Partial-Wave Analysis of Single Pseudo-scalar Photoproduction: Scope, Techniques, and Formalism

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(for the CLAS Collaboation)

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EXCITED BARYONS AND CQM PREDICTIONS

- The Constituent Quark Model allows theorists to investigate the baryon spectrum in the non-pQCD regime.
  - neglects possible contributions of sea quarks and gluons
- Quark Model calculations of excited nucleon states have been made by Capstick and Roberts.
- However, *many* of the states predicted have never been conclusively observed → *The Missing Baryon Problem*
- Possible explanations for non-observation:
  - Missing resonances exist, but cross-sections are smaller than current experimental sensitivity
  - Missing resonances exist, but do not couple (strongly) to $N\pi$ → most of the world's data
  - Corrections to quark model...

WHY STUDY $K^+\Lambda$?

- Self-analyzing Λ decay allows for measurement of all particle polarizations.
- 16 possible observables [Barker, Donnachie, Storrow. NP**B95**, (1975)]:
  - Differential Cross Section: $d\sigma/d\cos\theta_K^{c.m.}$
  - Single-polarization: Σ, T, P_Λ
  - Beam-recoil: O_x, O_z, C_x, C_z
  - Beam-target: E, F, G, H
  - Target-recoil: T_x, T_z, L_x, L_z
- Parity invariance of EM and strong interactions

  $\Rightarrow$ describe interaction with 4 complex amplitudes...
- These observables are not independent
  - Only 7 **carefully** chosen observables measured with infinite precision necessary [Chiang, Tabakin. PRC**55** (1997)]
  - Analyses of the constraints provided by realistic measurements have been performed...
    - Ireland, PRC**82** (2010) → information theory approach
    - Sandorfi, *et al.* arXiv:1010.4555 → sample amplitudes and test models

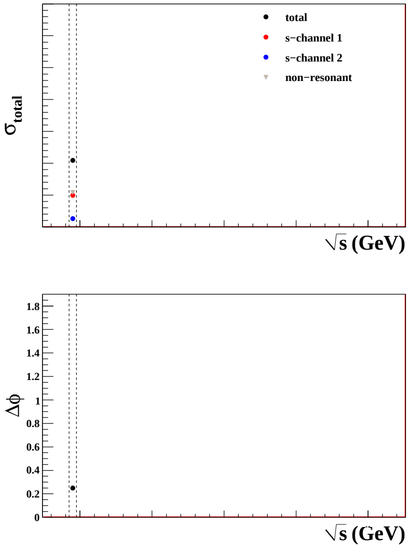
WHY STUDY $K^+\Lambda$?

- $I = \frac{1}{2}$ of the $K^+\Lambda$ final state couples only to N^*, NOT Δ^*
- Some N^* states have been observed to couple to $K\Lambda$:
  - $S_{11}(1650)^{***}$, $D_{13}(1700)^{**}$, $P_{11}(1710)^{**}$, $P_{13}(1720)^{**}$, ...
- Previous PWA have shown discrepant results (experimental discrepancies)
- Several groups have studied non-resonant production models
  - Kaon-MAID ⇒ iso-bar models
  - Ghent ⇒ Regge-ized K and K^* exchange
- New $K^+\Lambda$ photoproduction data resolves discrepancies in $d\sigma$
- New and forth-coming analyses present more polarization info.

MASS-INDEPENDENT PWA METHOD

- Data is binned in $\sqrt{s}$
- Independent fits in each bin
- Event-based and χ^2 fits
- Non-resonant model is locked
- *s*-channel and non-resonant interference
- many iterations, randomized parameters
- Patterns emerge in $\sqrt{s}$
- B-W descriptions show intensity shapes
- Fit phase differences ⇒ extract masses and widths
- Success with $\gamma p \longrightarrow \omega p$

  M. Williams, PRC **80**, 065209 (2009)



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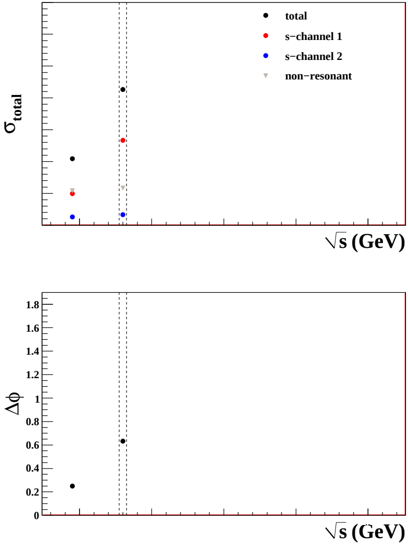

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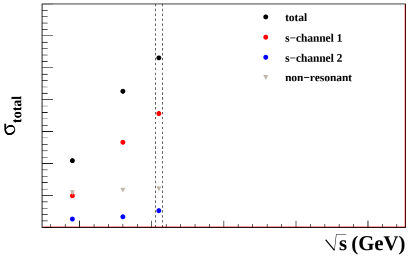

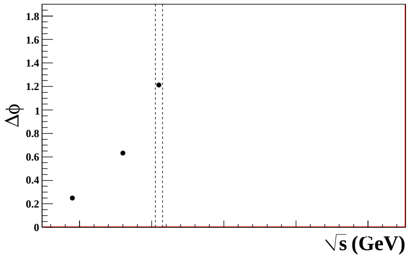

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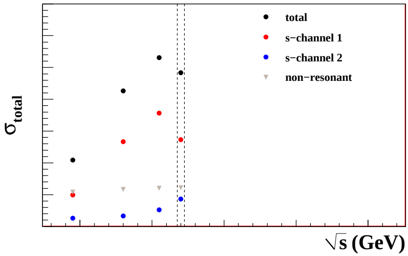

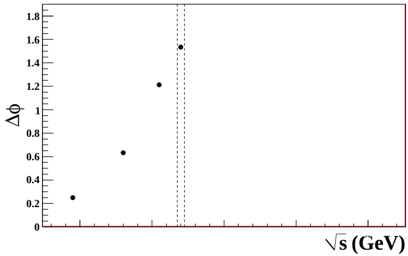

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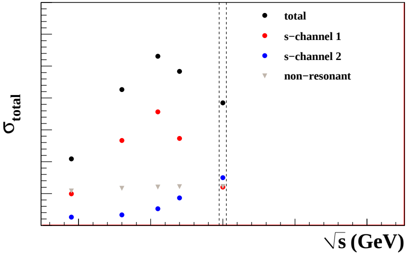

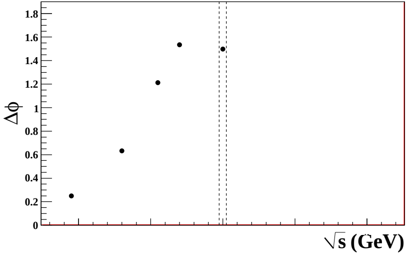

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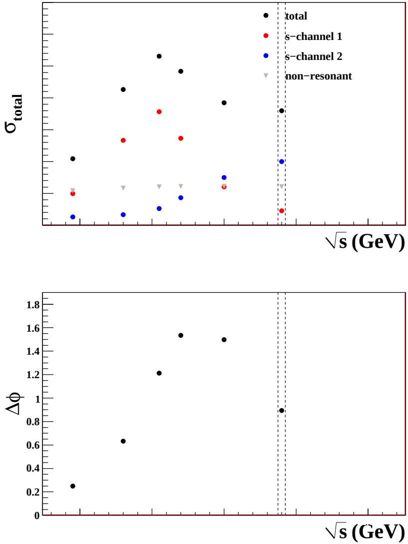

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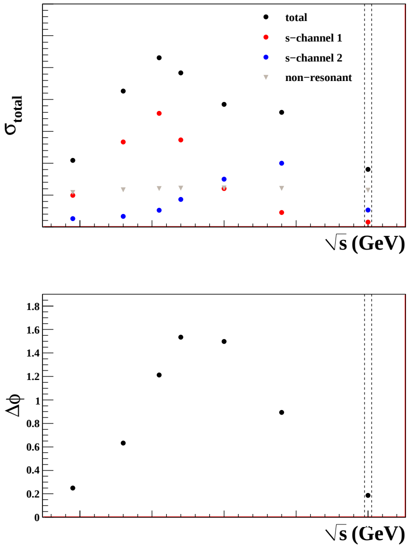

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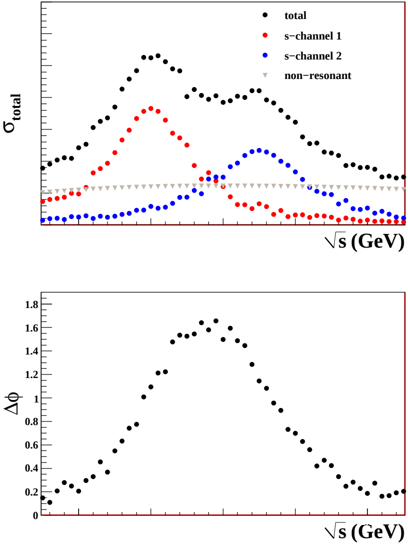

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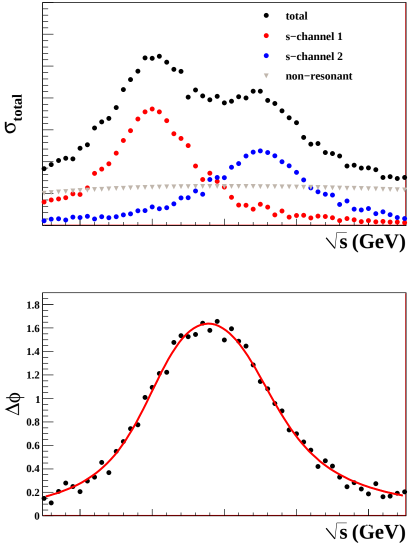
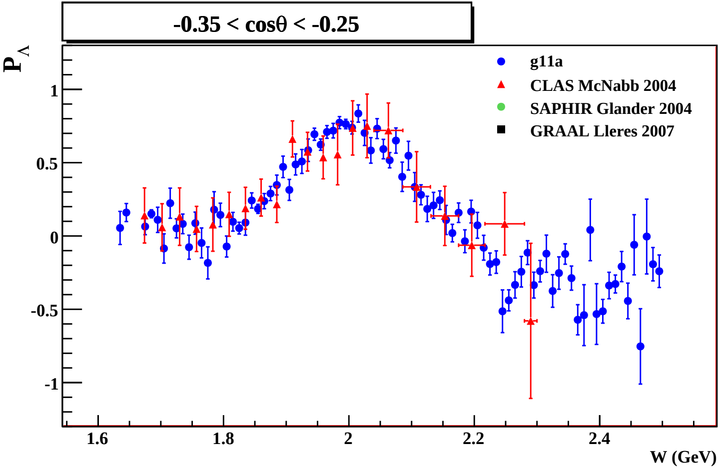


POLARIZATION INFORMATION IN $\gamma p \rightarrow K^+\Lambda$

We see much shape in polarization observables...



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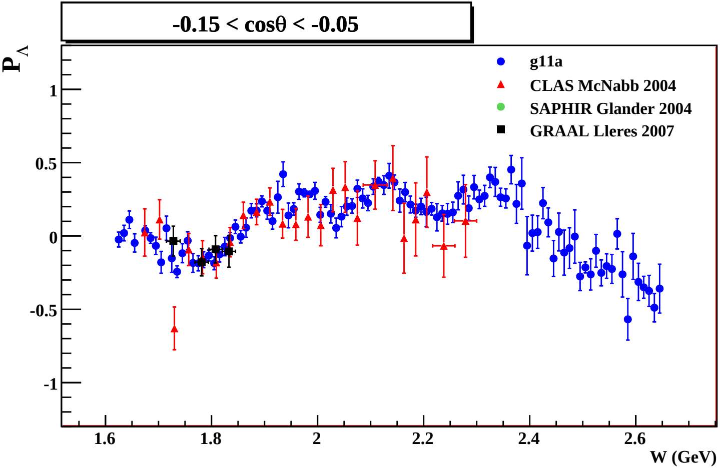

SINGLE PS MESON PHOTOPRODUCTION

Choose production amplitudes in the longitudinal basis

- Spin projections for all particles are along $\hat{z} \parallel \vec{p}_\gamma$
- Natural choice for description of observables in the same frame for all kinematics
- Calculable with `qft++` package
- Complex production amps then depend on (kinematics and) the spins of γ, target, and final-state baryon $\Rightarrow m_\gamma,\ m_t,\ m_b$
- Parity invariance allows us to relate amplitudes for positive and negative photon helicities

$$\mathcal{A}_{m_\gamma m_t m_b}:$$

$$\begin{aligned}
\mathcal{A}_{+++} &= +\mathcal{A}_{---} = L_1 \\
\mathcal{A}_{++-} &= -\mathcal{A}_{--+} = L_2 \\
\mathcal{A}_{+--} &= +\mathcal{A}_{-++} = L_3 \\
\mathcal{A}_{+-+} &= -\mathcal{A}_{-+-} = L_4
\end{aligned}$$

DENSITY MATRIX APPROACH

We follow the density matrix approach [Fasano, Tabakin, Saghai. PRC **46**, (1997)]

- 2×2 matrix J_{m_γ} of the amplitude for transition from $|m_t\rangle$ to $|m_b\rangle$

$$\langle m_b | J_{m_\gamma} | m_t \rangle = \mathcal{A}_{m_\gamma m_t m_b}$$

- Intensity profile is then proportional to $\mathrm{Tr}[\rho_{out} J \rho_{in} J^\dagger]$
- Write the density matrices of fermions as $\rho = \frac{1}{2}(1 + \vec{P} \cdot \vec{\sigma})$
- Photon density matrix is written as (FTS,AS)

$$\rho^\gamma = \frac{1}{2}\left[\begin{array}{cc} 1 + P^S_z & P^S_x - iP^S_y \\ P^S_x + iP^S_y & 1 - P^S_z \end{array}\right]$$
$$= \frac{1}{2}\left[\begin{array}{cc} 1 + P^\gamma_C & -P^\gamma_L \exp(-2i(\theta - \varphi)) \\ -P^\gamma_L \exp(2i(\theta - \varphi)) & 1 - P^\gamma_C \end{array}\right]$$

- $\varphi \Rightarrow$ angle btw. photon pol. and reaction plane
- $\theta \Rightarrow$ angle btw. photon pol. and lab frame reference
- Careful treatment of theory and experiment

DENSITY MATRIX APPROACH

- Write the intensity profile as $\mathcal{I} = \mathcal{I}_0 \left(\frac{\mathrm{Tr}[\rho^b J \rho^i \rho^\gamma J^\dagger]}{\mathrm{Tr}[JJ^\dagger]} \right)$
- To simplify, we treat $\vec{\sigma}$ and $\vec{P}$ as 4-component objects

$$\{\sigma_0, \sigma_1, \sigma_2, \sigma_3\} \equiv \{I, \sigma_x, \sigma_y, \sigma_z\}$$
$$\{P_0, P_1, P_2, P_3\} \equiv \{1, P_x, P_y, P_z\}$$

- Define $T_{lmn} \equiv \frac{\mathrm{Tr}[\sigma_n^b J \sigma_m^i \sigma_l^\gamma J^\dagger]}{\mathrm{Tr}[JJ^\dagger]}$
- Allows us to write $\mathcal{I} \sim \sum_{lmn \in \{0,1,2,3\}} P_l^S P_m^i P_n^b \, T_{lmn}$
- Polarization observables are then equivalent to specific T_{lmn}

$$P = \frac{\sigma^{(0,0,+y)} - \sigma^{(0,0,-y)}}{\sigma^{(0,0,+y)} + \sigma^{(0,0,-y)}} \Rightarrow P = T_{002}$$
$$F = \frac{\sigma^{(r,+x,0)} - \sigma^{(r,-x,0)}}{\sigma^{(r,+x,0)} + \sigma^{(r,-x,0)}} \Rightarrow F = T_{310}$$

DENSITY MATRIX APPROACH

- Trace calculation gives a 64-term expression in terms of particle pols.
- 32 terms disappear due to parity-invariance considerations
- Can then relate to experiments that observe two polarizations...
- Example: Beam-recoil experiment ($\vec{P}^t = \vec{0}$)
  $$\begin{aligned} \sigma^{(\gamma,0,b)} = \sigma_0\{1 - P_L^\gamma \Sigma \cos(2\phi) + P_y^b(P - P_L^\gamma T \cos(2\phi)) \\ +P_x^b \left(P_C^\gamma C_x + P_L^\gamma O_x \sin(2\phi)\right) \\ +P_z^b \left(P_C^\gamma C_z + P_L^\gamma O_z \sin(2\phi)\right)\} \end{aligned}$$
- ϕ is the angle btw. linear photon pol. and reaction plane

POLARIZATION OBSERVABLES FROM $\mathcal{A}$

We can then project polarization observables from the $\mathcal{A}_{m_\gamma m_t m_b}$

- Calculations are simple the traces with appropriate Pauli operators
- Sum over the polarization states of each particle
- $$\begin{aligned} P &= \mathrm{Tr}[\sigma_y^b J J^\dagger] \\ &= \sum \langle m_b|\sigma_y|m_b^{'}\rangle\langle m_b^{'}|J_{m_\gamma}|m_t\rangle\langle m_t|J^\dagger_{m_\gamma}|m_b\rangle \\ &= \sum_{m_\gamma m_t} -2\mathrm{Im}(\mathcal{A}_{m_\gamma m_t +}\mathcal{A}^*_{m_\gamma m_t -}) \end{aligned}$$
- $$\begin{aligned} L_x &= \mathrm{Tr}[\sigma_x^b J \sigma_z^i J^\dagger] \\ &= \sum \langle m_b|\sigma_x|m_t\rangle\langle m_t|J_{m_\gamma}|m_t'\rangle\langle m_t'|\sigma_z|m_b'\rangle\langle m_b'|J^\dagger_{m_\gamma}|m_b\rangle \\ &= \sum_{m_\gamma} 2\mathrm{Re}\left(\mathcal{A}_{m_\gamma +-}\mathcal{A}^*_{m_\gamma ++} - \mathcal{A}_{m_\gamma --}\mathcal{A}^*_{m_\gamma -+}\right) \end{aligned}$$

OUTLOOK

- Density matrix method paper arXiv:1010.4978
- Several channels ready for independent PWA:
  $\gamma p \to K^+\Lambda$, $K^+\Sigma^0$, ηp, $\eta^{'} p$
- For KY, available polarizations
  - P for $\sqrt{s} \in [1.63 \text{ GeV}, 2.84 \text{ GeV}]$ (CLAS, SAPHIR, GRAAL)
  - Σ for $\sqrt{s} \in [1.64 \text{ GeV}, 2.4 \text{ GeV}]$ (GRAAL, LEPS)
  - T for $\sqrt{s} \in [1.64 \text{ GeV}, 1.92 \text{ GeV}]$ (GRAAL)
  - O_x, O_z for $\sqrt{s} \in [1.64 \text{ GeV}, 1.92 \text{ GeV}]$ (GRAAL)
  - C_x, C_z for $\sqrt{s} \in [1.65 \text{ GeV}, 2.53 \text{ GeV}]$ (CLAS)
- Forthcoming measurements:
  - P, T, Σ, O_x, O_z for $\sqrt{s} \in [1.70 \text{ GeV}, 2.15 \text{ GeV}]$ (CLAS g8)
  - Access to other (B-T, T-R) observables with CLAS FROST data
- Comprehensive/consistent treatment of polarizations will allow for confident analysis