Near Threshold $\pi^0$ Electroproduction at High $Q^2$

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Outline

- Motivation
- Historical background
- Theoretical predictions
- Results from CLAS
- Summary
Motivation: Theoretical and Experimental

Processes

\[ e + p \rightarrow e + \pi^0 + p \]
\[ e + p \rightarrow e + \pi^+ + n \]

- **Theoretical:**
  - New extensions of low energy theorems at high \( Q^2 \) (Braun et al)
  - New generalized form factors \( G_1(Q^2) \) and \( G_2(Q^2) \)
  - Axial form factor \( G_A(Q^2) \): Fourier transform of the axial charge distribution in the proton that is probed via axial current

\[ J_{5\mu}^a \sim \bar{q}\gamma_{\mu}\gamma_5\lambda^aq \]

- **Experimental:**
  - Previous experiments limited to \( Q^2 < 1 \text{ GeV}^2 \)
  - No data exists for \( Q^2 \) between 1 – 10 GeV\(^2\)
Threshold

\[ W \in (W_{th}, 1.16) \text{ GeV} \]

\[ Q^2 \sim 2 - 5 \text{ GeV}^2 \] (CLAS)
For $Q^2 = 0 \text{ GeV}^2$

- **Low-Energy Theorems** (LETs) (Kroll-Ruderman, 1954)
- *Axial form factor* - $G_A$

\[ Q^2 \gtrapprox 10 \text{ GeV}^2 \]

\[ Q^2 \sim 1-10 \text{ GeV}^2 \]

\[ Q^2 \rightarrow \infty \]

Predictions of spatial distribution of the axial charge ($G_A$) and two new generalized form factors $G_{\pi N1}$ and $G_{\pi N2}$ (Braun et al., 2008)
For $Q^2 = 0$ GeV$^2$

- Low-Energy Theorems (LETs) (Kroll-Ruderman, 1954)
- Axial form factor - $G_A$

For $Q^2 \gtrsim 10$ GeV$^2$

- Perturbative QCD (pQCD) factorization methods (Pobylitsa et al, 2001)
Historical Background: Threshold Physics

- For $Q^2 = 0$ GeV$^2$
  - Low-Energy Theorems (LETs) (Kroll-Ruderman, 1954)
  - Axial form factor - $G_A$

- For $Q^2 \gtrsim 10$ GeV$^2$
  - Perturbative QCD (pQCD) factorization methods (Pobylitsa et al., 2001)

- For $Q^2 \sim 1 - 10$ GeV$^2$
  - Light Cone Sum Rule (LCSR) approach
  - Reproduce LET predictions for $Q^2 \sim 1$ GeV$^2$ and pQCD predictions for $Q^2 \to \infty$
  - Predictions of spatial distribution of the axial charge ($G_A$) and two new generalized form factors $G_1^{\pi N}$ and $G_2^{\pi N}$ (Braun et al, 2008)
At Threshold

Correlation function:

\[ \int dx \ e^{-i qx} \langle N(P')\pi(k)|j_{\mu}^{em}(0)|p(P)\rangle \]

At threshold, only S-wave contribution

\[ \langle N(P')\pi(k)|j_{\mu}^{em}(0)|p(P)\rangle \propto \tilde{N}(P') \gamma_5 \left\{ (\gamma_\mu q^2 - q_\mu q_\nu) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} p(P) \]

- S-wave: generalized form factors from LCSR \( G_1^{\pi N} \) and \( G_2^{\pi N} \)
- Related to S-wave multipoles: \( E_{0+} \) and \( L_{0+} \)

At threshold, the differential cross section in terms of S-wave multipoles

\[
\left. \frac{d\sigma_{\gamma^*}}{d\Omega_\pi} \right|_{\text{th}} \propto \left[ (E_{0+}^{\pi N})^2 + \epsilon \left( \frac{Q^2}{(\omega_{\gamma^*}^{\text{th}})^2} \right)^2 (L_{0+}^{\pi N})^2 \right]
\]

Relate the multipoles to the generalized form factors

\[
E_{0+}^{\pi N} \propto \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^{\pi N} \quad \text{and} \quad L_{0+}^{\pi N} \propto \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^{\pi N}
\]
At threshold, the differential cross section in terms of S-wave multipoles

\[ \left. \frac{d\sigma_{\gamma^*}}{d\Omega_{\pi}} \right|_{\text{th}} \propto \left[ (E_{0+}^\pi)^2 + \epsilon \frac{Q^2}{(\omega_{\gamma^*})^2} (L_{0+}^\pi)^2 \right] \]

Relate the multipoles to the generalized form factors

\[
\begin{align*}
E_{0+}^\pi &\propto \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^\pi N \\
L_{0+}^\pi &\propto \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^\pi N
\end{align*}
\]

LET expressions for the form factors at threshold can be related to the elastic form factors and the axial form factor:

\[
\begin{align*}
\frac{Q^2}{m_N^2} G_1^{\pi 0_p} &= \frac{g_A}{2} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^p \\
\frac{Q^2}{m_N^2} G_1^{\pi + n} &= \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{G_A}{\sqrt{2}} \\
G_2^{\pi 0_p} &= \frac{2g_A m_N^2}{Q^2 + 2m_N^2} G_E^p \\
G_2^{\pi + n} &= \frac{2\sqrt{2}g_A m_N^2}{Q^2 + 2m_N^2} G_E^n
\end{align*}
\]
At threshold, the differential cross section in terms of S-wave multipoles

\[
\left. \frac{d\sigma}{d\Omega_\pi} \right|_{\text{th}} \propto \left( E^{\pi N}_{0^+} \right)^2 + \epsilon \frac{Q^2}{(\omega_{\gamma^*})^2} \left( L^{\pi N}_{0^+} \right)^2
\]

Relate the multipoles to the generalized form factors

\[
E^{\pi N}_{0^+} \propto \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^{\pi N}
\]

\[
L^{\pi N}_{0^+} \propto \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^{\pi N}
\]

LET expressions for the form factors at threshold can be related to the elastic form factors and the axial form factor:

\[
\frac{Q^2}{m_N^2} G_1^{\pi 0 P} = \frac{g_A}{2} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^p
\]

\[
\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{G_A}{\sqrt{2}}
\]

\[
G_2^{0 P} = \frac{2g_A m_N^2}{Q^2 + 2m_N^2} G_E^p
\]

\[
G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{Q^2 + 2m_N^2} G_E^n
\]

- Obtained in the chiral limit \( m_\pi = 0 \)
- Only valid at threshold for \( Q^2 \sim 1 \text{GeV}^2 \)
Correlation function:

\[ \int dx \, e^{-iqx} \langle N(P') \pi(k) | j^\text{em}_\mu(0) | p(P) \rangle \]

Near threshold, S and P wave contributions

\[ \langle N(P') \pi(k) | j^\text{em}_\mu(0) | p(P) \rangle \propto \bar{N}(P') \gamma_5 \left\{ (\gamma_\mu q^2 - q_\mu \not{q}) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} p(P) \]

\[ + \bar{N}(P') k \gamma_5 (\not{q}' + m_N) \left\{ F_1^P(Q^2) \left( \gamma_\mu - \frac{q_\mu \not{q}}{q^2} \right) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} F_2^P(Q^2) \right\} p(P) \]

- **S-wave**: generalized form factors from LCSR ($G_1^{\pi N}$ and $G_2^{\pi N}$)
- **P-wave**: electromagnetic form factors ($F_1^P$ and $F_2^P$)
- **Both S and P-wave multipoles are involved** ($l = 0, 1, \cdots$)

Experimental difficulties:

- Very small cross sections
- High Bethe-Heitler contamination
- Poor $\phi_\pi$ resolution because $\theta_\pi \approx \theta_\gamma^*$

Analysis Steps:

- Electron and proton identification
- Corrections - Kinematic and Acceptance
- Pion Identification
  - Missing Mass Technique ($epX$)
  - Bethe-Heitler Subtractions

$$ep \rightarrow ep\pi^0$$
Pion Identification

$$epX$$

W: 1.09 GeV

Entries: 12874
Mean: 0.001005
RMS: 0.01085
$$\chi^2 / \text{ndf}$$: 1028 / 132
Prob: 0
p0: 8291 ± 95.4
p1: -0.0394205 ± 0.0000232
p2: 0.0621111 ± 0.000027
p3: 1248 ± 53.7
p4: 0.01243 ± 0.00039
p5: 0.065131 ± 0.000212
p6: 4.745 ± 0.305
p7: -63.06 ± 10.96

W: 1.11 GeV

Entries: 13187
Mean: 0.002646
RMS: 0.01106
$$\chi^2 / \text{ndf}$$: 993.3 / 151
Prob: 0
p0: 7618 ± 98.1
p1: -0.0044577 ± 0.000029
p2: 0.062508 ± 0.000039
p3: 2575 ± 74.7
p4: 0.0129 ± 0.0002
p5: 0.000221 ± 0.000185
p6: 4.504 ± 0.246
p7: -56.82 ± 7.56
Assuming elastic scattering, we can compute $\theta_{proton}$ independent of incident and scattered electron energies.

Pre-Radiation

\[
\tan \theta^1_p = \frac{1}{\left(1 + \frac{E'}{M - E' \cos \theta'_e}\right) \tan \frac{\theta'_e}{2}}
\]

Post-Radiation

\[
\tan \theta^2_p = \frac{1}{\left(1 + \frac{E}{M}\right) \tan \frac{\theta'_e}{2}}
\]

Calculate deviation of measured proton angle from elastic scattering process

\[
\Delta \theta^P_{1,2} \equiv \theta_{calc} - \theta_{meas}
\]
Bethe-Heitler Subtraction (continued)

$e \pi X$

$W: 1.09 \text{ GeV}$

$W: 1.11 \text{ GeV}$
Select pions with $|M_X^2 - \mu| < 3\sigma$
Lose pions with BH subtraction cuts
Use simulation to estimate % of lost pions
Correct for this loss for each kinematic bin at the cross section level
Cross Section Descriptions

Measure differential cross section for $ep \rightarrow ep\pi^0$:

$$\frac{d\sigma}{dE'd\Omega'_e d\Omega^*_\pi} = \Gamma \frac{d\sigma}{d\Omega^*_\pi}$$

Extract structure functions from reduced differential cross section:

$$\frac{d\sigma_{\gamma^* p \rightarrow p\pi^0}}{d\Omega^*_\pi} = \frac{p^*_\pi}{k^*_\gamma} \left( \frac{d\sigma_T}{d\Omega^*_\pi} + \varepsilon_L \frac{d\sigma_L}{d\Omega^*_\pi} + \varepsilon \frac{d\sigma_{TT}}{d\Omega^*_\pi} \cos 2\phi^*_\pi + \sqrt{2\varepsilon_L (\varepsilon + 1)} \frac{d\sigma_{LT}}{d\Omega^*_\pi} \cos \phi^*_\pi \right)$$

- Obtain multipoles $E_{0+}$ and $L_{0+}$
- Obtain form factors $G_{1N}^\pi$ and $G_{2N}^\pi$
Differential Cross Sections - $d\sigma/d\Omega$

$W = 1.09$ GeV, $Q^2 = 2.75$ GeV$^2$

Red points: experiment (dashed curve fit) · Blue curve: MAID 2007
Magenta curve: Braun 2008 · Green curve: Aznauryan 2009

Preliminary
W = 1.23 GeV, $Q^2 = 2.75$ GeV$^2$

Red points: experiment (dashed curve fit) · Blue curve: MAID 2007
Green curve: Aznauryan 2009

Preliminary
Integrated Cross Sections $\sigma(\pi^0 p)$

Preliminary

Integrated Cross Sections - $Q^6 \sigma(\pi^0 p)$

\[ Q^6 \times \sigma(\pi^0 p) [\mu b \times \text{GeV}^6] \]

\[ W = 1.09 \text{ GeV} \]

\[ W = 1.11 \text{ GeV} \]

\[ W = 1.23 \text{ GeV} \]

Preliminary

\[
\frac{d\sigma_T}{d\Omega} + \varepsilon \frac{d\sigma_L}{d\Omega} = \frac{d\sigma_{TT}}{d\Omega} = \frac{d\sigma_{LT}}{d\Omega}
\]

\[W = 1.09 \text{ GeV}\]

Preliminary

Pion electroproduction near threshold

- Test the applicability of LETs in $Q^2 \sim 1 - 10 \text{ GeV}^2$
- Very low statistics and small cross sections - difficult
- Differential and integrated cross sections have been obtained
- Structure functions have been extracted
- Extract S-wave multipoles $E_{0+}$ and $L_{0+}$ that are directly related to the generalized form factors, $G_1^{\pi N}$ and $G_2^{\pi N}$, near threshold
“The cake is a lie.” - Portal

- rederived low energy theorems to $O(m^2_\pi)$
- used chiral perturbation theory
- yields better results at $Q^2 = 0$
  (Vainshtein, Zakharov, 1970s; Scherer, Koch, 1990s)
LCSR: Differential cross section

\[ \frac{d\sigma_{\gamma^*}}{d\Omega_{\pi}} = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1 + \epsilon)}\sigma_{LT} \cos(\phi_{\pi}) + \epsilon\sigma_{TT} \cos(2\phi_{\pi}) + \lambda \sqrt{2\epsilon(1 - \epsilon)}\sigma'_{LT} \sin(\phi_{\pi}) \]

- \( \sigma_T \rightarrow G_1^{\pi N}, G_M^2 \)
- \( \sigma_L \rightarrow G_2^{\pi N}, G_E^2 \)
- \( \sigma_{LT} \rightarrow G_M, G_E, ReG_1^{\pi N}, ReG_2^{\pi N} \)
- \( \sigma_{TT} = 0 \)
- \( \sigma'_{LT} \rightarrow G_M, G_E, ImG_1^{\pi N}, ImG_2^{\pi N} \)

- \( \sigma_{TT} = 0 \): D-wave contribution neglected
- \( \sigma'_{LT} \): related to single-spin symmetry
Beam line = (0.09, −0.345, z) cm

All sector midplanes are corrected to line up with the beamline
Beam line = (0.09, −0.345, z) cm

All sector midplanes are corrected to line up with the beamline

Cut on $V_z \in (-8.0, -0.8)$ cm

Cut on $|\Delta V_z (e - p)| < 0.74$ cm
Very loose cut on vertex $v_z \in (-8.0, -0.8) \text{ cm}$

Include all cuts from analysis

Total charge collected

$$\text{FCUP}_{\text{empty target}} = 2.214 \text{ mC}$$
$$\text{FCUP}_{\text{production}} = 21.287 \text{ mC}$$

Total contamination $\sim 2 - 5\%$
BH cleanup cuts throw away good pions

- Estimate the pion loss using simulation: MAID 2007 model
- Ratio = Thrown / Kept
- Ratio is highest at $\cos \theta^*_\pi \rightarrow 1$ and $\phi^*_{\text{proton}} \approx 180^\circ$ (i.e., $\phi^*_\pi \approx 0^\circ$)
- Apply correction to each kinematic bin near threshold
Inclusive reaction: $ep \rightarrow eX$


Good agreement within $\pm 5\%$
- Exclusive reaction: \( ep \rightarrow ep \)
- Proton detection inefficiencies
- Ad hoc overall correction \( \sim 10\% \)
The cross sections vary inside bin
Center of bin is not true center
Correction factor:

\[ R_{W,Q^2,\cos\theta,\phi} = \frac{\sigma_{\text{center}}}{\sigma_{\text{average}}} \]

Average overall correction \sim 10\%
\[ \sigma_{\text{born}} = \frac{\sigma_{\text{meas}}}{\text{RC}} = \sigma_{\text{meas}} \left( \frac{\text{RAD}_{\text{gen}}}{\text{RAD}_{\text{rec}}} \right) \times \left( \frac{\text{NORAD}_{\text{gen}}}{\text{RAD}_{\text{gen}}} \right) \] (1)

- Corrections obtained from EXCLURAD using MAID 2007 model
- Radiative correction independent of vcut near threshold
- Correction is largest at \( \cos \theta \sim -0.9 \) because pion cross section goes to zero and BH dominates; is \( \phi \) dependent
- About 20% overall correction
\[
\frac{d\sigma}{d\Omega^{*}_{\pi}} = A + B\varepsilon \cos 2\phi^{*}_{\pi} + C[2\varepsilon L(\varepsilon + 1)]^{1/2} \cos \phi^{*}_{\pi}
\]

12 points in $\phi^{*}_{\pi} - 3$ parameters = 9 d.o.f.
Total number of fits = 5 $Q^{2}$ bins $\times$ 10 $\cos \theta^{*}_{\pi}$ bins = 50.