

# Radiative Corrections in SIDIS

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# Outline

- 1 Introduction
- 2 HAPRAD 2
- 3 Nuclear Effects
- 4 Proton Production
- 5 Conclusions
- 6 Backup

# Why do we need Radiative Corrections

$$e(k_1) + A(P) \rightarrow e'(k_2) + h(p_h) + X(p_X)$$

- Born cross section:

$$\sigma_B = \frac{2E_h d\sigma}{d^3p_h dx dy} = \frac{2\pi\alpha^2 My}{Q^4} L_{\mu\nu} W_{\mu\nu}$$

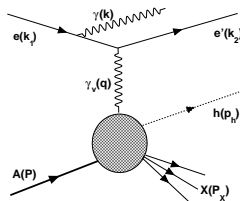
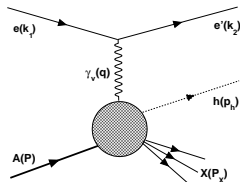
- Observed cross section:

$$\sigma_{obs} \sim \sigma_B + \alpha \int \sigma_\gamma \frac{dk}{k_0}$$

$$Q_r^2 \simeq Q^2 + 2k(k_1 - k_2) \sim Q^2 \left[ 1 - \frac{E_\gamma}{E_1} \right]$$

$$W_{\mu\nu} \sim F(Q_r^2) \sim 1/Q_r^6$$

- Radiative tail for open kinematics reactions



## History of SIDIS RC

- First explicitly calculated for SIDIS in (code POLRAD 2, with SIRAD option):

A.V. Soroko, N.M. Shumeiko, Sov.J.Nucl.Phys. 49, 838 (1989), Yad.Fiz. 49, 1348 (1989); Sov.J.Nucl.Phys. 53, 628 (1991), Yad.Fiz. 53, 1015 (1991); I. Akushevich, A. Ilyichev, N. Shumeiko, A. Soroko and A. Tolkachev, Comp.Phys.Com.104, 201 (1997)

for polarized and unpolarized cross sections integrated in  $p_T$  and  $\phi$ , including only diagonal SFs.

- Later extended in (HAPRAD):

I. Akushevich, A. Soroko, N. Shumeiko, EPJ C10, 681 (1999)

for fully differential unpolarized cross section.

- Further extended in (HAPRAD 2):

I. Akushevich, A. Ilyichev and M. Osipenko, Phys.Lett. B672, 35 (2009)

by adding the exclusive radiative tail and modeling explicitly  $\langle \cos\phi \rangle$  and  $\langle \cos 2\phi \rangle$  terms.

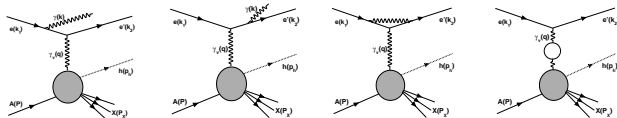
# State of Art

- unpolarized cross section at order  $\alpha^3$ :

$$\sigma_{RC} = \sigma_B e^{\delta_{soft,inf}} (1 + \delta_{VR,fin} + \delta_{vac}) + \sigma_{si.tail} + \sigma_{ex.tail}$$

- modeling of SIDIS and exclusive cross sections in a broader kinematic range:

$$\sigma_{si,ex.tail} \sim \alpha \int \sigma_{B,ex} \frac{dk}{k_0}$$

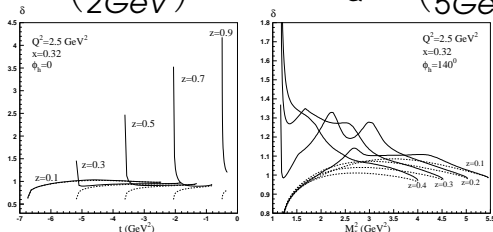


## Exclusive tail

$$\frac{d\sigma_{ex}}{dx dy dz dt d\phi} = \frac{1}{(4\pi)^7} \frac{sy^2}{\left(1 - \frac{m_p^2}{E_1^2}\right)(y^2 + \frac{Q^2}{E_1^2})} \int_0^{2\pi} \frac{2(kP)|M|^2}{1 + \tau - \frac{kp_h}{kP}} d\tau d\phi_k$$

where  $\tau = \frac{kq}{kP}$  and  $\sigma_{ex}^B$  MAID modified at large  $W$  and large  $Q^2$  by Cornell parametrization<sup>1</sup>

$$C_W = \left(\frac{W}{2\text{GeV}}\right)^{-0.002\theta_{CM}^2}, \quad C_{Q^2} = \left(\frac{Q^2}{5\text{GeV}^2}\right)^{-1.15} \quad (1)$$



<sup>1</sup>A. Browman et al., Phys. Rev. Lett. 35 (1975) 1313.

## Modeling SIDIS

$$\frac{d^5\sigma}{dx dy dz dt d\phi} = \frac{2\pi\alpha^2 s z}{Q^4} \sqrt{\kappa} \zeta \left[ (\epsilon \mathcal{H}_1 + \mathcal{H}_2)(1 + B \cos \phi + C \cos 2\phi) \right]$$

- LO pQCD-like parametrization for  $\phi$ -independent part (PDFlib and Kretzer FFs), Gaussian  $p_T$ -model:

$$\mathcal{H}_2 = \sqrt{1 - \frac{(M + m_h)^2}{M_X^2}} \frac{e^{-\frac{p_T^2 \vee p_T^2 + 2p_{\parallel}^2}{\langle p_T^2 \rangle}}}{\pi \langle p_T^2 \rangle} \sum_i e_i^2 x q_i(x) D_i^h(z)$$

- longitudinal to transverse ratio  $R$  is taken<sup>1</sup>  $R = 0.12$ :

$$\mathcal{H}_1 = (1 + \gamma^2) \frac{\mathcal{H}_2}{2x(1 + R)}$$

<sup>1</sup>C.J. Bebek et al., *Phys. Rev. Lett.* **38**, 1051 (1977).

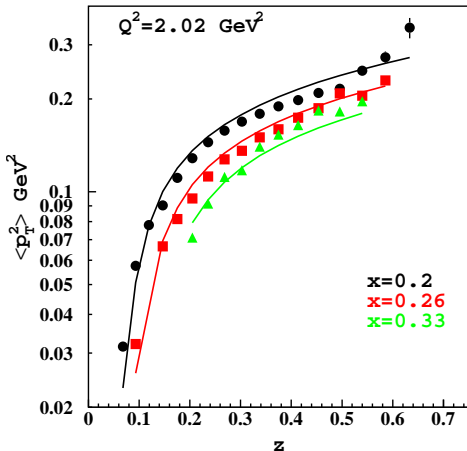
## Modeling $p_T$ -distribution

- mean  $p_T^2$  linear in  $W$ , instead of linear in  $s$  as in<sup>1</sup> and  $\sqrt{z}$  instead of  $a + bz^2$  from<sup>2</sup>:

$$\langle p_T^2 \rangle_0 = \frac{0.12W\sqrt{z}}{1 + (3.2\frac{m_h}{\nu z})^4}$$

- low- $z$  phase space shrinkage:

$$\langle p_T^2 \rangle = \frac{\langle p_T^2 \rangle_0}{1 + \frac{\langle p_T^2 \rangle_0}{p_h^2}}$$



<sup>1</sup>P. Schweitzer et al., Phys.Rev. D81, 094019 (2010)

<sup>2</sup>M. Anselmino et al., Phys.Rev. D71, 074006 (2005)

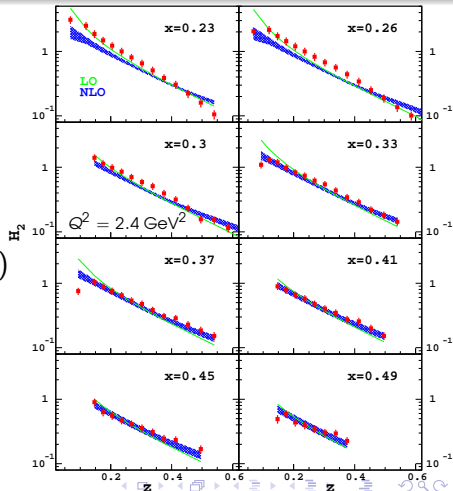


# Modeling xyz-distributions

- except for very low- $z$  LO calculations reproduce data fairly well.

$$H_2 = \int \mathcal{H}_2 dp_T^2 = \sum_i e_i^2 x q_i(x) D_i^h(z)$$

- CTEQ5 PDF and Kretzer FF are shown.



## Modeling $\phi$ -distribution

- Cahn effect<sup>1</sup>:

$$\langle \cos \phi \rangle = -\sqrt{\frac{p_T^2}{Q^2} \frac{2 \times 0.25z}{0.2 + 0.25z^2} \frac{\sqrt{1-y}(2-y)}{1 + (1-y)^2}}$$

$$\langle \cos 2\phi \rangle = \frac{p_T^2}{Q^2} \frac{2 \times 0.25^2 z^2}{(0.2 + 0.25z^2)^2} \frac{1-y}{1 + (1-y)^2}$$

- Berger effect<sup>1</sup>:

$$\langle \cos \phi \rangle = \sqrt{\frac{p_T^2}{Q^2} \frac{z l_1 (l_2 - \frac{p_T^2}{Q^2} l_1)}{l_2^2 + 4 \frac{p_T^2}{Q^2} z^2 l_1^2 + \frac{p_T^4}{Q^4} l_1^2 + \epsilon \frac{l_2^2 + \frac{p_T^4}{Q^4} l_1^2}{2x}} \frac{2-y}{\sqrt{1-y}}}$$

$$\langle \cos 2\phi \rangle = \frac{p_T^2}{Q^2} \frac{-l_1 l_2}{l_2^2 + 4 \frac{p_T^2}{Q^2} z^2 l_1^2 + \frac{p_T^4}{Q^4} l_1^2 + \epsilon \frac{l_2^2 + \frac{p_T^4}{Q^4} l_1^2}{2x}}$$

<sup>1</sup>R.N. Cahn, Phys.Rev. D40, 3107 (1989)

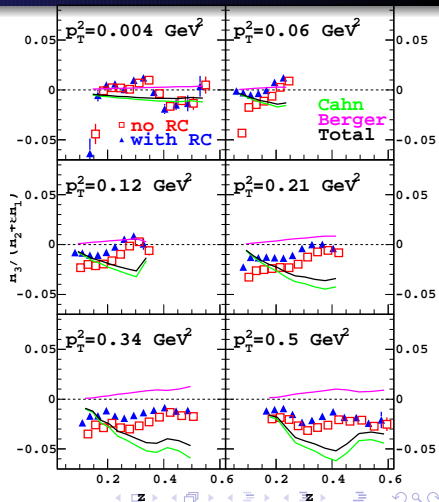
<sup>2</sup>E. Berger, Z.Phys. C4, 289 (1980)

# Effect on $\langle \cos\phi \rangle$

In JLab kinematics<sup>1</sup>:

- positive RC,
- decreasing with  $z$ ,
- increasing with  $p_T^2$ ,
- ranges from 0.003 up to 0.015 with average of 0.01.

<sup>1</sup>M. Osipenko et al., Phys.Rev. D80 (2009) 032004

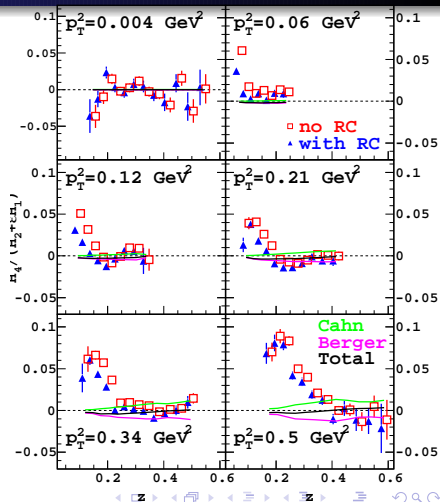


# Effect on $\langle \cos^2\phi \rangle$

In JLab kinematics<sup>1</sup>:

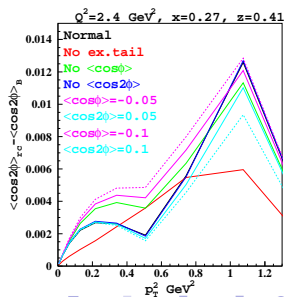
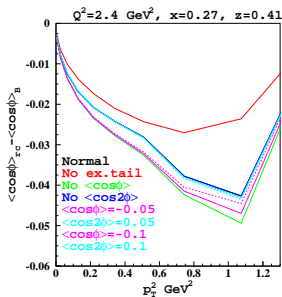
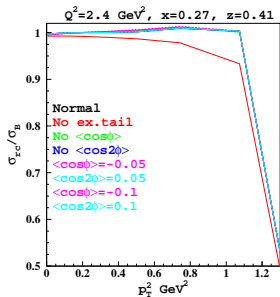
- negative RC,
- decreasing with  $z$ ,
- increasing with  $p_T^2$ ,
- ranges from -0.002 up to -0.02 with average of -0.007.

<sup>1</sup>M. Osipenko et al., Phys.Rev. D80 (2009) 032004



# Importance of different terms

- exclusive tail is important for large  $p_T^2$  or small  $M_X$ ,
- exclusive tail is important for azimuthal moments extraction,
- models of  $\langle \cos\phi \rangle$  and  $\langle \cos 2\phi \rangle$  terms are important only for  $\langle \cos 2\phi \rangle$  evaluation.

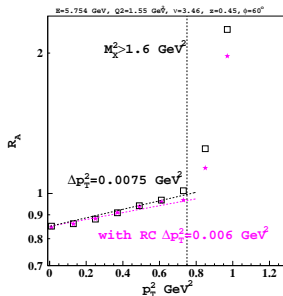
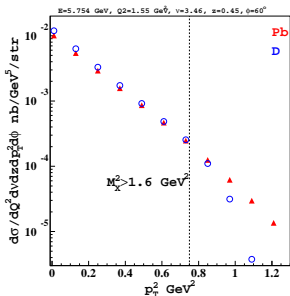


# Fermi Motion

- Non-relativistic Fermi gas for few nuclei (D, C, Fe, Pb):

$$\sigma^{nucl} = \int f(|\vec{P}_N|) \sigma^N(\vec{P}_N) d\vec{P}_N$$

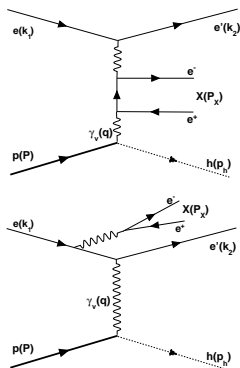
- Restricting missing mass  $M_X^2 < 2 \text{ GeV}^2$  allows to avoid significant effects.



## $\alpha^4$ Contribution

$$e(k_1) + p(P) \rightarrow e'(k_2) + p'(p_h) + X(p_X)$$

- $\alpha^2$  suppression is compensated by the nucleon elastic form-factor growth  $\sim 1/Q_r^6$ ,
- appears as a peak in  $\phi$  distribution at 180 degrees,

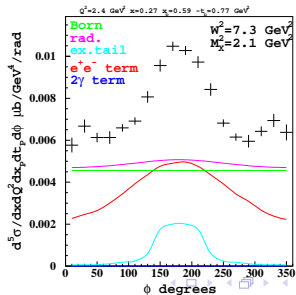
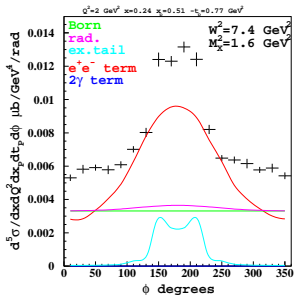


## $\alpha^4$ Contribution cont.

- comparison with CLAS preliminary data:

$$x_p = \frac{q(P - p_h)}{qP}, \quad t_p = (P - p_h)^2$$

- Contribution decreases with missing mass  $M_X$ , but even at 1-3 GeV<sup>2</sup> it is significant.





# Summary

## Unpolarized SIDIS:

- RC for fully differential cross section, including exclusive tail (HAPRAD 2),
- $\alpha^4$  terms for proton SIDIS,
- no radiation from the detected hadron,
- no external RC.

## Polarized SIDIS:

- RC for double spin asymmetry integrated in  $\phi$  and  $p_T^2$  (POLRAD 2),
- only diagonal SFs.

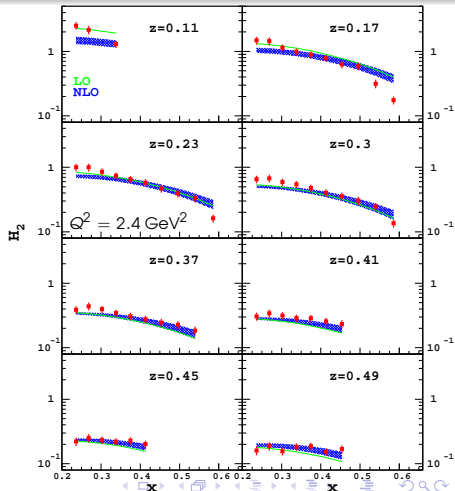
## Reasonably achievable:

- RC for SSAs,
- soft photon terms for radiation from the detected hadron.

# Backup Slides

# Modeling xyz-distributions

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