ as is outlined in section 4.

Measurements in 2-dimensions. Let us consider the experimental layout, where all drift chambers consist of axial wires only, but with charge division measurements along z. In this case, the matrix D in eq. (5) must be replaced by the transport matrix T, and the vector $\Delta\vec{V}_i = d_{meas,i} - d_i(\vec{q}_0)$ by

$$\Delta\vec{V}_i = \begin{pmatrix} 0 \\ 0 \\ 0 \\ d_{meas,i} - d_i(\vec{q}_0) \\ z_{meas,i} - z_i(\vec{q}_0) \end{pmatrix} \tag{43}$$

The weight matrix W in eq. (5) now represents the 2x2 covariance matrix of the measurements on plane i.

8. Test results

We have made extensive tests with Monte Carlo generated events, using the GEANT simulation of the CLAS detector. Realistic particle trajectories traversing the CLAS magnetic field were simulated. The magnetic field was calculated only once at the node points of a 3-dimensional grid. During actual particle tracing, the field inside of each elementary grid volume is approximated using a second order interpolation between the grid points. The coordinates (x_i, y_i, z_i) of hits generated by a charged particle passing through a cell of the drift chamber were digitized. The digitization yields the wire number and the minimum drift time. The drift time was smeared according to the expected position resolution of the measurement, 200 μm. The track fitting was tested over the full range of momenta and angles. As an example, for a $1GeV$ track the fit gives a momentum resolution of $\delta p/p \simeq 0.1\%$ (without multiple scattering),and $\delta p/p \simeq 0.25\%$ when both the position resolution and multiple scattering contribution were included.

Execution times. The results of timing depend essentially on the step size and on the track length. On a VAX8700 computer, for the step size of 1 cm (about 400 steps per track), the total execution time is ~290 ms per iteration. About 55% of this time is used for computing the magnetic field and an orbit propagation. The rest of the total time is spent for the calculation of transport matrix T and actual fitting procedure.

Convergence properties. The convergence depends on the starting values of the track parameters used for the first iteration, and also slightly on the non-uniformity of the magnetic field, especially close to the superconducting coils (deep angle $|\lambda| > 15°$). As an example, in Table I we show the results for a $1GeV$ particle starting at the origin and with angles: $\lambda_0 = 10°$ and $\varphi_0 = 50°$. One can see that the fit converges rapidly even for bad starting values (iteration 0): $\vec{x}_0 = (5.0,0.0,0.1)$, $p = 1.1GeV$, $\lambda_0 = 0°$ and $\varphi_0 = 49°$.

Table I. Convergence of the fit

Iteration Number	p [GeV]	λ_0 [Deg]	φ_0 [Deg]	d_0 [cm]	z_0 [cm]	χ^2
0	1.1000	0.0000	49.0000	-3.7740	0.1000	0.128E+06
1	0.9864	9.1578	50.4323	0.3937	-0.3253	0.180E+04
2	0.9874	10.0186	50.0326	0.0416	-0.0349	0.813E+02
3	1.0003	10.0614	50.0005	0.0065	-0.0054	0.643E+00
4	1.0002	10.0349	50.0022	0.0085	-0.0071	0.644E+00

For typical starting values given by the CLAS pattern recognition program ($\Delta\lambda, \Delta\varphi \leq 1°$ and $\Delta p/p \leq 5\%$), one needs no more than three iterations to converge to the point of minimum of χ^2.

9. Conclusion

From the tests we have done, the accuracy of the method depends only on the step size of the linked helical segments, and on the representation of the inhomogeneous magnetic field. The fit converges nearly as rapidly as in the case of a uniform magnetic field, but needs somewhat more time per iteration (~ 40%) due to calculation of the transport matrix and the magnetic field from the grid points. The method could be speeded up by a more efficient representation of the magnetic field and optimalization of the code itself.

We conclude that track reconstruction in a very inhomogeneous magnetic field is possible with easily available computer resources.

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