

Small angle (Low Q^2) GDH sum rule experiments

A. Deur
Jefferson Lab.

Outline:

- * Sum rules, GDH, Bjorken, spin polarizability sum rules;
- * Usefulness and context;
- * Measurements (published, being analyzed, to come);
- * What we learn practically;
- * Perspectives.

The Gerasimov-Drell-Hearn Sum Rule

A **sum rule** is a *rule* (e.g. “=”) that relates a *sum* (e.g. moment of structure functions) to a quantity characterizing the target.

GDH sum rule:

$$\frac{16\alpha\pi^2}{Q^2} \int_0^1 g_1 dx = \frac{-2\alpha\pi^2 \kappa^2}{M^2}$$

First spin structure function

Bjorken scaling variable

anomalous magnetic moment

α : fine structure constant

Valid for any kind of target (nucleon, nuclei, ...)

The Gerasimov-Drell-Hearn Sum Rule

A **sum rule** is a *rule* (e.g. “=”) that relates a *sum* (e.g. moment(s) of structure function) to a quantity characterizing the target.

GDH sum rule:

$$\frac{16\alpha\pi^2}{Q^2} \int_0^1 g_1 dx = \frac{-2\alpha\pi^2 \kappa^2}{M^2}$$

Originally derived for photo-absorption ($Q^2=0$) by Gerasimov, Drell, Hearn and others.

Generalized to $Q^2>0$ by Ji and Osborne: $\frac{16\alpha\pi^2}{Q^2} \int_0^1 g_1 dx = 2\alpha\pi^2 S_1$  spin-dep. DDVCS

Famous Bjorken sum rule is one aspect of generalized GDH sum rule:

At large Q^2 : $\text{GDH}(\text{proton}) - \text{GDH}(\text{neutron}) \propto Q^2 \times \text{Bjorken Sum Rule}$

$$\int g_1^p - g_1^n dx = \frac{1}{6} g_a \left(1 + \frac{\alpha_s(\ln(Q^2))}{\pi} + \dots \right) + \dots$$

Spin polarizabilities sum rules

A **sum rule** is a *rule* (e.g. “=”) that relates a *sum* (e.g. moment(s) of structure function) to a quantity characterizing the target.

Sum rules with higher moments exist, e.g. spin polarizabilities sum rules:

Generalized forward spin polarizability:

$$\gamma_0 = \frac{4e^2M^2}{\pi Q^6} \int x^2 \left(g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right) dx$$

Longitudinal-Transverse polarizability:

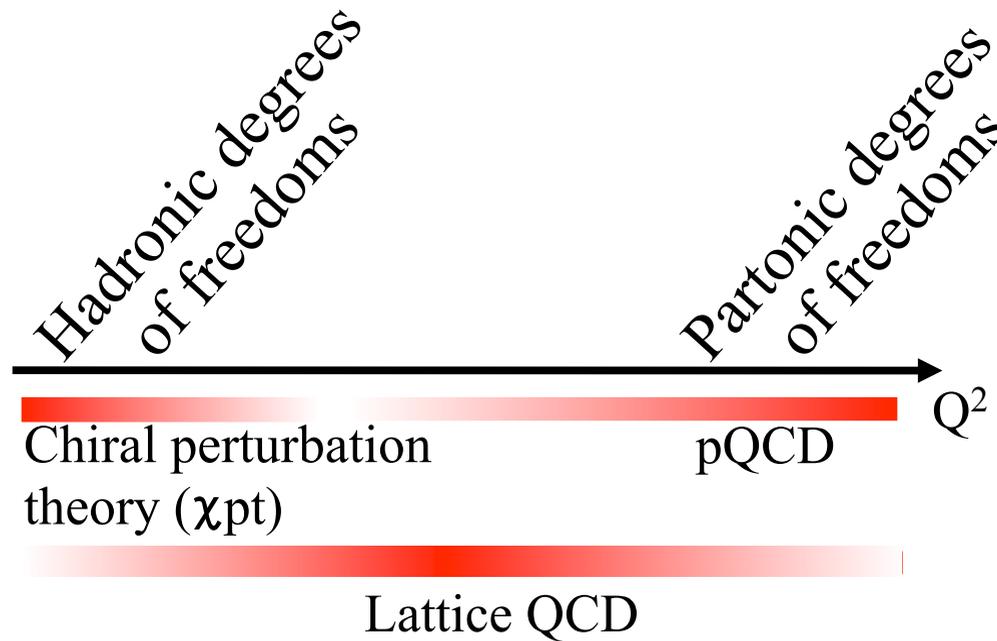
$$\delta_{LT} = \frac{4e^2M^2}{\pi Q^6} \int x^2 (g_1 + g_2) dx$$

x^2 -weighting favors the large- x reactions (resonances, important at low Q^2).

Interest of the generalized GDH sum rule

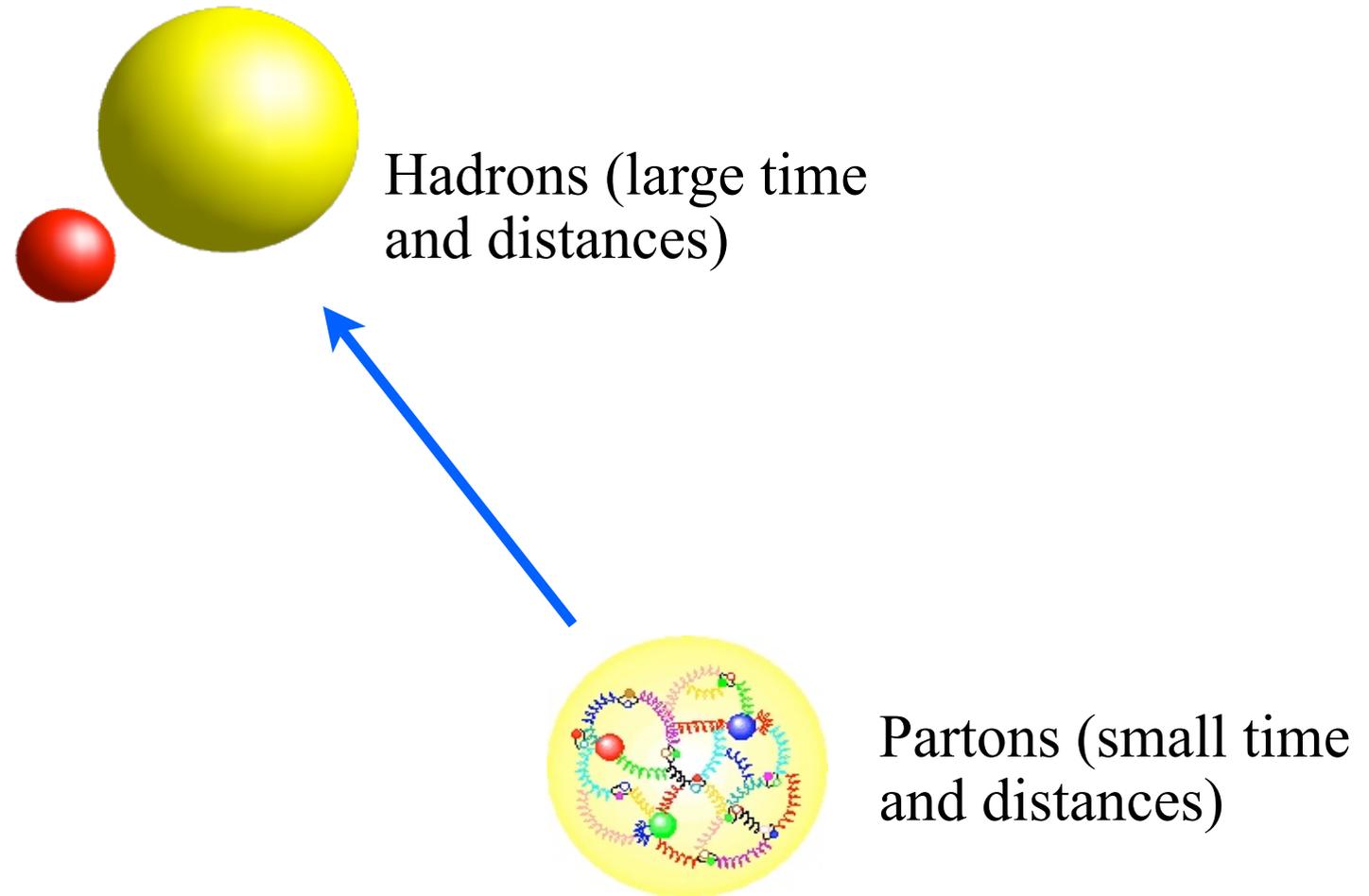
Sum rule valid at all Q^2 :

We can measure $\int g_1 dx$ at different Q^2 and compute the right hand side of the sum rule using different technics:



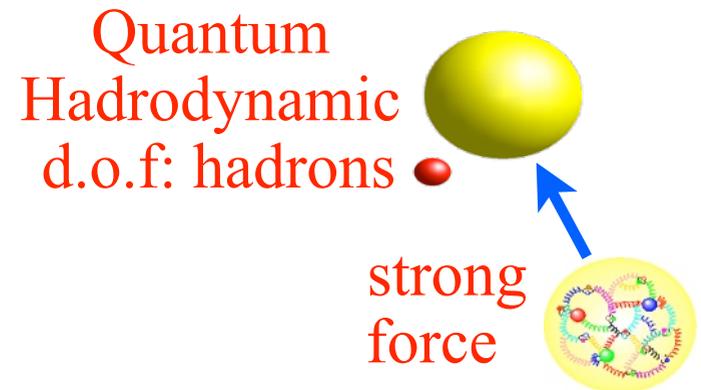
⇒ Study transition from hadronic to partonic description of strong interaction

i.e. study how effective degrees of freedom (hadrons) emerge from fundamental ones (partons).

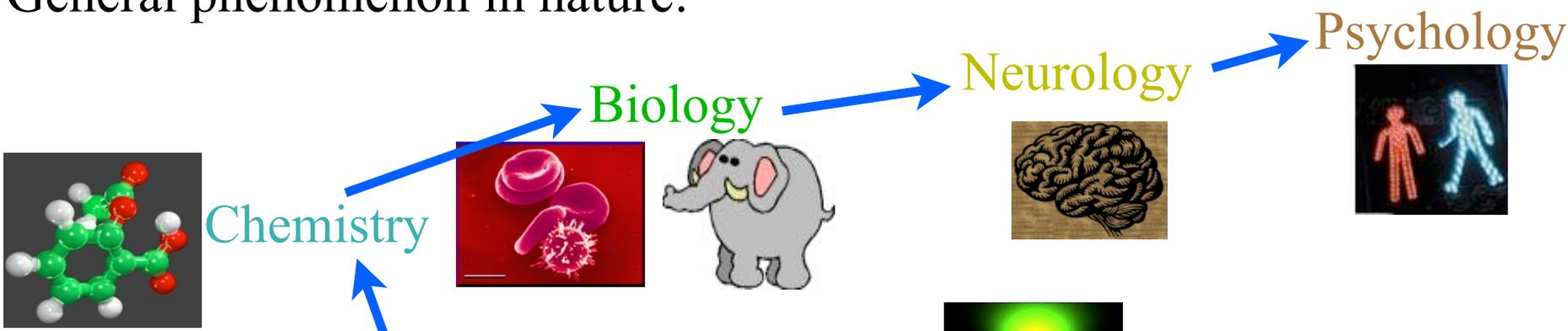


This is a general phenomenon in nature: When complexity makes the basic degrees of freedom too cumbersome to use, effective ones are used.

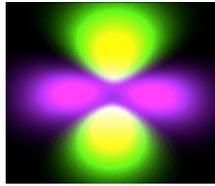
In addition to their practicality, effective descriptions provide a legitimate description of nature as long as the connection to the basic description is understood.



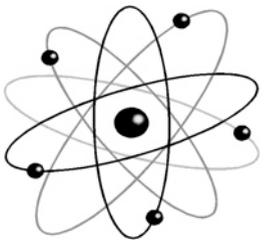
General phenomenon in nature:



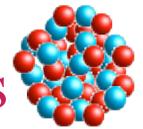
Molecular physics
d.o.f: atoms, Van der Waals force



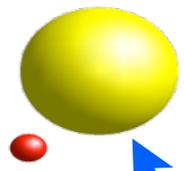
Atomic physics
d.o.f: electrons, nuclei, EM field



Nuclear physics
d.o.f: hadrons



Quantum
Hadrodynamic
d.o.f: hadrons



Fundamental forces: electromagnetic gravitation weak strong
Fundamental particles: quarks, electrons, neutrinos,...



Existing data

$Q^2=0$:
Mainz-Bohn
LEGS

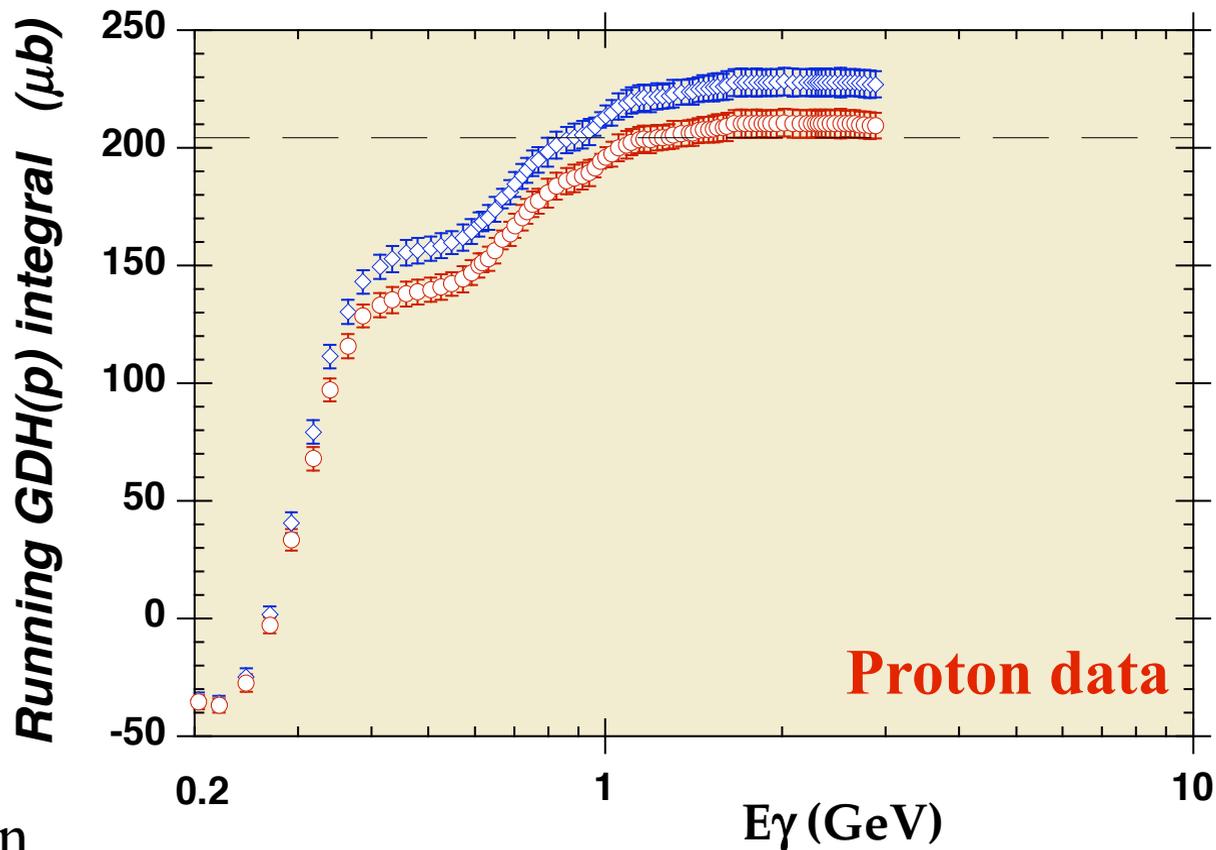
Intermediate Q^2 :
Jlab Hall A, B & C

Large Q^2 :
CERN, SLAC,
DESY (Hermes)



Existing data

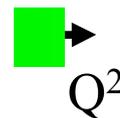
- ◇ INT[GDH(H)] Mainz+Bonn (μb)
- INT'[GDH(H)] = {Mainz+Bonn}+LEGS π^0 correction



$Q^2=0$:
Mainz-Bohn
LEGS

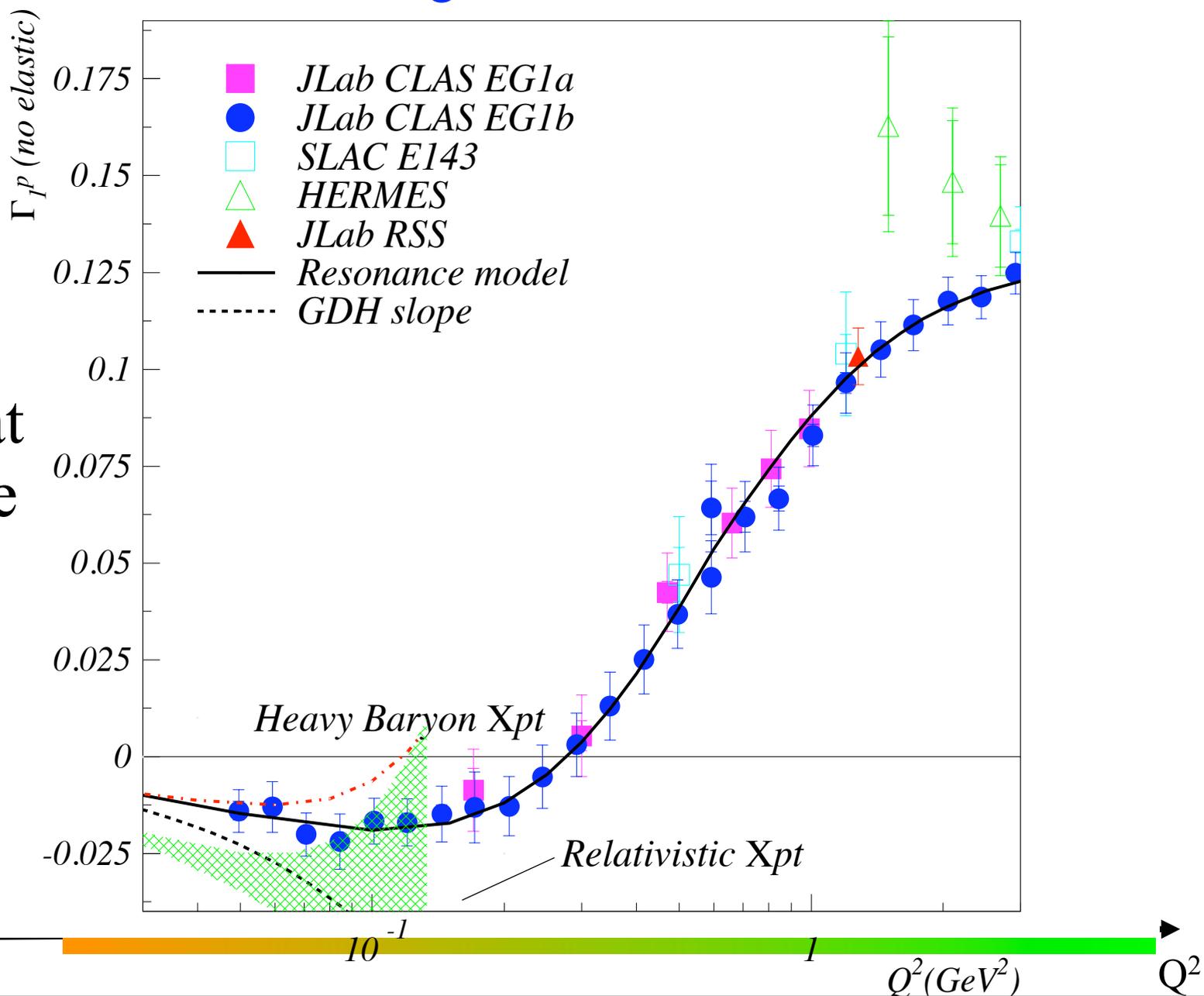
**GDHp [combined Mainz+Bonn+LEGS]
= 208 \pm 6(stat) \pm 14(sys) μb**

PRL 102, 172002(09)



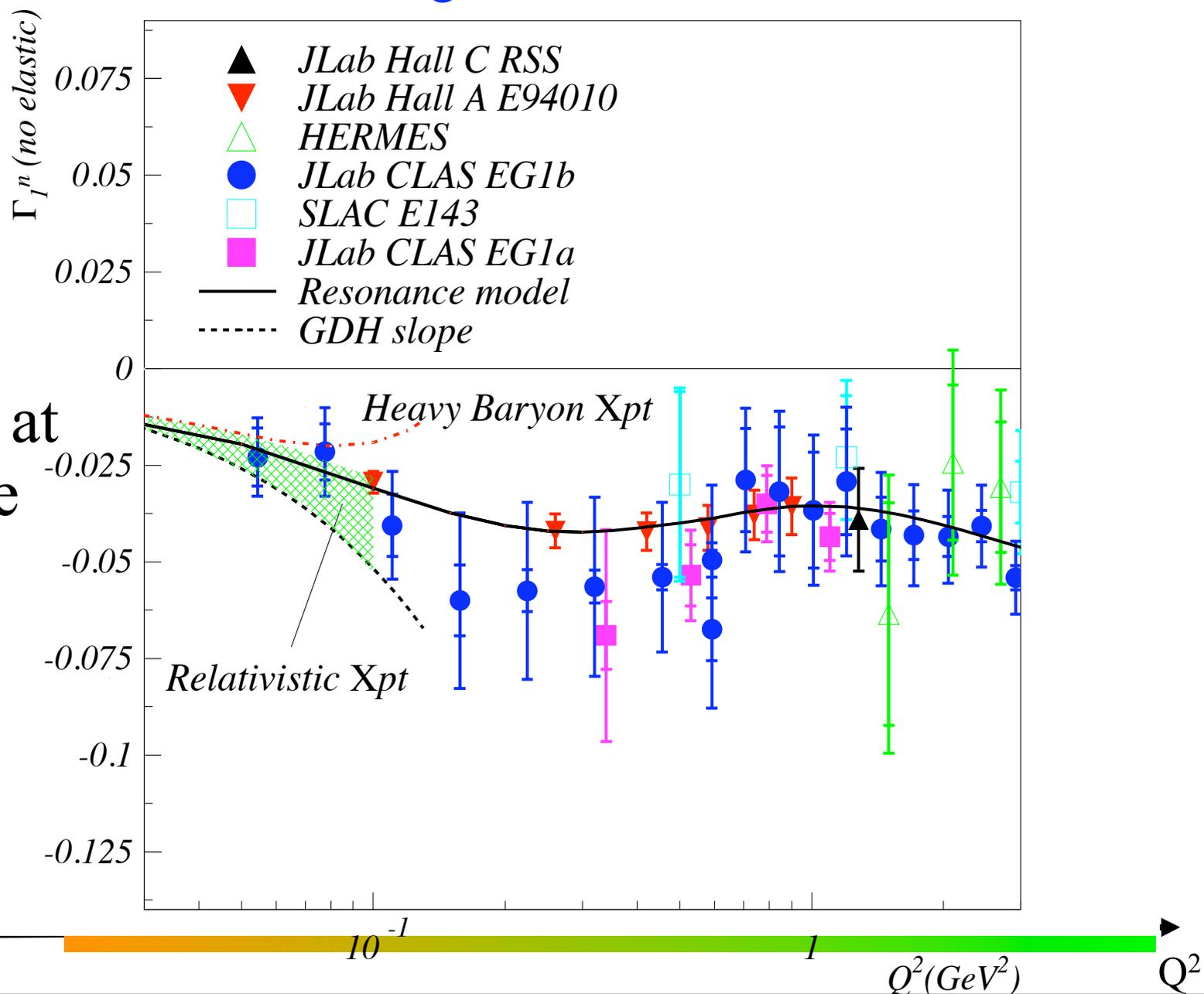
Existing data

$\int g_1 dx$
on **proton** at
intermediate
 Q^2 .



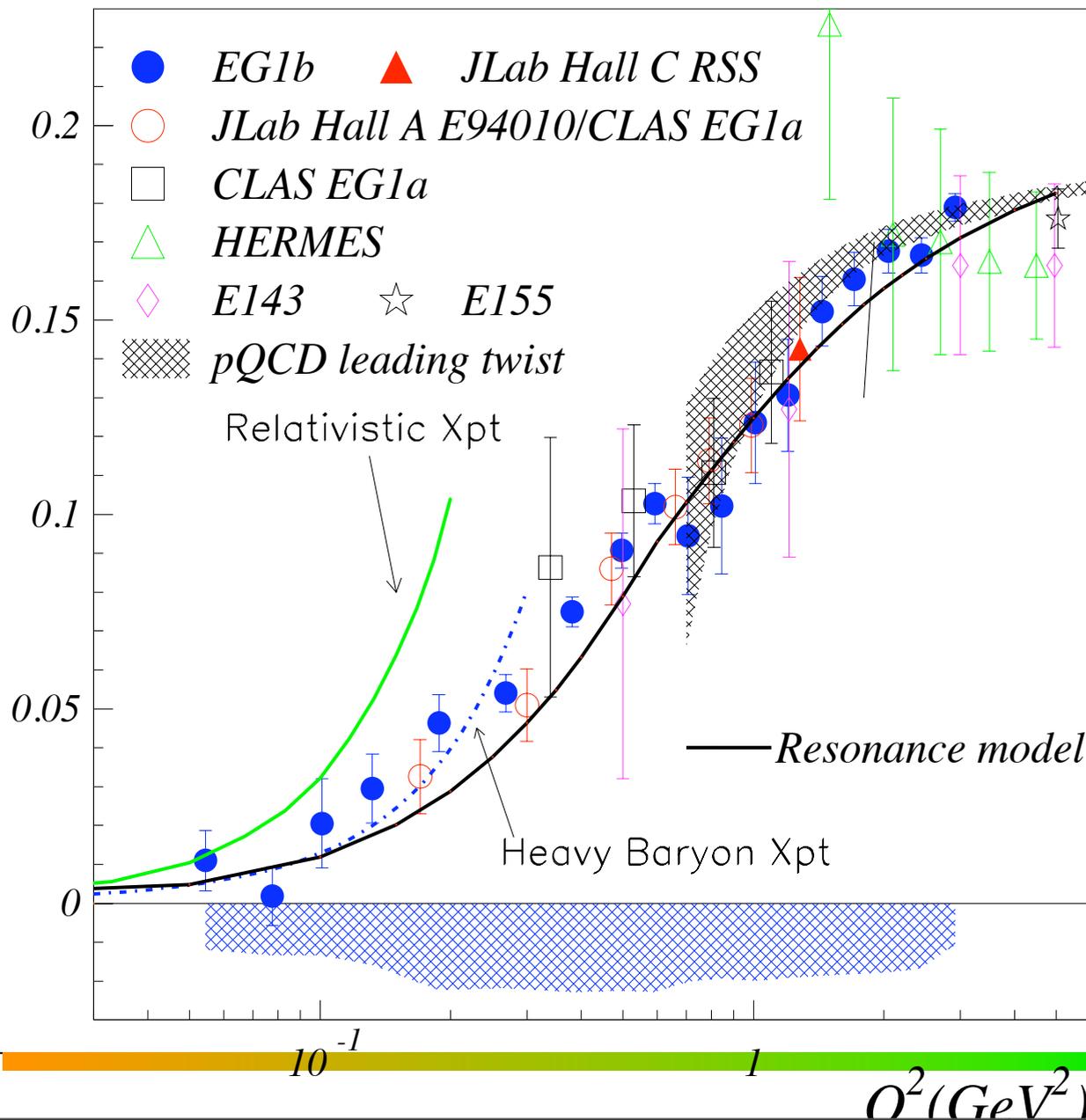
Existing data

$\int g_1 dx$
on **neutron** at
intermediate
 Q^2 .



Existing data

Γ_{I}^{p-n}

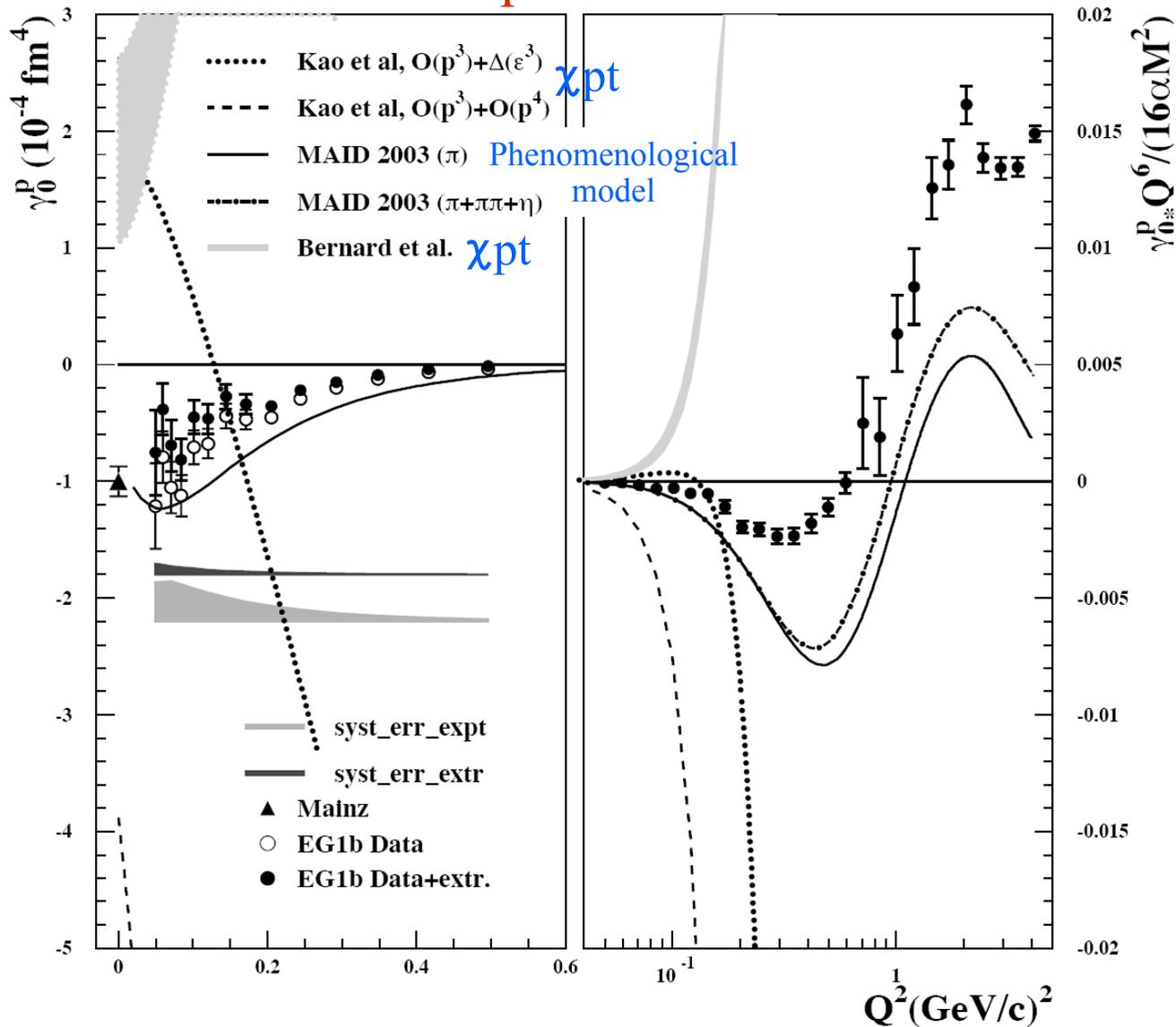


Bjorken Sum

Existing data

Spin Polarizabilities

proton

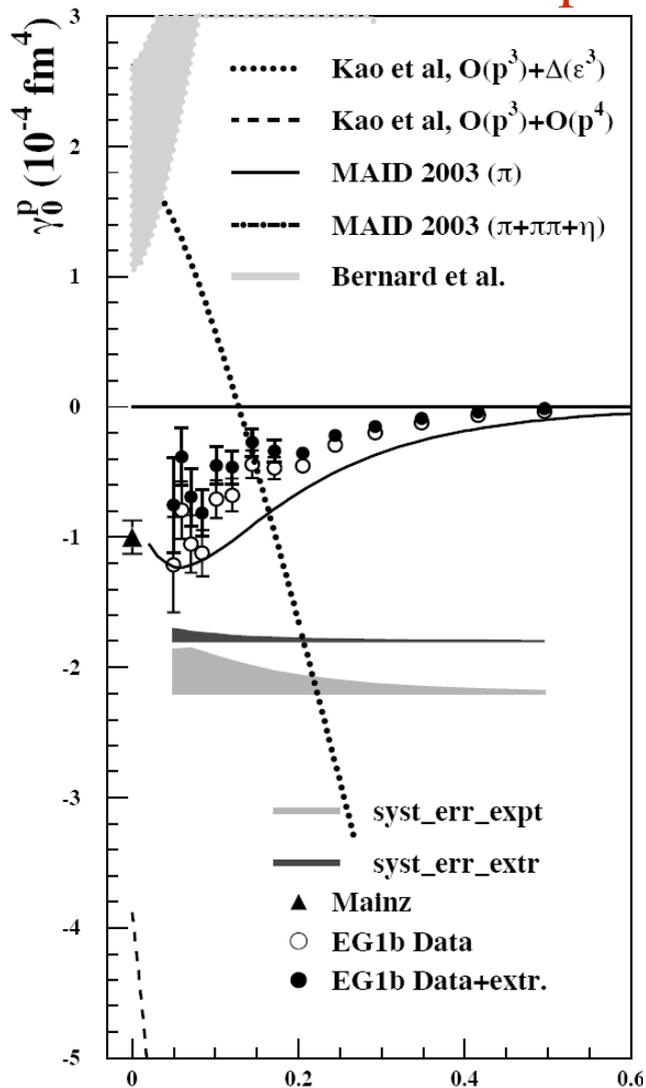


Large Q^2 :
CERN, SLAC,
DESY (Hermes)

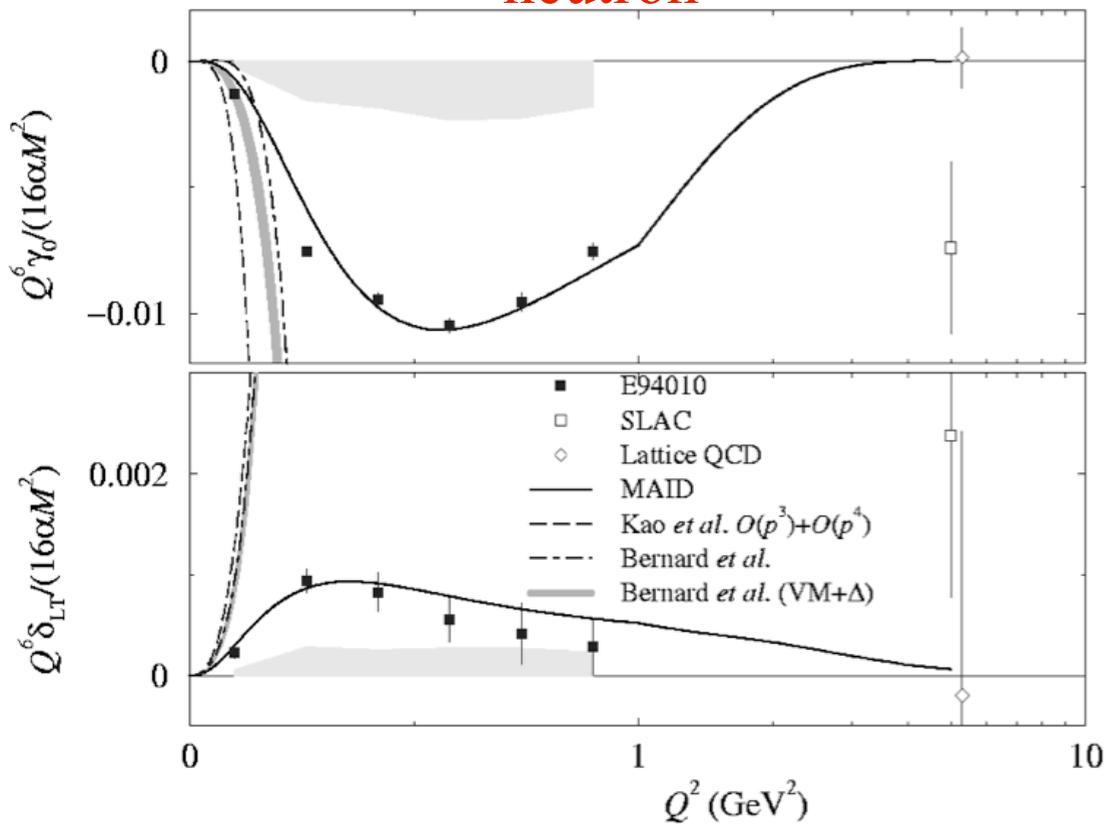
Existing data

Spin Polarizabilities

proton



neutron



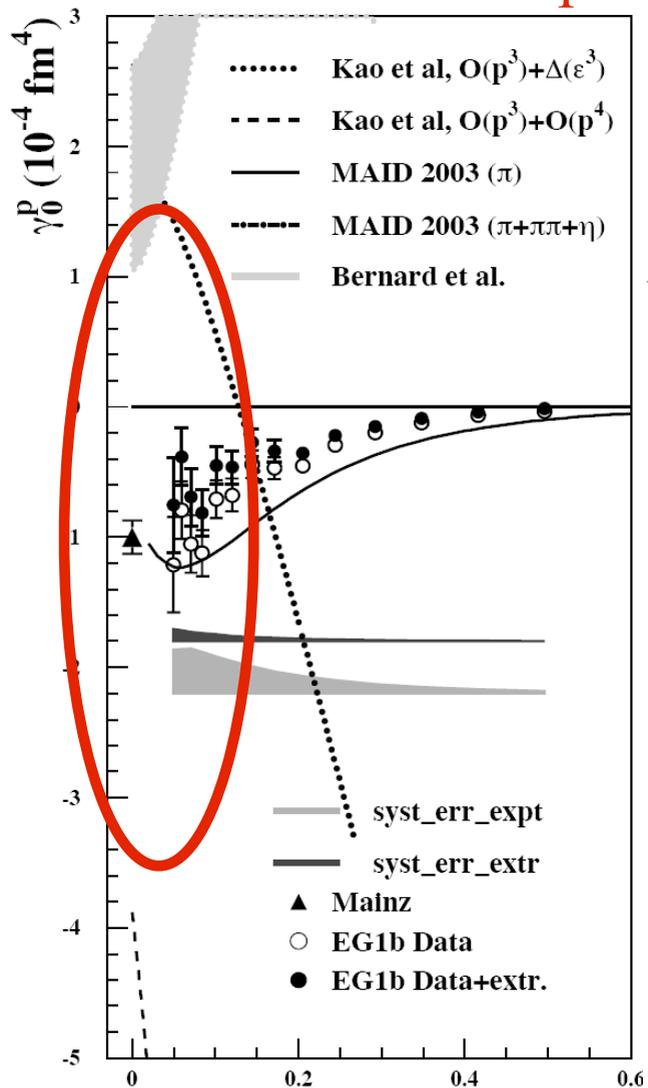
Large Q^2 :
CERN, SLAC,
DESY (Hermes)



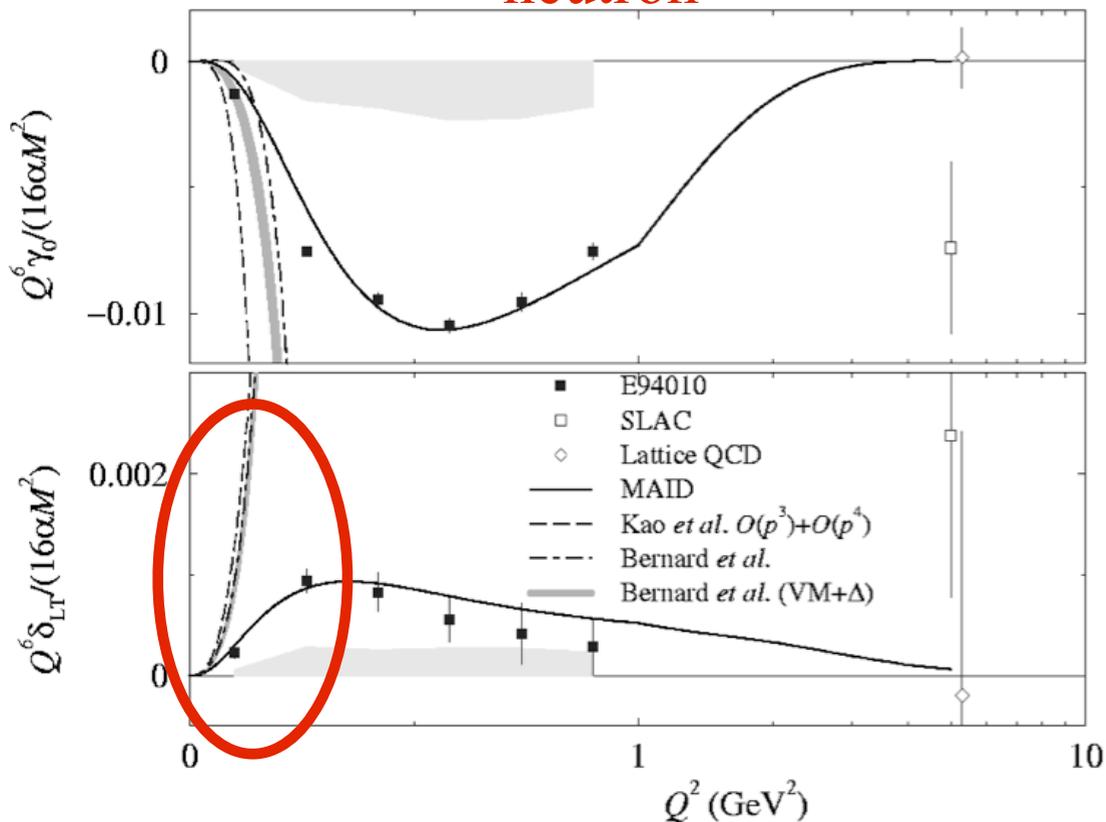
Existing data

Spin Polarizabilities

proton



neutron



Large Q^2 :
 CERN, SLAC,
 DESY (Hermes)



GDH at low Q^2 in:

- *Hall A: E97110, neutron(^3He)
- *Hall B: EG4, proton & neutron(D)

Upcoming results
bridge the gap:

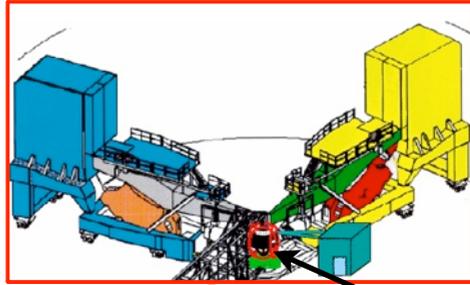
$Q^2=0$:
Mainz-Bohn
LEGS

Intermediate Q^2 :
Jlab Hall A, B & C

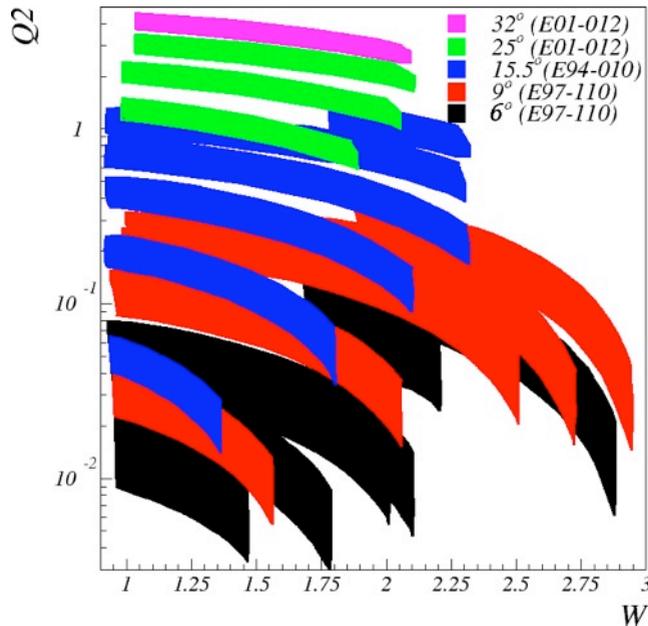
Large Q^2 :
CERN, SLAC,
DESY (Hermes)



Kinematic coverage

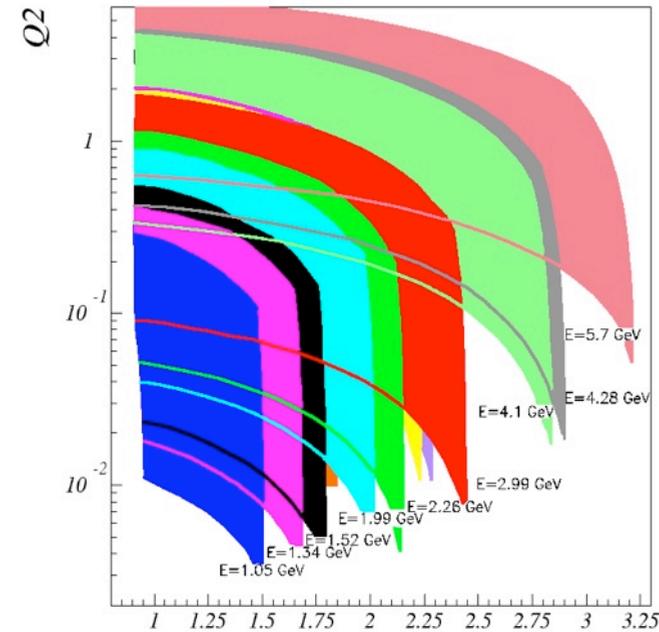
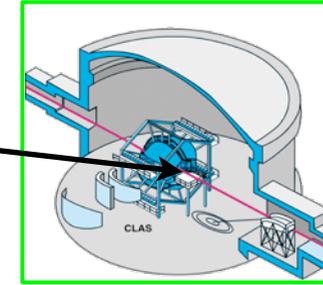


Polarized ^3He target



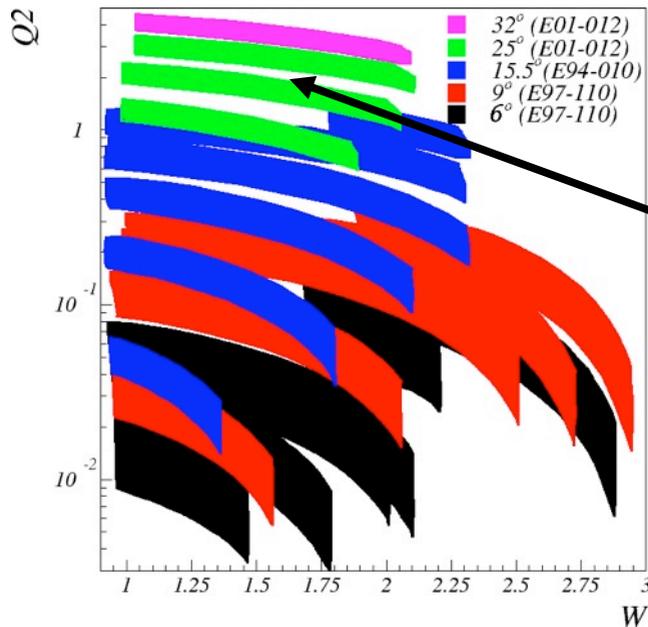
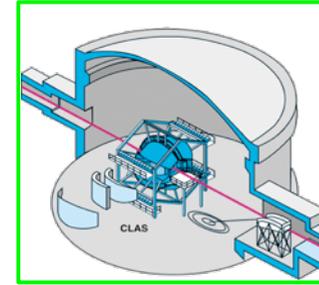
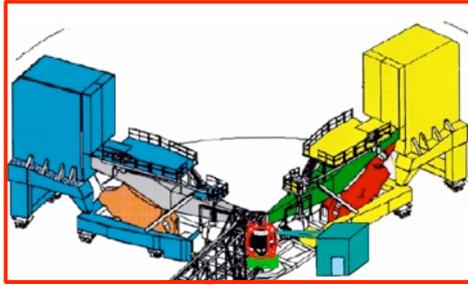
E94-010 E97-110 & E01012 experiments

Polarized NH_3 and ND_3 targets



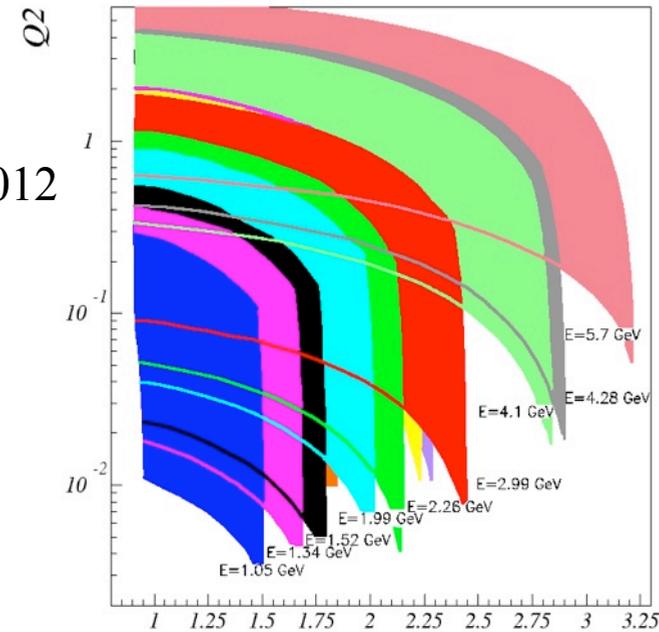
EG1 & EG4 experiments^W

Kinematic coverage



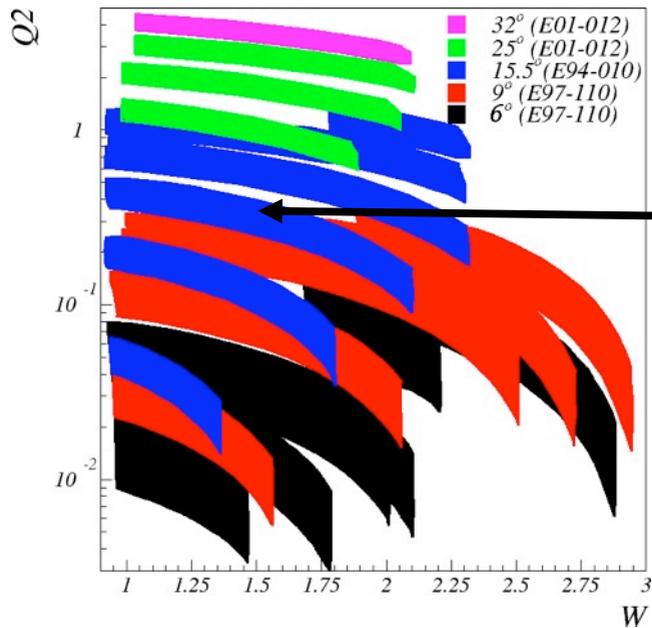
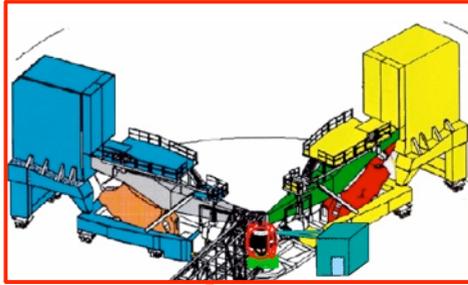
Experiment E01-012
(Spin-Duality)

E94-010 E97-110 & E01012 experiments



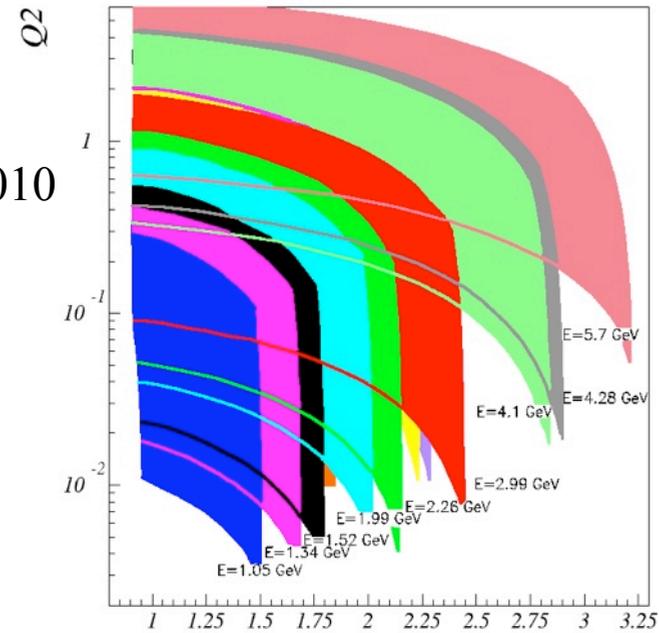
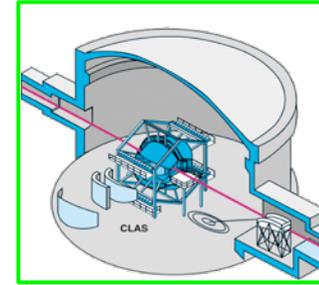
EG1 & EG4 experiments^W

Kinematic coverage



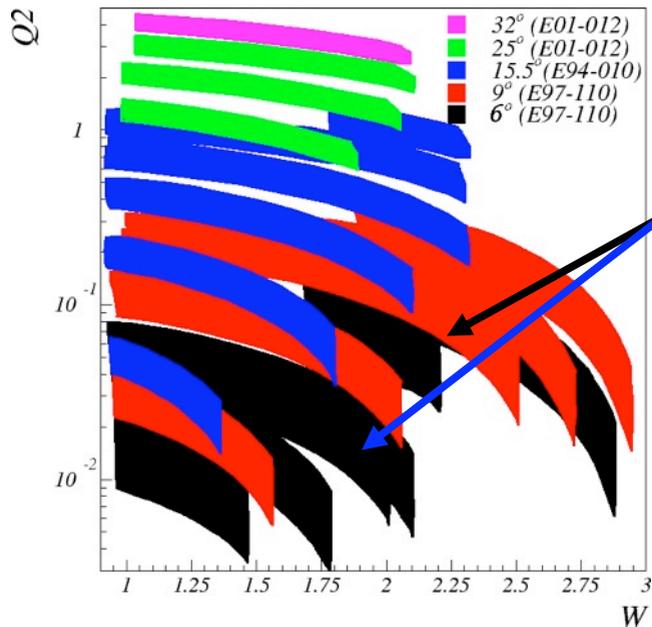
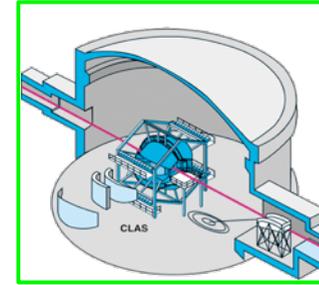
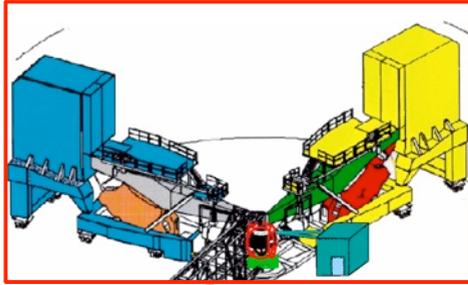
Experiment E94-010

E94-010 E97-110 & E01012 experiments



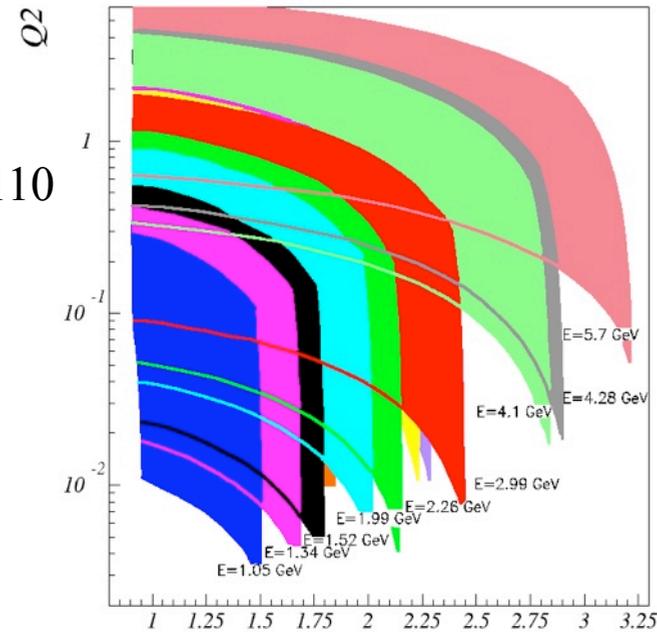
EG1 & EG4 experiments^W

Kinematic coverage



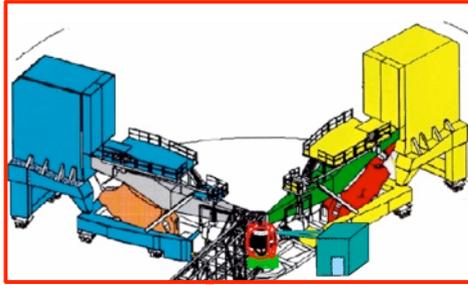
Experiment E97-110

E94-010 E97-110 & E01012 experiments

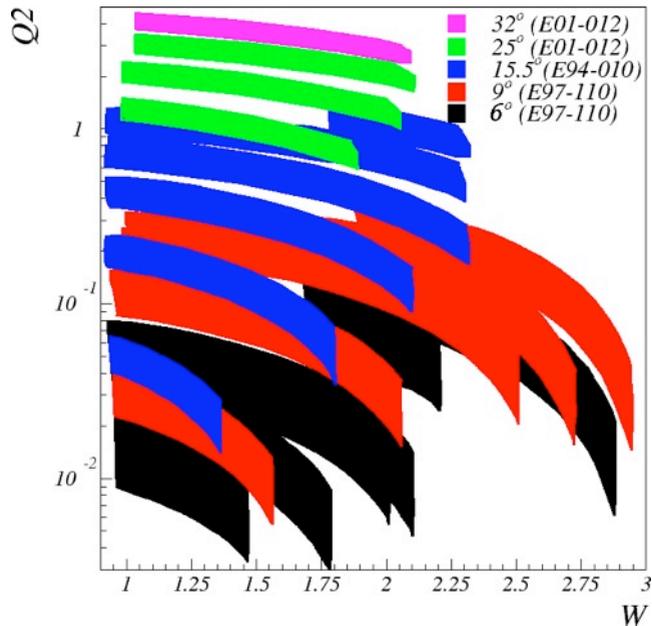
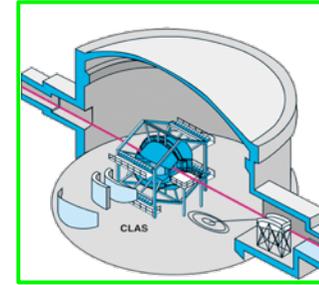


EG1 & EG4 experiments^W

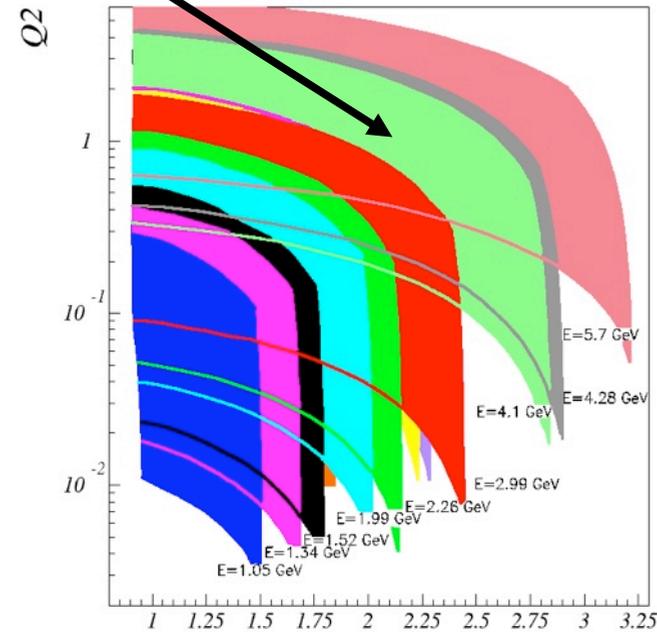
Kinematic coverage



Experiment Group **EG1**:
 NH_3 & ND_3

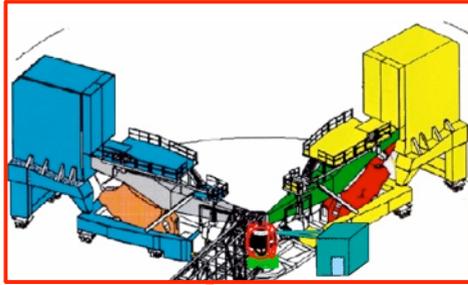


E94-010 E97-110 & E01012 experiments

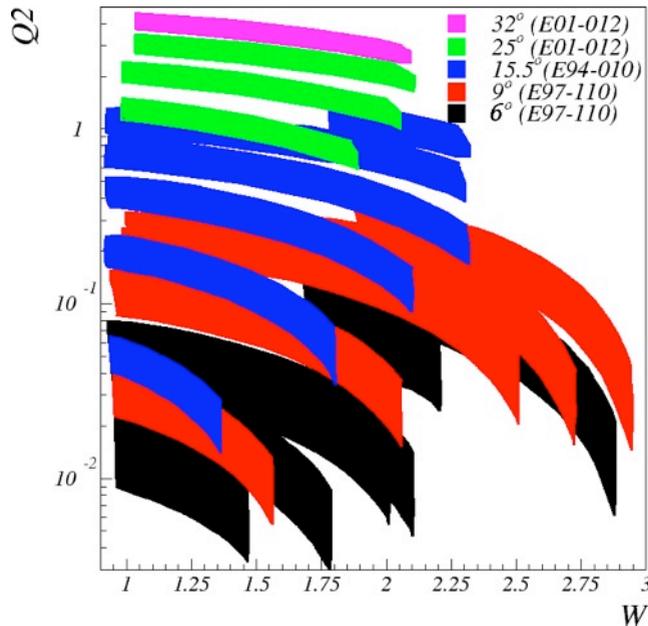
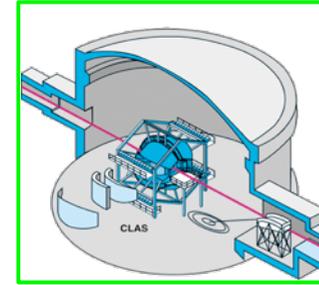


EG1 & EG4 experiments^W

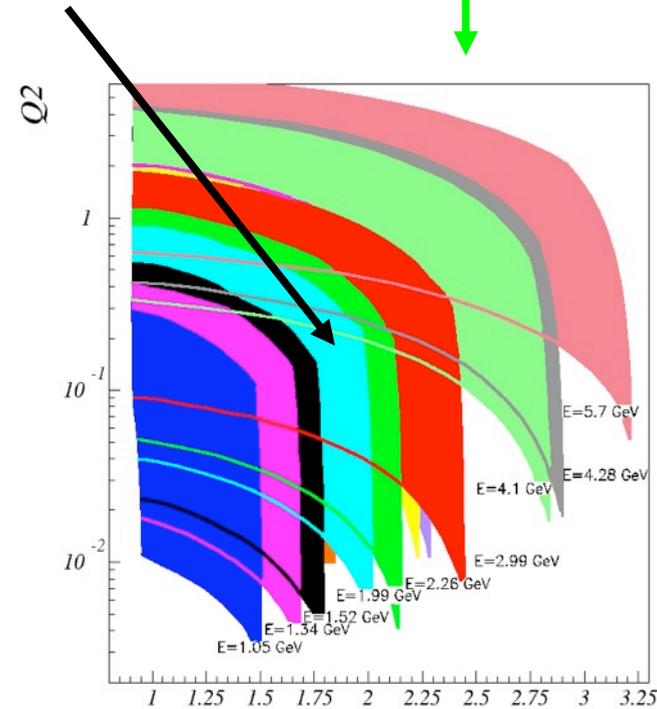
Kinematic coverage



Experiment Group EG4:
 NH_3 & ND_3



E94-010 E97-110 & E01012 experiments



EG1 & EG4 experiments^W

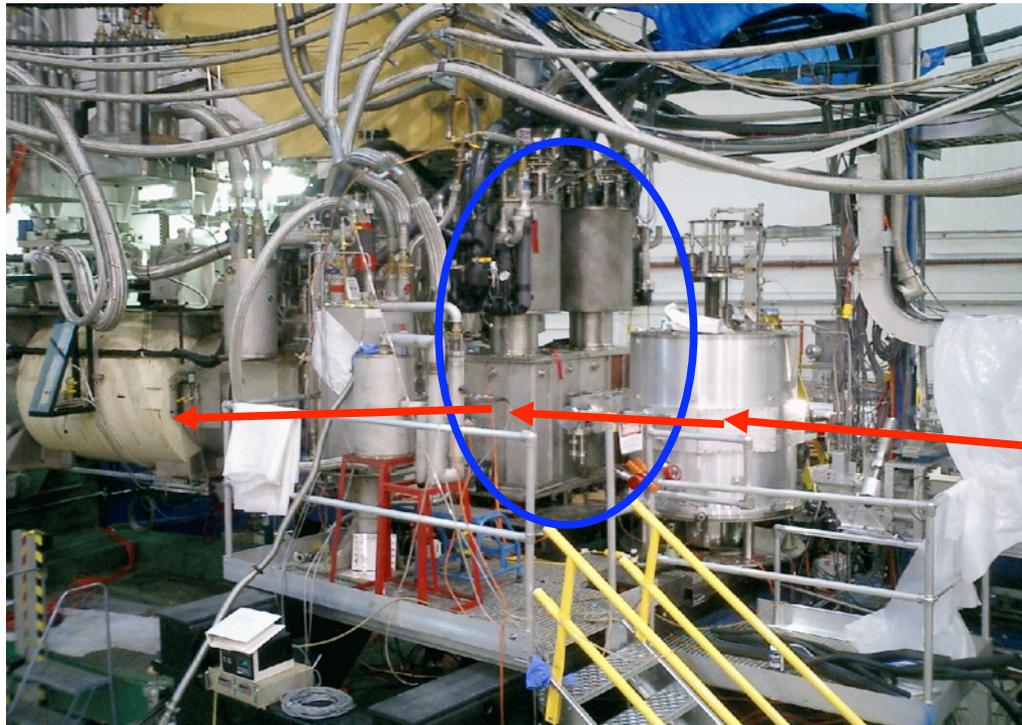
Going to low Q^2

Low Q^2 : $\begin{cases} \text{low beam energy} \\ \text{and/or} \\ \text{forward angle} \end{cases}$

Because we need integrals, our only choice is to work at forward angles.

In Hall A:

Septum magnets:



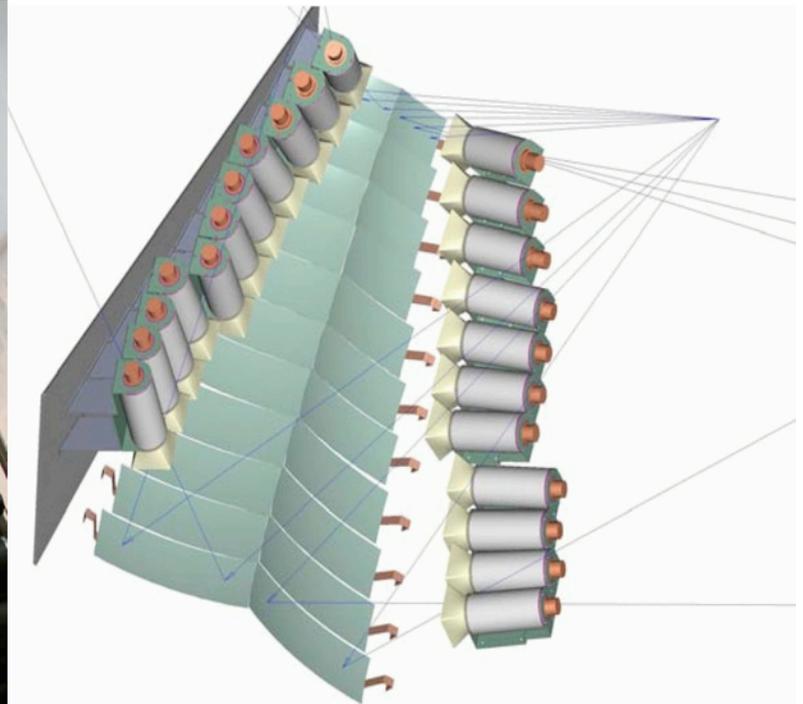
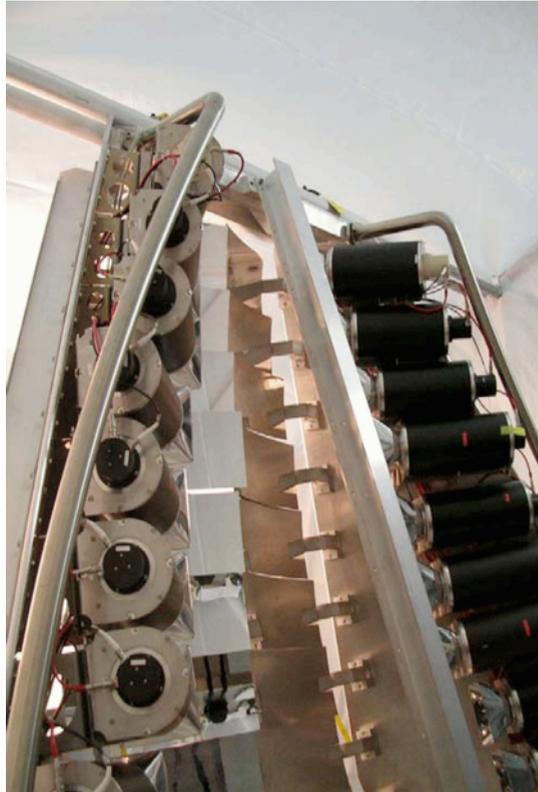
Going to low Q^2

Low Q^2 : $\left\{ \begin{array}{l} \text{low beam energy} \\ \text{and/or} \\ \text{forward angle} \end{array} \right.$

Because we need integrals, our only choice is to work at forward angles.

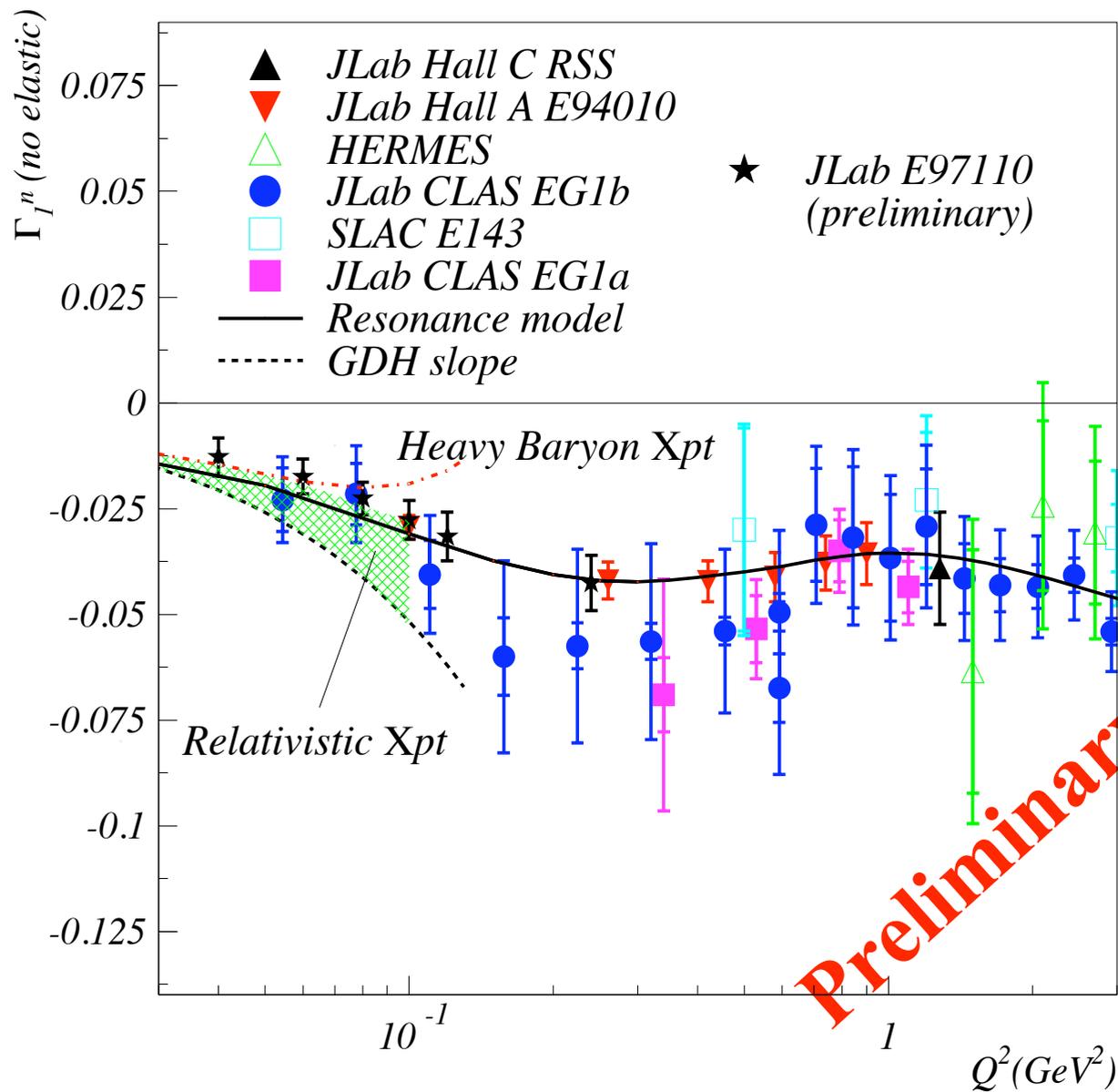
In Hall B:

New Cerenkov
detector:



Preliminary results on $\int g_1 dx$

Hall A (neutron):

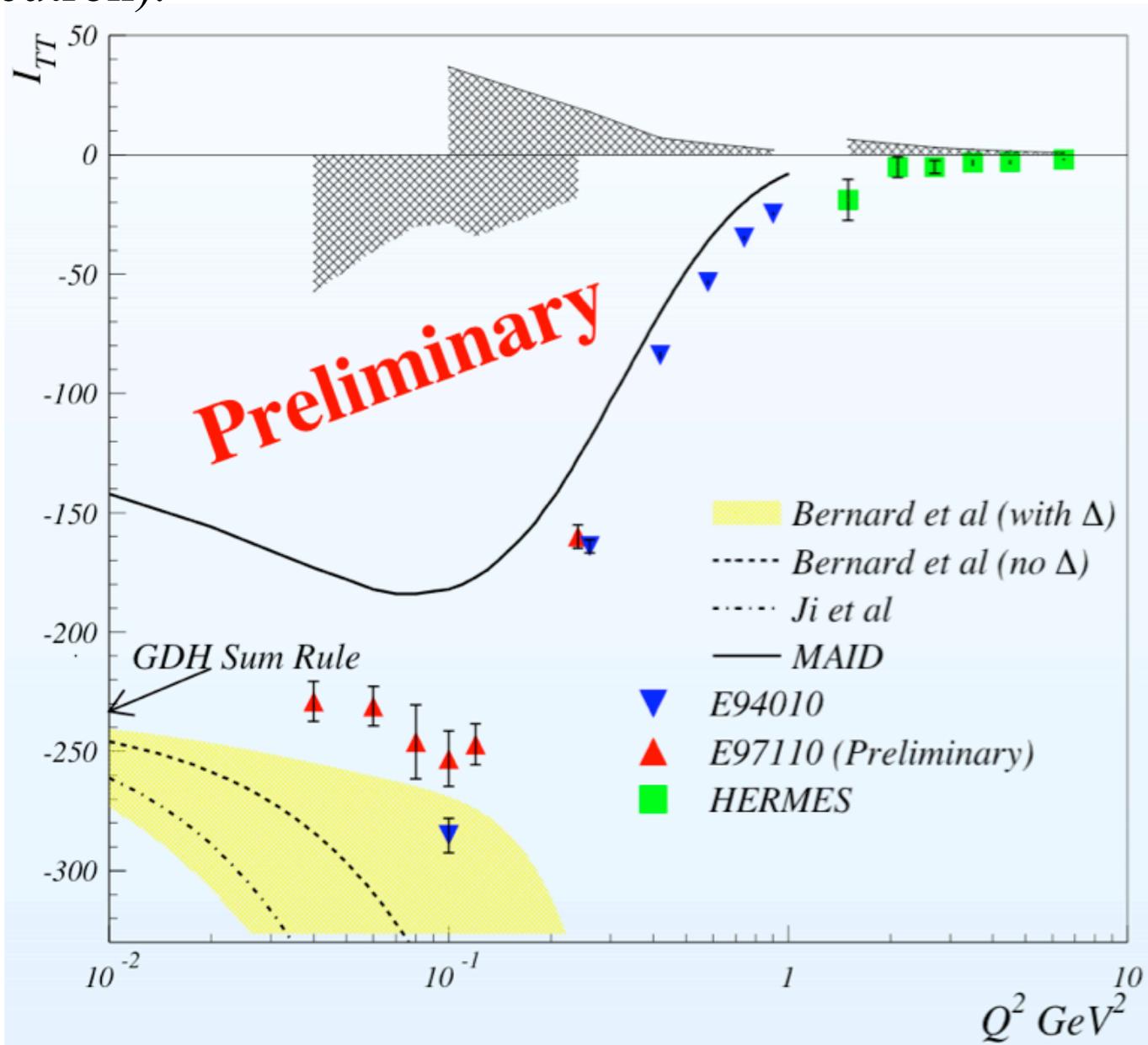


Remaining tasks:

- Finalized:
 - Radiative corrections;
 - Target polarimetry;
 - Acceptance study.
- Analyze the lowest Q^2 points.

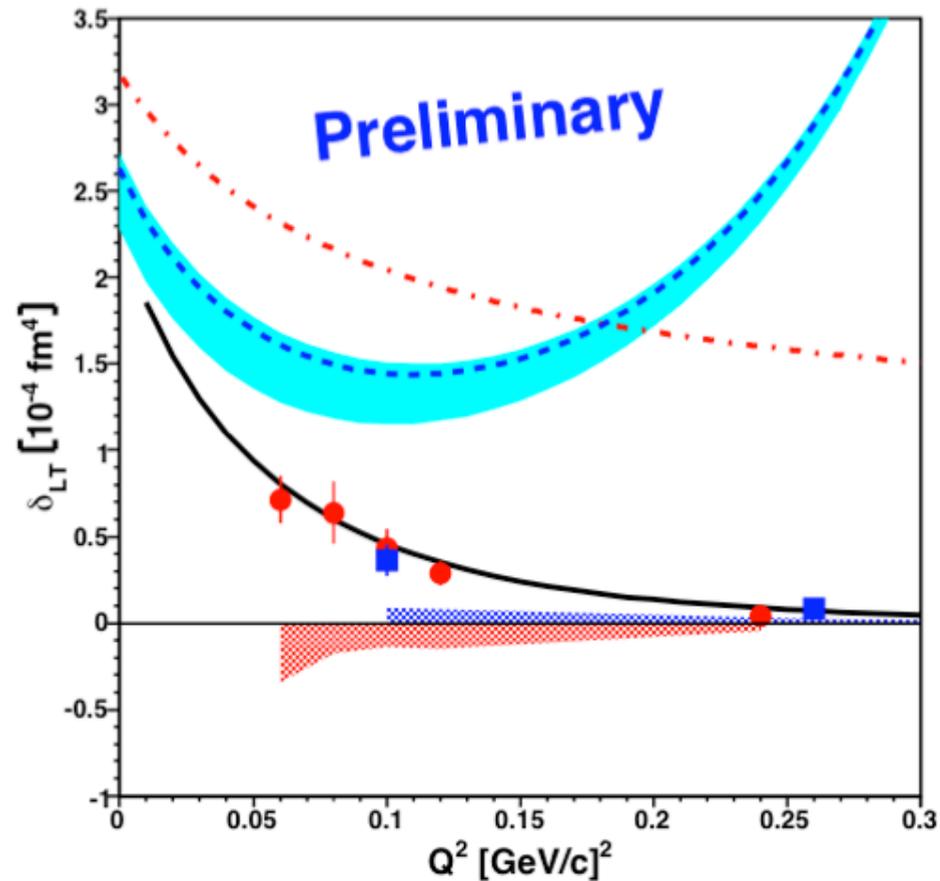
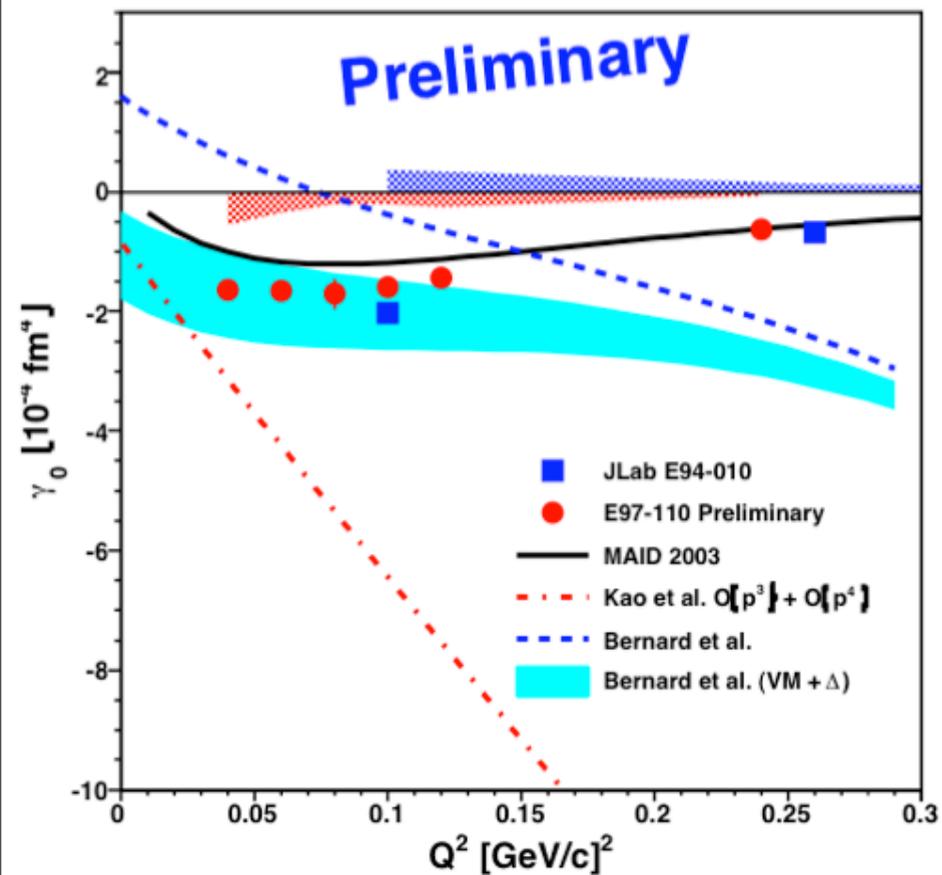
Preliminary results on GDH sum

Hall A (neutron):



Preliminary results on neutron spin polarizabilities

Hall A (neutron):



Hall B: Preliminary results on difference of polarized cross sections

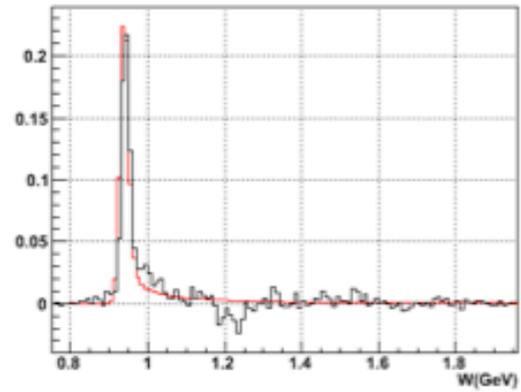
One step away from g_1 : $\sim g_1 \propto \sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}$.

Proton, 2.3 GeV

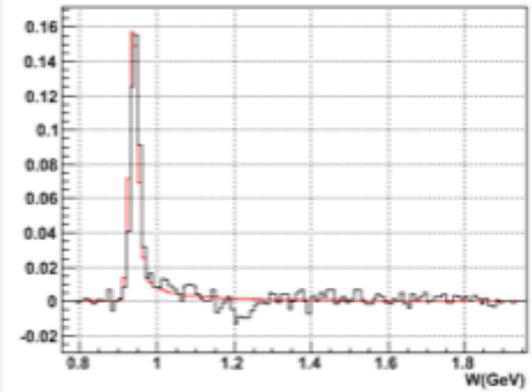
Data (black) normalized to elastic simulation (red)

μ_B

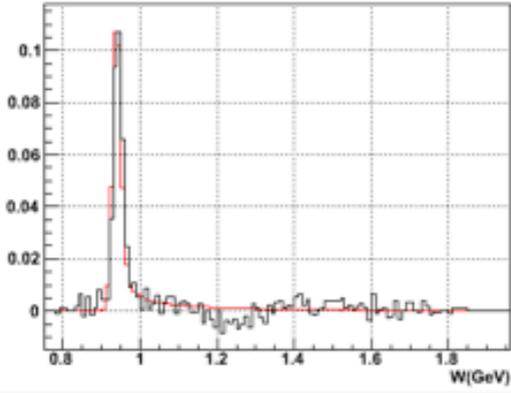
W, Simulated, 2.260GeV, $\langle Q^2 \rangle = 0.2050 \text{ GeV}^2$



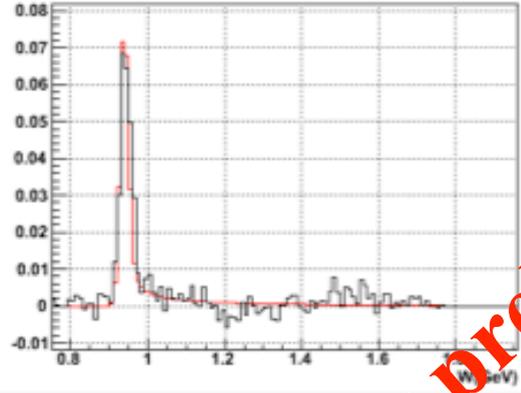
W, Simulated, 2.260GeV, $\langle Q^2 \rangle = 0.2440 \text{ GeV}^2$



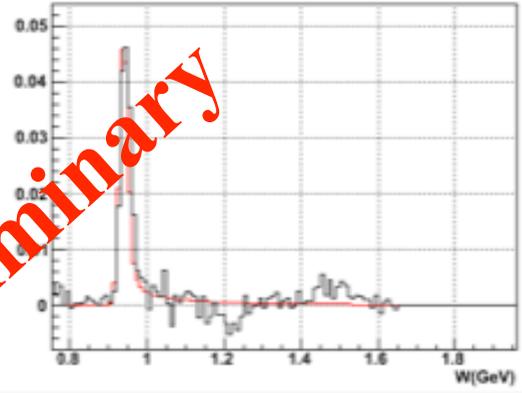
W, Simulated, 2.260GeV, $\langle Q^2 \rangle = 0.2920 \text{ GeV}^2$



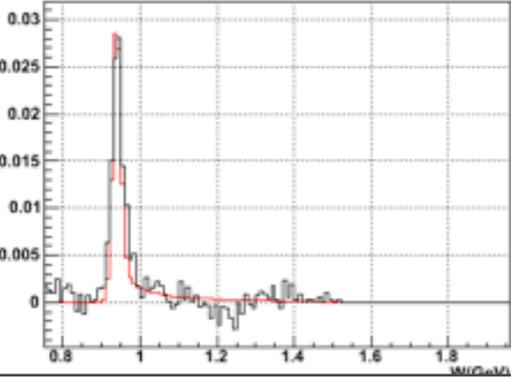
W, Simulated, 2.260GeV, $\langle Q^2 \rangle = 0.3480 \text{ GeV}^2$



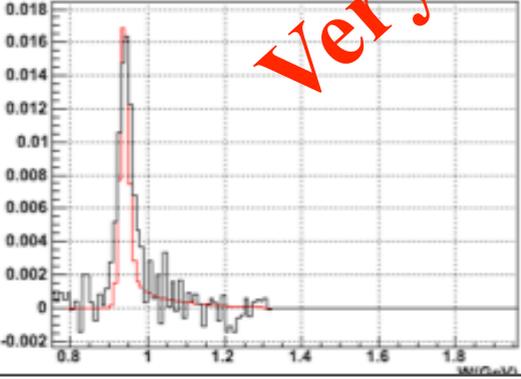
W, Simulated, 2.260GeV, $\langle Q^2 \rangle = 0.4160 \text{ GeV}^2$



W, Simulated, 2.260GeV, $\langle Q^2 \rangle = 0.4960 \text{ GeV}^2$



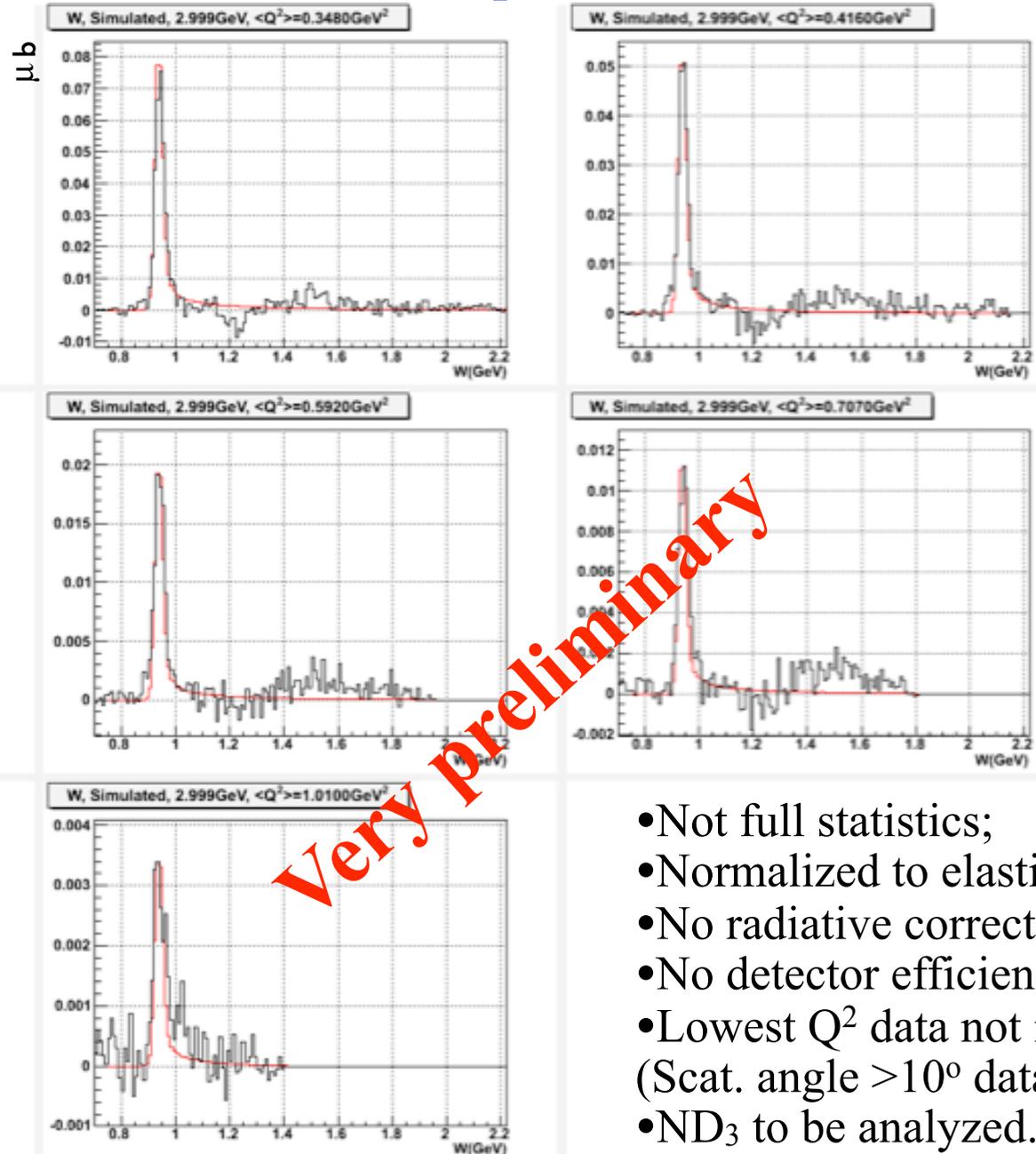
W, Simulated, 2.260GeV, $\langle Q^2 \rangle = 0.5920 \text{ GeV}^2$



Very preliminary

Hall B: Preliminary results on difference of polarized cross sections

Proton, 3.0 GeV



- Not full statistics;
- Normalized to elastic;
- No radiative corrections;
- No detector efficiency;
- Lowest Q^2 data not included (Scat. angle $> 10^\circ$ data);
- ND_3 to be analyzed.

GDH: "Study transition from hadronic to partonic description of strong force"

Examples of practical applications:

- Low Q^2 : extraction of χ_{pt} series coefficients;

GDH: "Study transition from hadronic to partonic description of strong force"

Examples of practical applications:

- Low Q^2 : extraction of χ_{pt} series coefficients;

Ex. Bjorken sum:

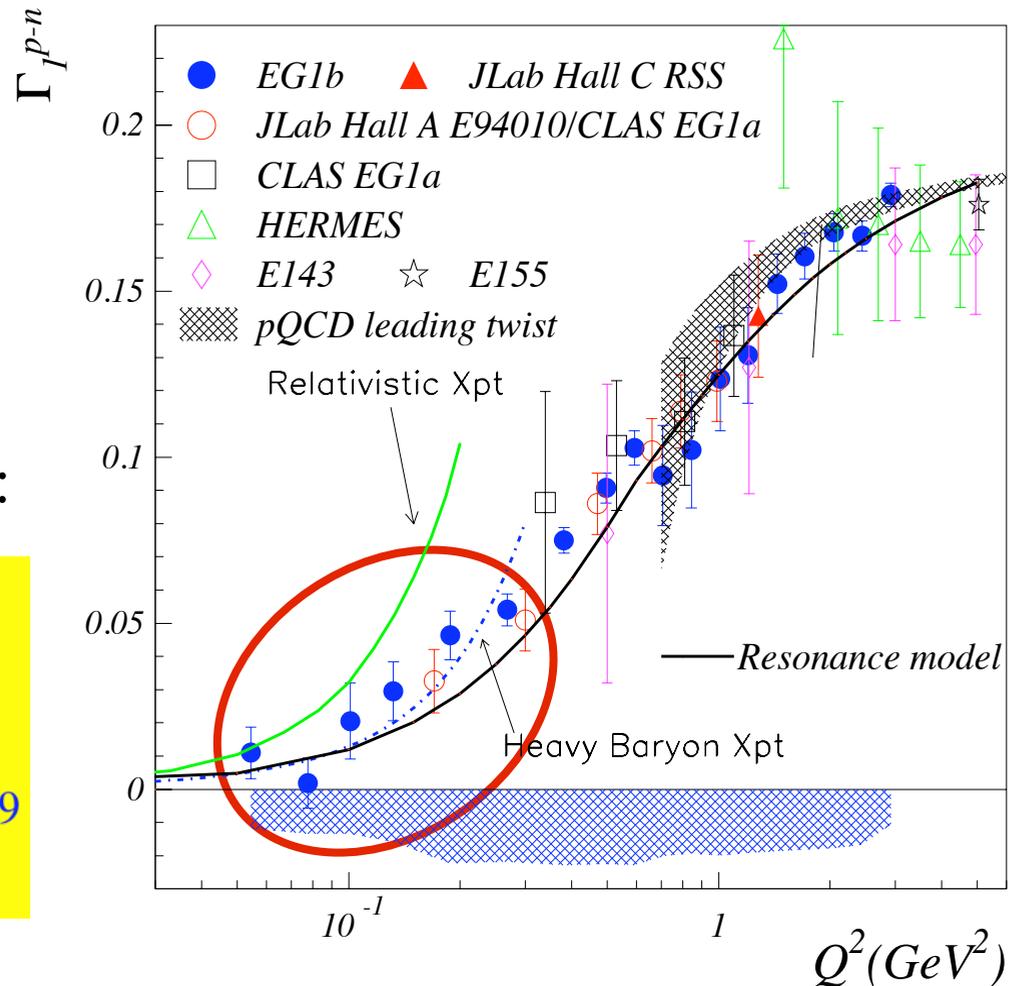
Low Q^2 fit:

$$\Gamma_1^{p-n} = \frac{K_n^2 - K_p^2}{8M^2} Q^2 + aQ^4 + bQ^6$$

$$a = 0.80 \pm 0.07 \pm 0.23, \quad b = -1.13 \pm 0.16 \pm 0.39$$

$$a^{\chi_{pT, Ji}} = 0.74, \quad a^{\chi_{pT, B.}} = 2.4$$

Q^2 upper limit: $0.4 \pm 0.1 \text{ GeV}^2$



GDH:

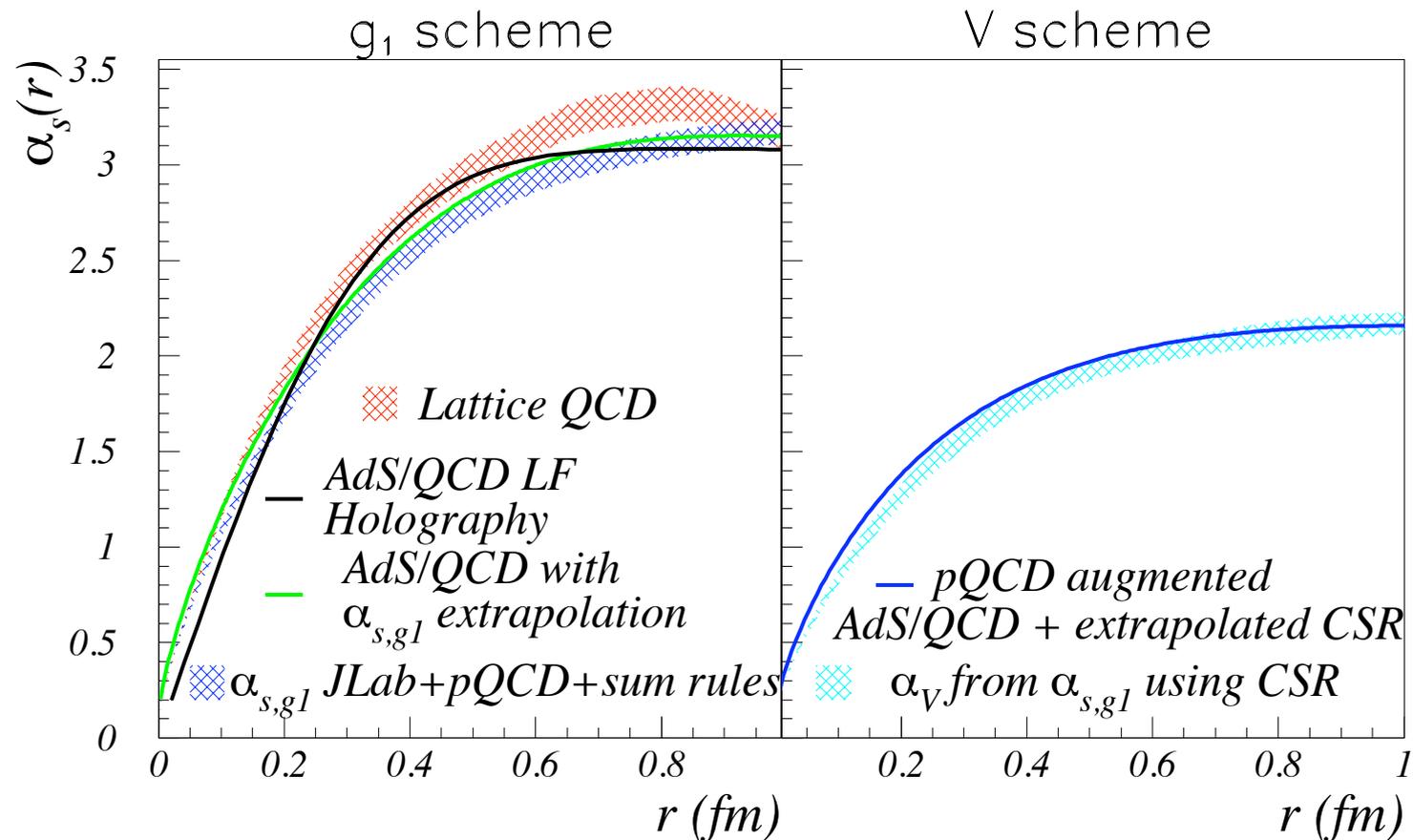
Examples of practical applications:

- Low Q^2 : extraction of χ_{pt} series coefficients;
- High Q^2 : extraction of pQCD series coefficients (higher twists);

GDH:

Examples of practical applications:

- Low Q^2 : extraction of χ_{pt} series coefficients;
- High Q^2 : extraction of pQCD series coefficients (higher twists);
- α_s , the strong coupling constant at large distances (>0.2 fm, i.e. low Q^2).



Summary/Perspective

Two experiments in Hall A & B fill the last gap (low Q^2).

Data are being analyzed:

- Hall A data ($0.05 < Q^2 < 0.25 \text{ GeV}^2$) will be available in upcoming months;
- Data at $0.02 < Q^2 < 0.05 \text{ GeV}^2$ requiring longer analysis;
- Hall B preliminary data on moments should be available this year.

One last missing piece: low Q^2 data on transversally polarized proton. Goal of the next Hall A experiment E08-027 (starts Nov. 2011.)

Low Q^2 data are of high precision. Higher precision data at intermediate *and large* Q^2 desirable.

- Hall B EG1b (intermediate Q^2) and EG1dvcs (higher Q^2);
- Hall C Sane;
- 12 GeV, important for the large Q^2 part of this program:
 - Convergence of sum rules;
 - minimize low-x issue;
 - increase Q^2 range;
 - higher precision extraction of higher twists.

Polarized HD target in Hall B: precision check of GDH sum rule ($Q^2=0$) and may provide a transverse target for electroproduction ($Q^2>0$).

GDH: "Study transition from hadronic to partonic description of strong force"

Examples of practical applications:

- Low Q^2 : extraction of χ_{pt} series coefficients;

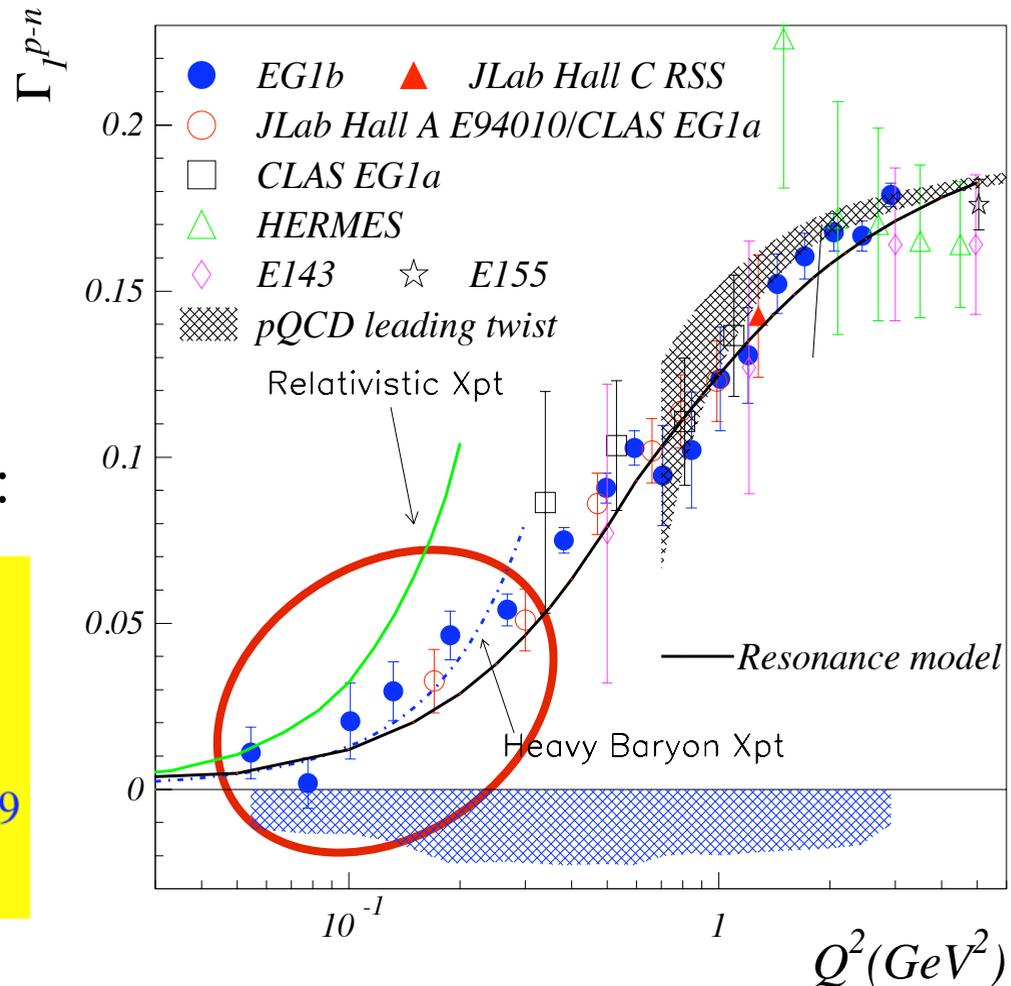
Ex. Bjorken sum:

Low Q^2 fit:

$$\Gamma_1^{p-n} = \frac{K_n^2 - K_p^2}{8M^2} Q^2 + aQ^4 + bQ^6$$

$$a = 0.80 \pm 0.07 \pm 0.23, \quad b = -1.13 \pm 0.16 \pm 0.39$$

$$a^{\chi_{pT, Ji}} = 0.74, \quad a^{\chi_{pT, B.}} = 2.4$$

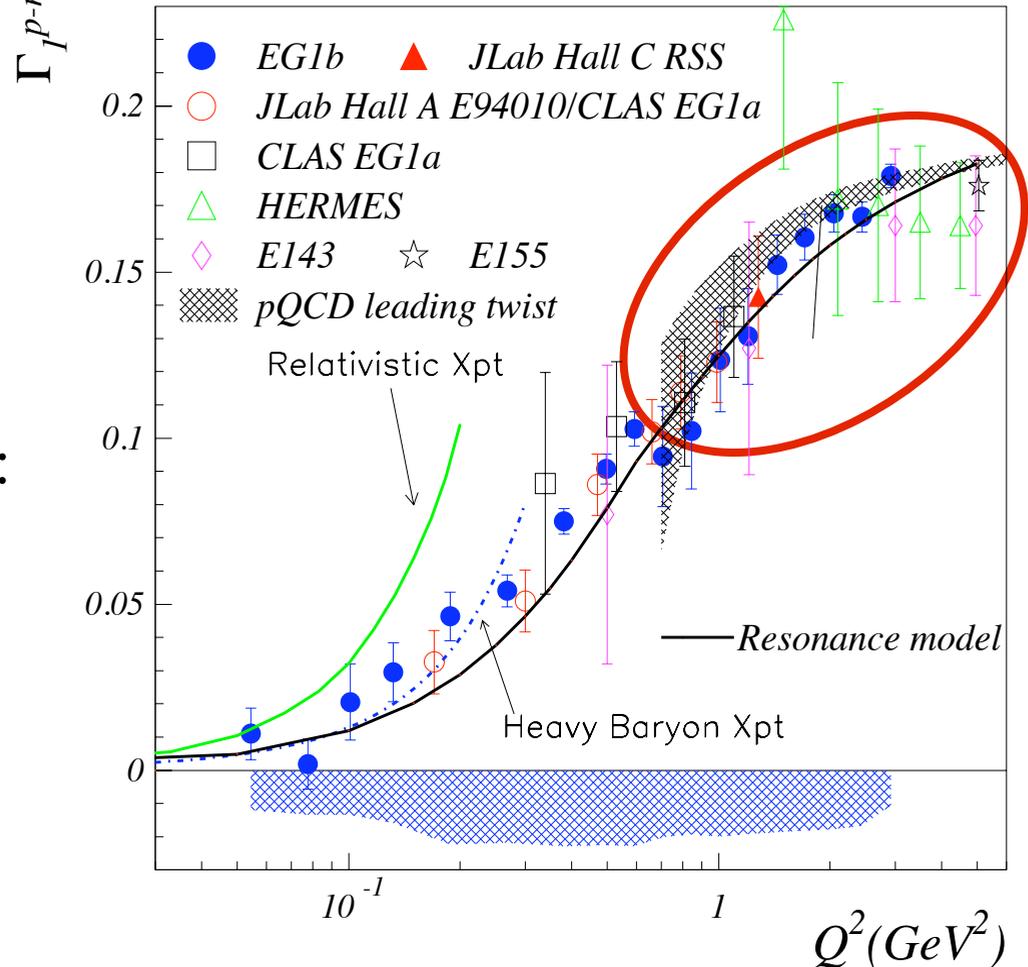


GDH: "Study transition from hadronic to partonic description of strong force"

Examples of practical applications:

- Low Q^2 : extraction of χ_{pt} series coefficients;
- High Q^2 : extraction of pQCD series coefficients;

Ex. Bjorken sum:



GDH: "Study transition from hadronic to partonic description of strong force"

Examples of practical applications:

- Low Q^2 : extraction of χ_{pt} series coefficients;
- High Q^2 : extraction of pQCD series coefficients;

$$\int g_1 dx = \sum_{\text{twist}=2,4,\dots}^{\mu} \frac{\mu^{\text{twist}}}{Q^{\text{twist}-2}}$$

Ex. Bjorken sum:
$$\int g_1^{p-n} dx = \underbrace{\frac{1}{6} g_a \left(1 + \frac{\alpha_s(\ln(Q^2))}{\pi} + \dots \right)}_{\text{Leading twist (twist 2)}} + \underbrace{\frac{\mu^4}{Q^2} + \frac{\mu^6}{Q^4} + \dots}_{\text{Higher twists}}$$

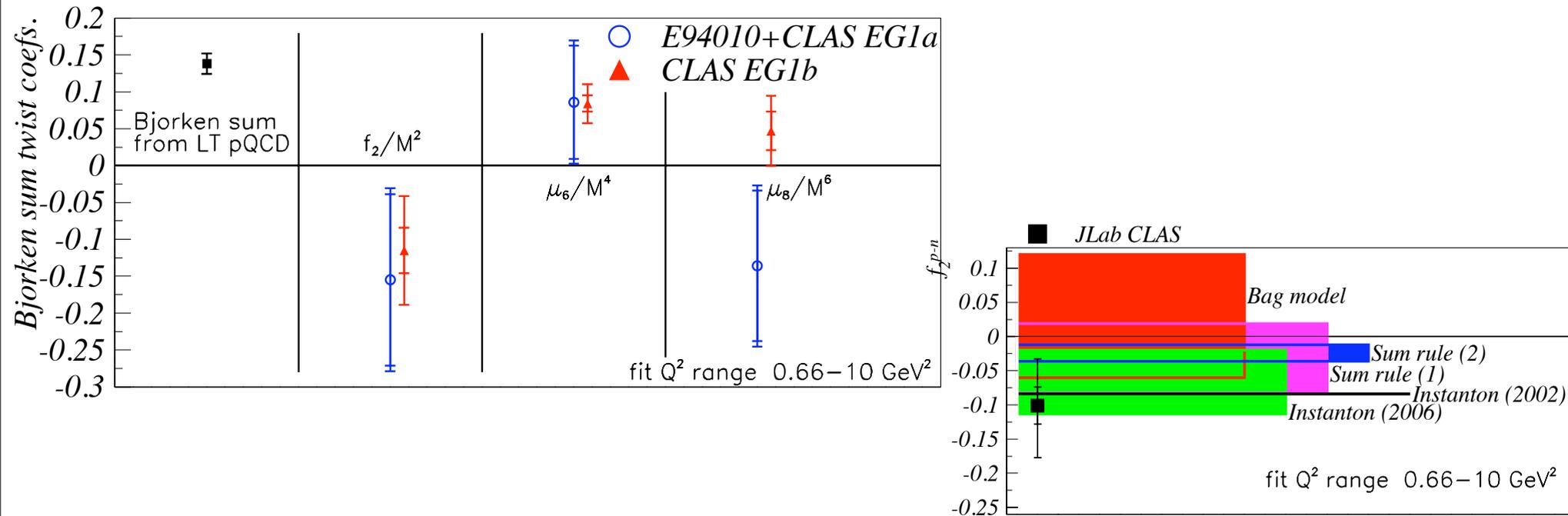
$$\frac{\mu^4}{Q^2} = \frac{M^2}{9} (a_2 + 4d_2 + 2f_2)$$

Leading twist (known from high energy data) → a_2
Twist 3 (known from high energy data) → d_2
Twist 4 → f_2

GDH: "Study transition from hadronic to partonic description of strong force"

Examples of practical applications:

- Low Q^2 : extraction of χ_{pt} series coefficients;
- High Q^2 : extraction of pQCD series coefficients;



f_2 large (similar to leading twist at $Q^2 = 1 \text{ GeV}^2$) in accordance to intuition.

μ_6/M^4 similar size as f_2 but opposite sign.

Overall, higher twist contribution small at $Q^2 = 1 \text{ GeV}^2$.