Production of Resonances Using CLAS at Jefferson Lab

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November 6, 2014

Resonance Workshop at Catania

Outline

I. Introduction to Jefferson Lab and CLAS

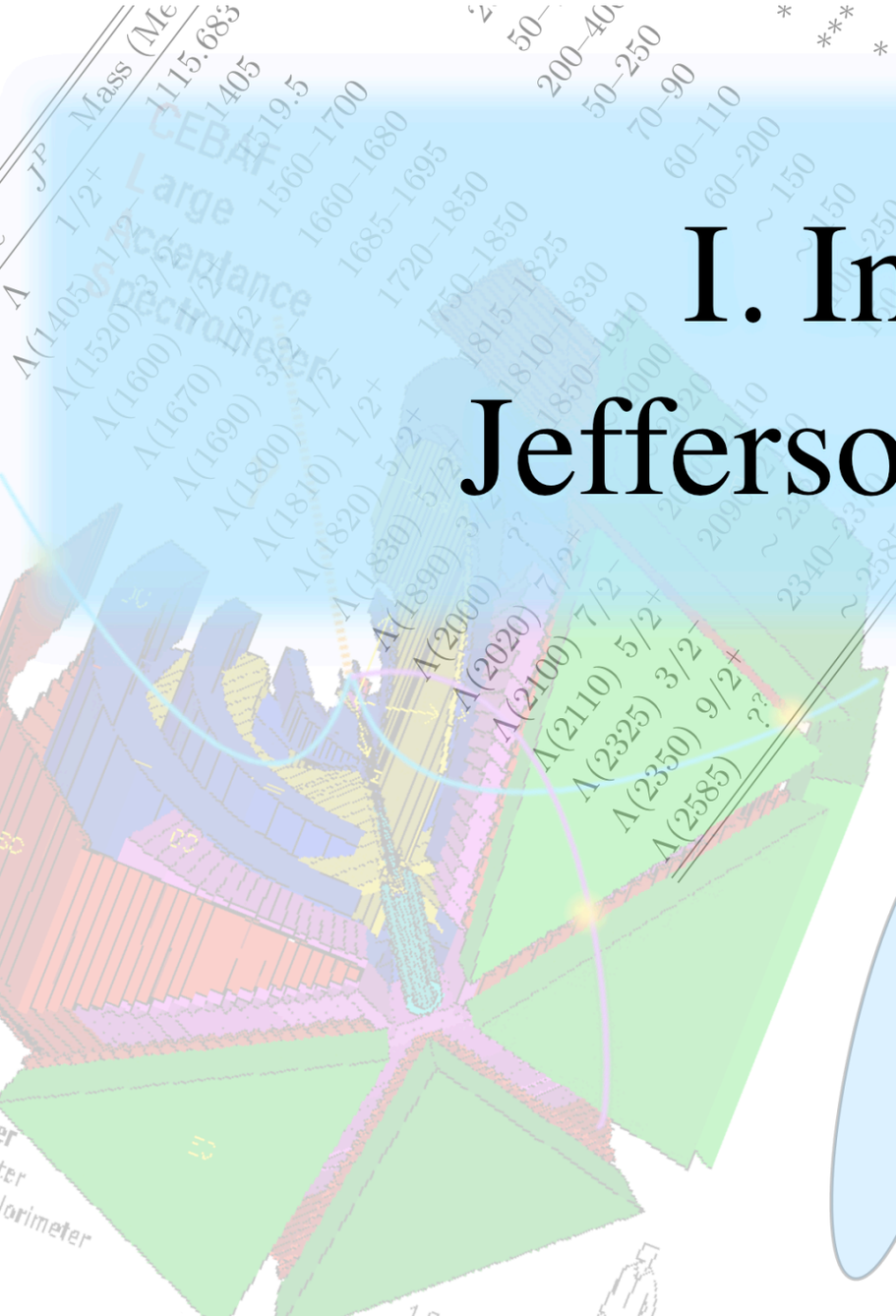
II. Analyses of Resonances at CLAS

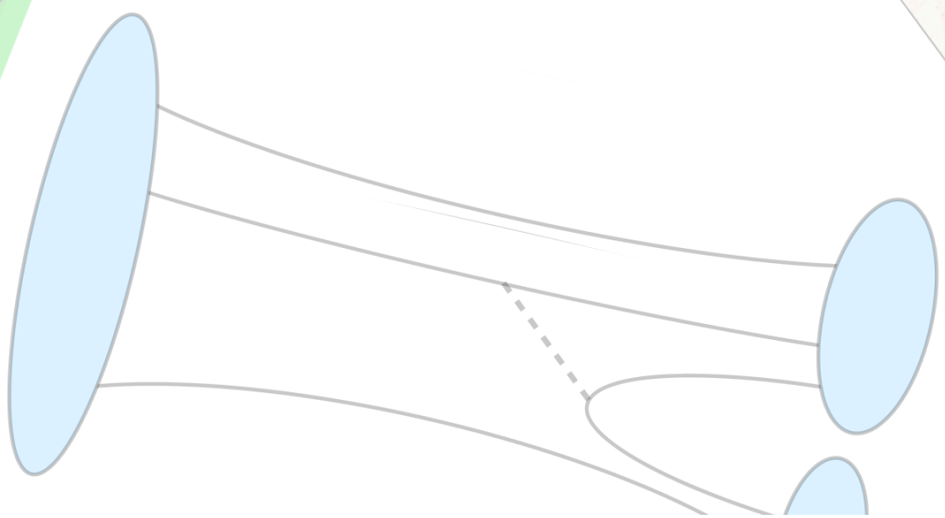
(1) $\eta, \eta', \omega, \phi, K^+ \Lambda, K^+ \Sigma^0$

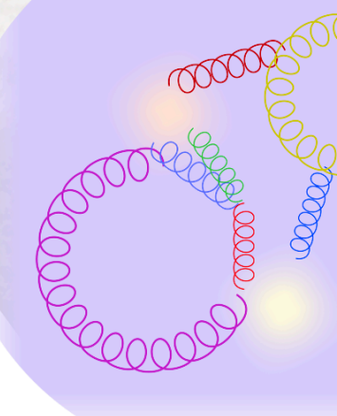
(2) hyperon resonances ($\Sigma^0(1385)$, $\Lambda(1405)$, $\Lambda(1520)$)

III. Prospects and Conclusion

I. Introduction to Jefferson Lab and CLAS

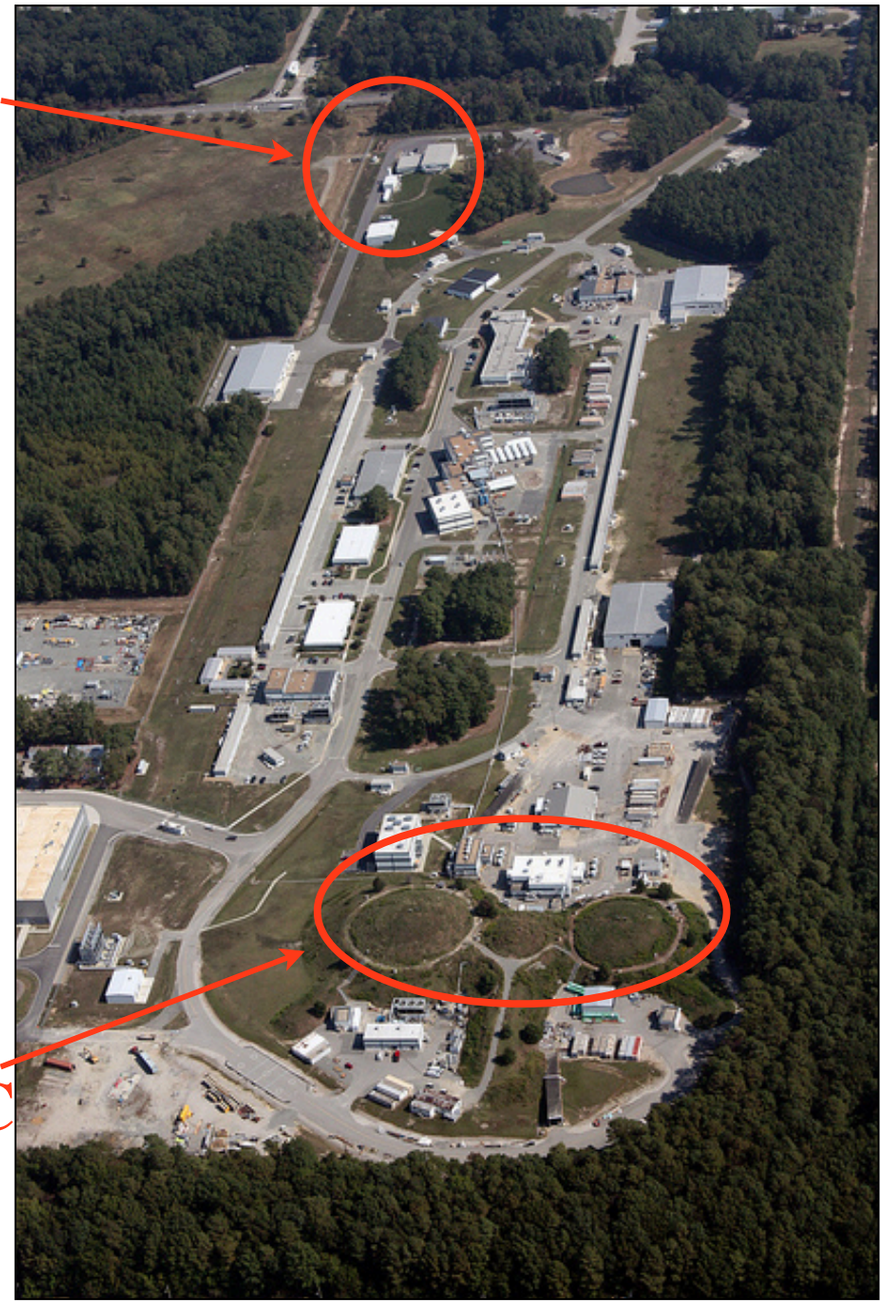






Jefferson Lab

- Located in Newport News, VA
- CEBAF accelerator provides electron beam every 2 ns
- $E_{e^-} < 6$ GeV until now, upgrading to 12 GeV
- New Hall D will house GlueX experiment for photoproduction of hadron resonances



https://www.jlab.org/

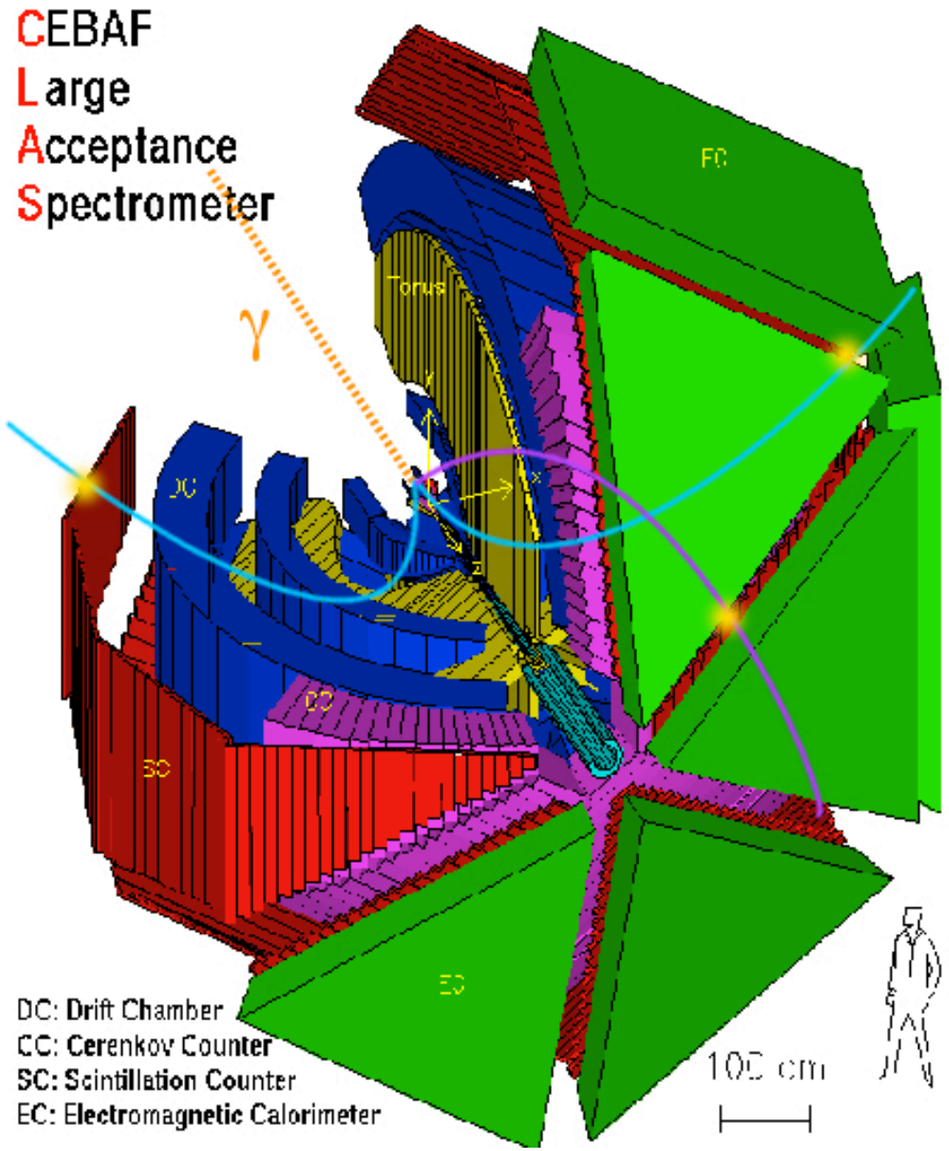
Resonances @JLab

- Electron beam of < 6 GeV ideal for production of hadron resonances
- Production of mesons:
  - π, η, η', K — *pseudoscalar*
  - ω, ϕ, K^* — *vector*
  - $f_0(980)$, ... — *scalar*
- Production of baryons:
  - $N(1440)$ — $S = 0$
  - $K^+ \Lambda, K^+ \Sigma^0, K^0 \Sigma^+, K^+ \Sigma^0(1385), K^+ \Lambda(1405), K^+ \Lambda(1520)$ — $S = -1$
  - $K^+K^+\Xi^-, K^+K^+\Xi^-(1530)$ — $S = -2$

Refs: http://www.jlab.org/Hall-B/shifts/index.php?display=utils&task=publications

The CLAS Detector

- CLAS was in Hall B of Jefferson Lab
- Took data for 14 years
- Photon tagging system used for photoproduction
- 3-layer drift chamber with $\delta p/p \sim 0.5\%$
- Liquid H_2, D_2, nuclear targets
- Start Counter around target
- Scintillator TOF paddles for PID

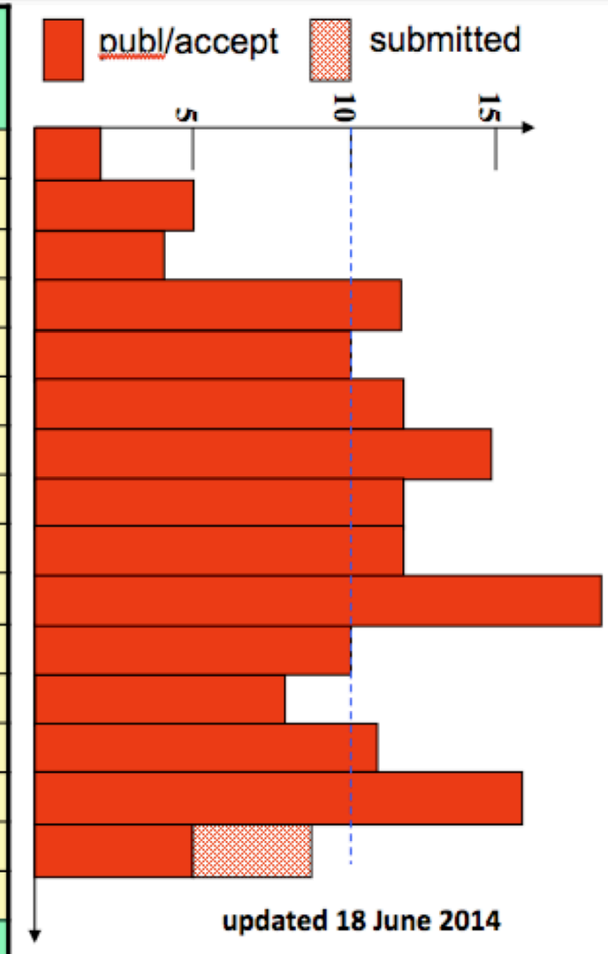


CLAS Publications

- Experiments on hadron spectroscopy, nuclear structure functions, nuclear processes

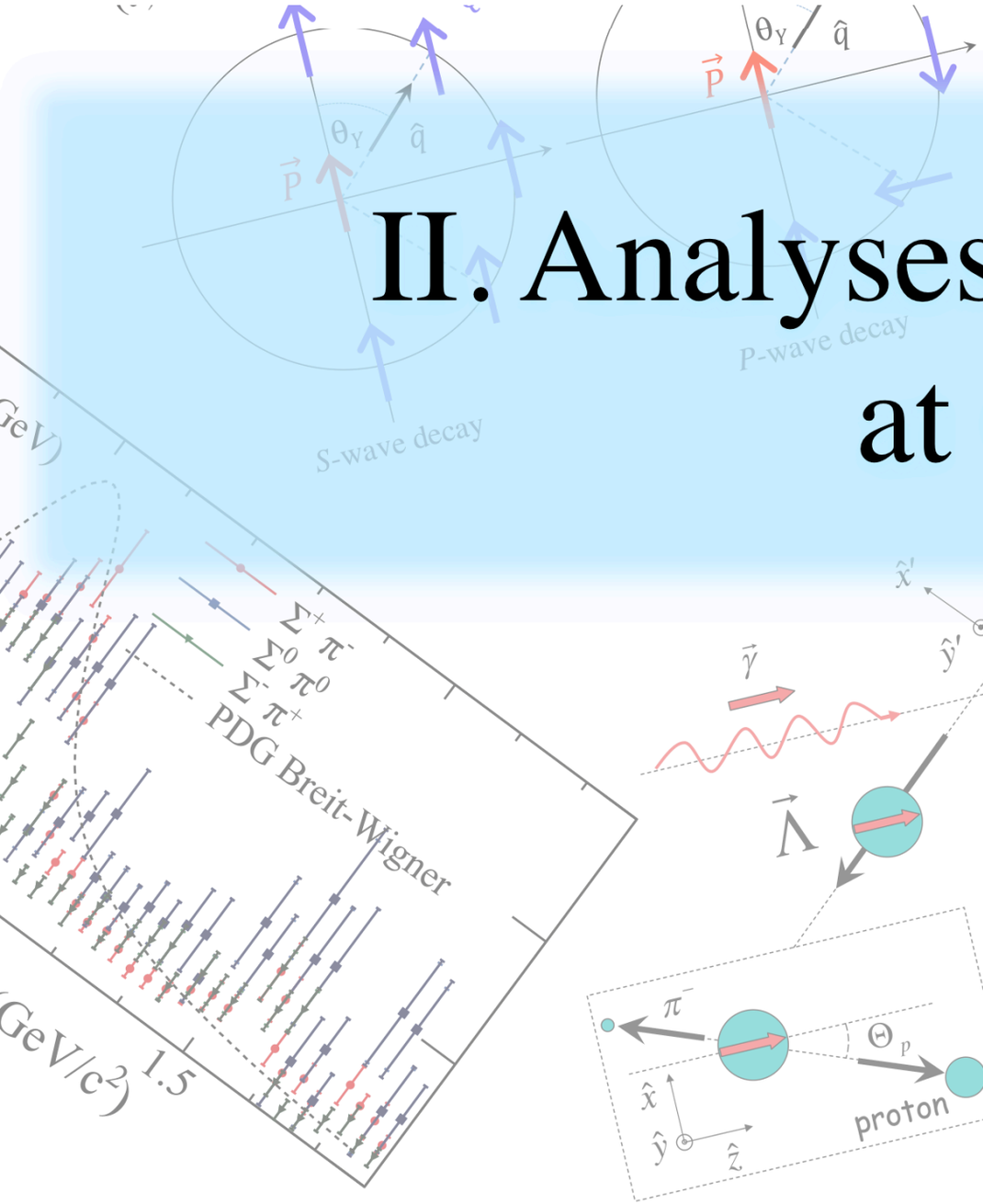
Hall B Exp. Physics Publications (refereed Journals)

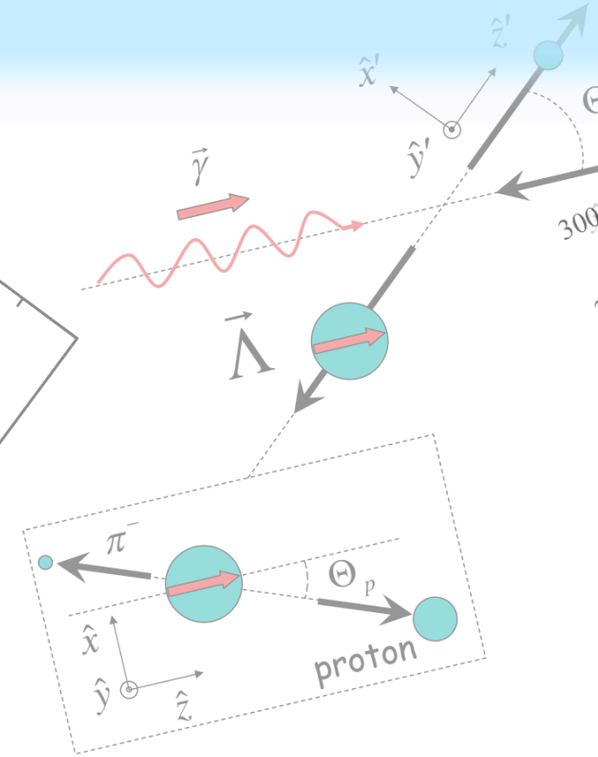
	Hadron Spectroscopy & Structure	Hard Processes & Str. Functions	Nuclear Processes	ALL
2000	-	1	1	2
2001	2	3	-	5
2002	3	-	1	4
2003	7	4	1	12
2004	3	3	4	10
2005	7	3	2	12
2006	8	4	3	15
2007	7	2	3	12
2008	4	6	2	12
2009	8	7	4	19
2010	4	2	4	10
2011	3	1	4	8
2012	6	3	2	11
2013	8	6	2	16
2014	3	2		5
SUM	71	47	33	151

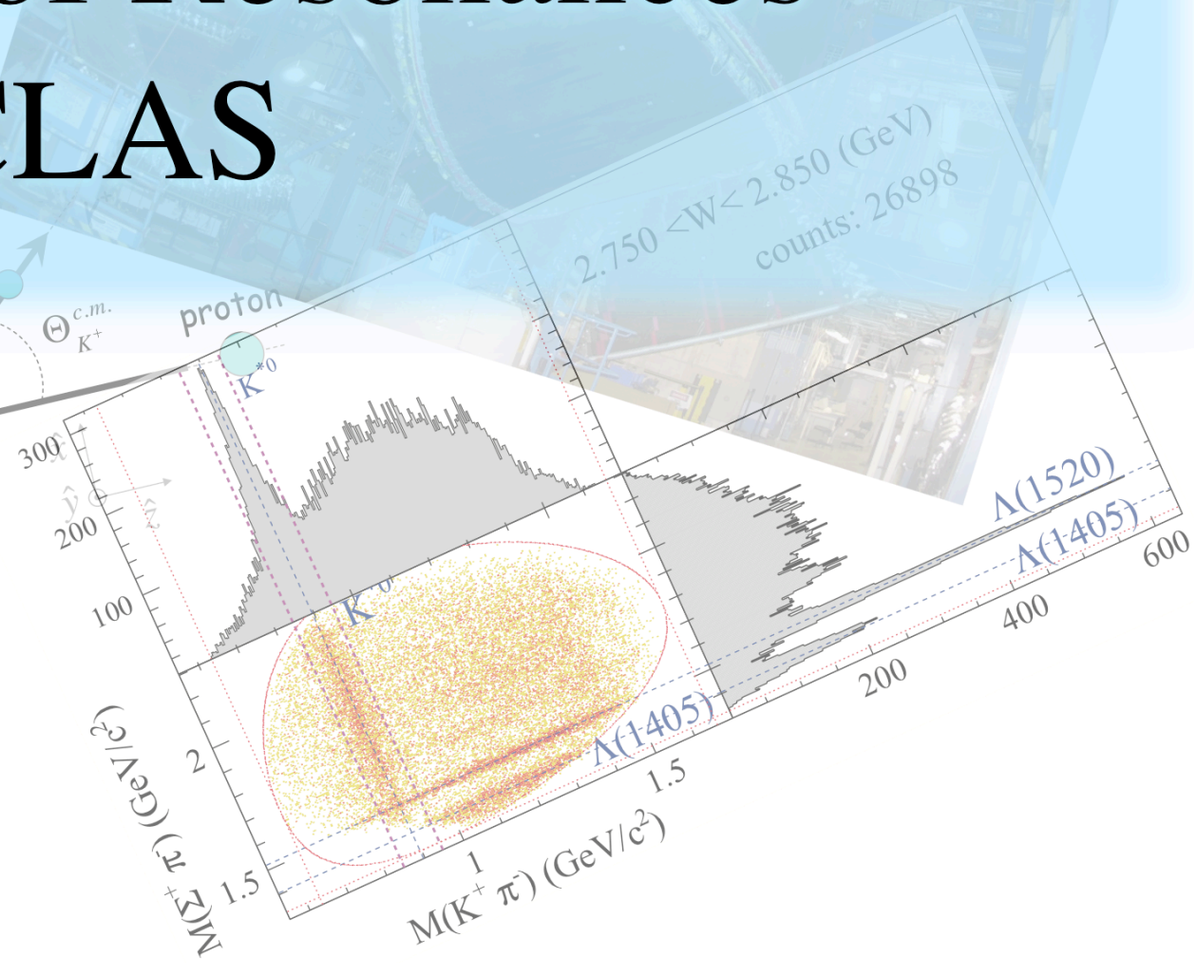


from http://www.jlab.org/Hall-B/pubs-web/

II. Analyses of Resonances at CLAS



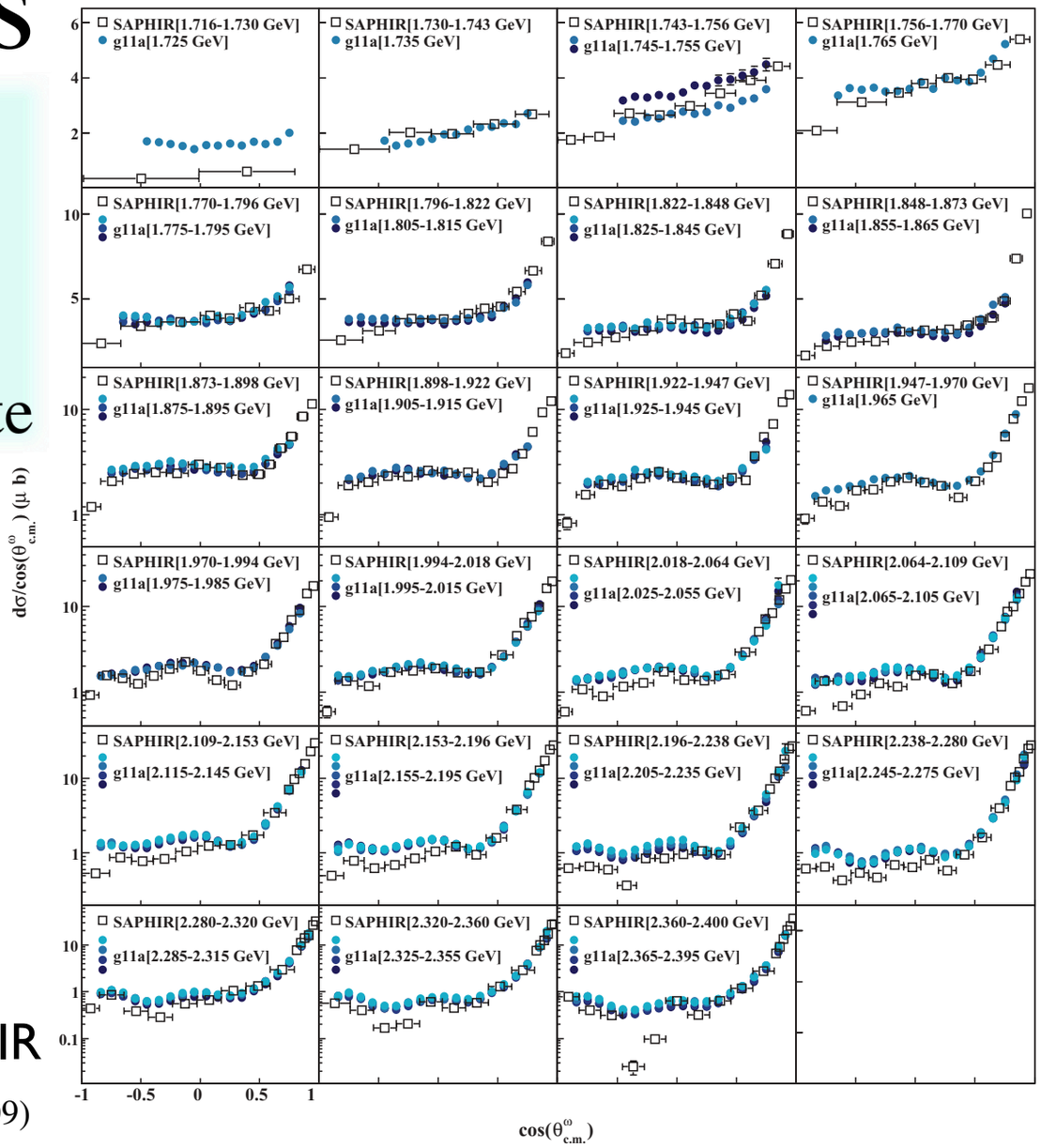




CLAS 6 GeV Results

- CLAS and others have published many photoproduction results
- Can be used to find intermediate resonances in $\gamma + p \rightarrow (N^*) \rightarrow$ final state

- $\gamma + p \rightarrow \omega\, p$
  - near threshold to W=2.8 GeV
  - differential cross sections
  - SDMEs



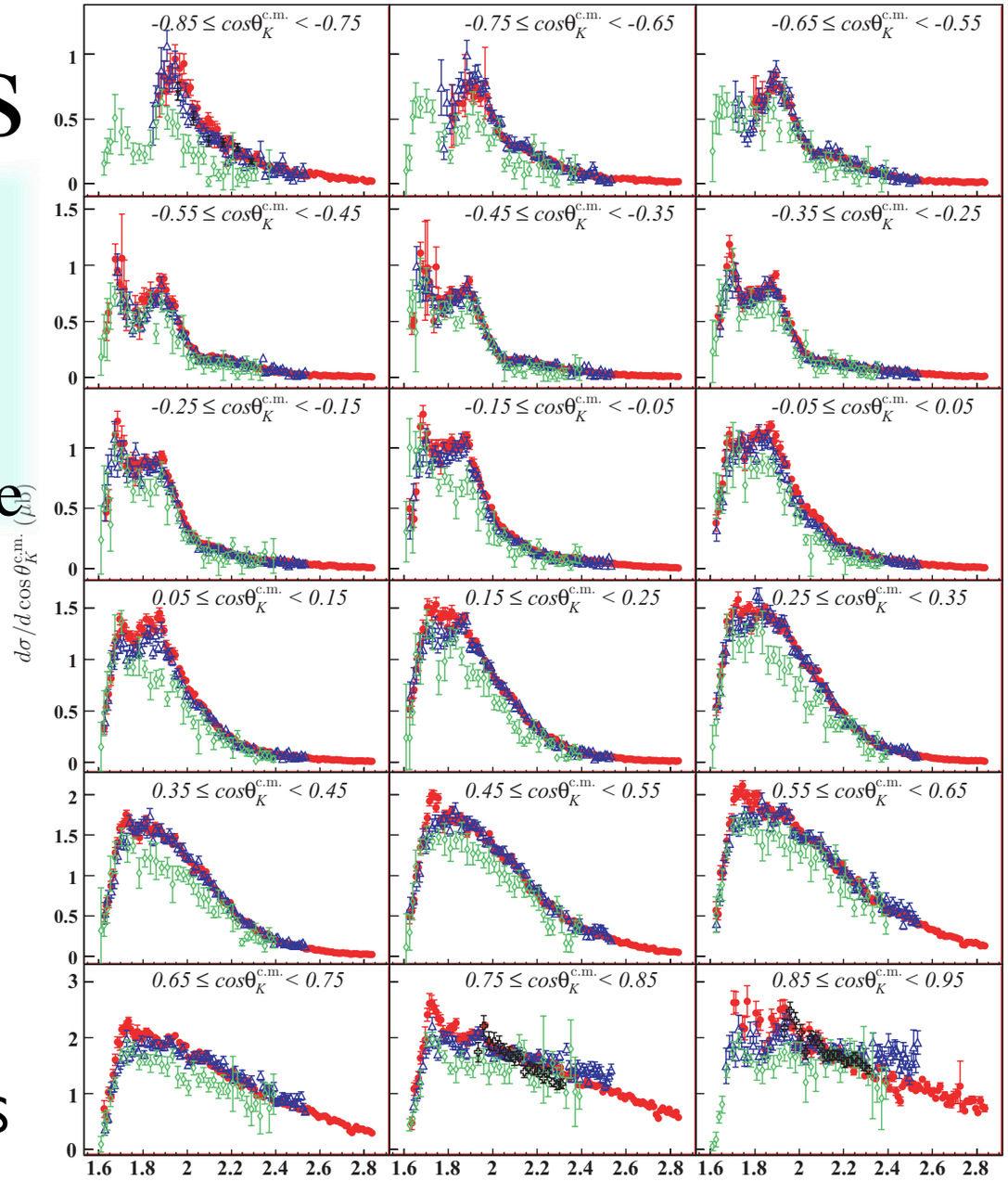
○CLAS □SAPHIR

M Williams (CLAS) PRC80, 065208 (2009)

CLAS 6 GeV Results

- CLAS and others have published many photoproduction results
- Can be used to find intermediate resonances in $\gamma + p \rightarrow (N^*) \rightarrow$ final state

- $\gamma + p \rightarrow K^+ \Lambda$
  - near threshold to W=2.8 GeV
  - differential cross sections
  - recoil polarizations



○CLAS(2010) △CLAS(2006) ◇SAPHIR ✚LEPS

M McCracken (CLAS) PRC81, 025201 (2010)

CLAS 6 GeV Results

- CLAS and others have published many photoproduction results
- Can be used to find intermediate resonances in $\gamma + p \rightarrow (N^*) \rightarrow$ final state

- $\gamma + p \rightarrow K^+ \Sigma^0$
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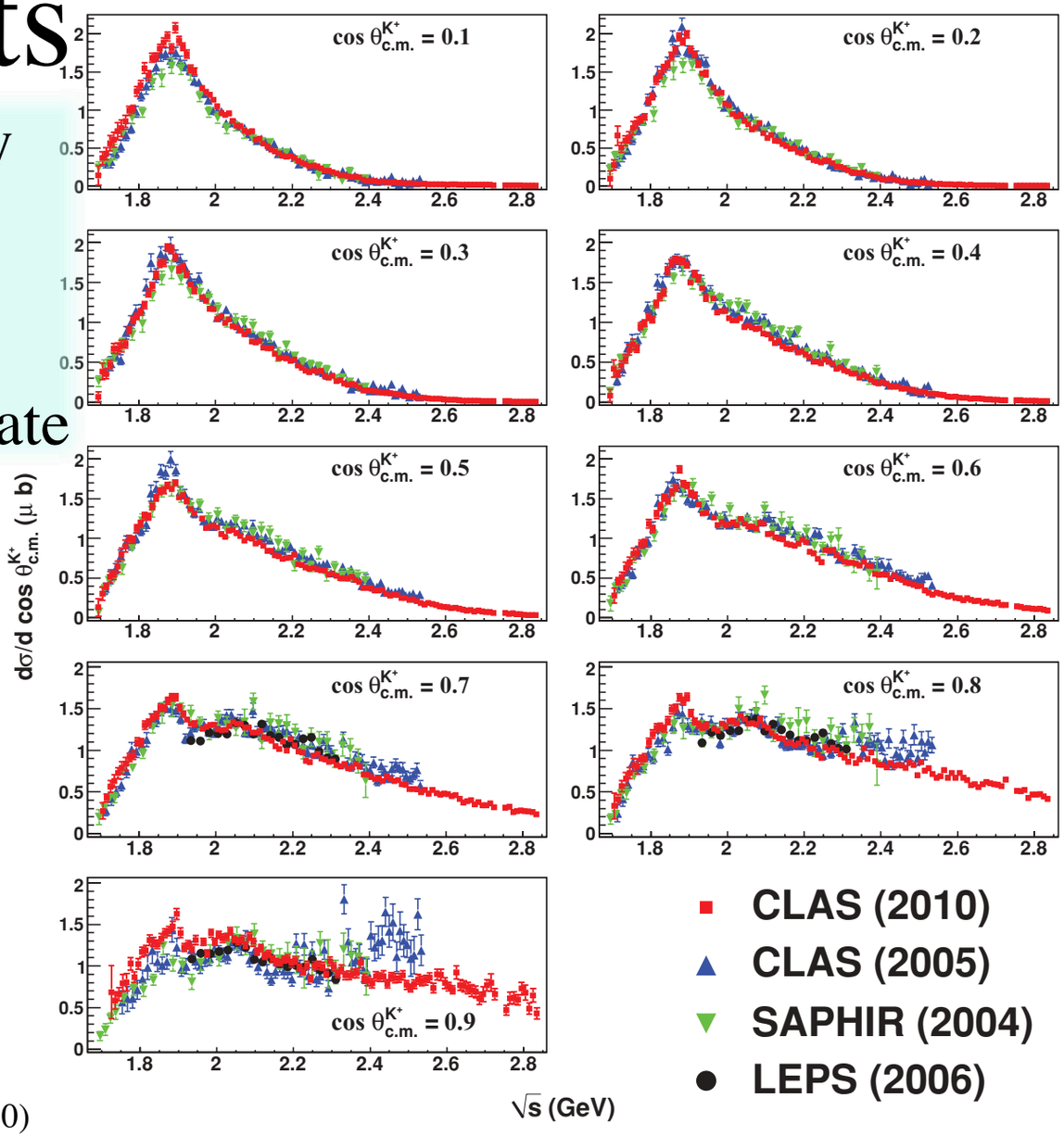
  - recoil polarizations

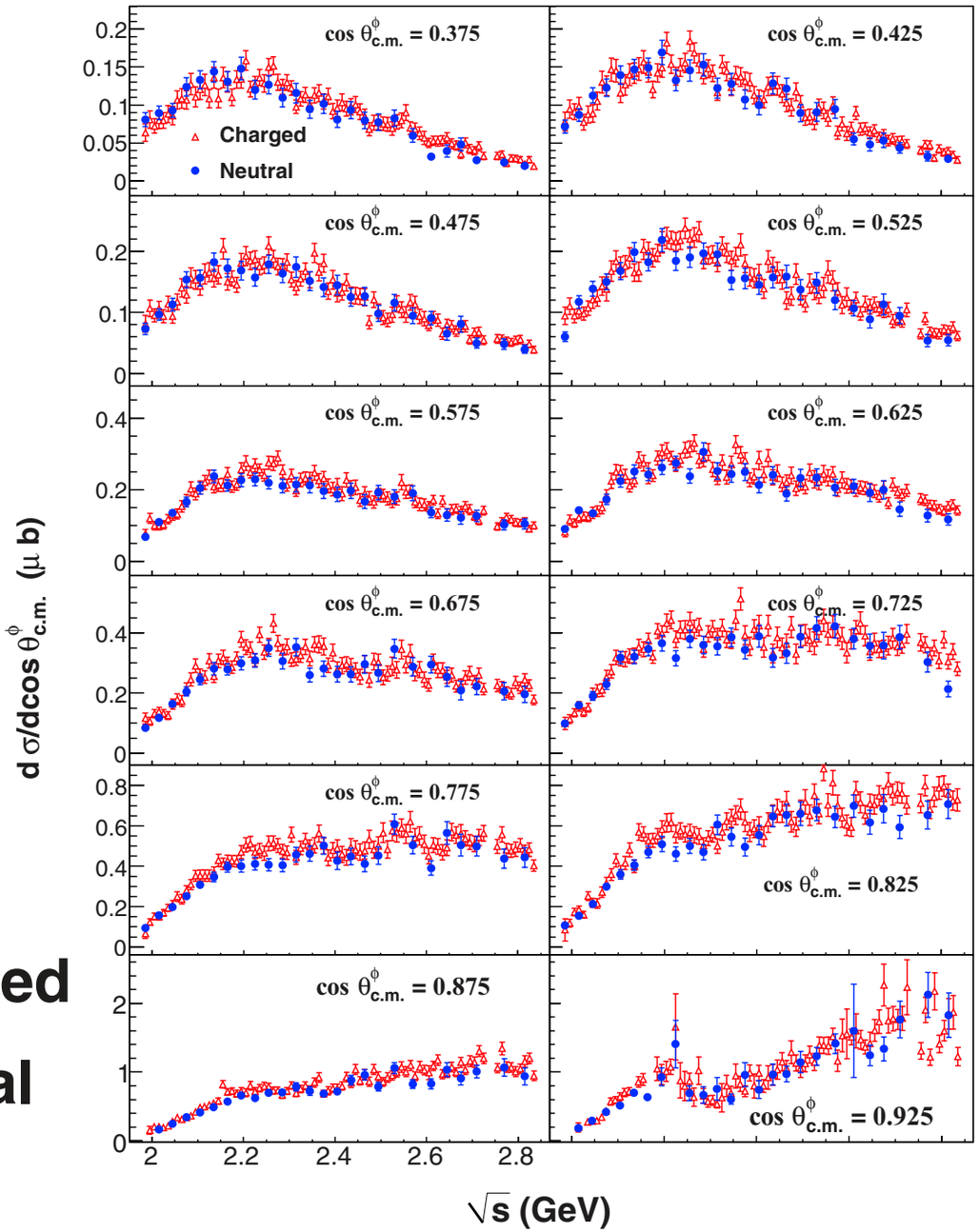


B Dey (CLAS) PRC82, 025202 (2010)

CLAS 6 GeV Results

- CLAS and others have published many photoproduction results
- Can be used to find intermediate resonances in $\gamma + p \rightarrow (N^*) \rightarrow$ final state

- $\gamma + p \rightarrow \phi\, p$
  - near threshold to W=2.8 GeV
  - differential cross sections
  - SDMEs



△ **Charged**

● **Neutral**

B Dey (CLAS) PRC89, 055208 (2014)

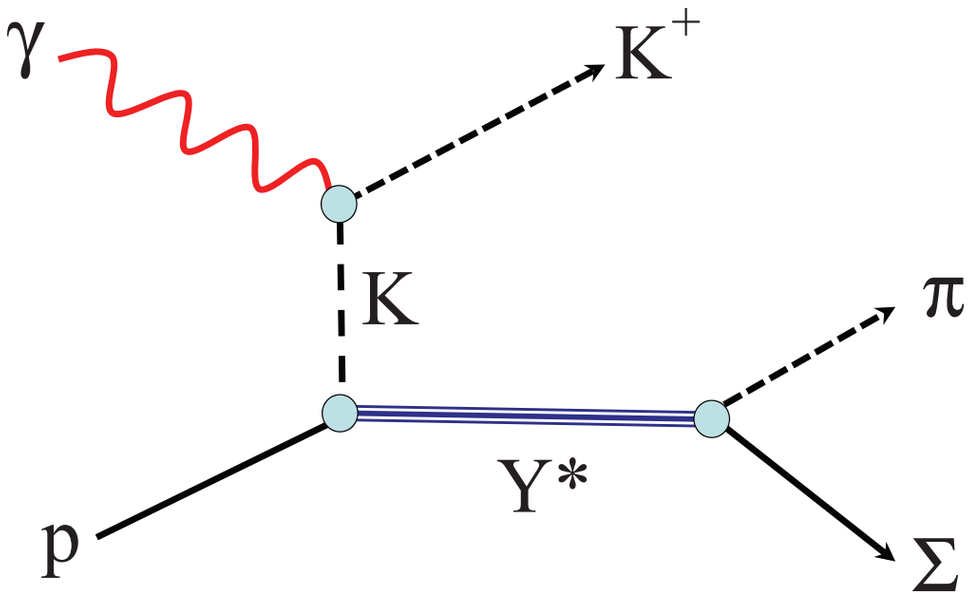
N^* States

- Recent results and analyses from CLAS and elsewhere have lead to updates in PDG
- Strange baryons remain unchanged, future work at Jefferson Lab 12 GeV

N^*	J^P $(L_{2I,2J})$	2010	2012	Δ	J^P $(L_{2I,2J})$	2010	2012
p	$1/2^+$ (P_{11})	****	****	$\Delta(1232)$	$3/2^+$ (P_{33})	****	****
n	$1/2^+$ (P_{11})	****	****	$\Delta(1600)$	$3/2^+$ (P_{33})	***	***
$N(1440)$	$1/2^+$ (P_{11})	****	****	$\Delta(1620)$	$1/2^-$ (S_{31})	****	****
$N(1520)$	$3/2^-$ (D_{13})	****	****	$\Delta(1700)$	$3/2^-$ (D_{33})	****	****
$N(1535)$	$1/2^-$ (S_{11})	****	****	$\Delta(1750)$	$1/2^+$ (P_{31})	*	*
$N(1650)$	$1/2^-$ (S_{11})	****	****	$\Delta(1900)$	$1/2^-$ (S_{31})	**	**
$N(1675)$	$5/2^-$ (D_{15})	****	****	$\Delta(1905)$	$5/2^+$ (F_{35})	****	****
$N(1680)$	$5/2^+$ (F_{15})	****	****	$\Delta(1910)$	$1/2^+$ (P_{31})	****	****
$N(1685)$			*				
$N(1700)$	$3/2^-$ (D_{13})	***	***	$\Delta(1920)$	$3/2^+$ (P_{33})	***	***
$N(1710)$	$1/2^+$ (P_{11})	***	***	$\Delta(1930)$	$5/2^-$ (D_{35})	***	***
$N(1720)$	$3/2^+$ (P_{13})	****	****	$\Delta(1940)$	$3/2^-$ (D_{33})	*	**
$N(1860)$	$5/2^+$		**				
$N(1875)$	$3/2^-$		***				
$N(1880)$	$1/2^+$		**				
$N(1895)$	$1/2^-$		**				
$N(1900)$	$3/2^+$ (P_{13})	**	***	$\Delta(1950)$	$7/2^+$ (F_{37})	****	****
$N(1990)$	$7/2^+$ (F_{17})	**	**	$\Delta(2000)$	$5/2^+$ (F_{35})	**	**
$N(2000)$	$5/2^+$ (F_{15})	**	**	$\Delta(2150)$	$1/2^-$ (S_{31})	*	*
~~$N(2080)$~~	D_{13}	**		$\Delta(2200)$	$7/2^-$ (G_{37})	*	*
~~$N(2090)$~~	S_{11}	*		$\Delta(2300)$	$9/2^+$ (H_{39})	**	**
$N(2040)$	$3/2^+$		*				
$N(2060)$	$5/2^-$		**				
$N(2100)$	$1/2^+$ (P_{11})	*	*	$\Delta(2350)$	$5/2^-$ (D_{35})	*	*
$N(2120)$	$3/2^-$		**				
$N(2190)$	$7/2^-$ (G_{17})	****	****	$\Delta(2390)$	$7/2^+$ (F_{37})	*	*
~~$N(2200)$~~	D_{15}	**		$\Delta(2400)$	$9/2^-$ (G_{39})	**	**
$N(2220)$	$9/2^+$ (H_{19})	****	****	$\Delta(2420)$	$11/2^+$ $(H_{3,11})$	****	****
$N(2250)$	$9/2^-$ (G_{19})	****	****	$\Delta(2750)$	$13/2^-$ $(I_{3,13})$	**	**
$N(2600)$	$11/2^-$ $(I_{1,11})$	***	***	$\Delta(2950)$	$15/2^+$ $(K_{3,15})$	**	**
$N(2700)$	$13/2^+$ $(K_{1,13})$	**	**				

V Crede, W Roberts, Rep. Prog. Phys. 76, 076301 (2013)

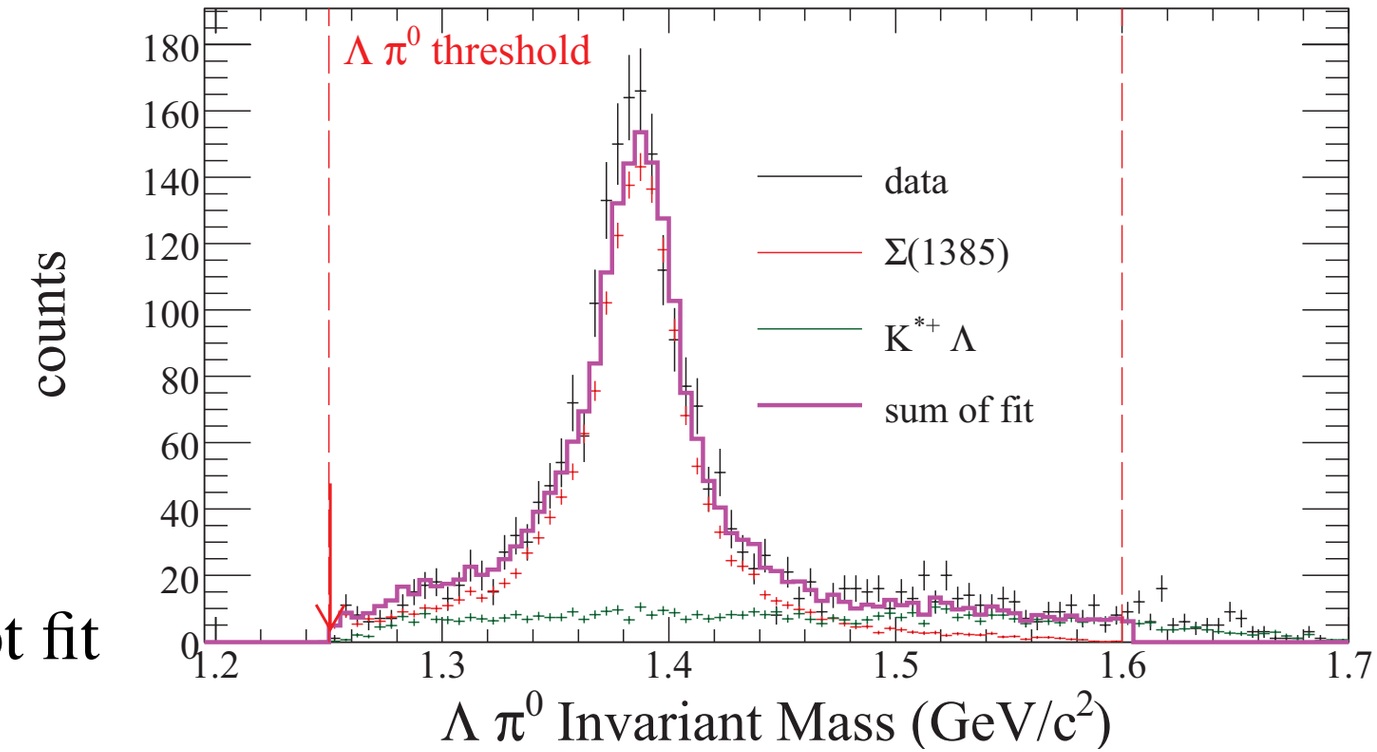
Photoproduction of Hyperon Resonances



- Production of $K^+ \Sigma^0(1385)$, $K^+ \Lambda(1405)$, $K^+ \Lambda(1520)$
- Interest is in properties of each state:
  - nature of each state - corresponce to constituent quark model states?
  - what is production mechanism?
  - does comparison give insight into composition?

$\gamma + p \rightarrow K^+ \Sigma^0(1385)$

- SU(3) flavor decuplet state
- $J^P = 3/2^+$
- Interest is in cross section
- Reconstructed through $\Sigma^0(1385) \rightarrow \Lambda\, \pi^0$ (87%)
- Line shape symmetric, does not fit well with mom.-dependent Breit-Wigner function:



$$\Gamma \propto \left(\frac{p}{p_0}\right)^{2L+1}$$

p: breakup mom. in rest frame

p_0: p at pole position

L: breakup angular momentum

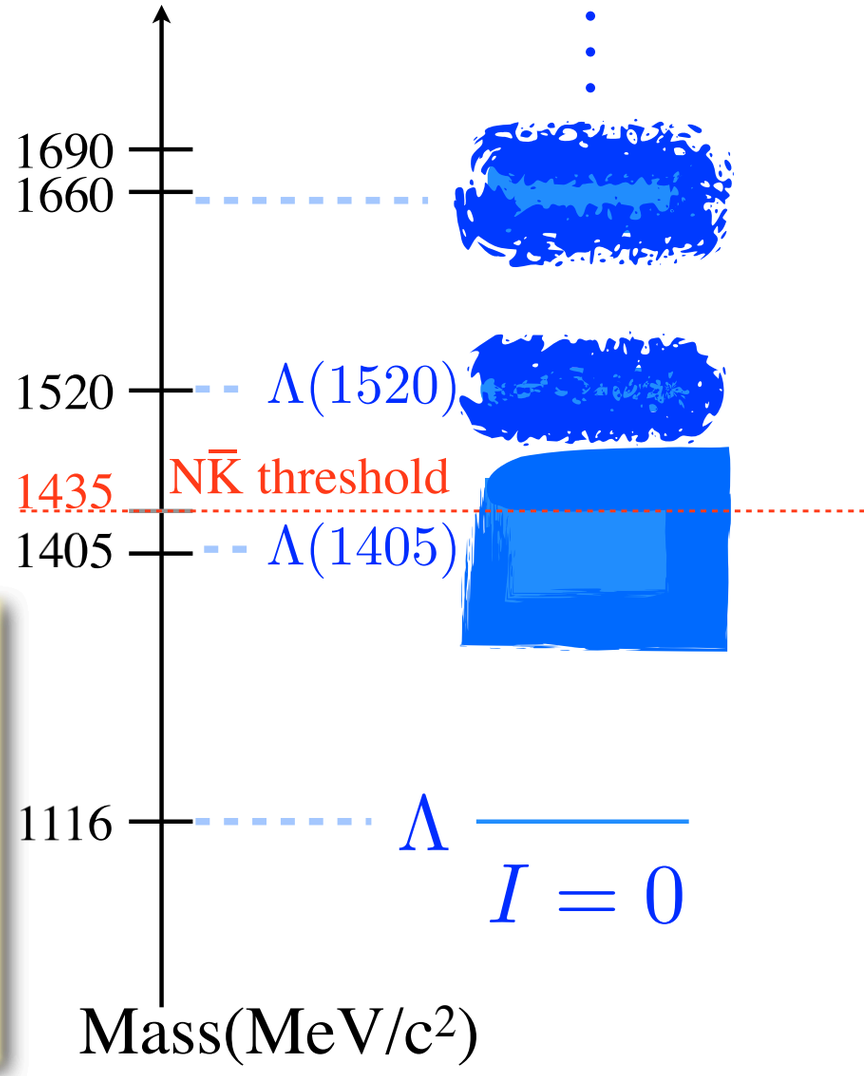
$\Rightarrow$ $\Gamma = \text{const.}$

$\gamma + p \rightarrow K^+ \Lambda(1405)$

1. N. Isgur and G. Karl, PRD18, 4187 (1978)
S. Capstick and N. Isgur, PRD34, 2809 (1986)
S. Capstick and W. Roberts, Prog.Part.Nucl.Phys. 45, S241 (2000)

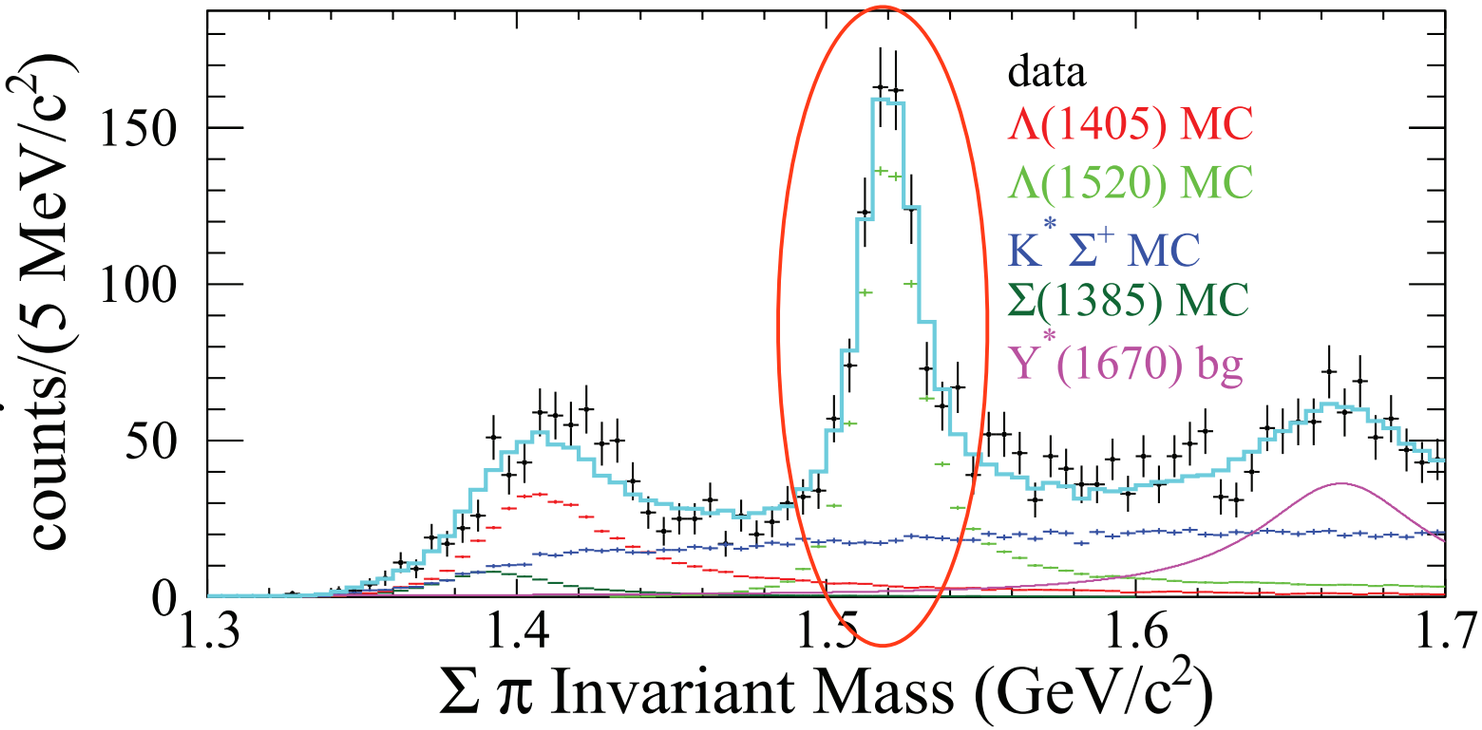
- **** state in PDG, first excited Λ state
- $M = 1405$ MeV/c^2, $\Gamma = 50$ MeV (PDG)
- Constituent Quark Model[1] has difficulty predicting mass
- Chiral unitary theory predicts two poles
- What is the line shape?

- Chiral Unitary Theory (χUT): combine chiral Lagrangians of low-mom. interactions + unitarity between channels
- "Fundamental" states (ground state pseudoscalar mesons, baryons) can "dynamically generate" resonances
- Within χUT, $\Lambda(1405)$ is textbook example



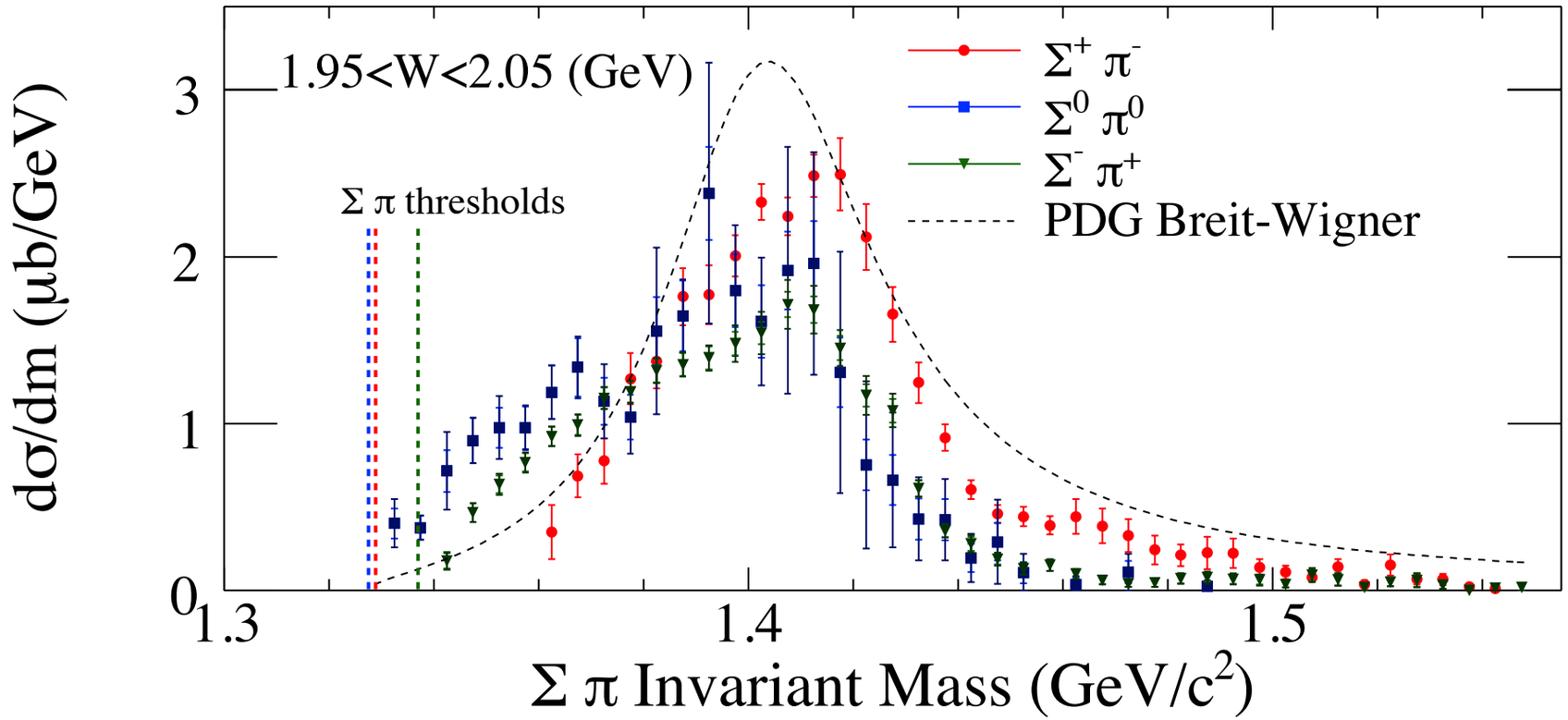
$\gamma + p \rightarrow K^+ \Lambda(1520)$

- Flavor SU(3) singlet state, partner of $\Lambda(1405)$?
- Interest is in cross section
- Reconstructed through $\Lambda(1520) \rightarrow \Sigma\pi$
- Fits with MC templates to determine contributions of each state



Line Shape of Λ(1405)

- Most precise measurement of Λ(1405)
- Measurement for all 3 Σπ channels
- Clear difference in shapes at low energies → small *I=1* amplitude



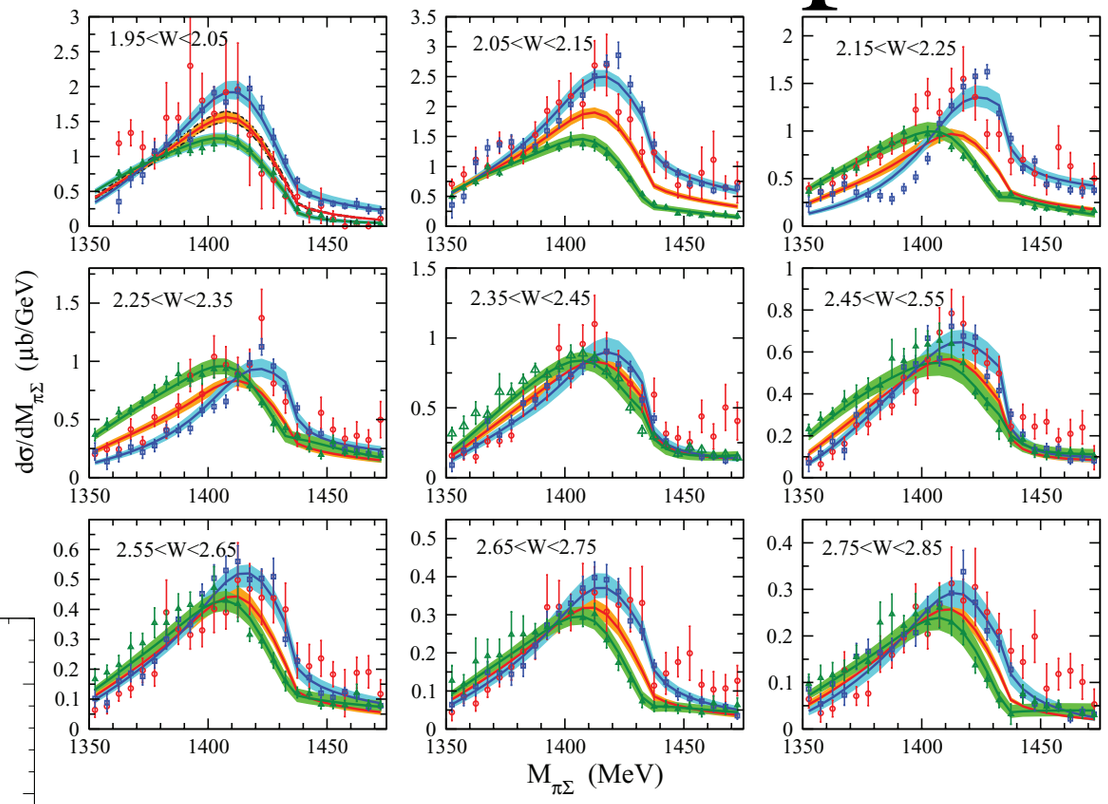
low energy -
below K* threshold

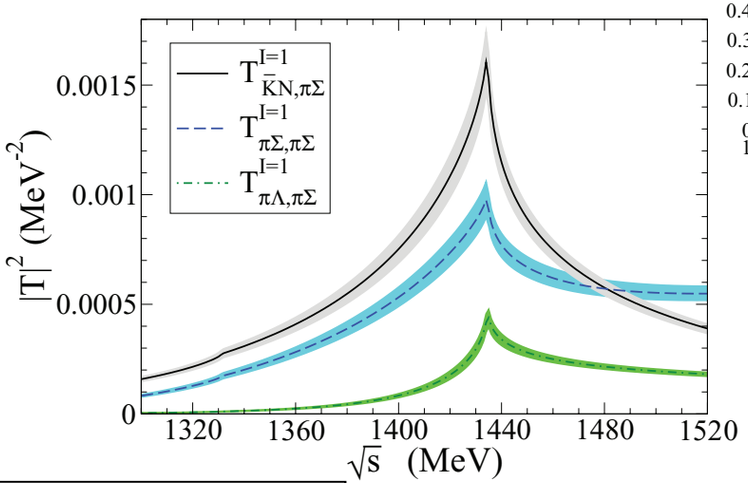
$\gamma + p \rightarrow K^+ \Lambda(1405)$

K. Moriya, R. Schumacher *et al*. [CLAS Collaboration]
PRC 87, 035206 (2013)

Discussion on Line Shapes

- Fits to data with chiral unitary amplitudes
- Interference of isospin *I=1* amplitude causes differences in Σπ channels



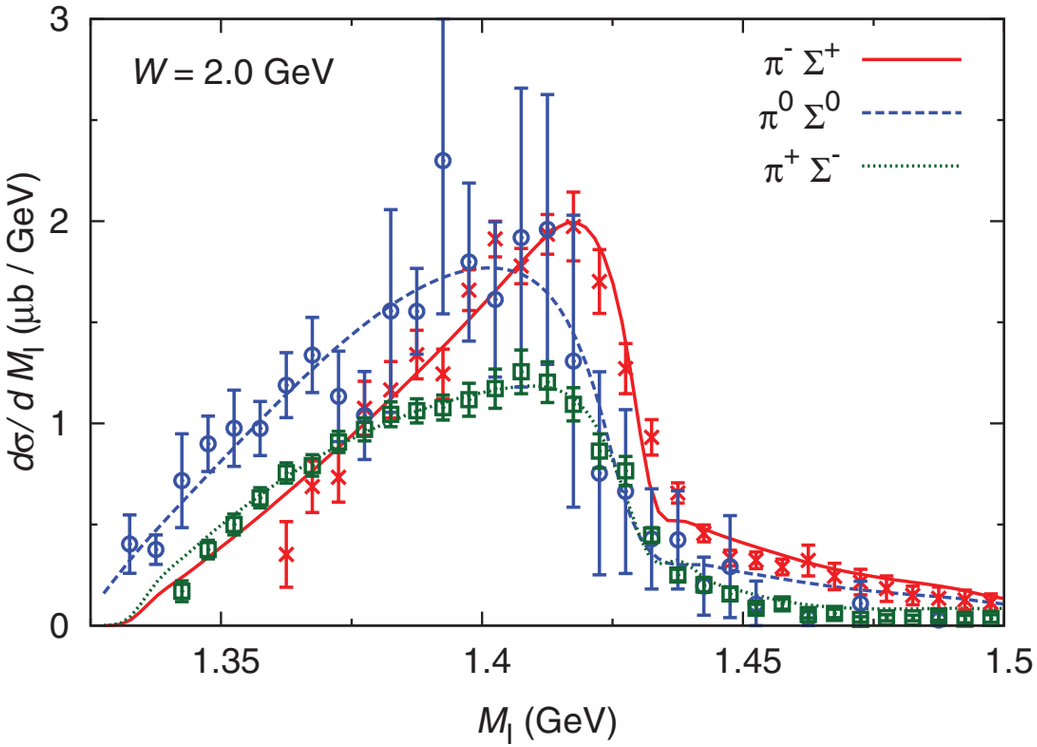


L. Roca, E. Oset, PRC 88, 055206 (2013), see also PRC 87, 055201 (2013)

- *I=1* amplitude produces cusp in the vicinity of the N$\bar{K}$ threshold - similar to $a_0(980)$

Discussion on Line Shapes

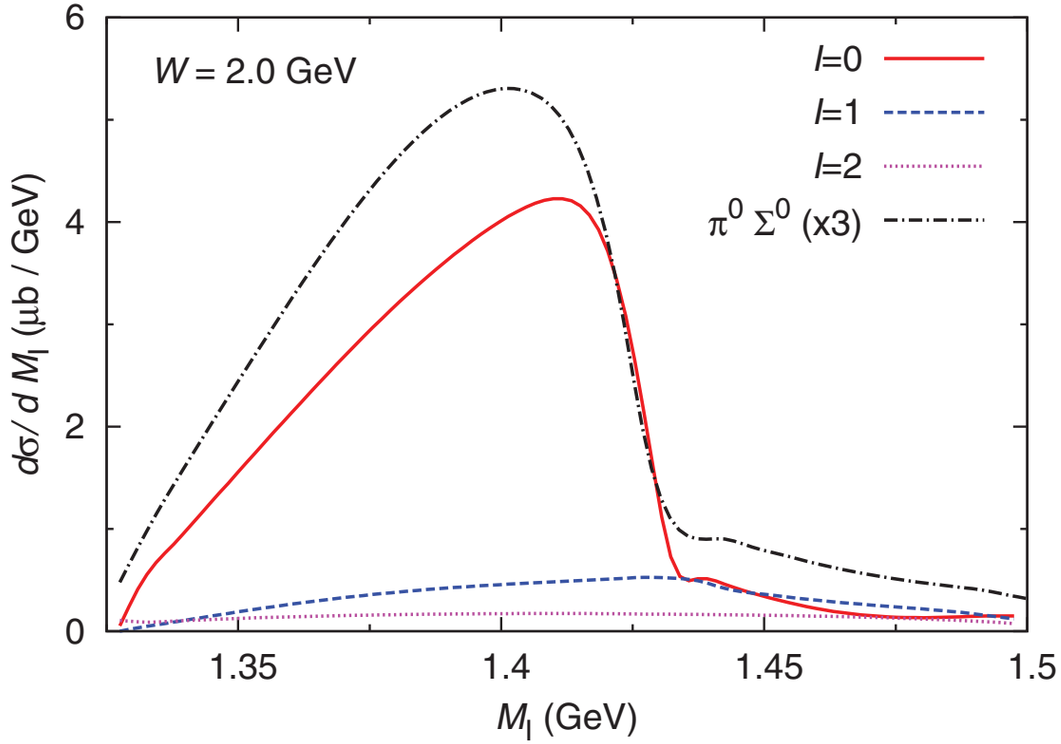
- S. X. Nakamura, D. Jido fit the line shapes and differential cross simultaneously using χUT amplitudes
  - Data fit well with χUT amplitudes
  - Fits are done in low energy region only where χUT assumptions hold



S. X. Nakamura, D. Jido
Prog. Theor. Exp. Phys. 2014, 023D01

Discussion on Line Shapes

- S. X. Nakamura, D. Jido fit the line shapes and differential cross simultaneously using χUT amplitudes
  - χUT amplitudes suggest higher mass pole contributes more
  - More extensive analysis planned



S. X. Nakamura, D. Jido
Prog. Theor. Exp. Phys. 2014, 023D01

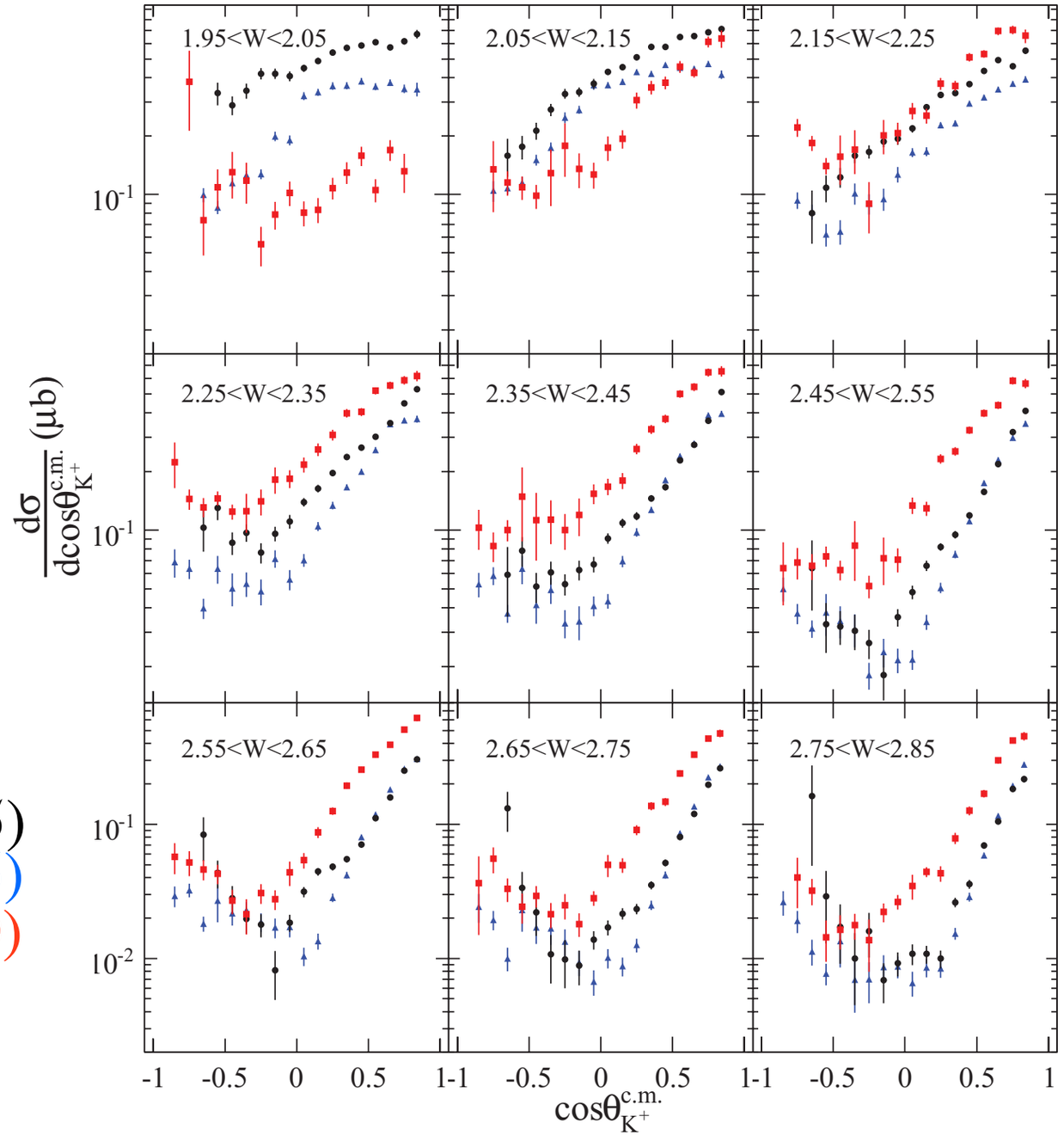
Cross Section Results

- Measure $\Lambda(1405)$ differential cross section results
  - Each $\Sigma\pi$ channel measured separately
- Also measure nearby
  - $\Sigma^0(1385)$ ($J^P = 3/2^+$)
  - $\Lambda(1520)$ ($J^P = 3/2^-$)
- Three excited hyperons, all with different characteristics

$\Lambda(1405)$	dynamically generated resonance? S-wave $\Sigma\pi$-$N\bar{K}$ coupling, or internal *P*-wave quark excitation?
$\Sigma(1385)$	flavor SU(3) decuplet, $J^P = 3/2^+$ with L=0 for quarks
$\Lambda(1520)$	flavor SU(3) singlet, $J^P = 3/2^-$ with L=1 for quarks

3 Hyperons

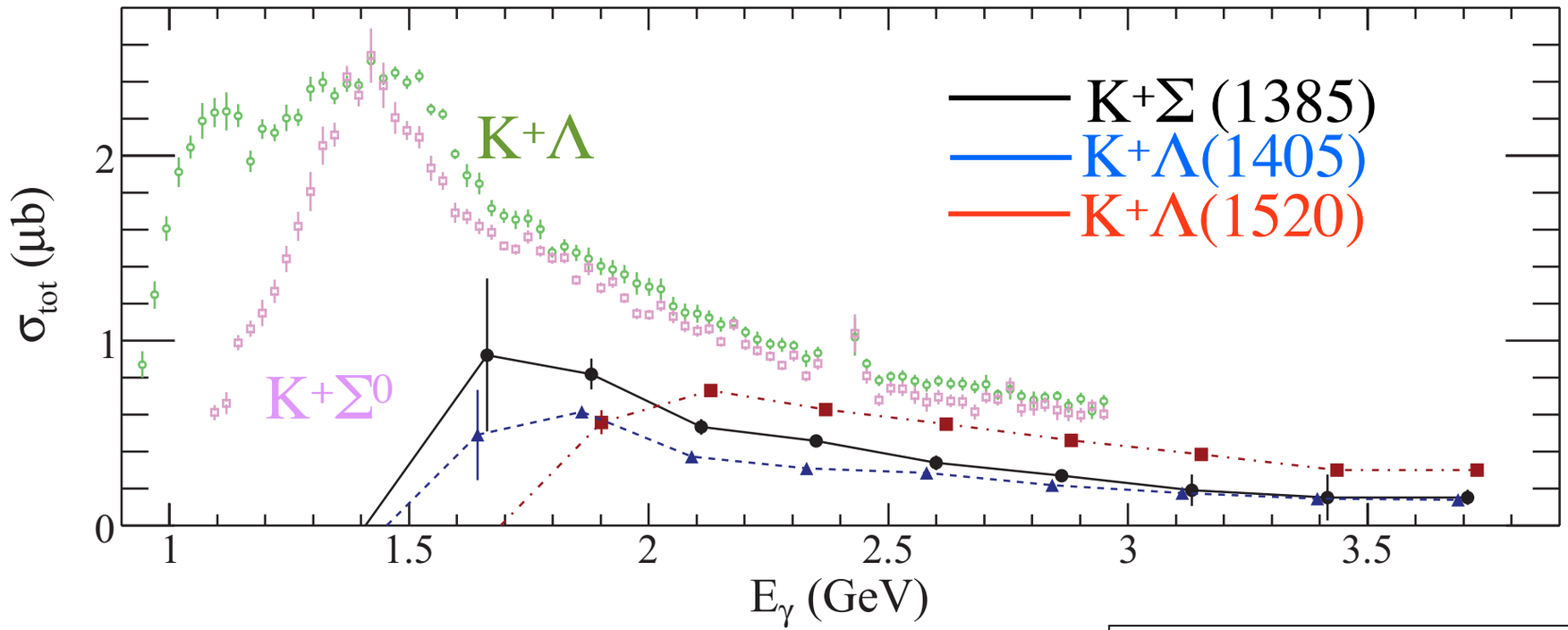
- 9 bins of energy
- All 3 hyperons have similar forward-peaked behavior
- Λ(1520): larger production at higher energies



K. Moriya, R. Schumacher *et al*. [CLAS Collaboration]
PRC 88, 045201 (2013)

Measurement of σ_{tot}

- Extrapolate to all angles, determine σ_{tot}
- Cross sections comparable to ground state hyperons

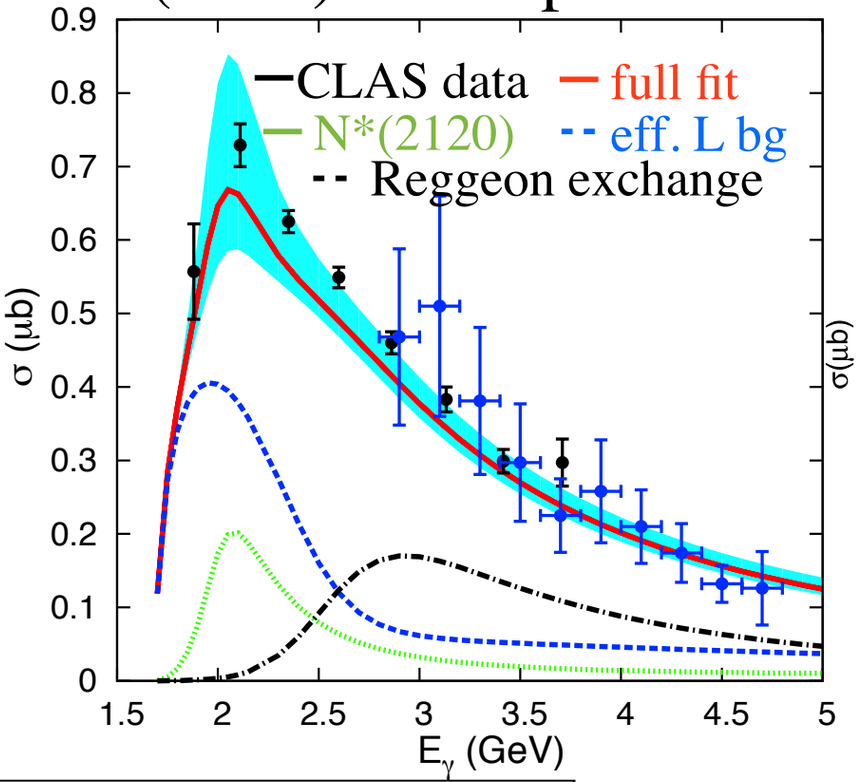


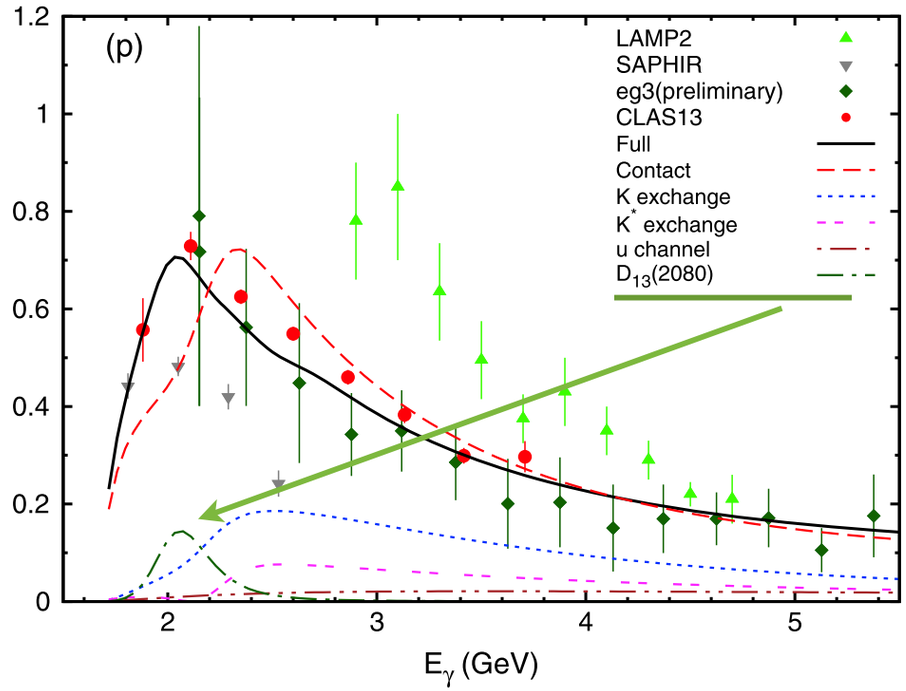
K. Moriya, R. Schumacher *et al*. [CLAS Collaboration]
PRC 88, 045201 (2013)

R. Bradford, R. Schumacher *et al*. [CLAS Collaboration]
PRC 73, 035202 (2006)
(ground state results)

Cross Section Discussion

- PDG lists ** N(2080) state (now N(2120))
- Fit to CLAS Λ(1520) results: Regge-plus-resonance effective approach shows N(2120) $3/2^-$ is preferred





En Wang, Ju-Jun Xie, Juan Nieves arXiv:1405.3142[nucl-th]
see also Phys. Rev. C 89, 015203 (2014)

Jun He,
Nucl. Phys. A 927, 24 (2014)

III. Future Prospects

"Complete" Experiments

- Ongoing analysis effort at CLAS
- Measure enough polarization combinations to completely determine amplitudes
- 4 complex amplitudes →16 real observables →need to measure 8
- Use polarized γ, p for $\vec{\gamma} + \vec{p} \rightarrow K^+ \vec{\Lambda}$ and measure final pol. of Λ
- Analyses also ongoing for other channels (η, ω production)

see backup for full list

Usual symbol	Helicity representation	Transversity representation	Experiment required [a]	Type
$d\sigma/dt$	$\|N\|^2+\|S_1\|^2+\|S_2\|^2+\|D\|^2$	$\|b_1\|^2+\|b_2\|^2+\|b_3\|^2+\|b_4\|^2$	$\{-;-;-\}$	S
$\Sigma\, d\sigma/dt$	$2\mathrm{Re}(S^{\dagger}S_2 - ND^*)$	$\|b_1\|^2+\|b_2\|^2-\|b_3\|^2-\|b_4\|^2$	$\{L(\frac{1}{2}\pi,0);-;-\}$ $\{-;y;y\}$	
$T\, d\sigma/dt$	$2\mathrm{Im}(S_1N^* - S_2D^*)$	$\|b_1\|^2-\|b_2\|^2-\|b_3\|^2+\|b_4\|^2$	$\{-;y;-\}$ $\{L(\frac{1}{2}\pi,0);0;y\}$	
$P\, d\sigma/dt$	$2\mathrm{Im}(S_2N^* - S_1D^*)$	$\|b_1\|^2-\|b_2\|^2+\|b_3\|^2-\|b_4\|^2$	$\{-;-;y\}$ $\{L(\frac{1}{2}\pi,0);y;-\}$	
$G\, d\sigma/dt$	$-2\mathrm{Im}(S_1S_2^* + ND^*)$	$2\mathrm{Im}(b_1b_3^* + b_2b_4^*)$	$\{L(\pm\frac{1}{4}\pi);z;-\}$	BT
$H\, d\sigma/dt$	$-2\mathrm{Im}(S_1D^* + S_2N^*)$	$-2\mathrm{Re}(b_1b_3^* - b_2b_4^*)$	$\{L(\pm\frac{1}{4}\pi);x;-\}$	
$E\, d\sigma/dt$	$\|S_2\|^2-\|S_1\|^2-\|D\|^2+\|N\|^2$	$-2\mathrm{Re}(b_1b_3^* + b_2b_4^*)$	$\{c;z;-\}$	
$F\, d\sigma/dt$	$2\mathrm{Re}(S_2D^* + S_1N^*)$	$2\mathrm{Im}(b_1b_3^* - b_2b_4^*)$	$\{c;x;-\}$	
$O_x\, d\sigma/dt$	$-2\mathrm{Im}(S_2D^* + S_1N^*)$	$-2\mathrm{Re}(b_1b_4^* - b_2b_3^*)$	$\{L(\pm\frac{1}{4}\pi);-;x'\}$	BR
$O_z\, d\sigma/dt$	$-2\mathrm{Im}(S_2S_1^* + ND^*)$	$-2\mathrm{Im}(b_1b_4^* + b_2b_3^*)$	$\{L(\pm\frac{1}{4}\pi);-;z'\}$	
$C_x\, d\sigma/dt$	$-2\mathrm{Re}(S_2N^* + S_1D^*)$	$2\mathrm{Im}(b_1b_4^* - b_2b_3^*)$	$\{c;-;x'\}$	
$C_z\, d\sigma/dt$	$\|S_2\|^2-\|S_1\|^2-\|N\|^2+\|D\|^2$	$-2\mathrm{Re}(b_1b_4^* + b_2b_3^*)$	$\{c;-;z'\}$	
$T_x\, d\sigma/dt$	$2\mathrm{Re}(S_1S_2^* + ND^*)$	$2\mathrm{Re}(b_1b_2^* - b_3b_4^*)$	$\{-;x;x'\}$	TR
$T_z\, d\sigma/dt$	$2\mathrm{Re}(S_1N^* - S_2D^*)$	$2\mathrm{Im}(b_1b_2^* - b_3b_4^*)$	$\{-;x;z'\}$	
$L_x\, d\sigma/dt$	$2\mathrm{Re}(S_2N^* - S_1D^*)$	$2\mathrm{Im}(b_1b_2^* + b_3b_4^*)$	$\{-;z;x'\}$	
$L_z\, d\sigma/dt$	$\|S_1\|^2+\|S_2\|^2-\|N\|^2-\|D\|^2$	$2\mathrm{Re}(b_1b_2^* + b_3b_4^*)$	$\{-;z;z'\}$	

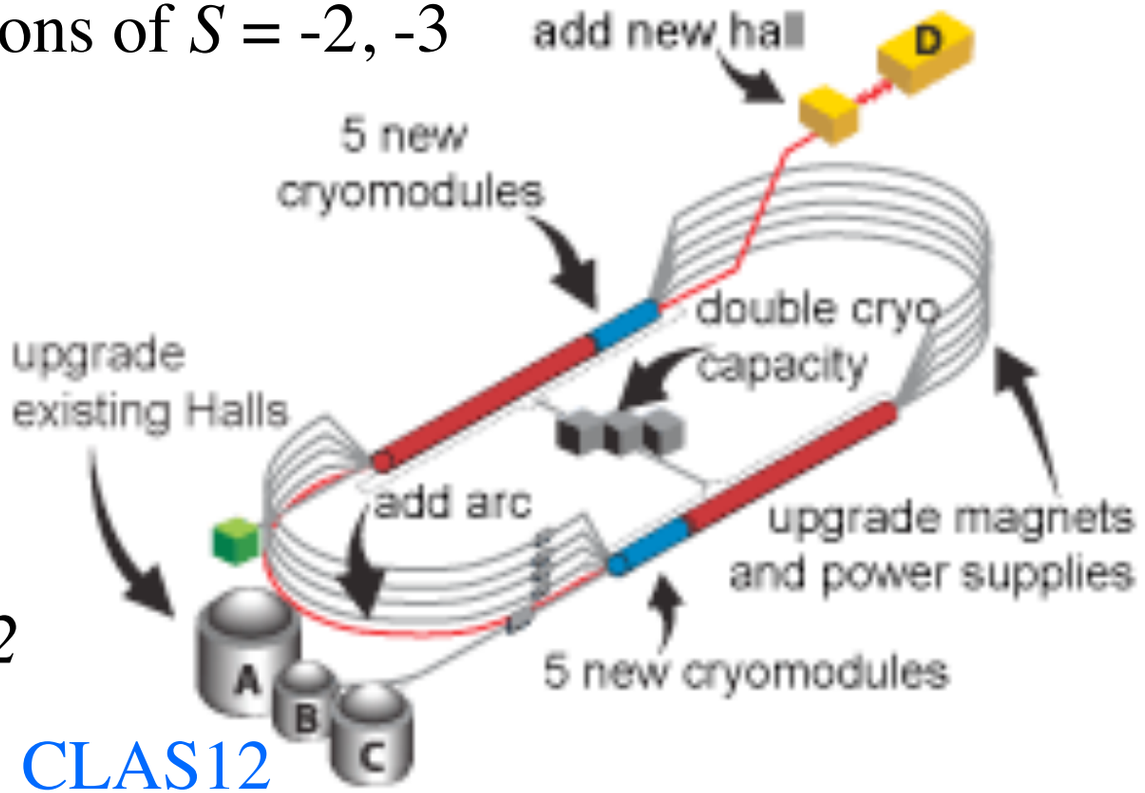
I.S. Barker, A. Donnachie, J.K. Storrow Nucl. Phys. B95, 347 (1975)

Hadronic Physics Programs @12 GeV

- CLAS12
  - Meson spectroscopy with low Q^2
  - Very Strange Collaboration: baryons of S = -2, -3

- GlueX
  - Mapping of meson spectrum
  - Search for J^{PC}-exotic mesons
  - Studies of baryons with S=-1, -2

GlueX

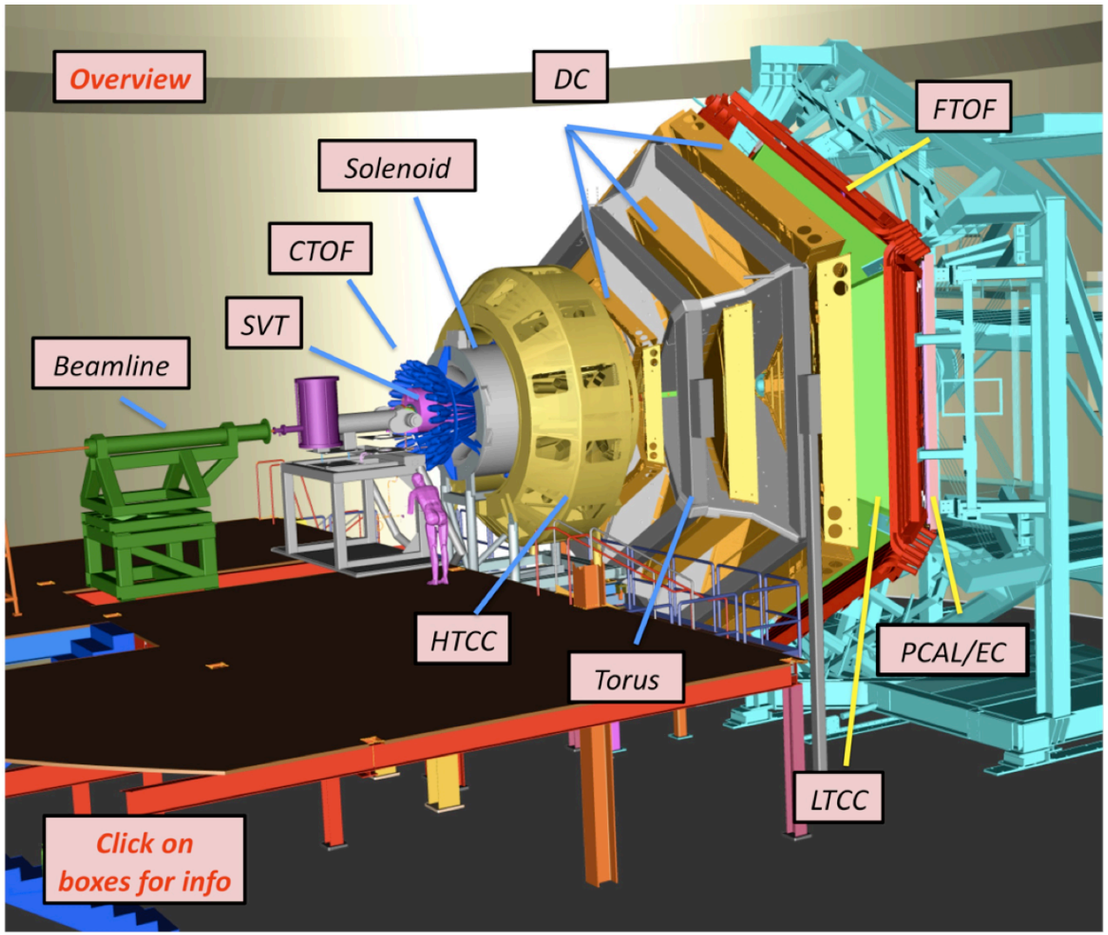


CLAS12

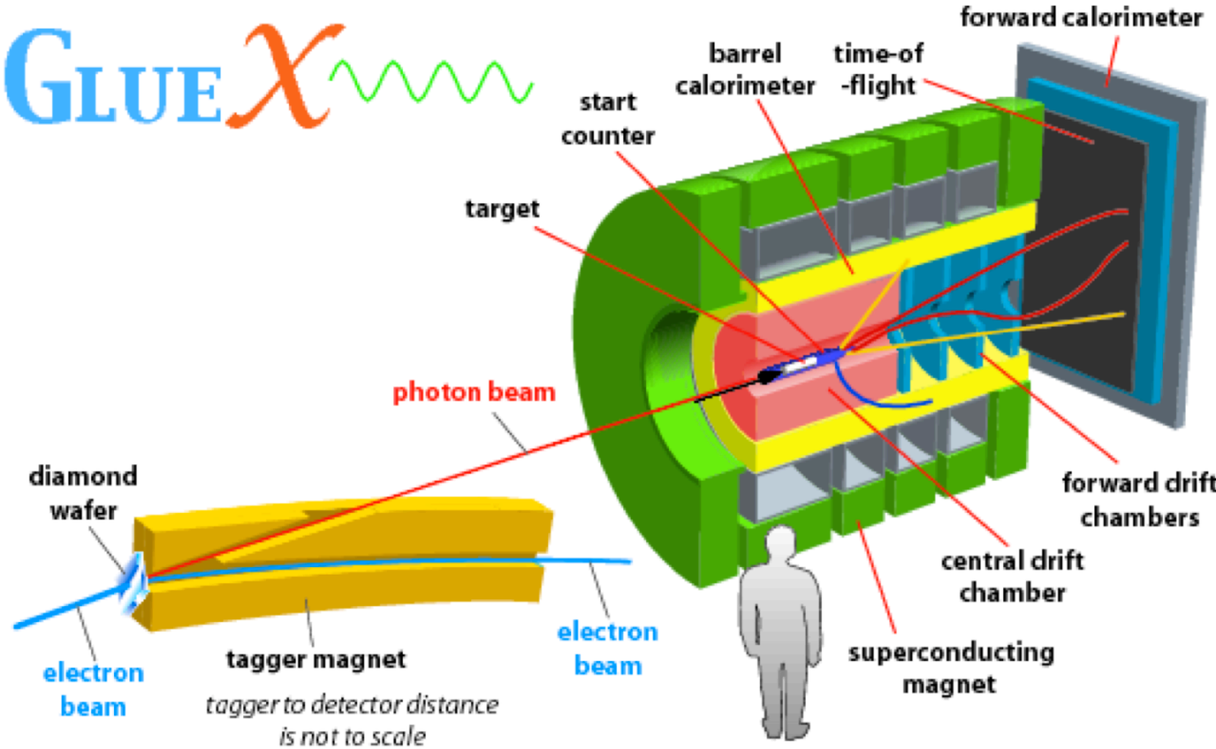
PAC proposals: CLAS12

http://www.jlab.org/exp_prog/proposals/11/PR12-11-005.pdf

http://www.jlab.org/exp_prog/proposals/12/PR12-12-008.pdf



GLUEX



PAC proposals:

http://arxiv.org/abs/1305.1523

http://arxiv.org/abs/1408.0215

Resonances with Strangeness -1, -2

- Jefferson Lab 12 GeV program will feature higher mass hyperon resonances
  - larger kinematic energy range
  - higher, more uniform acceptance, also for neutrals
- For $S = -1$, coupling of channels between $\Sigma\pi$, $\Lambda\pi$, $N\bar{K}$
- For $S = -2$, decay modes are $\Xi\pi$, $\Xi\pi\pi$, $\Lambda\bar{K}$, $\Sigma\bar{K}$
- Are there any states of interest, surprises in $S = -1, -2$?

Conclusions

- Jefferson Lab and CLAS has produced many detailed results on hadron production with < 6 GeV beams
- For all CLAS results see http://www.jlab.org/Hall-B/shifts/index.php?display=utils&task=publications
- Many more analyses under review for both mesons and baryons
- CLAS12 and GlueX will produce further electro- or photoproduction results at 12 GeV
  - exotic J^{PC} mesons
  - excited hyperons with $S = -1, -2, -3?$
- Commissioning has started for 12 GeV era, the future is almost here

Backup

CLAS Analyses on proton target

	σ	Σ	T	P	E	F	G	H	T_x	T_z	L_x	L_z	O_x	O_z	C_x	C_z	CLAS run Period
$p\pi^0$	✔	✓	✓	✓	✓	✓	✓	✓									g1, g8, g9
$n\pi^+$	✔	✓	✓	✓	✓	✓	✓	✓									g1, g8, g9
$p\eta$	✔	✓	✓	✓	✓	✓	✓	✓									g1, g11, g8, g9
$p\eta'$	✔	✓	✓	✓	✓	✓	✓	✓									g1, g11, g8, g9
$p\omega$	✔	✓	✓	✓	✓	✓	✓	✓									g11, g8, g9
$K^+\Lambda$	✔	✓	✓	✔	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✔	✔	g1, g8, g11
$K^+\Sigma^0$	✔	✓	✓	✔	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✔	✔	g1, g8, g11
$K^{0*}\Sigma^+$	✔										✓	✓			✓	✓	g1, g8, g11

Source: Volker Burkert

published, acquired, FroST(g9b) Butanol (C_4H_9OH) target

CLAS Analyses on neutron target

	σ	Σ	T	P	E	F	G	H	T_x	T_z	L_x	L_z	O_x	O_z	C_x	C_z	CLAS run Period
$p\pi^-$	✔	✓	✓		✓	✓	✓	✓									g2, g10, g13, g14
$p\rho^-$	✓	✓	✓		✓	✓	✓	✓									g2, g10, g13, g14
$K^0\Lambda$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	g13, g14
$K^0\Sigma^0$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	g13, g14
$K^+\Sigma^-$	✓	✓	✓		✓	✓	✓	✓									g10, g13, g14
$K^{0*}\Sigma^0$	✓	✓															g10, g13

Source: Volker Burkert

published, acquired, HD-ice HD target

Σ(1385) Line shape

- S.R. Borenstein et al., PRD9, 3006 (1974)
- $K^- + p \rightarrow \Lambda\pi^+\pi^-$ with p_K = 2.18 GeV/c at BNL 31-in. liquid hydrogen bubble chamber
- Fits are with Breit-Wigner curve with energy-independent width
- Note this is for the charged Σ(1385), where there is no Λ(1405)

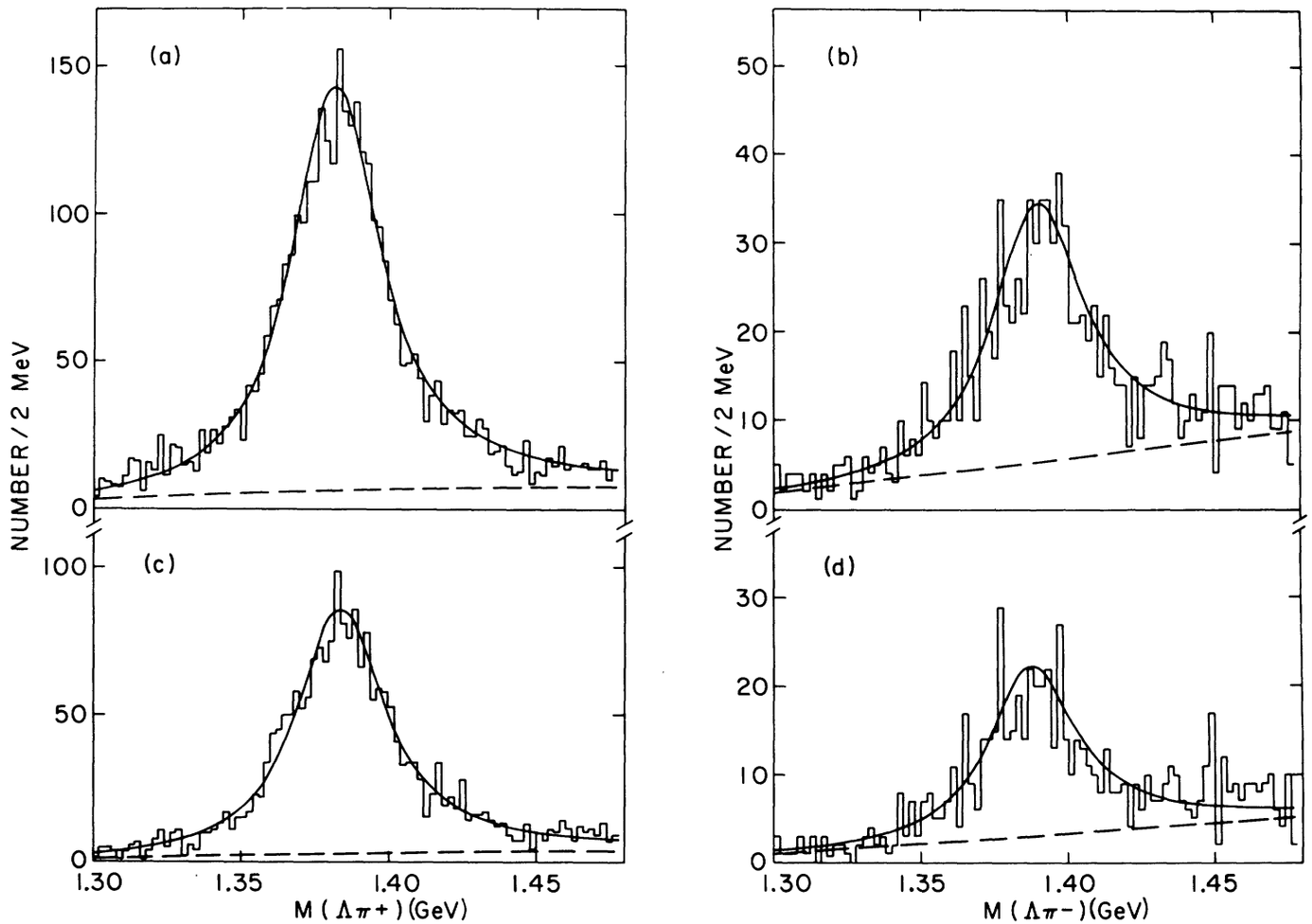


FIG. 1. (a) The $\Lambda\pi^+$ mass distribution and (b) the $\Lambda\pi^-$ mass distribution of the events from the final state $\Lambda\pi^+\pi^-$; (c) and (d) the same as (a) and (b) but with events excluded if $M(\pi^+\pi^-)$ falls in the ρ band, 660–860 MeV; the solid curve and the dashed background curve are the result of the fitting procedure described in the text.

$\Sigma(1385)$ Line shape

- W. Cameron et al., NPB143, 189 (1978)
- $K^- + p \rightarrow \Lambda\pi^+\pi^-$ with W = 1.775 - 1.957 GeV at CERN 2m HBC
- Fits are with relativistic Breit-Wigner curve with energy-independent width
- Good fits are not obtained when using a P-wave relativistic Breit-Wigner form

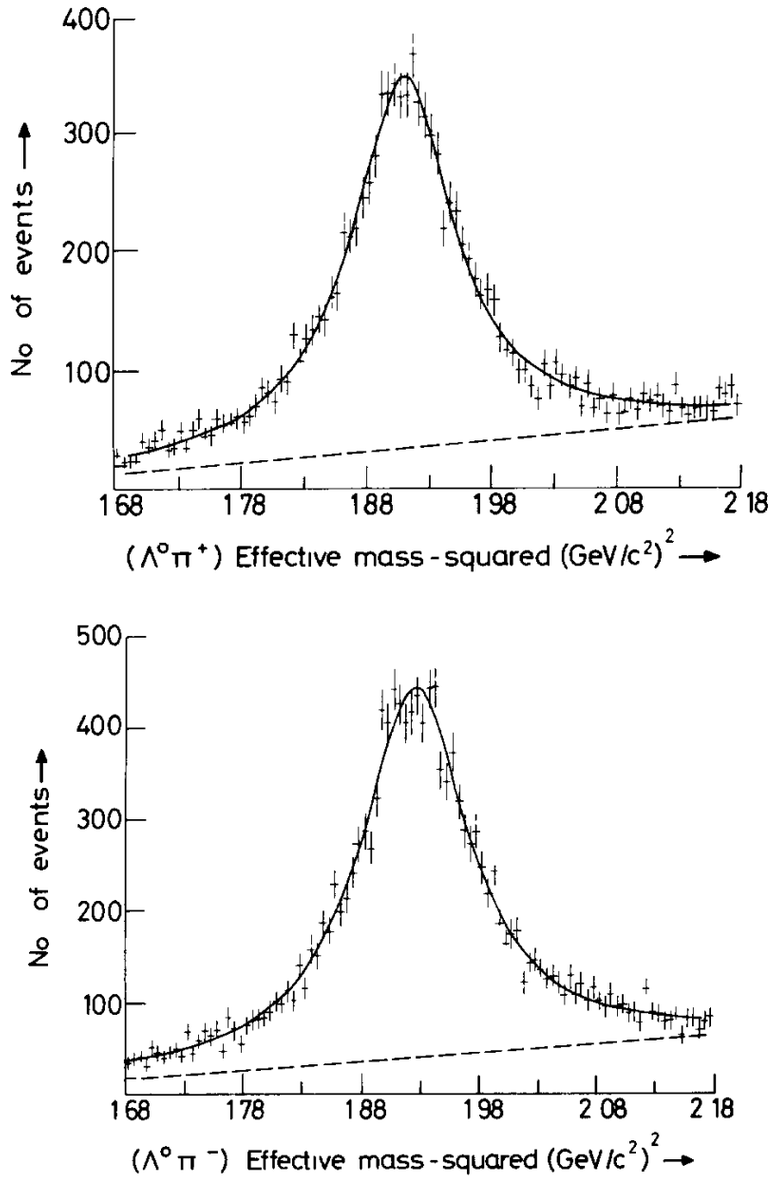
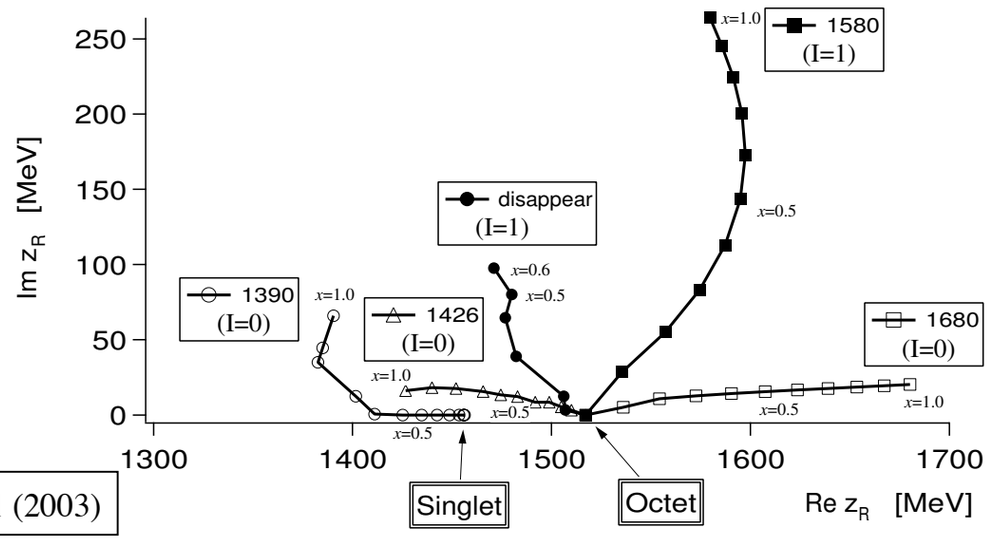


Fig. 4. (a) $\Lambda\pi^+$ and (b) $\Lambda\pi^-$ mass-squared distributions in the region of the $\Sigma(1385)$ resonance The curves are the result of the FIT 1 described in the text, the dashed line indicates the background under the resonance

Theory

- Chiral Unitary Theory (χUT): combine chiral Lagrangians of low-momentum interactions + unitarity between channels
- "Fundamental" states (ground state pseudoscalar mesons, baryons) can "dynamically generate" resonances
- Within χUT, Λ(1405) is textbook example
- Recent developments predict that there are 2 poles near the Λ(1405), each excited differently depending on the reaction

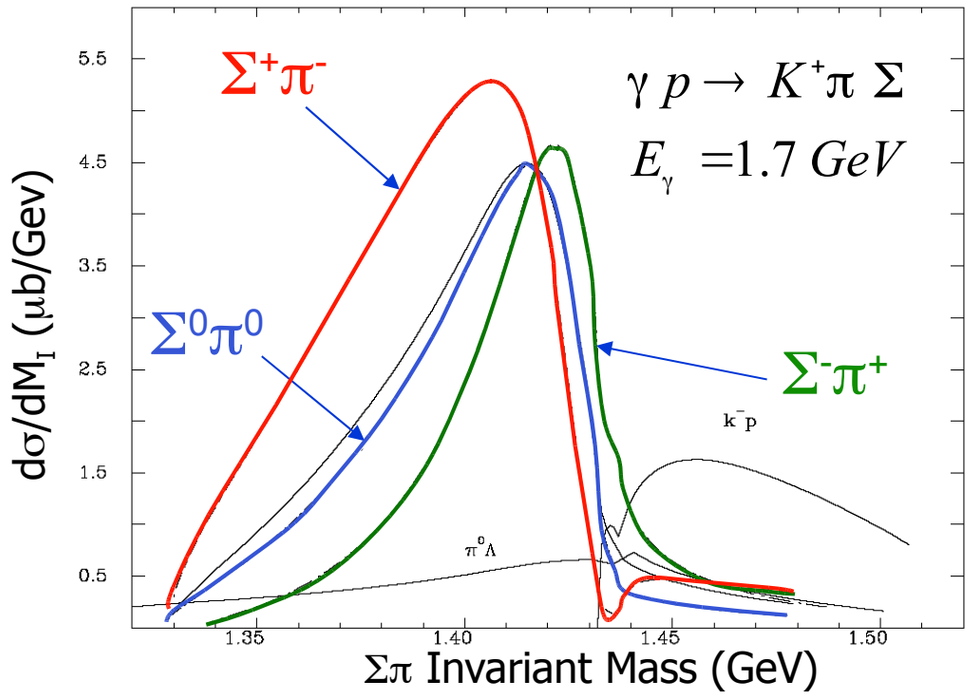
- Pole positions when breaking SU(3) symmetry
- 2 poles appear in the Λ(1405) mass region



D. Jido, J. A. Oller, E. Oset, A. Ramos, U.-G. Meissner Nucl. Phys. A 725, 181 (2003)

χUT Prediction

- χUT predicted photoproduction line shape of the Λ(1405)
- Distortion due to the $N\bar{K}$ and other channels
- Interference with small *I=1* amplitude causes differences in Σπ channels
- More recent summary by Hyodo, Jido (Prog. Part. Nucl. Phys. 67, 55)



J. C. Nacher, E. Oset, H. Toki, A. Ramos, Phys. Lett. B 455, 55 (1999)

I=1 *I=0* *interference of I = 0,1*

$$\frac{d\sigma(\pi^+\Sigma^-)}{dM_I} \propto \frac{1}{2}\left|T^{(1)}\right|^2 + \frac{1}{3}\left|T^{(0)}\right|^2 + \frac{2}{\sqrt{6}}\mathrm{Re}\left(T^{(0)}T^{(1)*}\right) + O\left(T^{(2)}\right)$$

$$\frac{d\sigma(\pi^-\Sigma^+)}{dM_I} \propto \frac{1}{2}\left|T^{(1)}\right|^2 + \frac{1}{3}\left|T^{(0)}\right|^2 - \frac{2}{\sqrt{6}}\mathrm{Re}\left(T^{(0)}T^{(1)*}\right) + O\left(T^{(2)}\right)$$

$$\frac{d\sigma(\pi^0\Sigma^0)}{dM_I} \propto \frac{1}{3}\left|T^{(0)}\right|^2 + O\left(T^{(2)}\right)$$