

# New Results on Spin Structure Functions at Very Low Momentum Transfers ( $Q^2$ ) from Jefferson Lab

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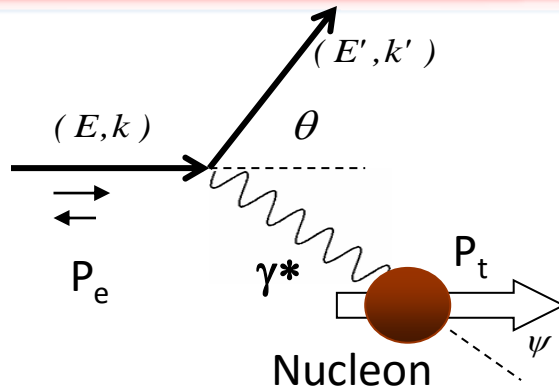
For the CLAS collaboration

- Spin structure at Low  $Q^2$  - Formalism and Motivation
- Recent Data
- Summary and Outlook



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UNIVERSITY

# Inclusive Lepton Scattering & Structure Functions



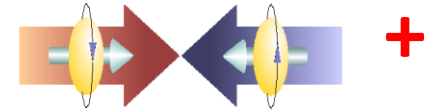
Kinematics

$$\nu = E - E'$$

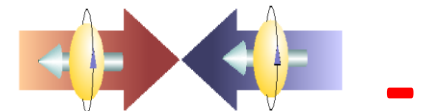
$$Q^2 = 4EE' \sin^2\left(\frac{\theta}{2}\right)$$

$$W^2 = M^2 + 2M\nu - Q^2$$

$$x = \frac{Q^2}{2M\nu}$$



Versus



**Cross-section:** In the case of a target polarization along the beam:

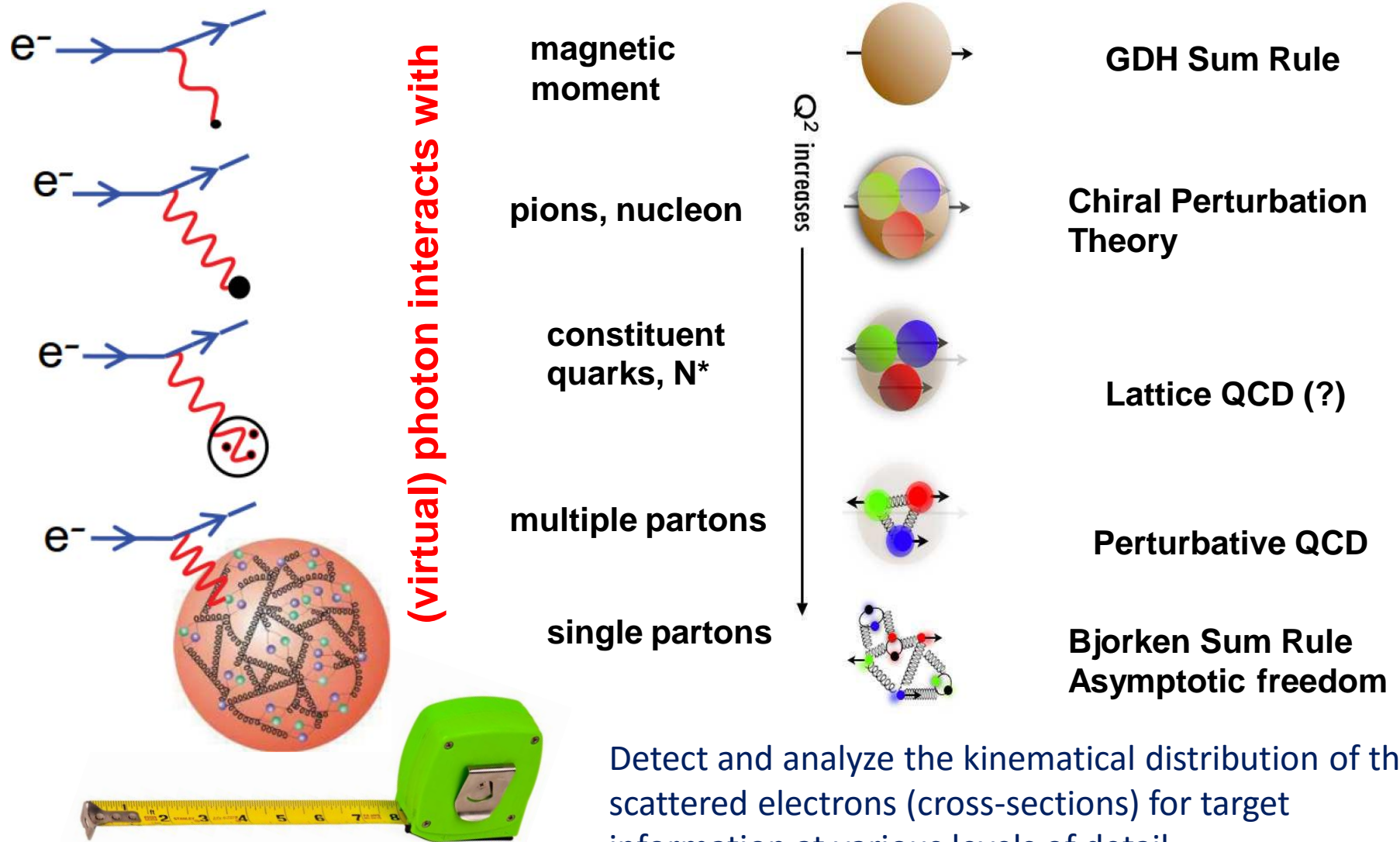
Spin independent

$$\frac{d^2\sigma}{dE' d\Omega} = \sigma_{\text{point}} \left[ \frac{2}{M} F_1(Q^2, \nu) \tan^2 \frac{\theta}{2} + \frac{F_2(Q^2, \nu)}{\nu} \right. \\ \left. \left[ \pm 2 \tan^2 \frac{\theta}{2} \left[ (E + E' \cos \theta) \frac{M^2}{\nu} g_1(Q^2, \nu) - \gamma^2 M^2 g_2(Q^2, \nu) \right] \right] \right]$$

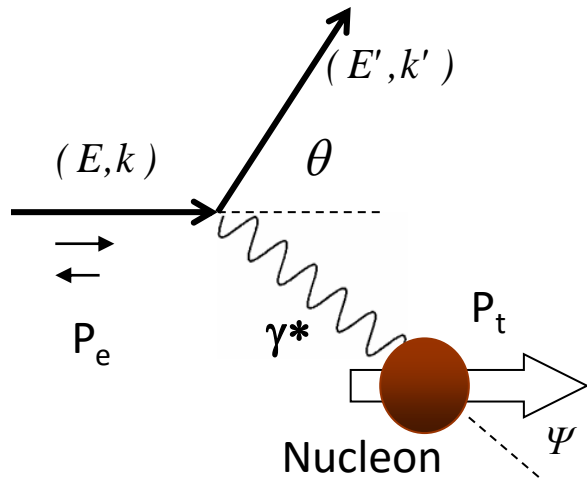
With  $\gamma^2 = \frac{Q^2}{\nu^2}$  and, **+/- for antiparallel/parallel** beam-target polarizations

In the DIS limit, structure functions are directly related to polarized and unpolarized quark distribution functions

# Nucleon Structure vs Distance Scale



# View in Terms of the Photo-absorption Cross Sections and Asymmetries



- Lepton scattering can be viewed as **the two step interaction** - emission of a virtual photon and then the absorption of the photon by the target.
- Therefore, the **spin structure functions are related with four independent virtual photo-absorption cross sections and their asymmetries**:

Certain combinations of the virtual photo-absorption cross sections provide us with certain structure functions, such as:

$$g_1 = \frac{MK}{8\pi^2\alpha(1+\gamma^2)} (\sigma_{1/2}^T - \sigma_{3/2}^T + 2\gamma\sigma_{1/2}^{TL})$$

photon-spin 1  $\Rightarrow$  nucleon-spin  $+1/2$   
 $\sigma_{3/2}$

photon-spin 1  $\Rightarrow$  nucleon-spin  $-1/2$   
 $\sigma_{1/2}$

→

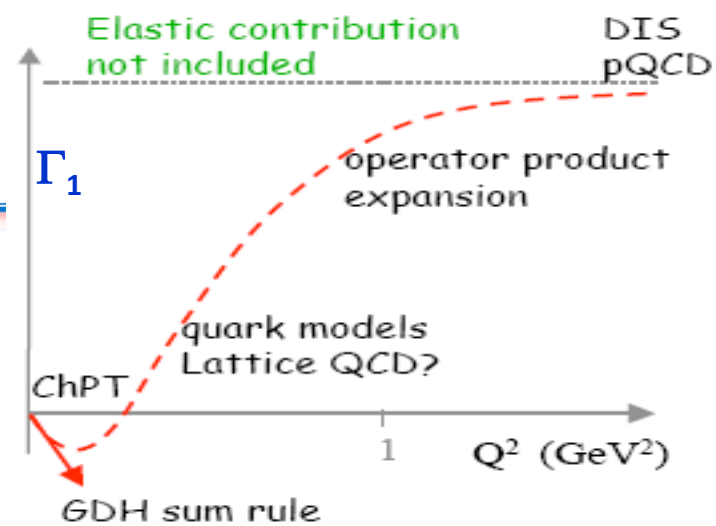
$A_1(x, Q^2) = \frac{\sigma_{1/2}^T - \sigma_{3/2}^T}{\sigma_{1/2}^T + \sigma_{3/2}^T} = \frac{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)}{F_1(x, Q^2)}$

$A_2(x, Q^2) = \frac{2\sigma_{1/2}^{TL}}{\sigma_{1/2}^T + \sigma_{3/2}^T} = \frac{\gamma[g_1(x, Q^2) + g_2(x, Q^2)]}{F_1(x, Q^2)}$

Virtual photon asymmetries

with  $\gamma^2 = \frac{4M^2x^2}{Q^2} = \frac{Q^2}{\nu^2}$

# Low $Q^2$ Motivation - Integrals



Interesting variation with  $Q^2$

Hadron  $\longrightarrow$  Parton

$$\bar{\Gamma}_1 = \int_0^{x_{th}} g_1(x, Q^2) dx$$

- Relativistic Baryon  $\chi$ PT with  $\Delta$ , Bernard, Hemmert, Meissner;
- Heavy Baryon  $\chi$ PT, Ji, Kao, Osborne; Kao, Spitzenberg, Vanderhaeghen
- Lensky, Alarcón, Pascalutsa, PRC90, 055202 (2014).
- Bernard, Epelbaum, Krebs, Ulf-G. Meißner, Phys. Rev. D 87, 054032 (2013)

$$\bar{I}_{TT} = \frac{2M^2}{Q^2} \int_0^{x_{th}} \left( g_1 - \frac{2M^2 x^2}{Q^2} g_2 \right) dx \xrightarrow{(Q^2 \rightarrow 0)} -\frac{\kappa^2}{4}$$

**GDH**

$$\gamma_0 = \frac{16\alpha M^2}{Q^6} \int_0^{x_{th}} \left( g_1 - \frac{2M^2 x^2}{Q^2} g_2 \right) x^2 dx$$

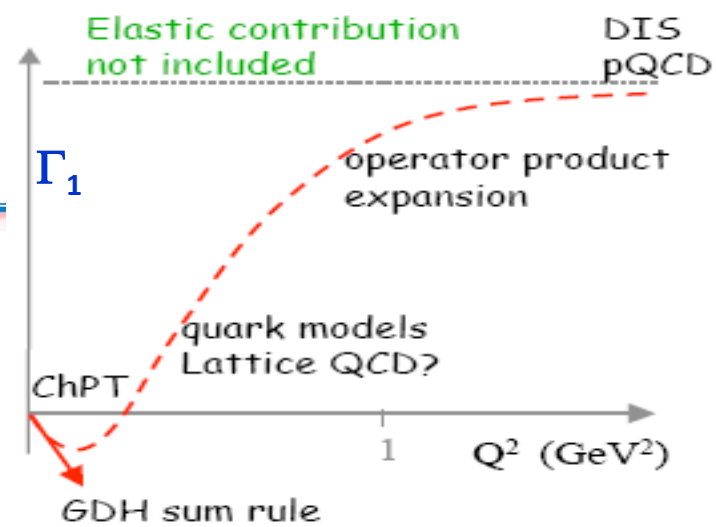
$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx$$

# Low $Q^2$ Motivation - First Moment of $g_1$

- At low momentum transfers ( $Q^2$ ), one can study the **transition from partonic (quark-gluon) to hadronic (nucleonic) descriptions of Strong interaction by testing & constraining effective theories** based on QCD such as Chiral Perturbation Theory ( $\chi$ PT).

- In the **parton model**, it is the **fraction of the nucleon spin** contributed by the quark helicities

- Enters directly into two historically important sum rules - **Ellis-Jaffe sum rule (Spin Crisis) and Bjorken sum rule (QCD validation)**.
- Some low  $Q^2$  predictions from  **$\chi$ PT** and **phenomenological models**



Interesting variation with  $Q^2$

Hadron  $\longrightarrow$  Parton

$$\bar{\Gamma}_1 = \int_0^{x_{th}} g_1(x, Q^2) dx$$

$$\bar{I}_{TT} = \frac{2M^2}{Q^2} \int_0^{x_{th}} \left( g_1 - \frac{2M^2 x^2}{Q^2} g_2 \right) dx \xrightarrow{(Q^2 \rightarrow 0)} -\frac{\kappa^2}{4}$$

**GDH**

$$\gamma_0 = \frac{16\alpha M^2}{Q^6} \int_0^{x_{th}} \left( g_1 - \frac{2M^2 x^2}{Q^2} g_2 \right) x^2 dx$$

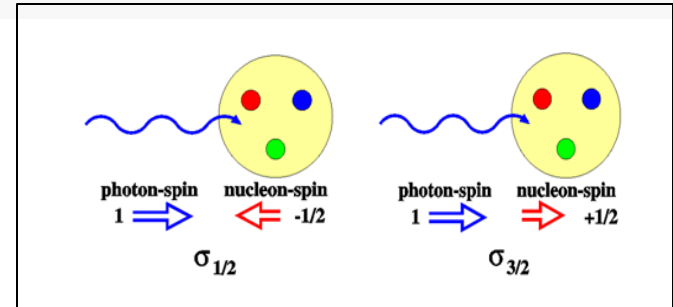
$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx$$

# Low $Q^2$ Motivation - Generalized GDH Integral

**Gerasimov, Drell and Hearn (GDH) Sum Rule** (PRL 16(1966) 908; Sov.J.Nucl Phys.2 (1966) 430)

$$\int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu} \left( \sigma^{3/2}(\nu) - \sigma^{1/2}(\nu) \right) = \frac{2\pi^2 \alpha \kappa^2}{M^2}$$

$\nu_{th}$  is pion production threshold,  $\kappa$  = anomalous magnetic moment



**First measurements at Mainz** (up to 800 MeV), **Bonn** (up to 3 GeV)

& **LEGS** (up to 421 MeV) agree with the predictions.

**Extension of GDH Sum Rule to  $Q^2 > 0$  (virtual photons)** using the dispersion relation for the **forward virtual photon Compton scattering amplitude  $S_1(\nu=0, Q^2 > 0)$** , just as the real photon GDH sum rule derived from the **dispersion relation for the invariant Compton amplitude  $S_1(\nu, Q^2 = 0)$** .

$$\bar{I}_{TT}(Q^2) = \frac{2M^2}{Q^2} \int_0^{x_{th}} \left( g_1 - \frac{2M^2 x^2}{Q^2} g_2 \right) dx \xrightarrow{(Q^2 \rightarrow 0)} -\frac{\kappa^2}{4}$$

**GDH**

The new sum rule **yields the GDH sum rule at  $Q^2=0$ , and the Bjorken sum rule in the DIS regime**, thus establishing a smooth connection between the hadronic worlds of quark-confinement & partonic world of asymptotic freedom.



# Low $Q^2$ Motivation - Generalized Nucleon Spin Polarizabilities



Forward spin polarizability

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right] dx$$

Sensitive to resonances

Longitudinal transverse spin polarizability

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx$$

Insensitive to the  $\Delta$  resonance

- **Generalized polarizabilities:** extensions polarizabilities to the case of virtual photon Compton scattering.
  - can be calculated in  $\chi$ PT at low  $Q^2$ .
  - **converge faster** than the first moments due to power weighting by  $1/v$  or  $x$  and **thus easier to determine from the low energy measurements.**
  - Thus, **reduced dependence on the extrapolations to the unmeasured regions at large  $v$  or small  $x$ , and higher sensitivity to the low energy behavior** of the cross sections (particularly the threshold behaviour), **hence better tools to test  $\chi$ PT and phenomenological model predictions**



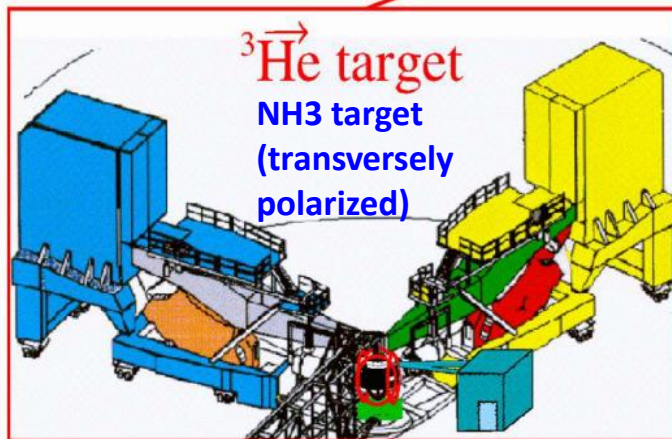
# Jefferson Lab Experimental Halls

6 GeV polarized  
CW electron beam  
Pol=85%, 200 $\mu$ A

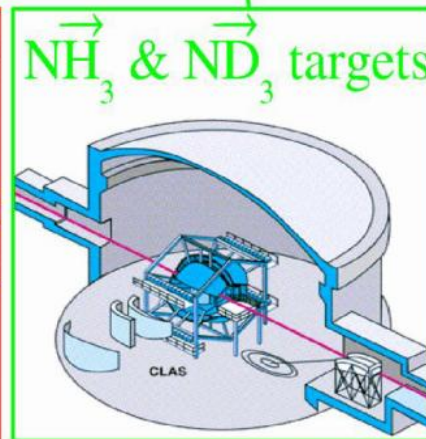


Now upgraded  
to 12 GeV

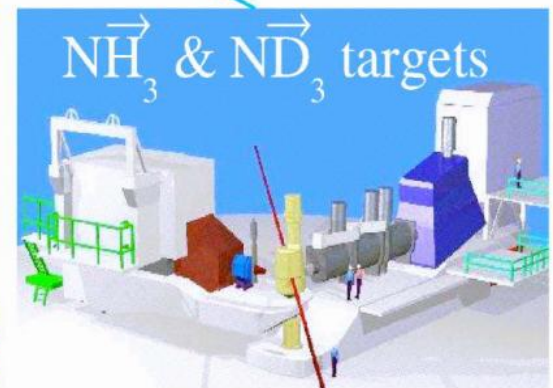
The electron beam can  
be delivered  
simultaneously to the  
three halls with high  
polarization



Hall A: two HRS'

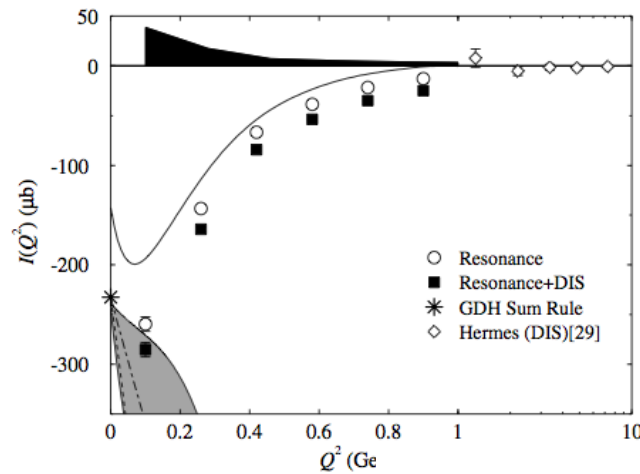
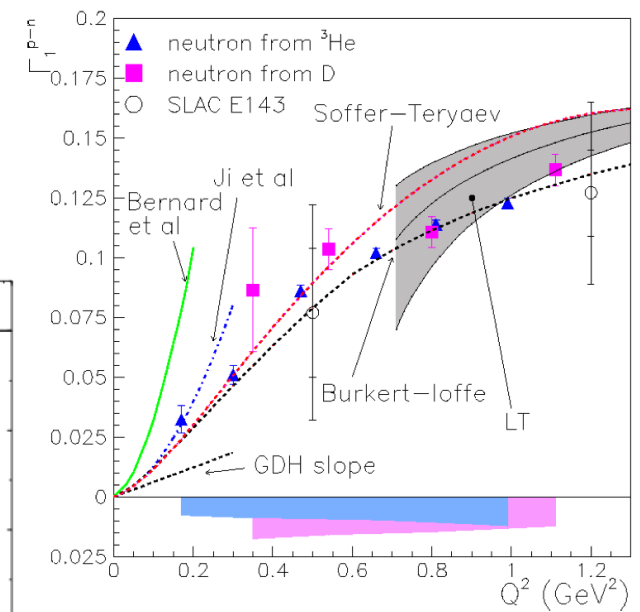


Hall B: CLAS

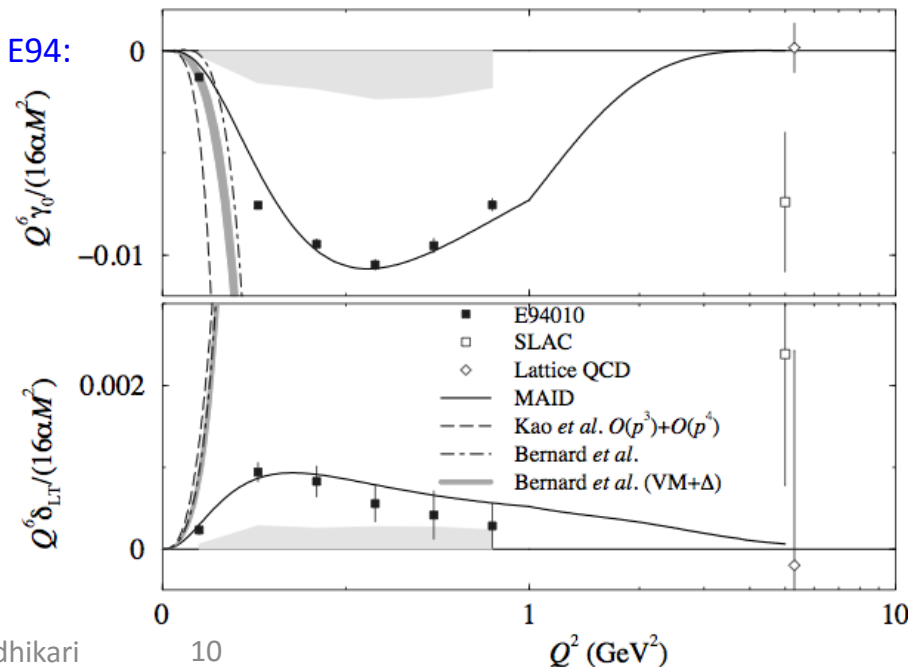
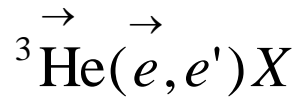


Hall C: HMS+SOS

# Hall A E94-010 Experiment



Hall A publications from E94:  
 PRL 101, 022303 (2008)  
 PRL 93, 152301 (2004)  
 PRL 92, 022301 (2004)  
 PRC 70, 065207 (2004)



# Hall-A E97-110: Low Q<sup>2</sup> Spin Structure

Spokesmen: J.-P. Chen, A. Deur, F. Garibaldi

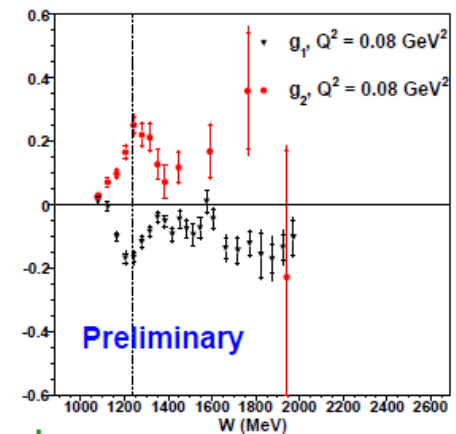
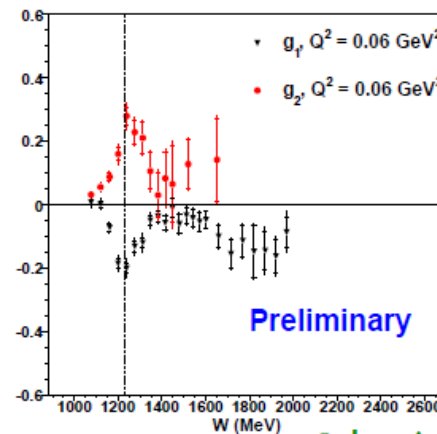
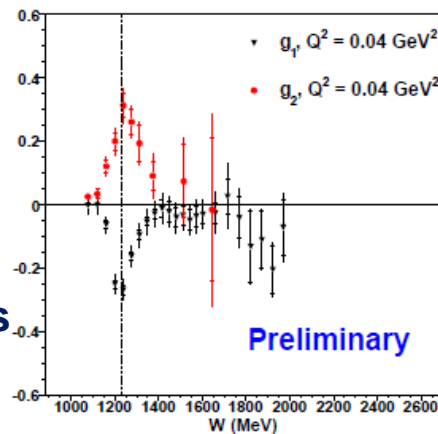
## $g_1, g_2$ results

➤ Inclusive measurement:  ${}^3\text{He}(e, e')X$

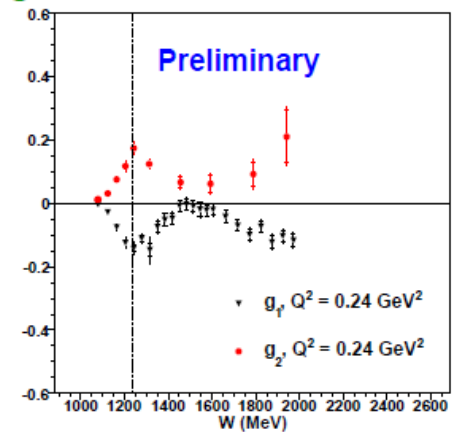
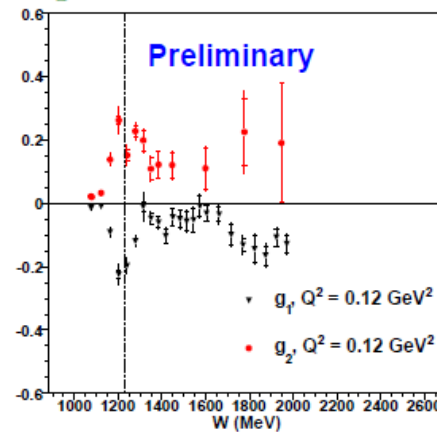
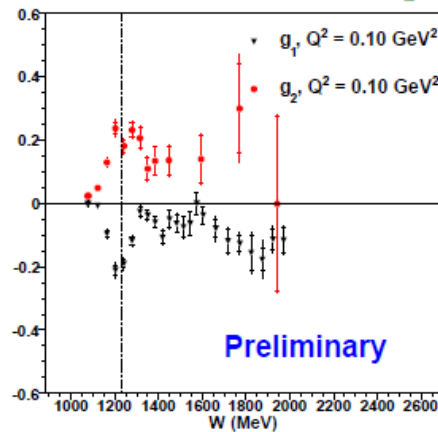
- Scattering angles of 6° and 9°
- Polarized (~ 75%) electron beam

➤ Pol. (~ 40%)  
 **${}^3\text{He}$  target**  
(para & perp)

➤ Measured  
polarized cross-  
section differences



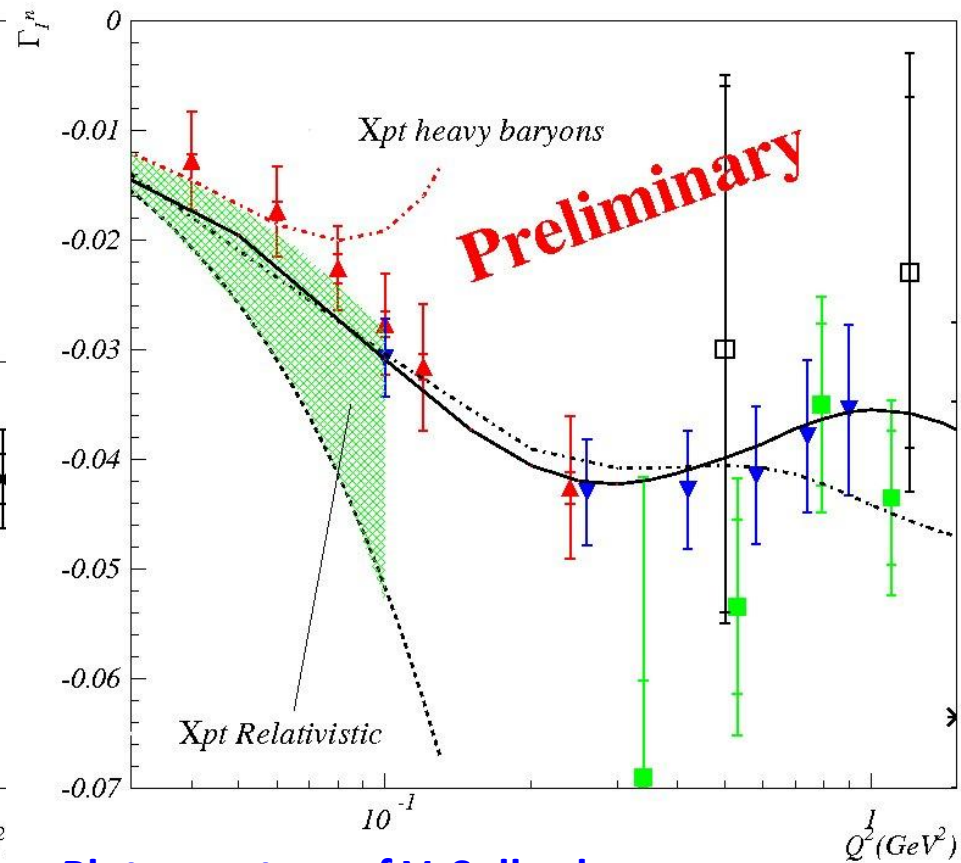
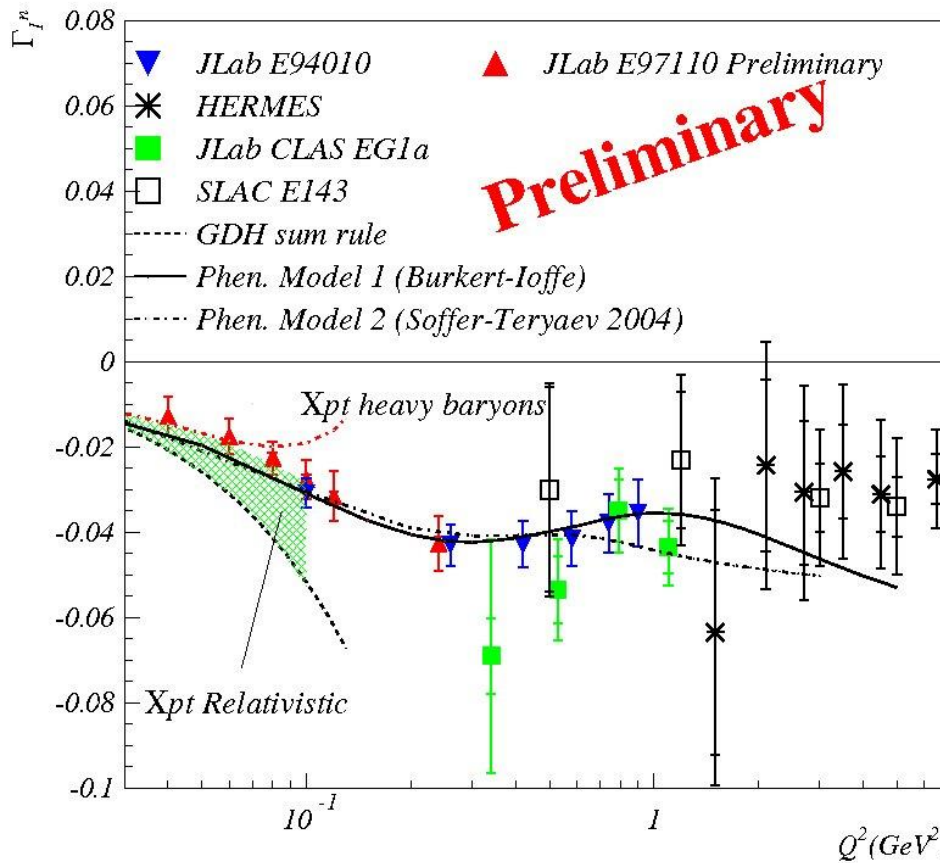
$g_2 \approx -g_1 \Rightarrow \sigma_{LT} \approx 0$  in  $\Delta$  region



Plots courtesy of  
V. Sulkosky

# E97-110: First Moment of $g_1^n$

$$\Gamma_1 = \int_0^{x_0} g_1(x, Q^2) dx$$



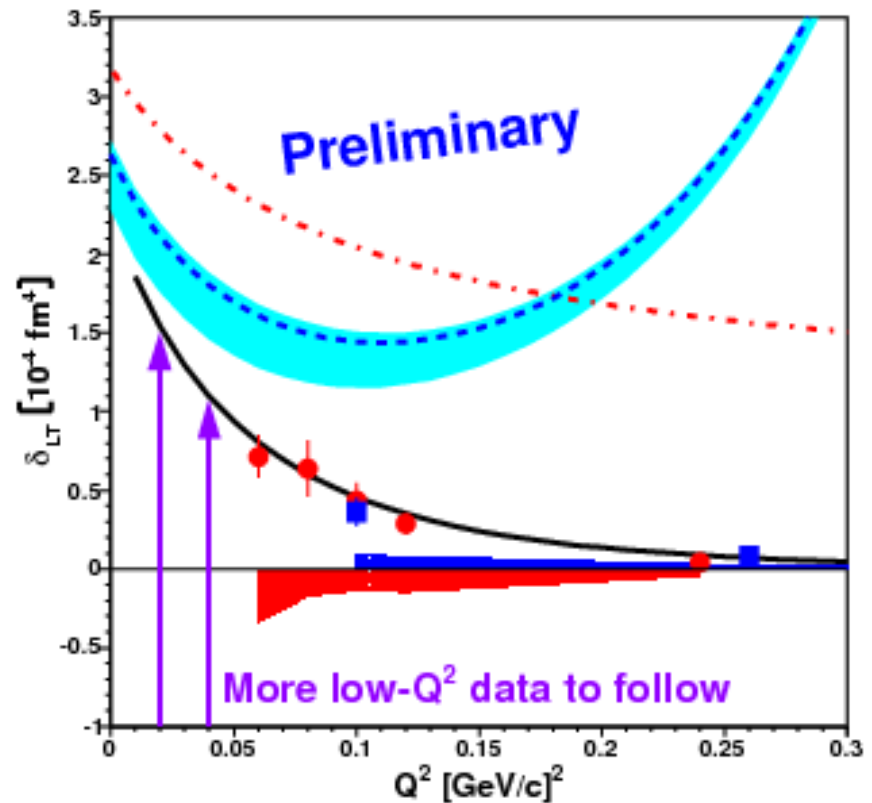
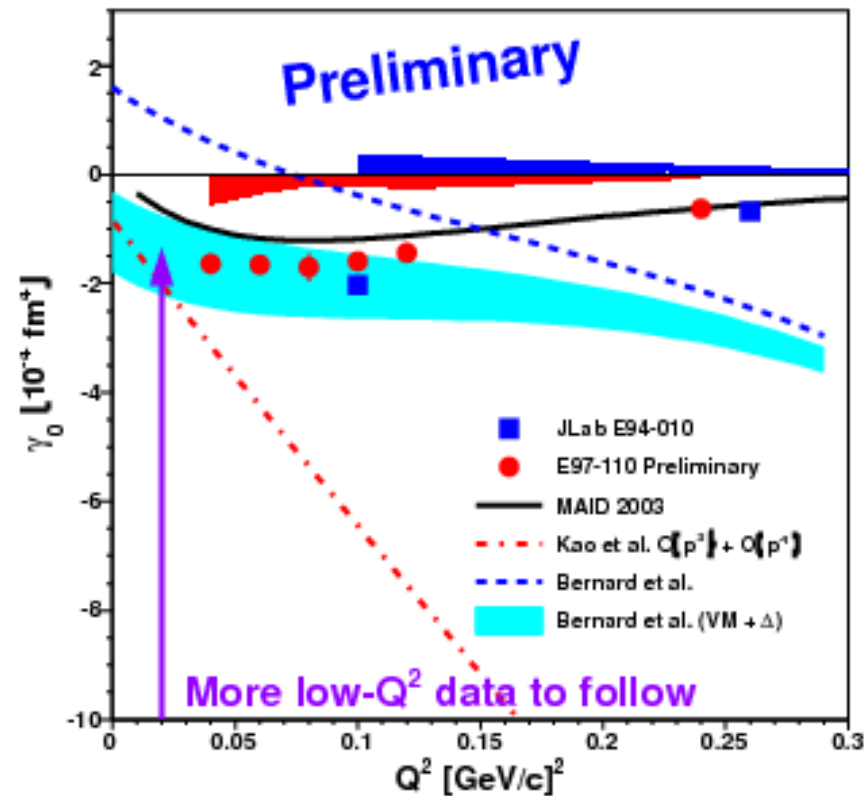
Plots courtesy of V. Sulkosky



# E97-110: New Results for Neutron Spin Polarizabilities

$$\gamma_0 = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right]$$

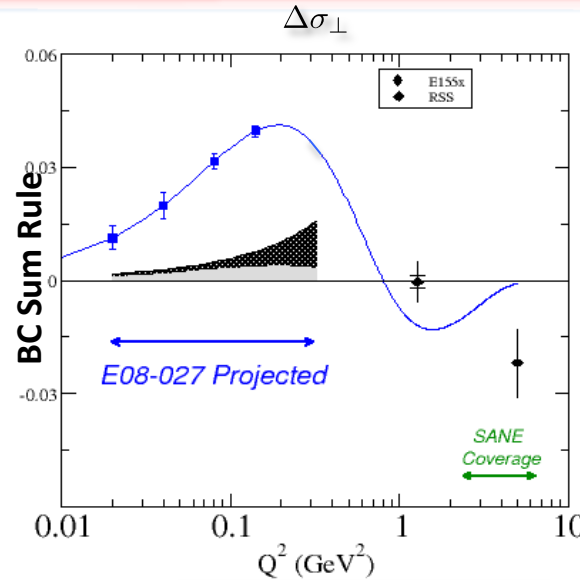
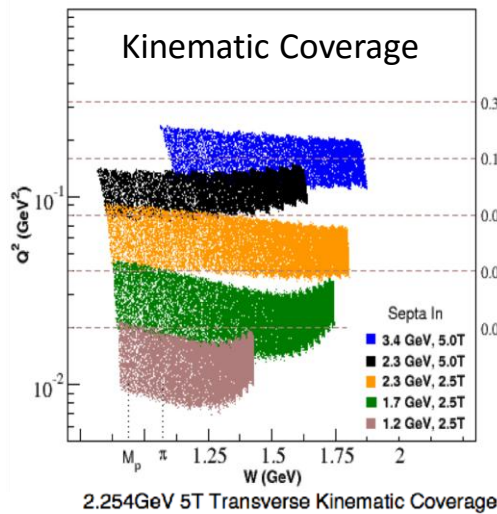
$$\delta_{LT} = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2]$$



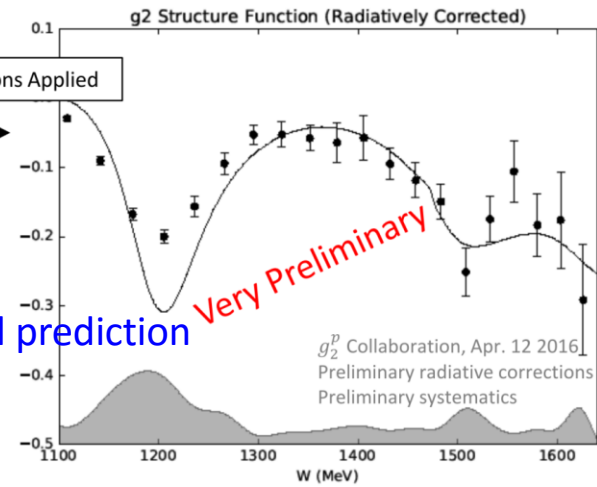
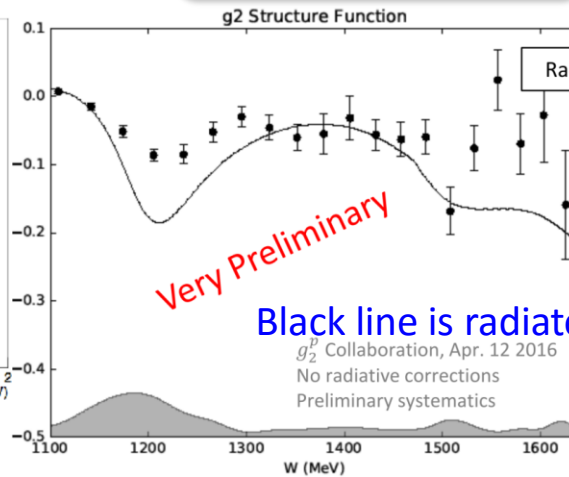
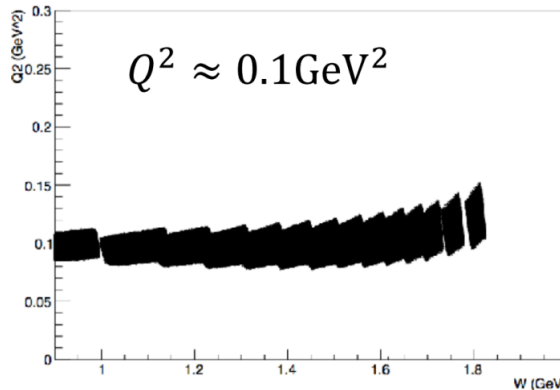
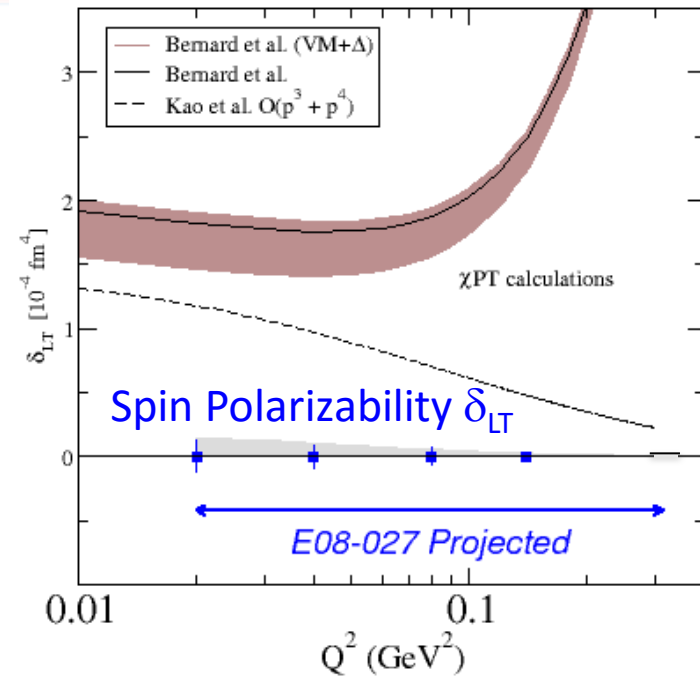
Surprising failure of  $\chi$ PT: no  $\Delta$  resonance contribution expected for  $\delta_{LT}$

# Hall A g2p: Proton Spin Structure Functions

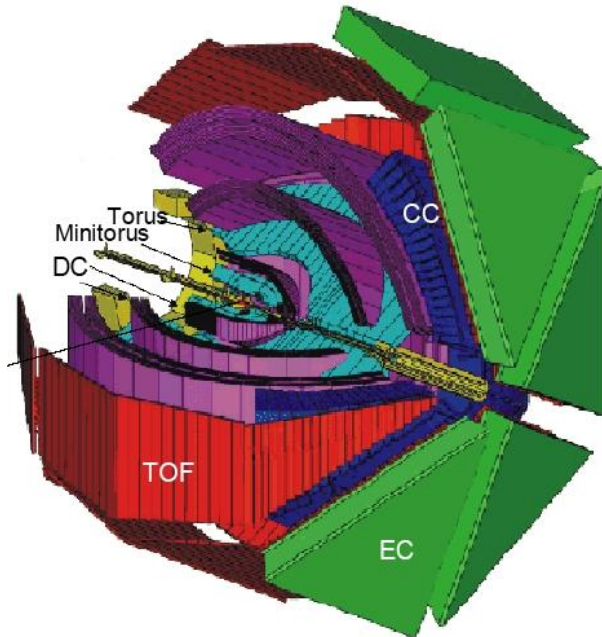
Plots courtesy of  
R. Zielinski & T. Badman



$M_p < W < 2$  GeV  
 $0.02 < Q^2 < 0.2$  GeV<sup>2</sup>



# Hall B CLAS Experiment EG1



Longitudinally polarized ( $\sim 70\%$ ) electrons

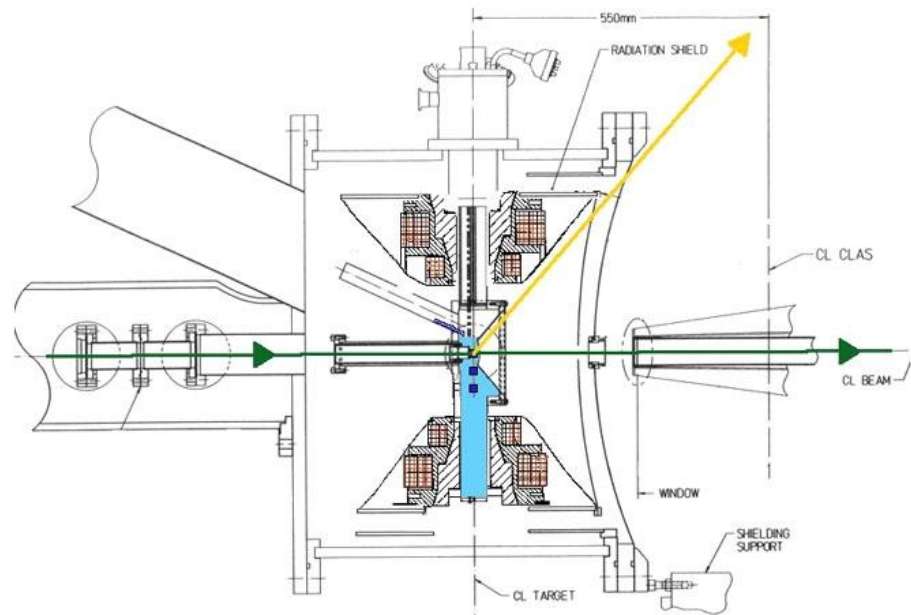
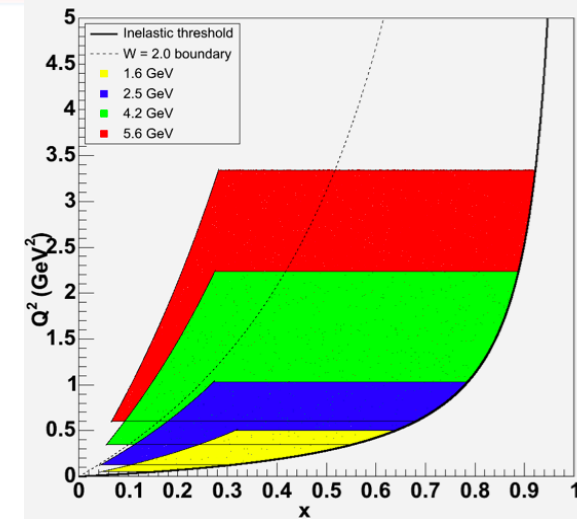
Beam energies: 1.6, 2.5, 4.2, 5.7 GeV

Polarized solid ammonia targets  $\text{NH}_3$ ,  $\text{ND}_3$

Measured large range in  $Q^2$  and  $W$

$0.05 < Q^2 < 5.0 \text{ GeV}^2$  in 39 bins

$W < 3.0 \text{ GeV}$  in 10 MeV 300 bins



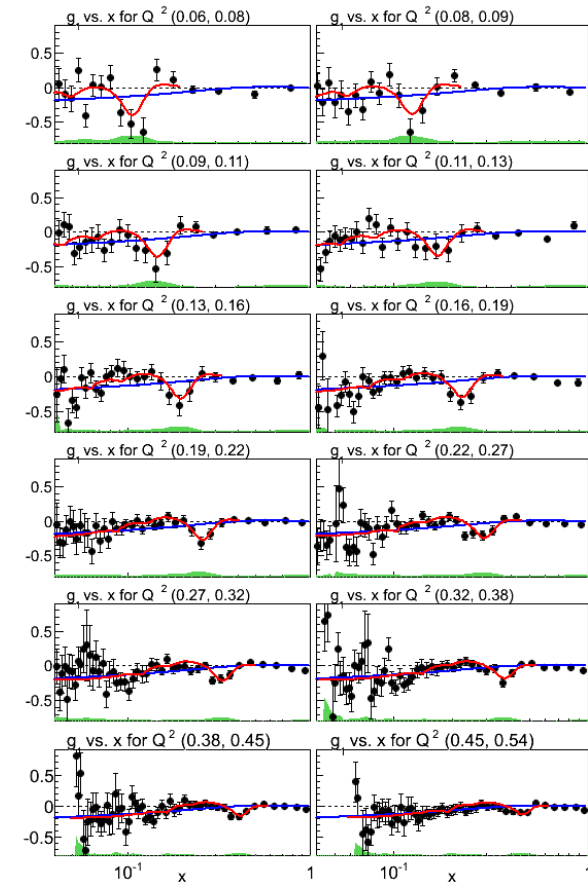
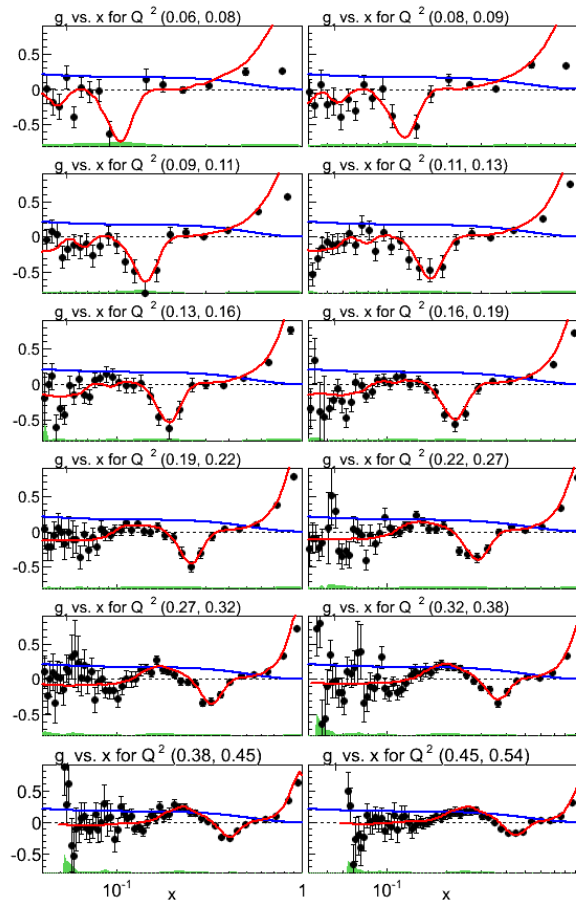
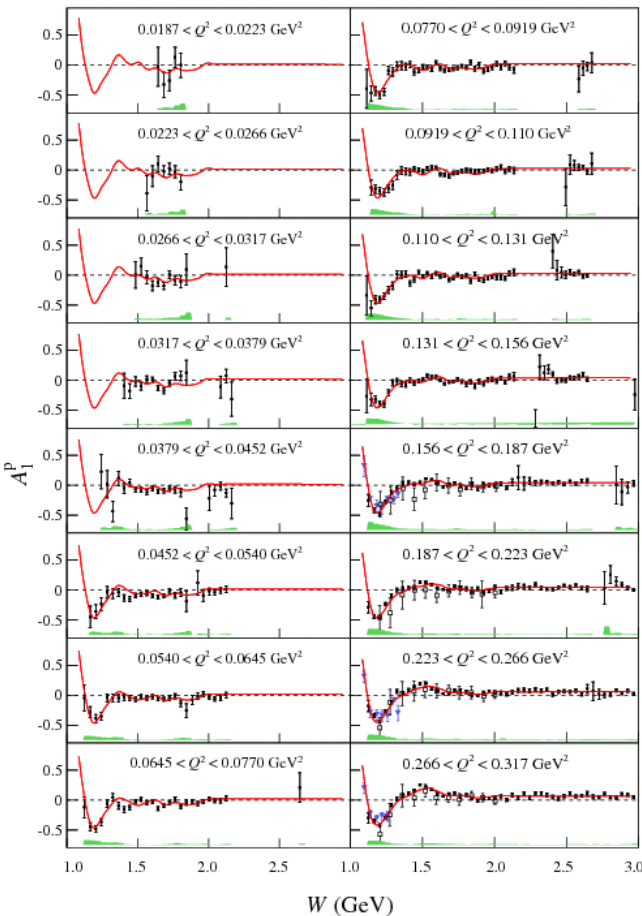


# EG1b: $g_1$ at Low $Q^2$

## Proton

## Deuteron

## Neutron



Plots courtesy of R. Fersch

N. Guler et al – PhysRevC.92.055201

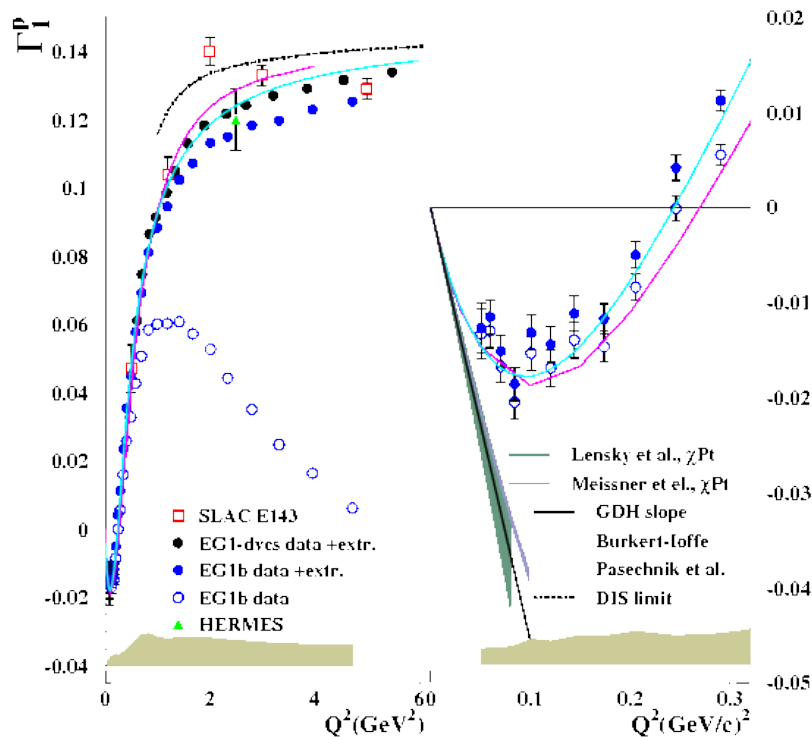
# EG1 results: $\Gamma_1 = \int g_1(x, Q^2) dx$

## Deuteron & neutron results from N. Guler

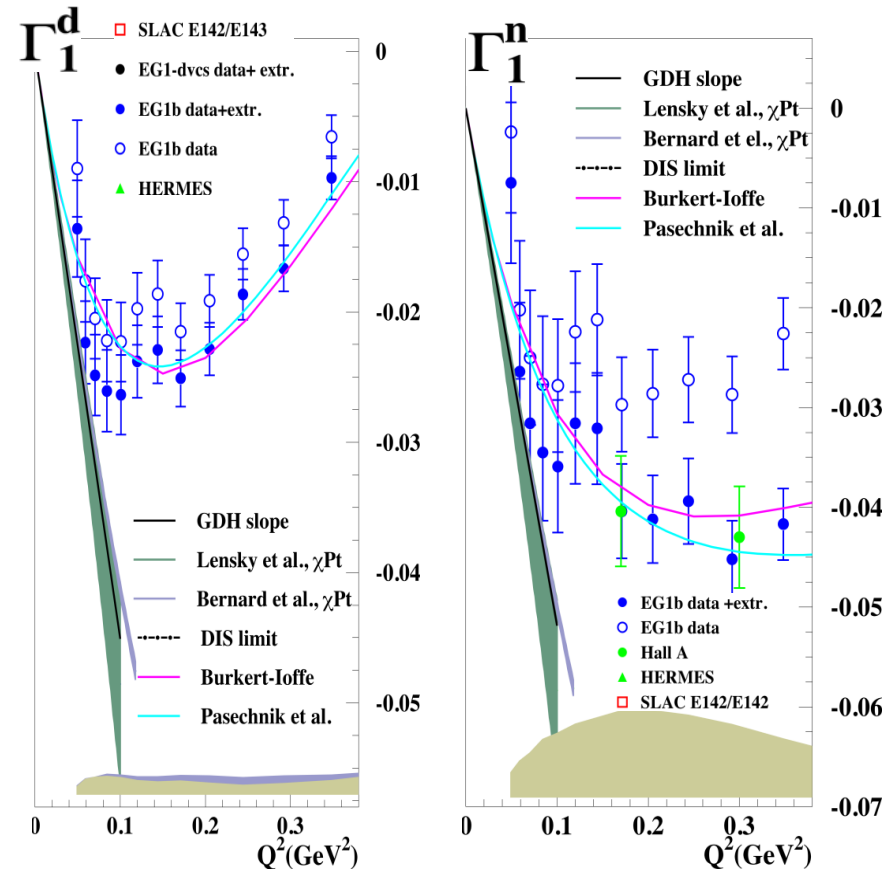
Shows expected trend toward DIS result at high  $Q^2$

At low  $Q^2$  observed a negative slope as expected from GDH Sum Rule.

Agreement with  $\chi$ PT at the lowest points.



## Proton results from R. Fersch



Deuteron analysis was repeated with full data set and more extended results were obtained for the  $\Gamma_1$  at low to moderate  $Q^2$  region.

# EG1: $\gamma_0$ from p,d, & n

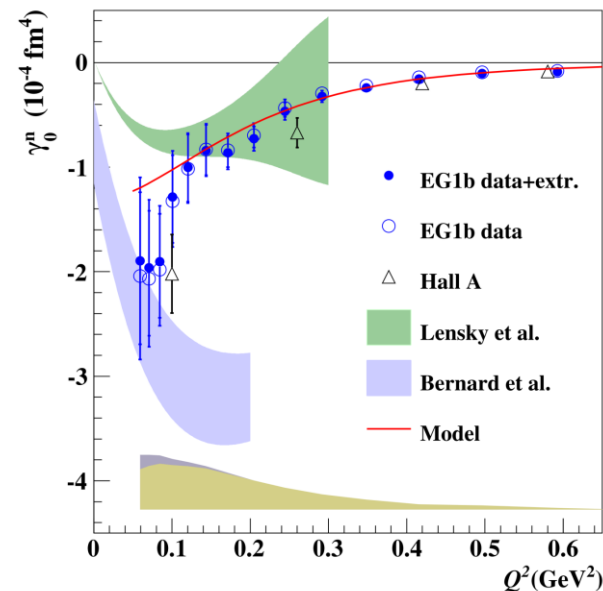
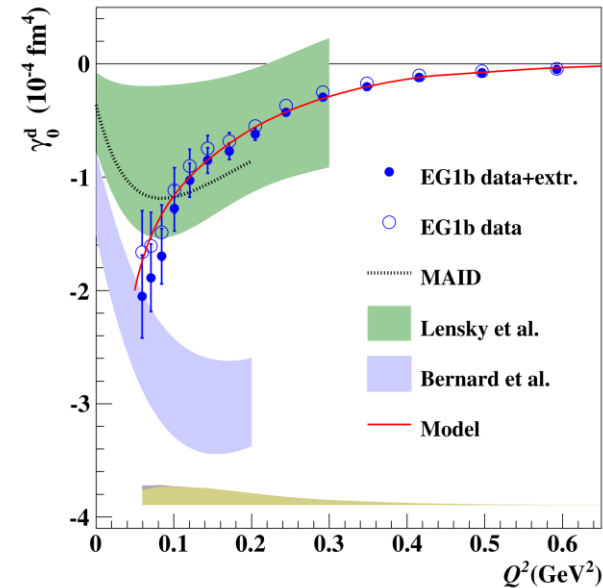
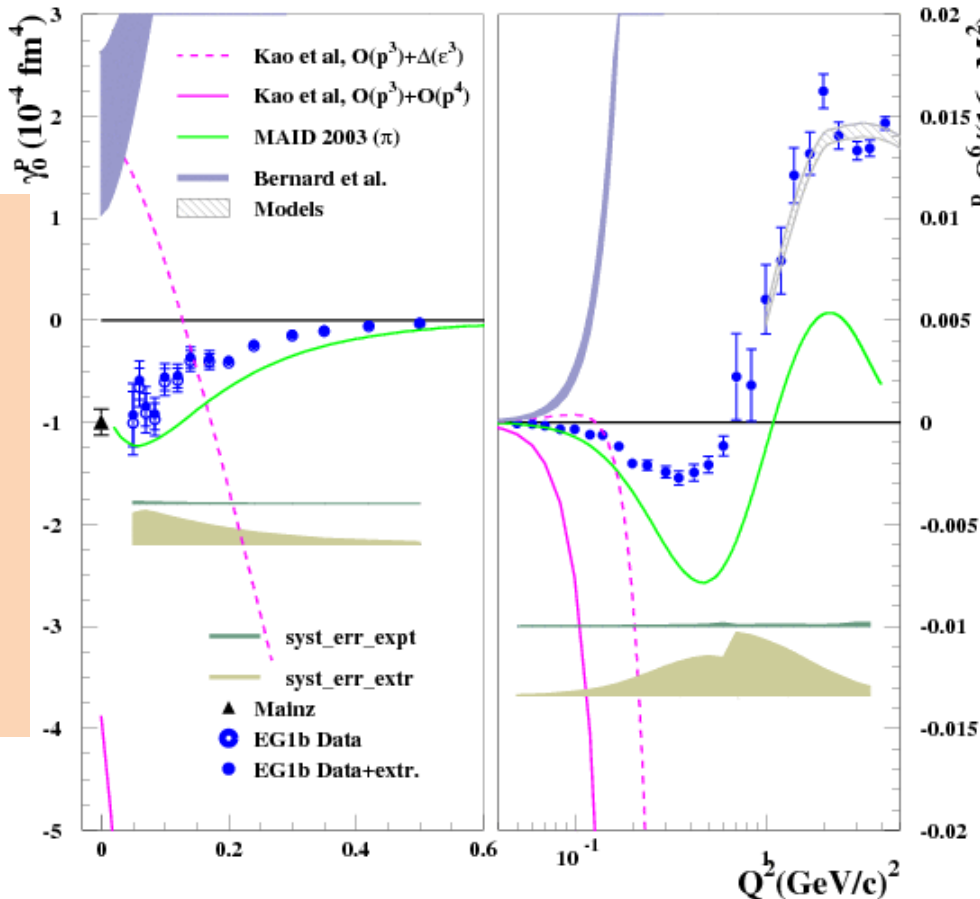
$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} A_1 F_1 x^2 dx$$

$F_1$  obtained from fit to world data

Discrepancy with MAID - mainly due to parametrizations of  $F_1$ .

No agreement with  $\chi$ PT, even at  $Q^2 = 0.05 \text{ GeV}^2$

Observed some evidence of expected  $Q^6$  scaling at  $Q^2 \approx 1.5 \text{ GeV}^2$



# Hall B CLAS Experiment: EG4

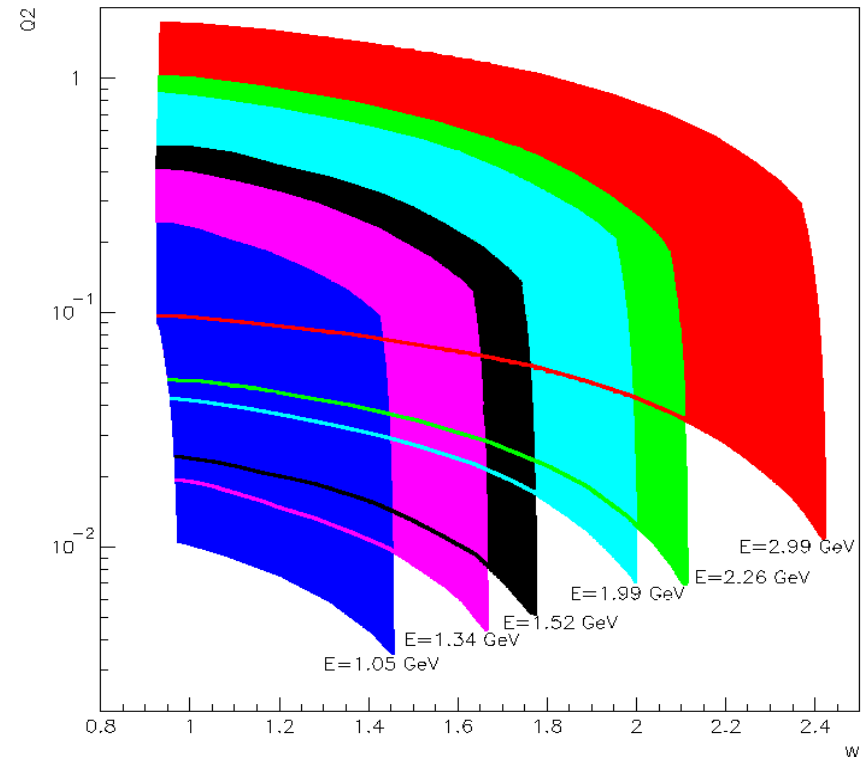
## “EG4”:

- Similar conditions to **EG1**
- Kinematical coverage extended down to  $Q^2 = 0.015 \text{ GeV}^2$  using
  - lower beam energies -1.0, 1.3, 2.0, 2.3, 3.0 GeV
  - electron outbending CLAS configuration
  - a new Cerenkov detector.
- Measurement of  $g_1$  at low  $Q^2$
- Test of  $\chi$ PT as  $Q^2 \rightarrow 0$
- Measured Absolute XS differences
- Goal : **Extended GDH Sum Rule**  
**Proton**  
**Deuteron**
- Ran in 2006

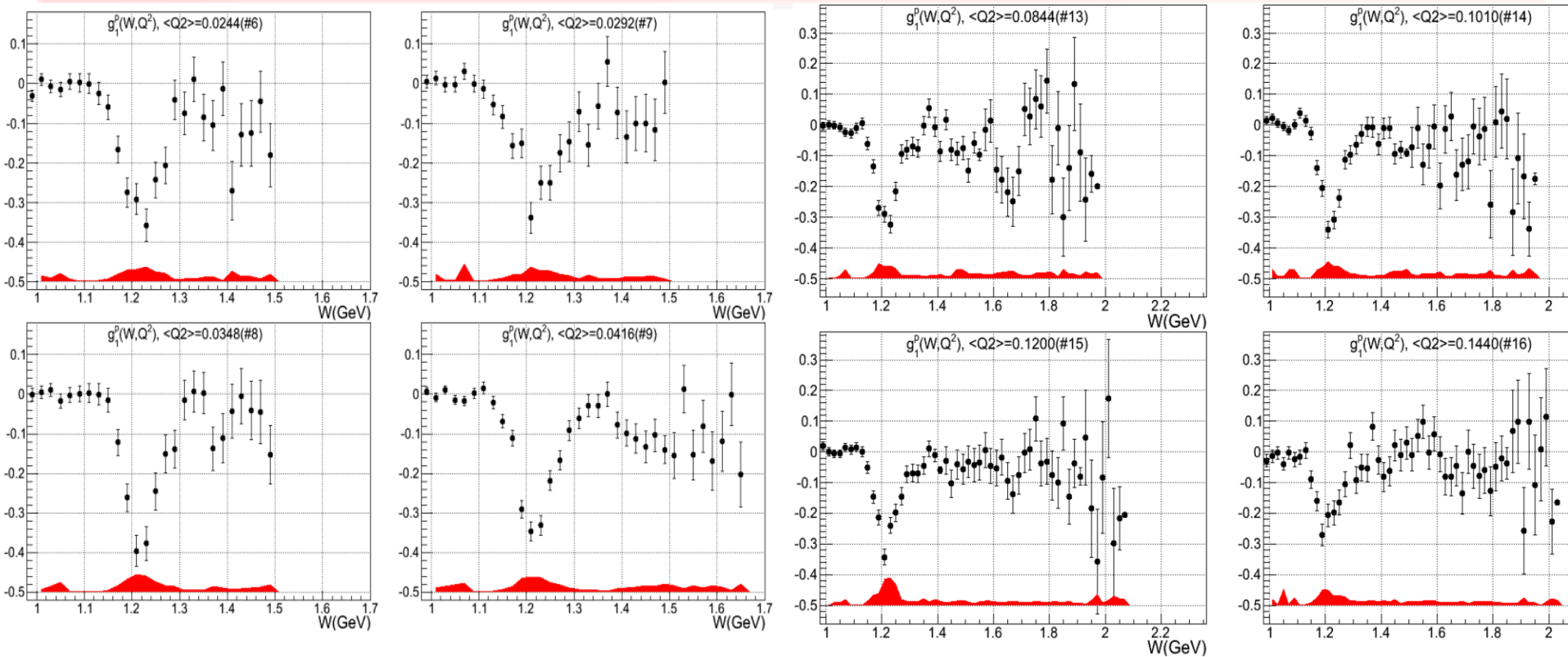
## Spokespersons:

**NH<sub>3</sub>:** M. Battaglieri, A. Deur, R. De Vita, M. Ripani

**ND<sub>3</sub>:** A. Deur, G. Dodge, K. Slifer



# EG4 – Proton $g_1$ Results



Red bands are total systematic uncertainties.

# Preliminary

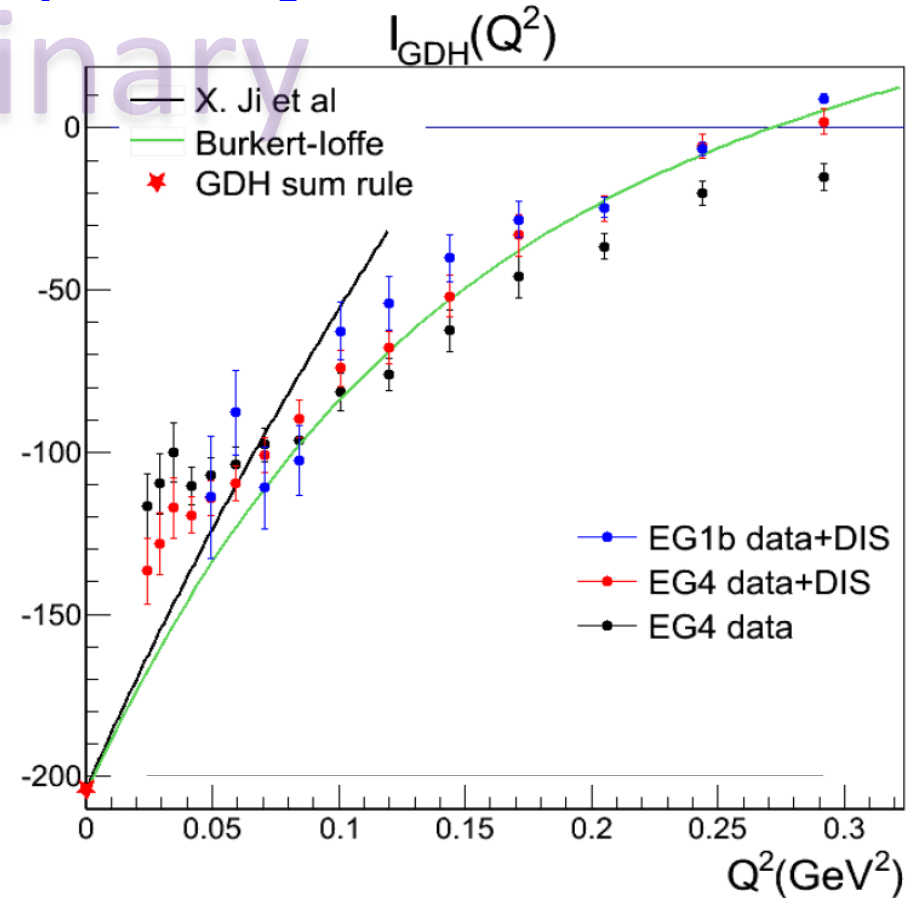
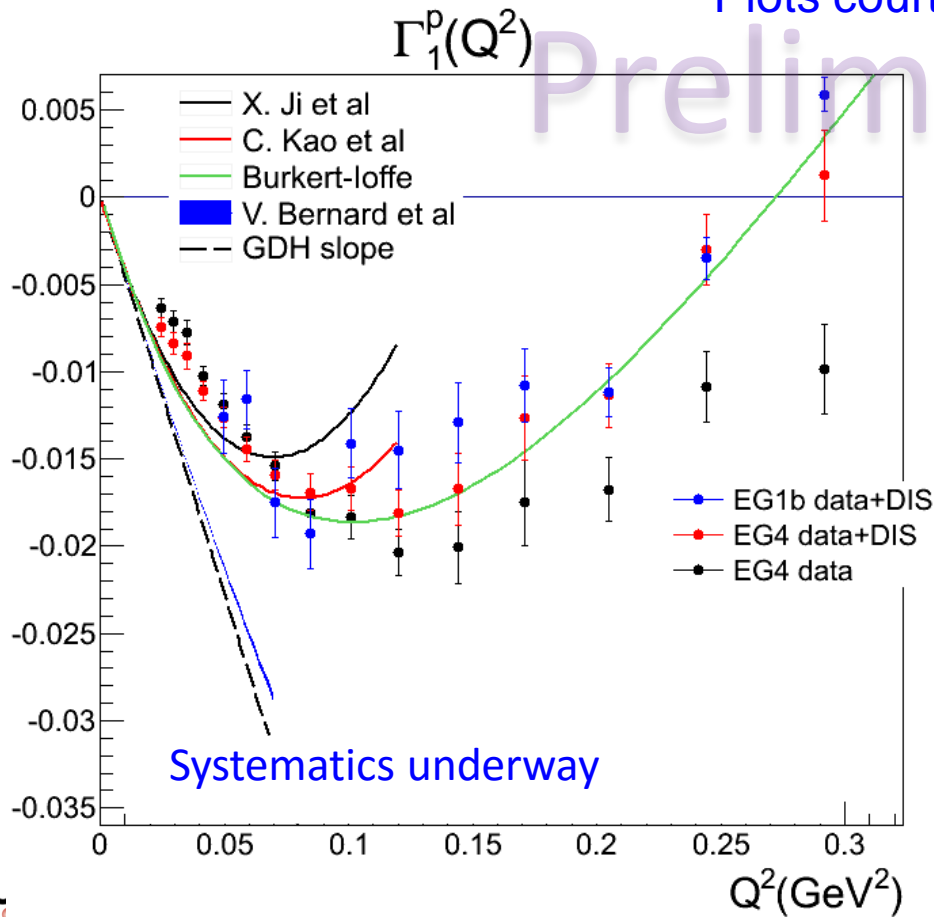
Plots courtesy of  
H. Kang

# EG4 Proton Results: First moment

$$\bar{\Gamma}_1 = \int_0^{x_{th}} g_1(x, Q^2) dx$$

$$\bar{I}_{TT} = \frac{2M^2}{Q^2} \int_0^{x_{th}} \left( g_1 - \frac{2M^2 x^2}{Q^2} g_2 \right) dx \xrightarrow{(Q^2 \rightarrow 0)} -\frac{\kappa^2}{4} \quad \text{GDH}$$

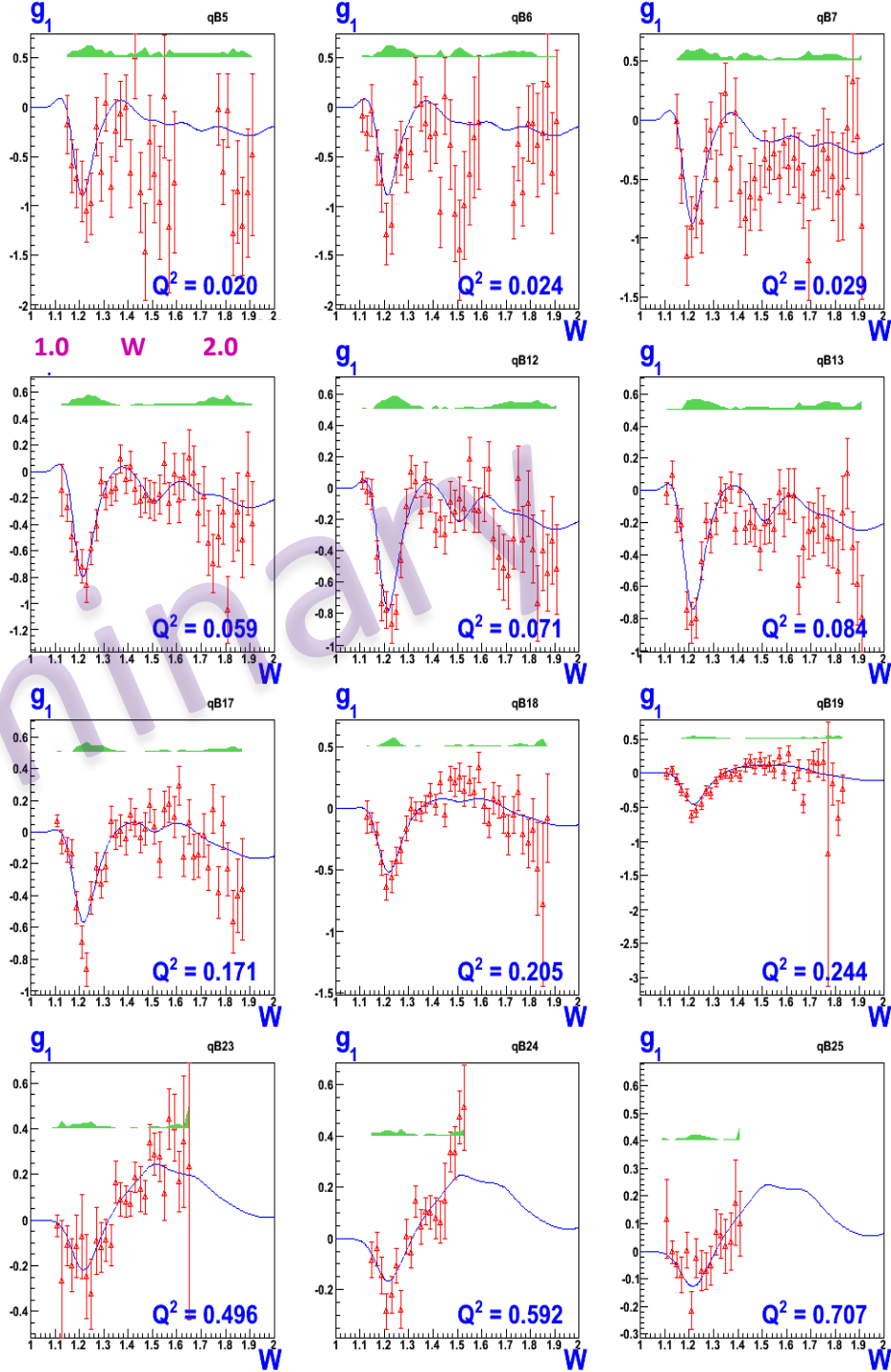
Plots courtesy of H. Kang





# EG4 Results: Extracted $g_1$ (Deuteron)

$g_1(\text{Model})$   
 $g_1(\text{comb.})$   
 Sys. Err.



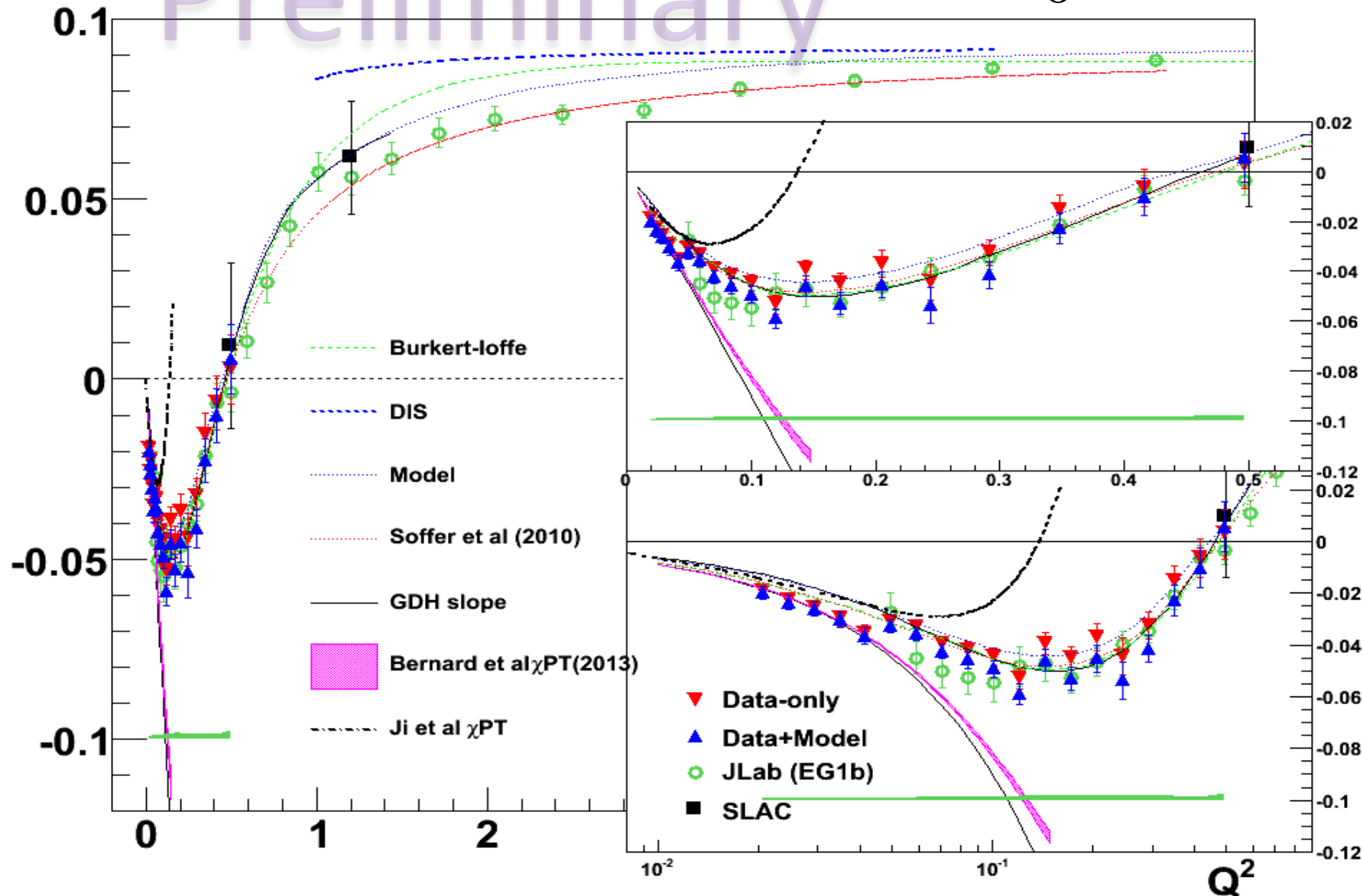


# Results: First Moment

$\Gamma_1^d$

$$\bar{\Gamma}_1 = \int_0^{x_{th}} g_1(x, Q^2) dx$$

Preliminary



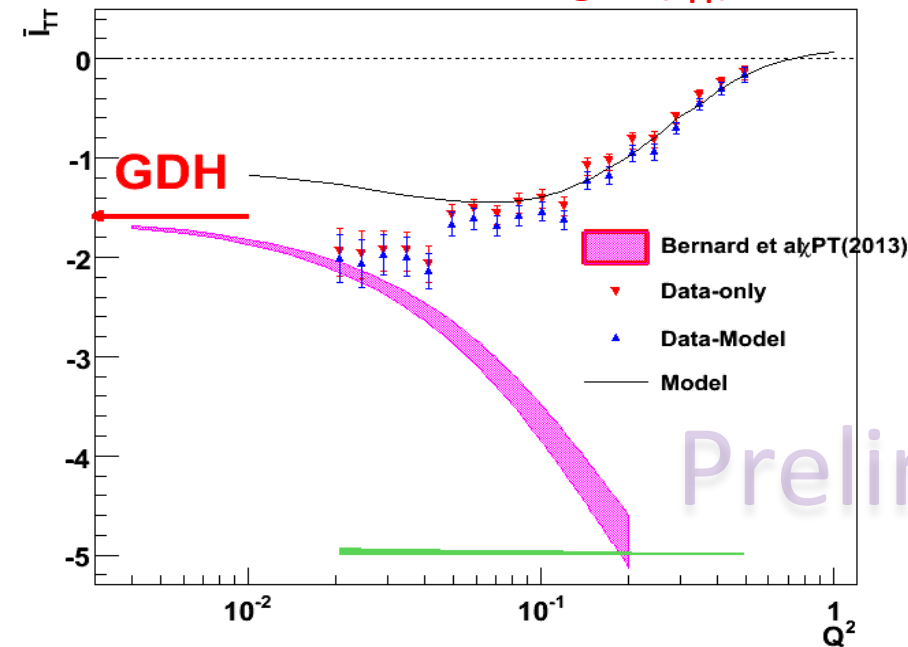
# Results: Generalized GDH integral ( $I_{TT}$ ) & Generalized Forward Spin polarizability ( $\gamma_0$ )



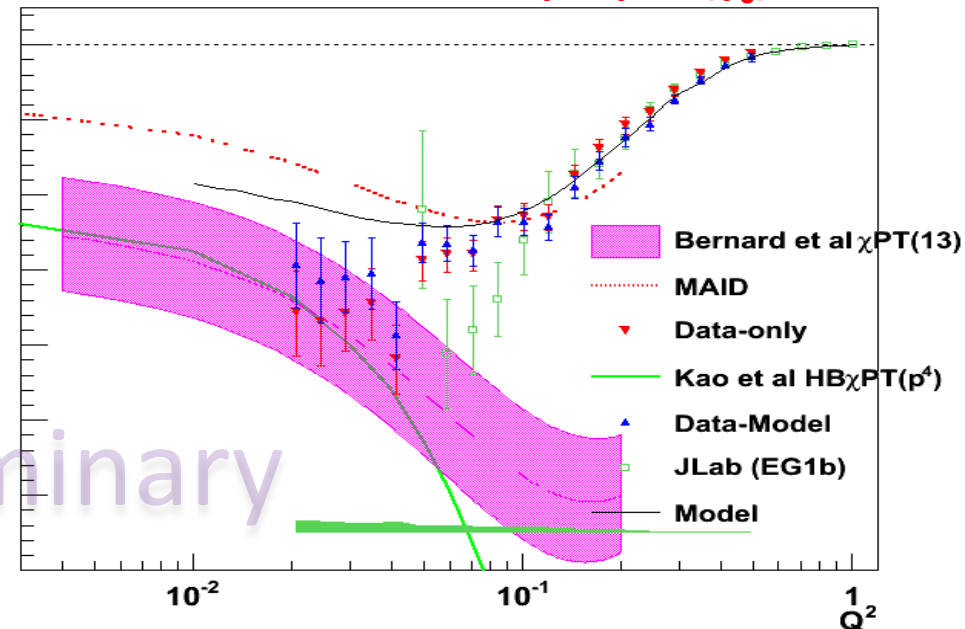
$$\bar{I}_{TT} = \frac{2M^2}{Q^2} \int_0^{x_{th}} \left( g_1 - \frac{2M^2 x^2}{Q^2} g_2 \right) dx \xrightarrow{(Q^2 \rightarrow 0)} -\frac{\kappa^2}{4} \text{ GDH}$$

$$\gamma_0 = (16\alpha M^2 / Q^6) \int_0^{x_{th}} \left( g_1 - \frac{2M^2 x^2}{Q^2} g_2 \right) x^2 dx$$

Generalized GDH integral ( $I_{TT}$ )

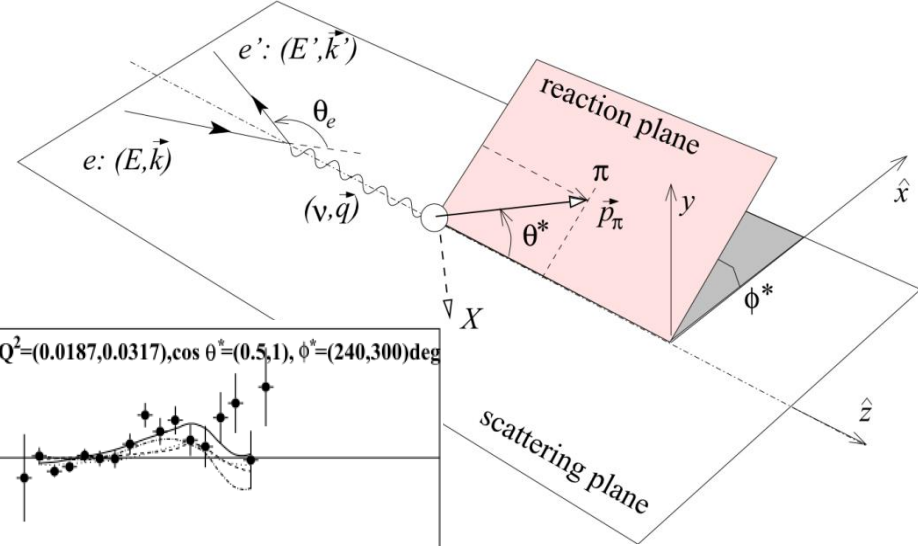


Generalized forward spin pol. ( $\gamma_0$ )

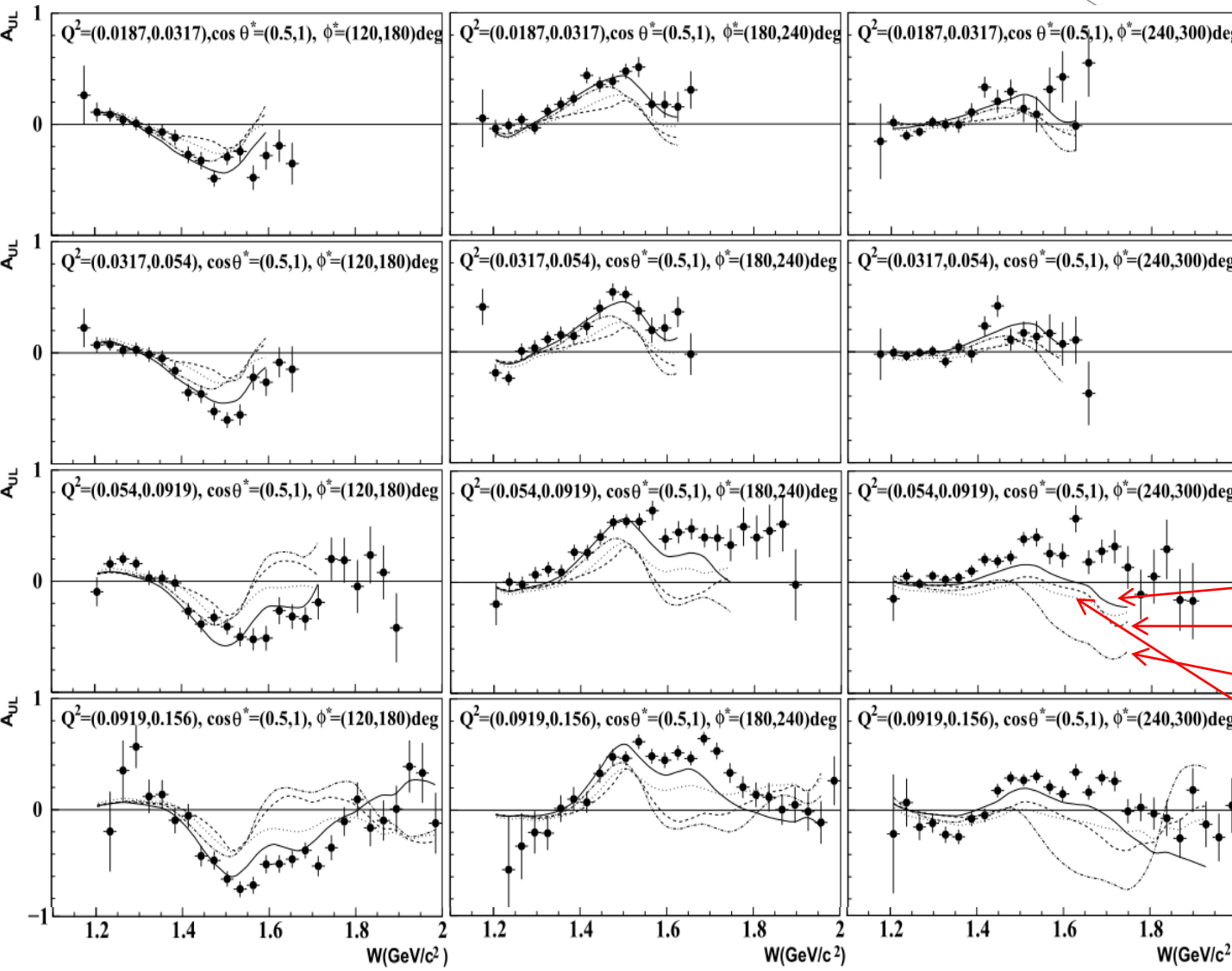


# EG4: Spin Asymmetry $A_{UL}$ Results on $p(e,e'\pi^+)n$

Plots courtesy of Xiaochao Zheng



The target spin  
asymmetries  $A_{UL}$   
for the  $\pi^+n$   
channel



**Models:**  
JANR (solid),  
MAID2007 (dashed),  
SAID (dash-dotted),  
DMT2001 (dotted)

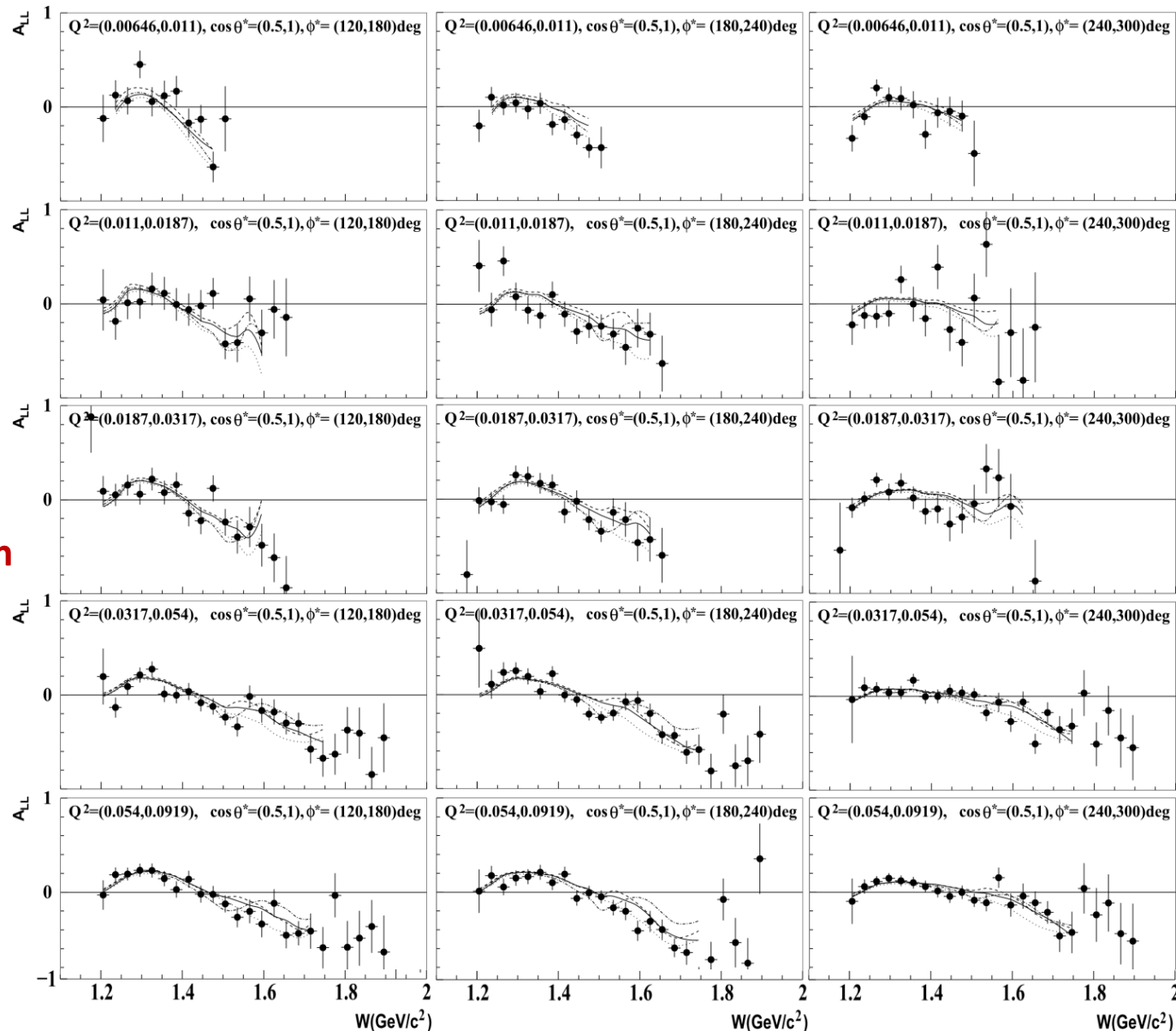


# EG4: Spin Asymmetry $A_{LL}$ Results on $p(e,e'\pi^+)n$

Plots courtesy of  
Xiaochao Zheng

Results on the double-  
spin symmetries  $A_{LL}$  for  
 $\pi^+n$  channel in different  
 $Q^2$  bins

The paper **Measurement  
of Target and Double-spin  
Asymmetries for  $ep \rightarrow e\pi^+(n)$   
Reaction in the  
Nucleon Resonance  
Region at Low  $Q^2$  nearly  
ready for publication.**



# Summary

- A wealth of new low  $Q^2$  data on the nucleon spin structure in the non-perturbative regime has been produced in Hall A, and B at Jefferson Lab as part of a broad spin physics program
- Nucleon polarizability  $\gamma_0$  is a more stringent test of  $\chi$ PT than  $\Gamma_1$ .  $\chi$ PT converge very slowly for the spin polarizabilities.
- Low  $Q^2$  analysis of EG4 data from Hall B and g2p data from Hall A are in the final stages.
- At very low  $Q^2$  the EG4 results show good agreement with other JLab results and with available  $\chi$ PT predictions.
- Neutron data extraction from EG4's deuteron and proton data is expected in near future.
- Ongoing 6 GeV data analyses and the future 12 GeV JLab measurements at low  $Q^2$  are expected to shed more light on the nucleon spin structure in the non-perturbative region.

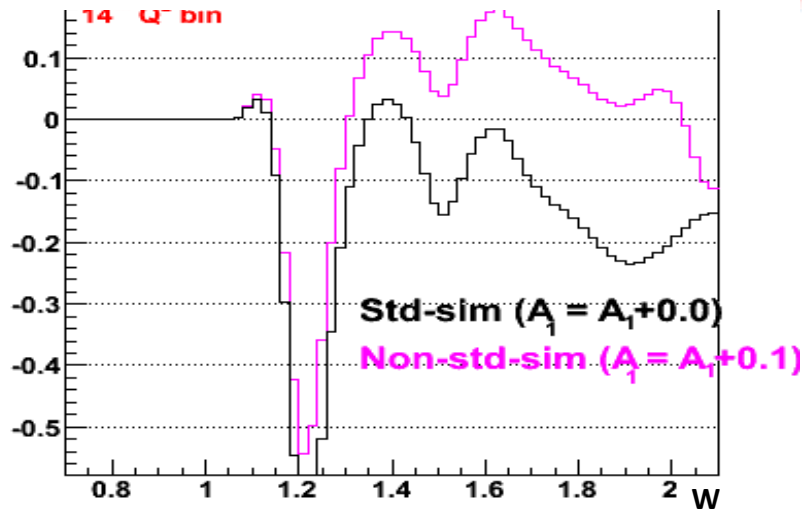
# Thank you!

# Method for $g_1$ calculation

$$\Delta N = (N^+ - N^-)$$

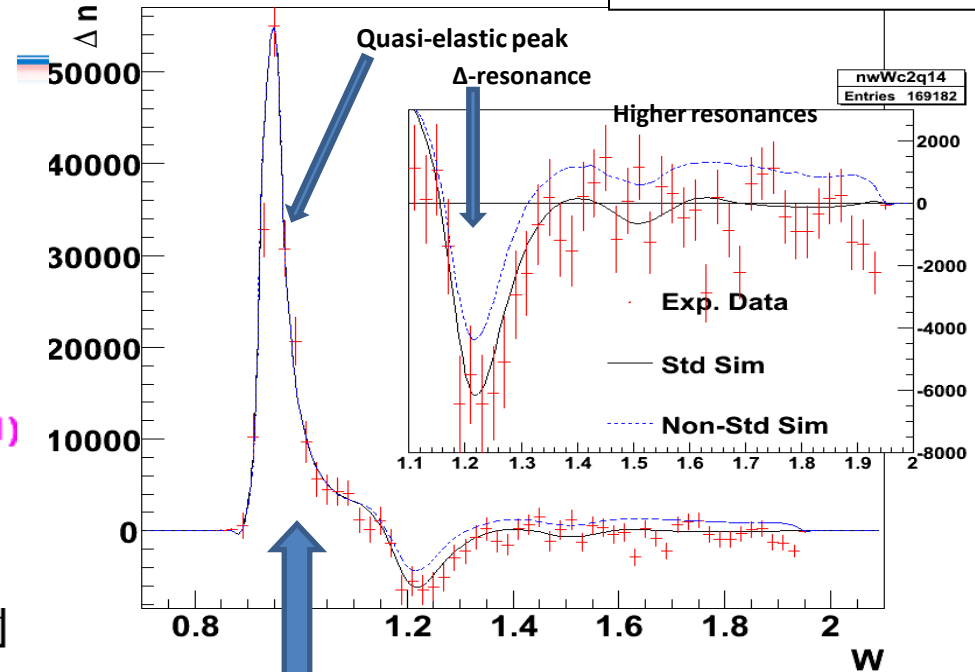
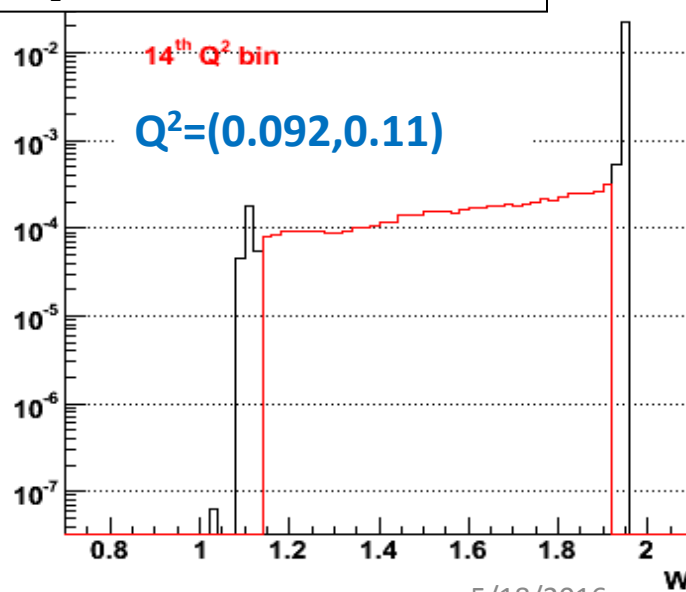
$$Q^2 = (0.092, 0.11)$$

Model  $g_1$  in  $Q^2 = (0.092, 0.11)$



$\Delta g_1 / \Delta(\Delta N)$  (proportionality factor)

Entries 70



First, simulated data was normalized w.r.t data by comparing in q.e. region.

$$g_1^{extr}(W, Q^2) = g_1^{Standard} + \frac{\Delta n^{data}(W, Q^2) - \Delta n^{standard}(W, Q^2)}{B(W, Q^2)}$$



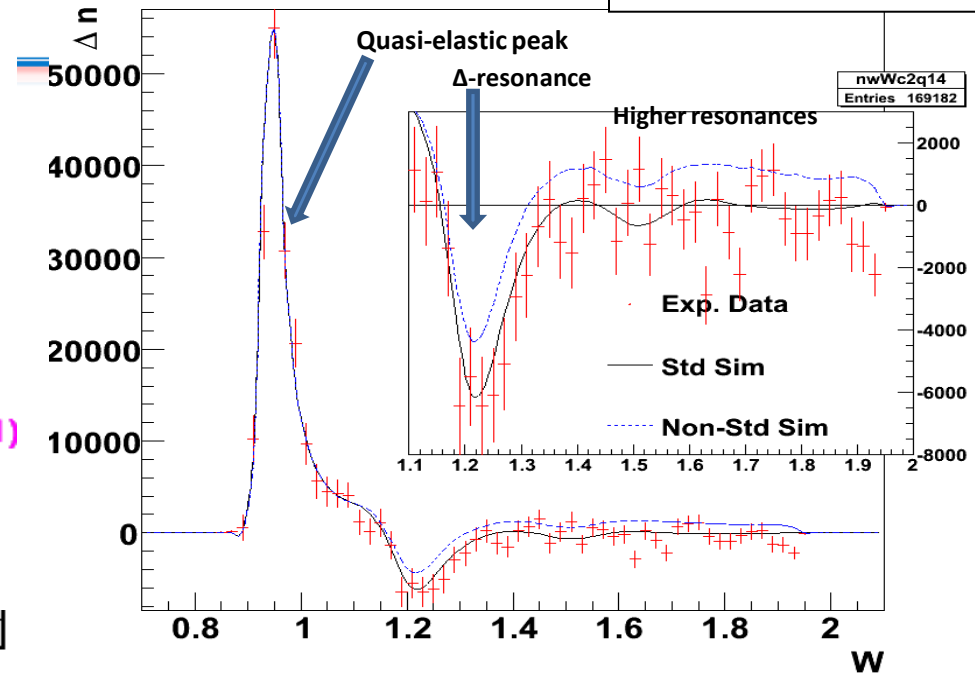
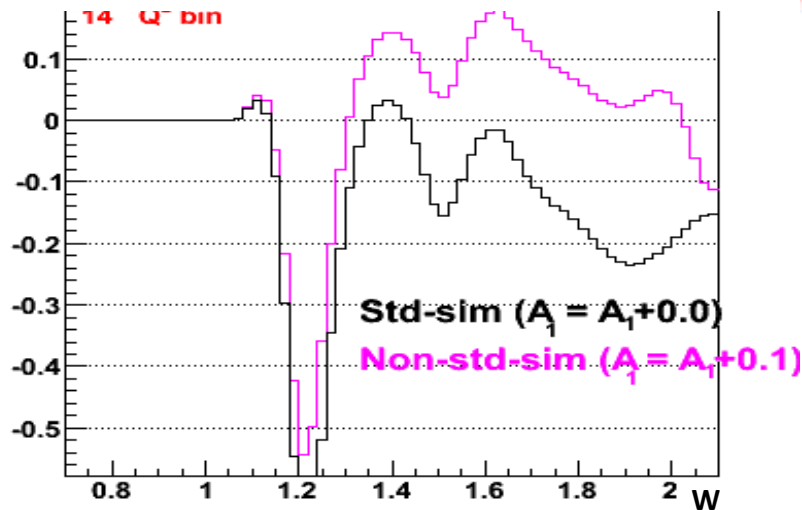
# Method for $g_1$ calculation



$$\Delta N = (N^+ - N^-)$$

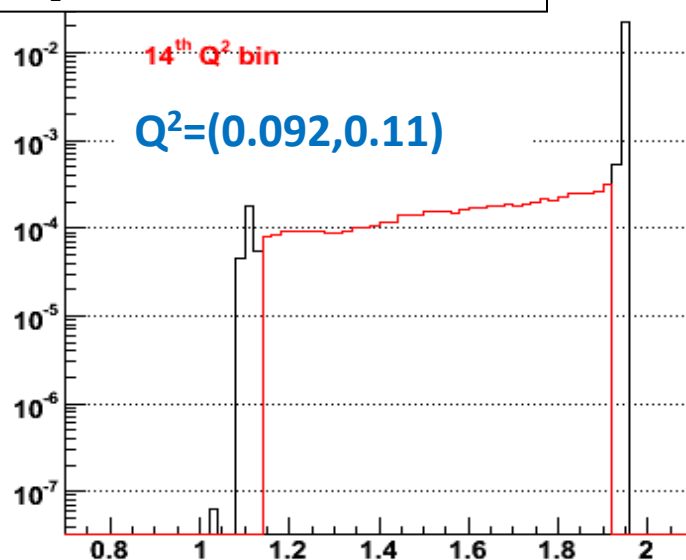
$$Q^2 = (0.092, 0.11)$$

Model  $g_1$  in  $Q^2 = (0.092, 0.11)$

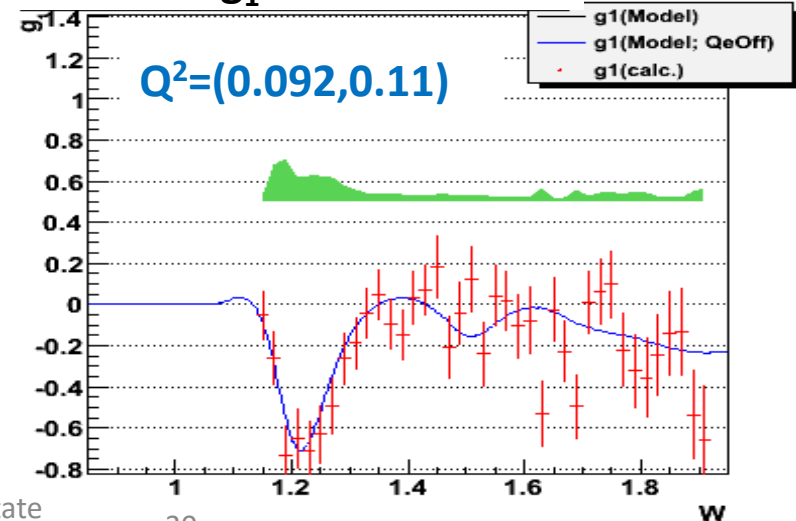


$\Delta g_1 / \Delta(\Delta N)$  (proportionality factor)

Entries 70



Calculated  $g_1$



# Sources of Systematic errors



- 1) Overall scaling factor (Mostly due to PbPt, Target length )
- 2) Radiative corrections
- 3) Model Uncertainties
- 4) Contaminations of polarized H in the target and  $\pi^-$  in the scattered electrons.
- 5) Beam energy measurement
- 6) CC-efficiency estimation
- 7)  $e^+e^-$  pair symmetric contamination

# Estimation of Systematic errors

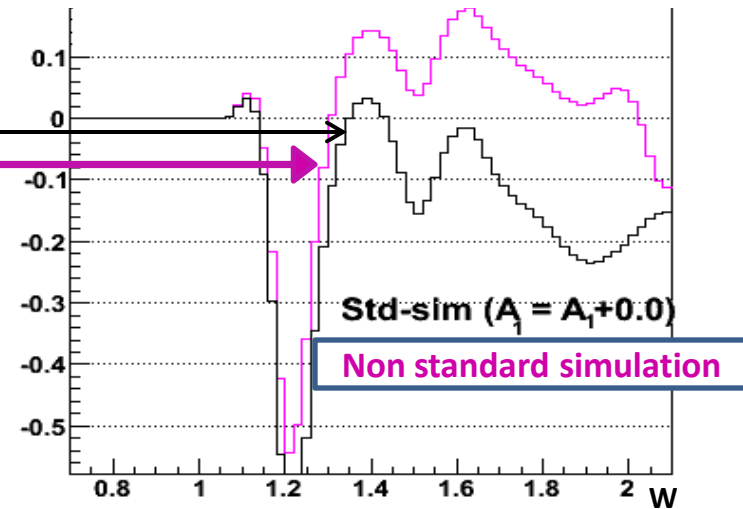
## One Example:

### Model Uncertainty Contribution

e.g., change  $A_1/A_2/F_1/R$  within fit errors

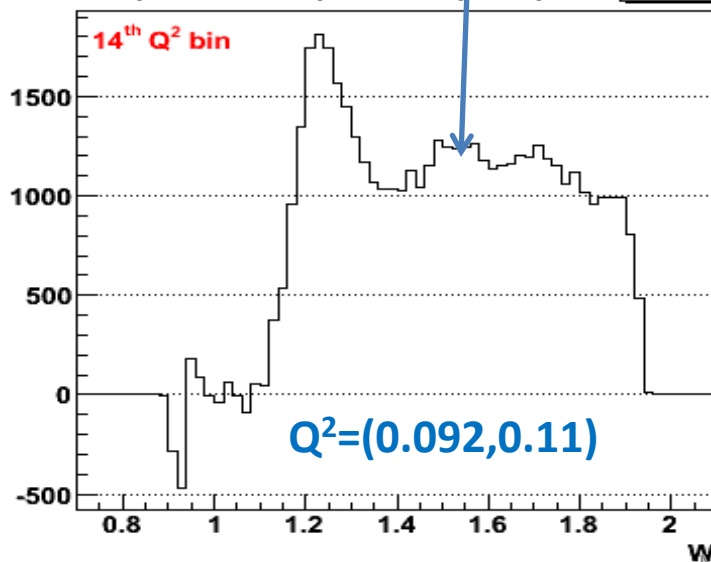
$$\Delta g_1^i(W, Q^2) = g_1^{standard} + \frac{\Delta n^i - \Delta n^{standard}}{B(W, Q^2)} - g_1^i$$

Model  $g_1$  in  $Q^2=(0.092,0.11)$

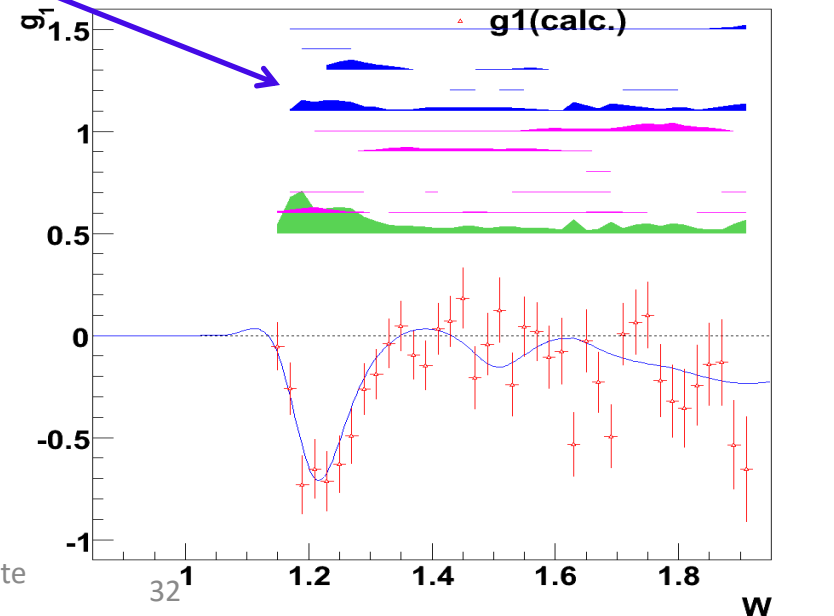


$\Delta N$  (non-std) -  $\Delta N$  (std)

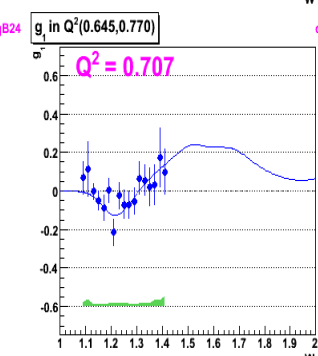
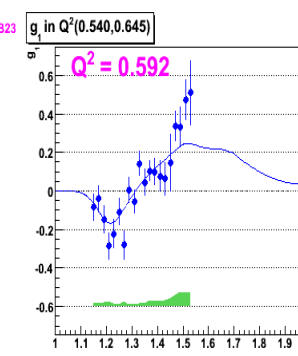
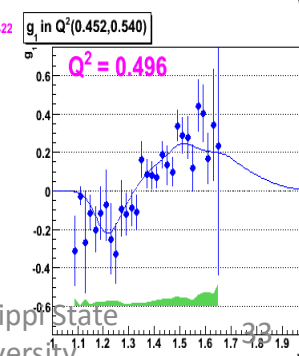
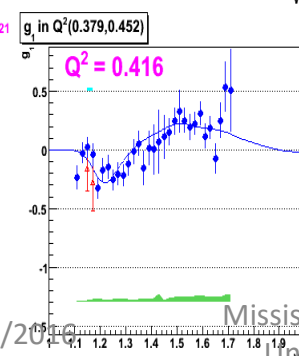
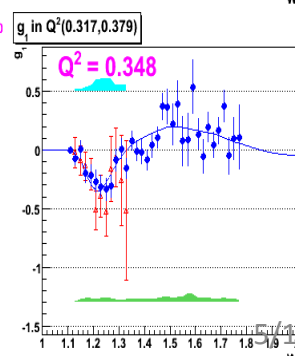
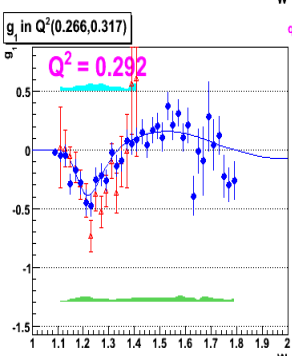
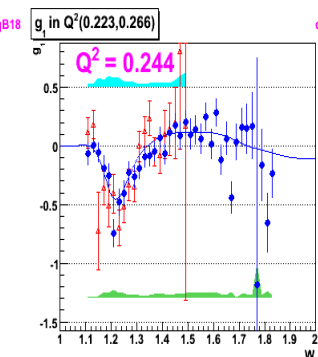
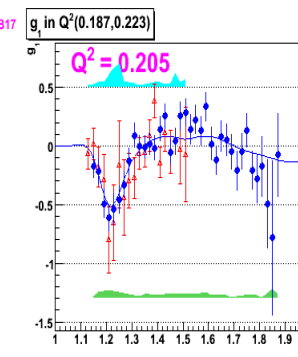
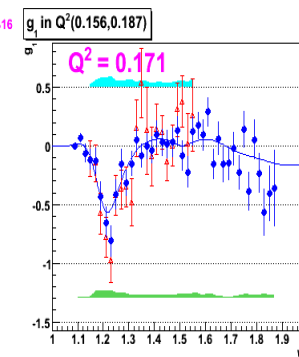
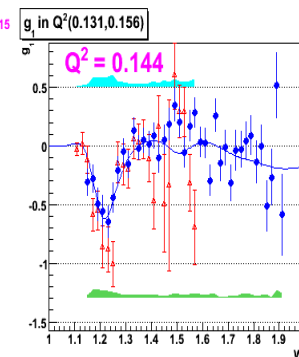
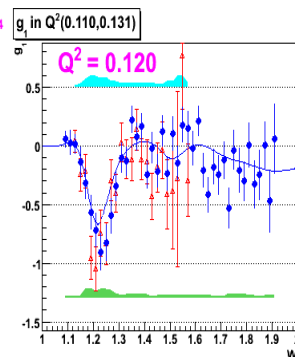
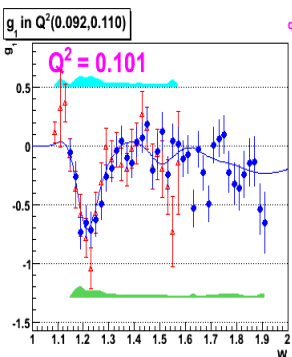
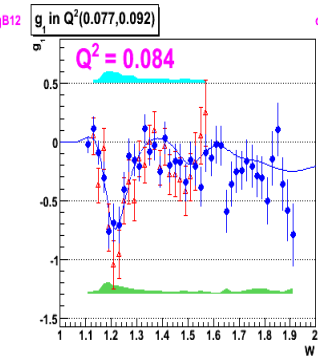
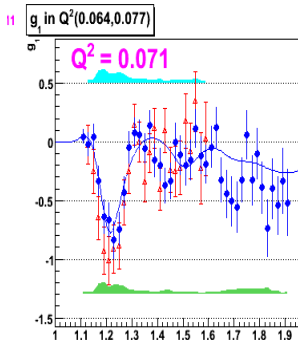
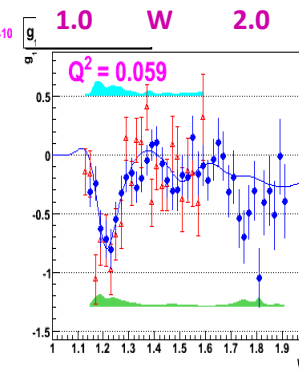
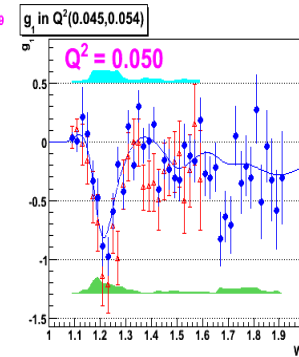
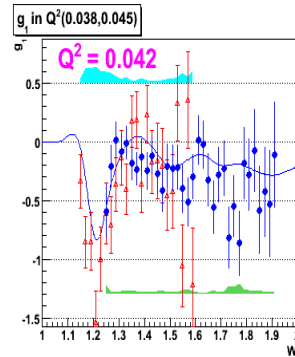
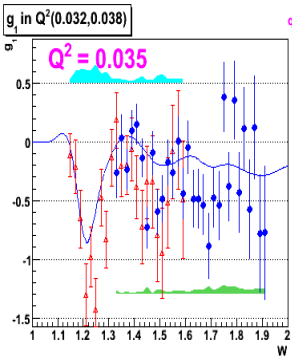
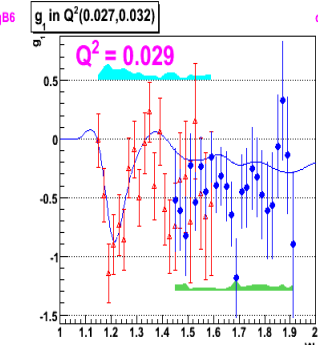
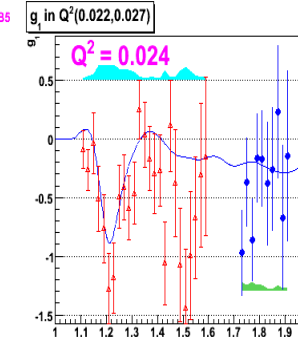
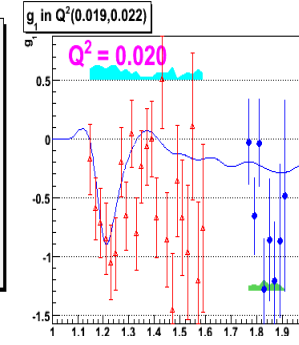
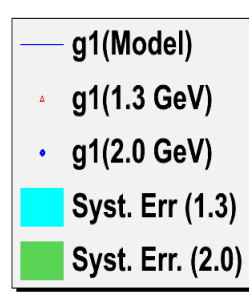
DDnSSvWq14  
Entries 70



$g_1$  for  $Q^2(0.092,0.110)$



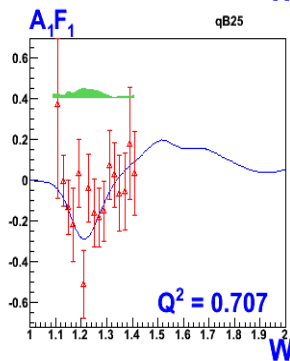
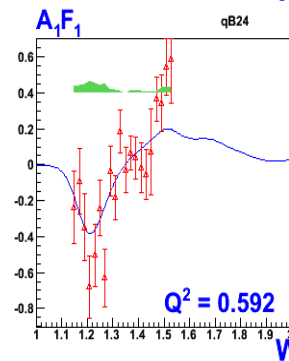
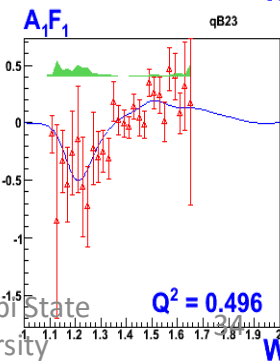
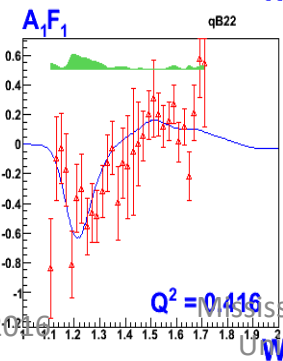
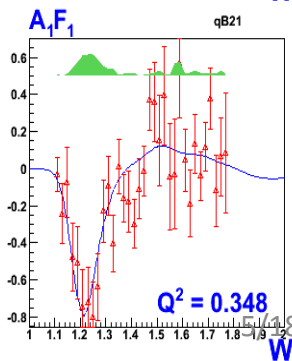
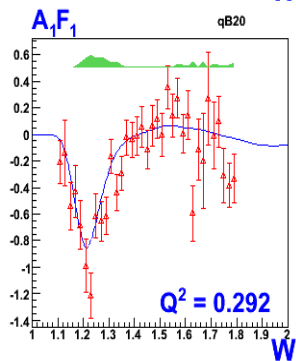
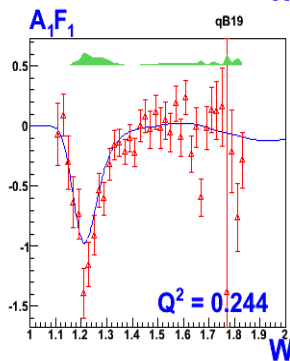
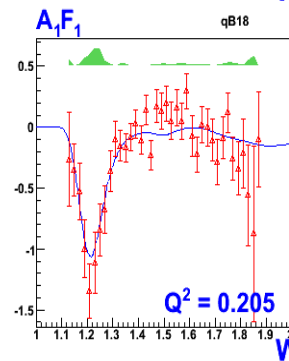
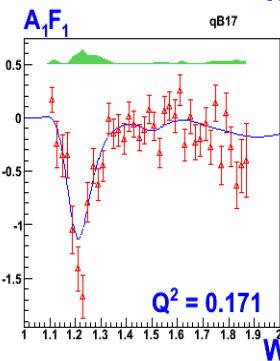
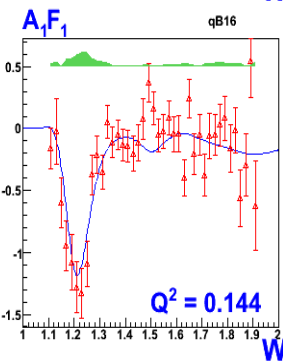
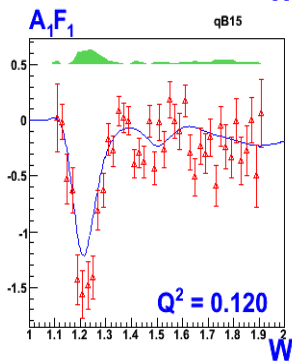
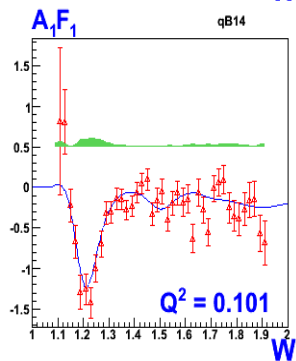
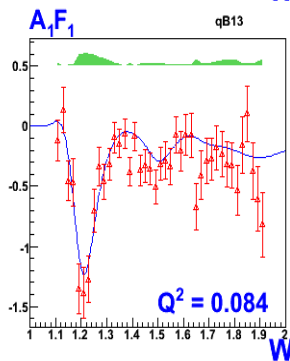
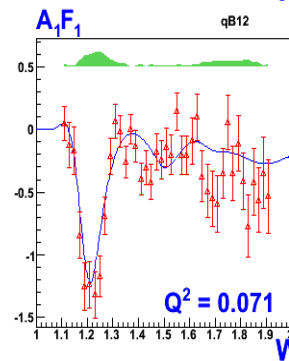
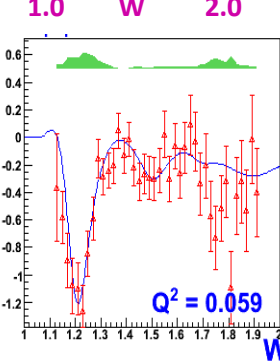
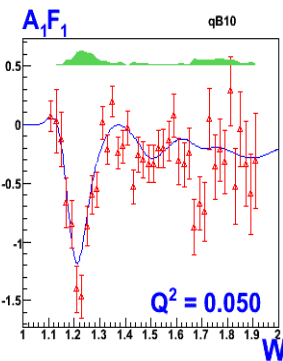
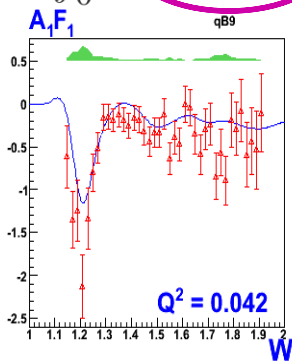
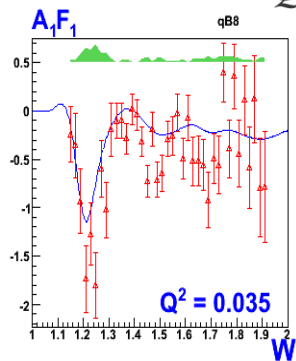
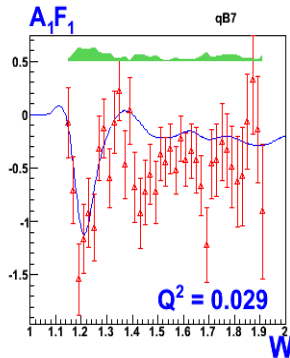
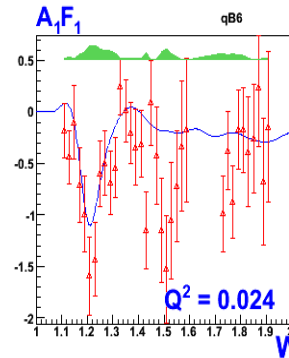
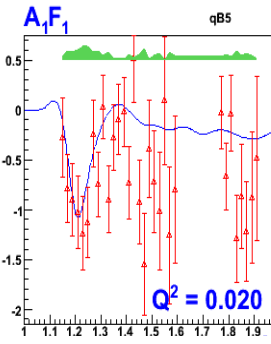
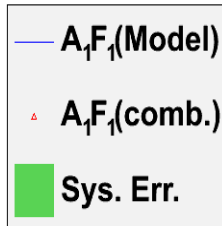
# Extracted $g_1$



$$A_1 F_1 = g_1 - (4M^2 x^2 / Q^2) g_2$$

$$\bar{I}_{TT} = \frac{2M^2}{Q^2} \int_0^{x_0(Q^2)} dx A_1 F_1(x, Q^2)$$

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} A_1 F_1(x, Q^2) x^2 dx$$



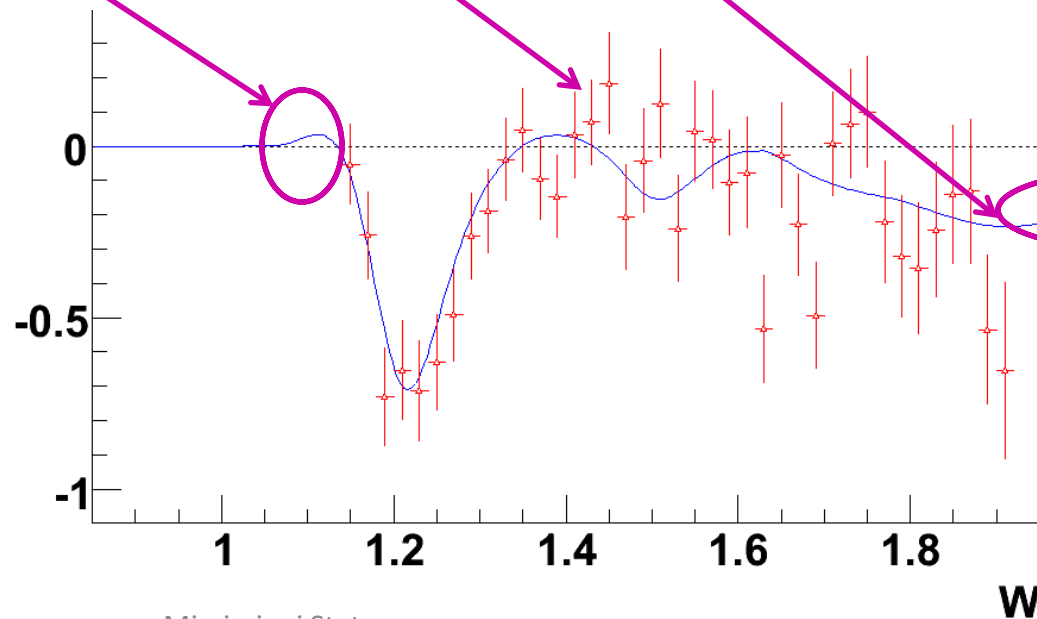
# Calculation of Integrals

$$\Gamma_1(Q^2) = \int_{x=0.001}^{x(W_{data})} g_1(x, Q^2) dx \quad \text{model}$$

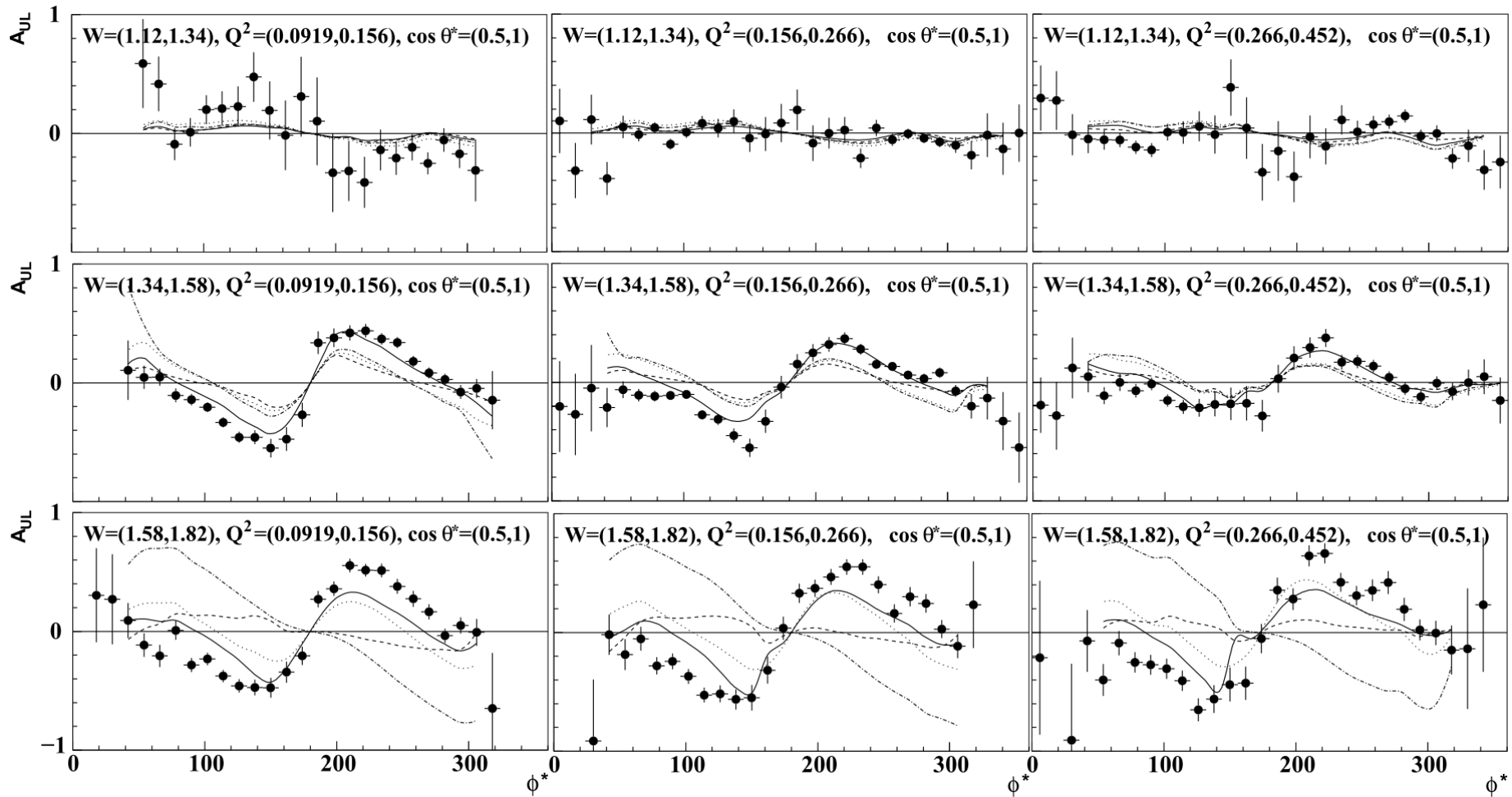
$$+ \int_{x(W_{data})}^{W=1.15} g_1(x, Q^2) dx \quad \text{data (or model for gaps)}$$

$$+ \int_{W=1.15}^{W=1.08} g_1(x, Q^2) dx \quad \text{model}$$

$$x = \frac{Q^2}{Q^2 + W^2 - M^2}$$



# EG4: Spin Asymmetry $A_{UL}$ Results on $p(e,e'\pi^+)n$



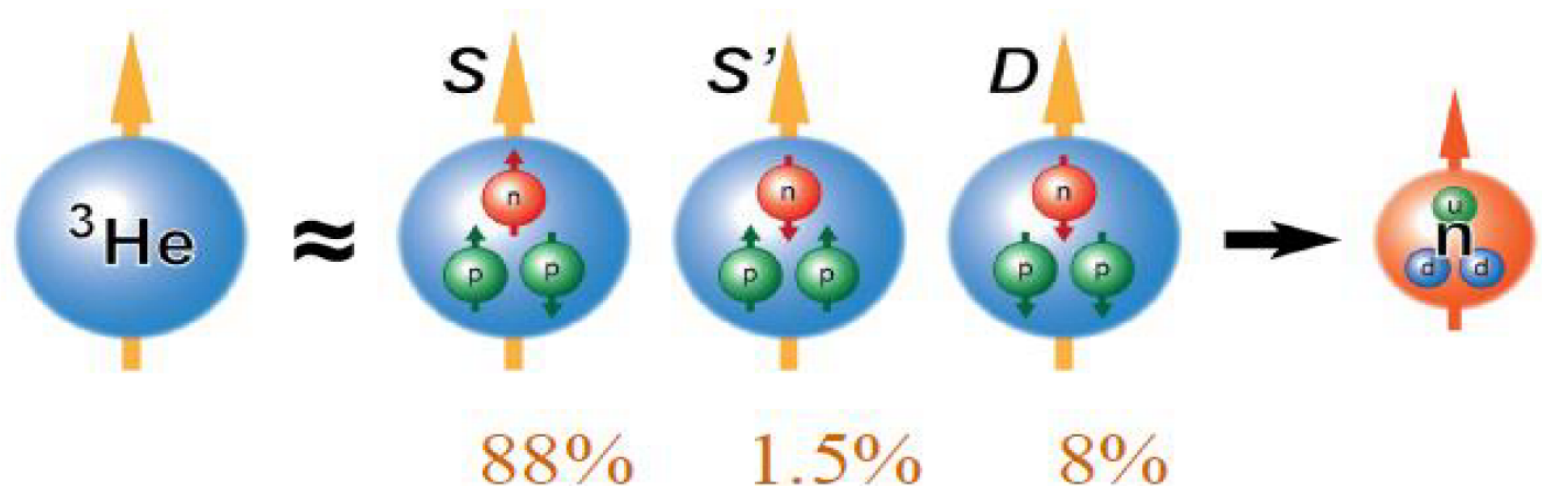
Target spin asymmetries  $A_{UL}$  for the  $\pi^+n$  channel as a function of the invariant mass  $\phi^*$

Plots courtesy of Xiaochao Zheng



# Neutron Target

- Neutron mean lifetime is just under 15 mins.
- $^3\text{He}$  nucleus has two protons whose spins are paired, and a single neutron that accounts for most of the nuclear spin.
- So,  $^3\text{He}$  is an effective polarized neutron target.



F. R. P. Bissey, A. W. Thomas, and I. R. Afnan, Phys. Rev. C64, 024004 (2001)