# Evaluating Polarization Data

NSTAR 2017

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August 20-23, 2017

# Introduction

We have a lot of data!

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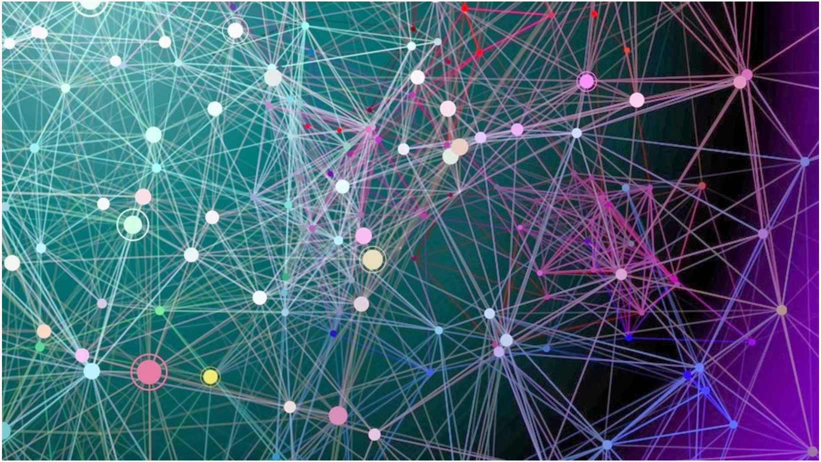
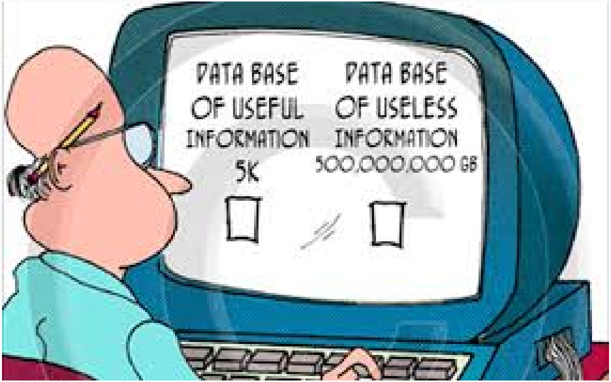


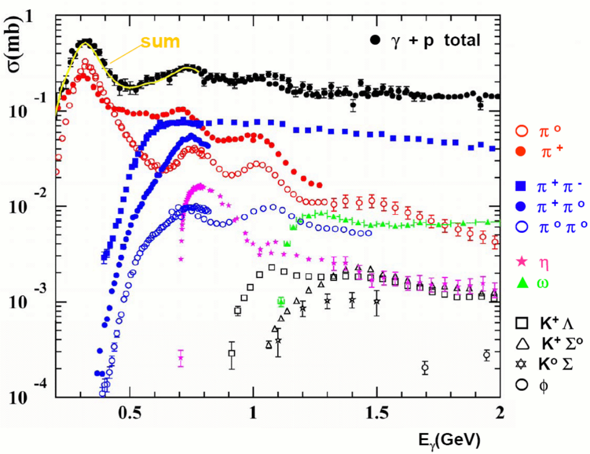
How do we make connections to understanding physics?

# Introduction



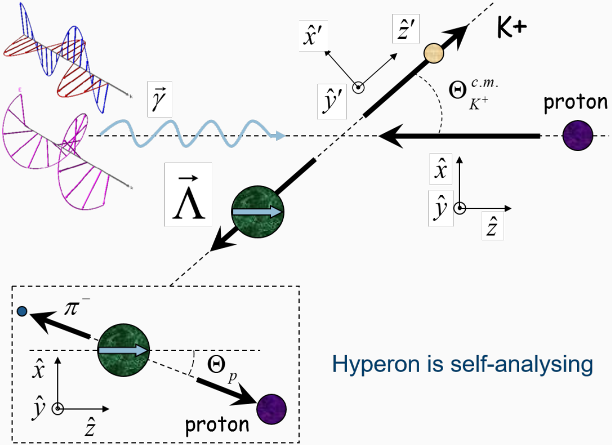

What if the data is junk?

# Pseudoscalar Meson Photoproduction




We have gone way beyond measuring cross-sections!

# Pseudoscalar Meson Photoproduction - $K\Lambda$ example




Hyperon is self-analysing

# The Transversity Basis

Transversity amplitudes $b_j$ ($j = 1, 2, 3, 4$): quantization axis perpendicular to reaction plane and the linear photon polarizations $J_x$ and $J_y$

$$\begin{aligned} b_1 &= {}_y\langle +|J_y|+\rangle_y, \\ b_2 &= {}_y\langle -|J_y|-\rangle_y, \\ b_3 &= {}_y\langle +|J_x|-\rangle_y, \\ b_4 &= {}_y\langle -|J_x|+\rangle_y. \end{aligned}$$

Normalized transversity amplitudes (NTA) $a_j$ ($j = 1, 2, 3, 4$)

$$a_j \equiv \frac{b_j}{\sqrt{|b_1|^2 + |b_2|^2 + |b_3|^2 + |b_4|^2}},$$

The $a_j$ are functions of $W$ (hadronic mass) and $\theta_{\text{c.m.}}$ (scattering angle)

# Pseudoscalar Meson Photoproduction - Observables

| Type | Observable | Transversity representation | Helicity representation |
|---|---|---|---|
| S | $\sigma$ | $\lvert a_1\rvert^2 + \lvert a_2\rvert^2 + \lvert a_3\rvert^2 + \lvert a_4\rvert^2$ | $\lvert h_1\rvert^2 + \lvert h_2\rvert^2 + \lvert h_3\rvert^2 + \lvert h_4\rvert^2$ |
| | $\Sigma$ | $\lvert a_1\rvert^2 + \lvert a_2\rvert^2 - \lvert a_3\rvert^2 - \lvert a_4\rvert^2$ | $2\Re(h_1 h_4^* - h_2 h_3^*)$ |
| | $P$ | $\lvert a_1\rvert^2 - \lvert a_2\rvert^2 + \lvert a_3\rvert^2 - \lvert a_4\rvert^2$ | $2\Im(h_1 h_3^* + h_2 h_4^*)$ |
| | $T$ | $\lvert a_1\rvert^2 - \lvert a_2\rvert^2 - \lvert a_3\rvert^2 + \lvert a_4\rvert^2$ | $2\Im(h_1 h_2^* + h_3 h_4^*)$ |
| BT | $E$ | $2\Re(a_1 a_3^* + a_2 a_4^*)$ | $\lvert h_1\rvert^2 - \lvert h_2\rvert^2 + \lvert h_3\rvert^2 - \lvert h_4\rvert^2$ |
| | $F$ | $2\Im(a_1 a_3^* - a_2 a_4^*)$ | $2\Re(h_1 h_2^* + h_3 h_4^*)$ |
| | $G$ | $2\Im(a_1 a_3^* + a_2 a_4^*)$ | $-2\Im(h_1 h_4^* + h_2 h_3^*)$ |
| | $H$ | $-2\Re(a_1 a_3^* - a_2 a_4^*)$ | $-2\Im(h_1 h_3^* - h_2 h_4^*)$ |
| BR | $C_x$ | $-2\Im(a_1 a_4^* - a_2 a_3^*)$ | $2\Re(h_1 h_3^* + h_2 h_4^*)$ |
| | $C_z$ | $2\Re(a_1 a_4^* + a_2 a_3^*)$ | $\lvert h_1\rvert^2 + \lvert h_2\rvert^2 - \lvert h_3\rvert^2 - \lvert h_4\rvert^2$ |
| | $O_x$ | $2\Re(a_1 a_4^* - a_2 a_3^*)$ | $-2\Im(h_1 h_2^* - h_3 h_4^*)$ |
| | $O_z$ | $2\Im(a_1 a_4^* + a_2 a_3^*)$ | $2\Im(h_1 h_4^* - h_2 h_3^*)$ |
| TR | $T_x$ | $2\Re(a_1 a_2^* - a_3 a_4^*)$ | $-2\Re(h_1 h_4^* + h_2 h_3^*)$ |
| | $T_z$ | $2\Im(a_1 a_2^* - a_3 a_4^*)$ | $-2\Re(h_1 h_2^* - h_3 h_4^*)$ |
| | $L_x$ | $-2\Im(a_1 a_2^* + a_3 a_4^*)$ | $2\Re(h_1 h_3^* - h_2 h_4^*)$ |
| | $L_z$ | $2\Re(a_1 a_2^* + a_3 a_4^*)$ | $\lvert h_1\rvert^2 - \lvert h_2\rvert^2 - \lvert h_3\rvert^2 + \lvert h_4\rvert^2$ |

# Extracting Observables

$$
\begin{aligned}
\sigma_{Total} = \sigma_0\{1 &- P_L^\gamma P_T^T P_y^R \sin(\phi)\cos(2\phi) + \Sigma(-P_L^\gamma \cos(2\phi) + P_T^T P_y^R \sin(\phi)) \\
&+ T(P_T^T \sin(\phi) - P_L^\gamma P_y^R \cos(2\phi)) + P(P_y^R - P_L^\gamma P_T^T \sin(\phi)\cos(2\phi)) \\
&+ E(-P_C^\gamma P_L^T + P_L^\gamma P_T^T P_y^R \cos(\phi)\sin(2\phi)) + F(P_C^\gamma P_T^T \cos(\phi) + P_L^\gamma P_L^T P_y^R \sin(2\phi)) \\
&- G(P_L^\gamma P_L^T \sin(2\phi) + P_C^\gamma P_T^T P_y^R \cos(\phi)) - H(P_L^\gamma P_T^T \cos(\phi)\sin(2\phi) - P_C^\gamma P_L^T P_y^R) \\
&- C_x(P_C^\gamma P_x^R - P_L^\gamma P_T^T P_z^R \sin(\phi)\sin(2\phi)) - C_z(P_C^\gamma P_z^R + P_L^\gamma P_T^T P_x^R \sin(\phi)\sin(2\phi)) \\
&- O_x(P_L^\gamma P_x^R \sin(2\phi) + P_C^\gamma P_T^T P_z^R \sin(\phi)) - O_z(P_L^\gamma P_z^R \sin(2\phi) - P_C^\gamma P_T^T P_x^R \sin(\phi)) \\
&+ L_x(P_L^T P_x^R + P_L^\gamma P_T^T P_z^R \cos(\phi)\cos(2\phi)) + L_z(P_L^T P_z^R - P_L^\gamma P_T^T P_x^R \cos(\phi)\cos(2\phi)) \\
&+ T_x(P_T^T P_x^R \cos(\phi) - P_L^\gamma P_L^T P_z^R \cos(2\phi)) + T_z(P_T^T P_z^R \cos(\phi) + P_L^\gamma P_L^T P_x^R \cos(2\phi))\}
\end{aligned}
$$

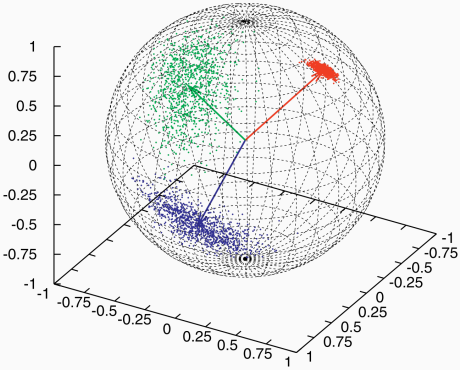
Cross section as a function of beam ($P^\gamma_{C,L}$), target ($P^T_{L,T}$) and recoil ($P^R_{x,y,z}$) polarization

# Constrained Parameter Estimation

The condition relating the normalized transversity amplitudes:

$$|a_1|^2+|a_2|^2+|a_3|^2+|a_4|^2 = 1$$

defines a unit sphere in $\mathbb{R}^8$.




- Can we map PDFs in observable space to PDF in amplitude space?
- If so, can we project amplitude PDF back into a joint observable PDF?

# Test Case: $\pi$-N Scattering

Two amplitudes, four observables:

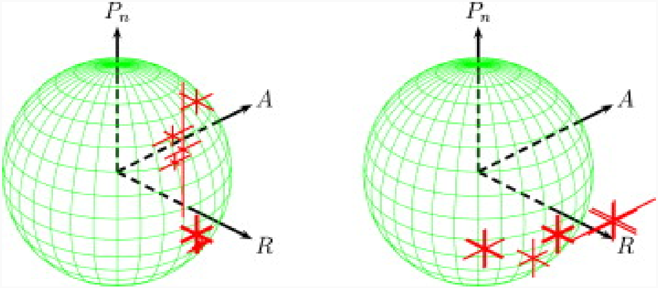
$$\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2$$
$$A = |f|^2 - |g|^2$$
$$R = -2\,\mathrm{Re}\,(fg^\star)$$
$$P = 2\,\mathrm{Im}\,(fg^\star)$$

Normalize:

$$|f|^2 + |g|^2 = 1$$

Constraint:

$$A^2 + R^2 + P^2 = 1$$




$\pi^- p$ (left) and $\pi^+ p$ (right) polarization observables

# Constrained Parameter Estimation - Kinematic Fitting

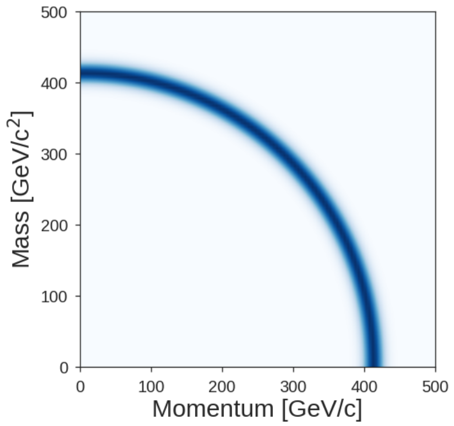
Measure energy $E \pm \delta E$ and momentum $p \pm \delta p$, identify particle type

Calculate measured mass:

$$m^2 = E^2 - p^2$$

**Example**

- Measured energy: $415 \pm 10$ MeV
- Measured momentum: $400 \pm 5$ MeV/c



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Measure energy $E \pm \delta E$
and momentum $p \pm \delta p$,
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**Example**

- Measured energy:
  $415 \pm 10$ MeV
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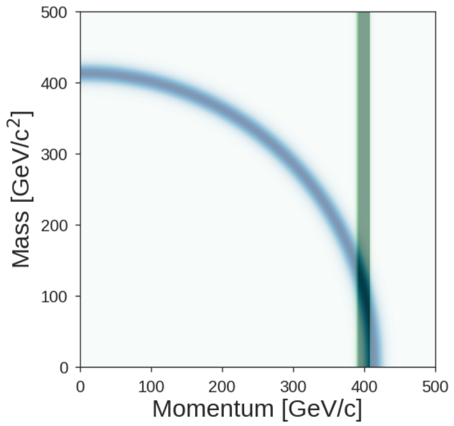
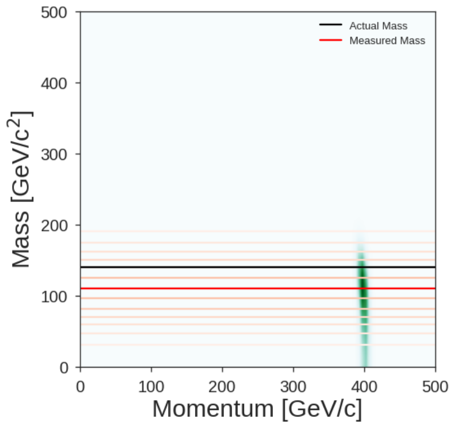
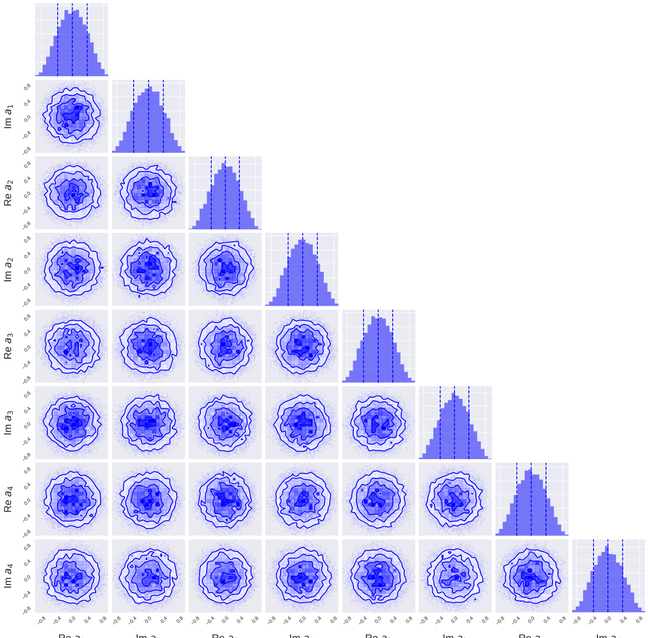


# Constrained Parameter Estimation - Kinematic Fitting

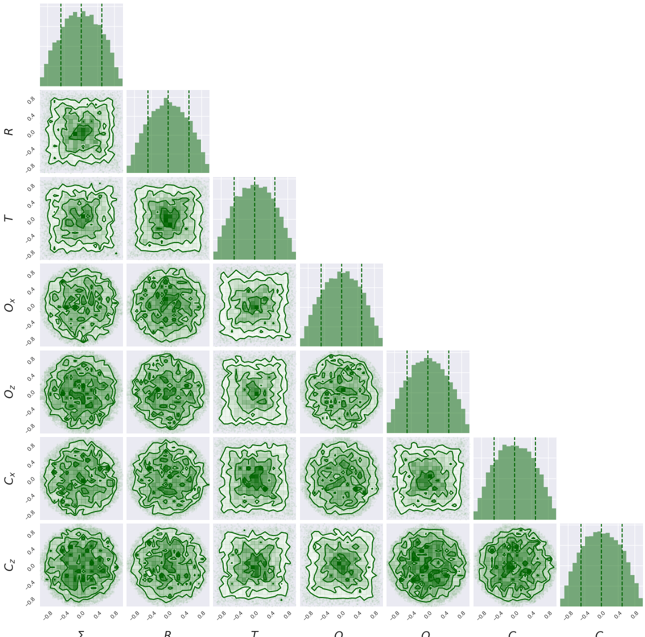
- Measured mass: $110 \pm 42$ MeV/c$^2$
- Identify particle as pion
- $\Rightarrow$ Adjusted energy: $421 \pm 10$ MeV
- $\Rightarrow$ Adjusted momentum: $397 \pm 5$ MeV/c



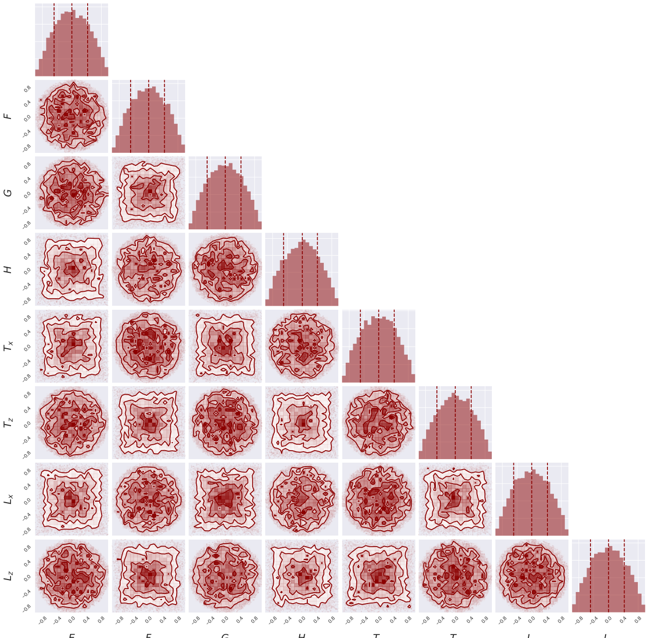
# Prior PDF for Amplitudes



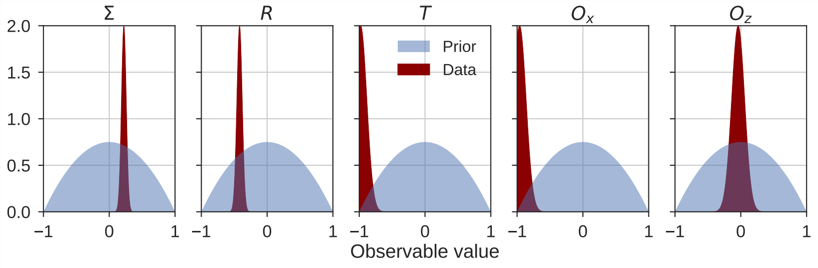
# Prior PDF for Observables to be Measured



# Prior PDF for Unmeasured Observables



# Data from One $W-\theta$ Bin

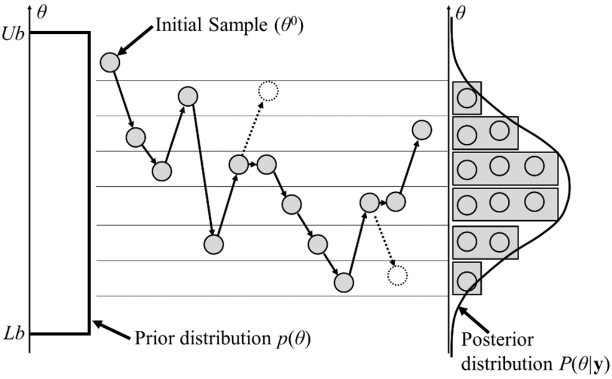



| Observable | Value | Uncertainty |
|---|---|---|
| $\Sigma$ | 0.222 | 0.037 |
| $R$ | -0.419 | 0.041 |
| $T$ | -0.979 | 0.095 |
| $O_x$ | -0.962 | 0.099 |
| $O_z$ | -0.040 | 0.099 |

# Calculating PDF in Amplitude Space

- Sample amplitudes $\{a_i\}$; $\quad i = 1, 2, 3, 4$
- Calculate observables $o_j = f_j(a_i)$; $\quad j = 1, ...16$
- Evaluate probability for each observable, based on ratio of gaussian PDF from "raw" data and (quadratic) prior PDF
- Use Markov Chain Monte Carlo (MCMC)

# Metropolis-Hastings MCMC Algorithm

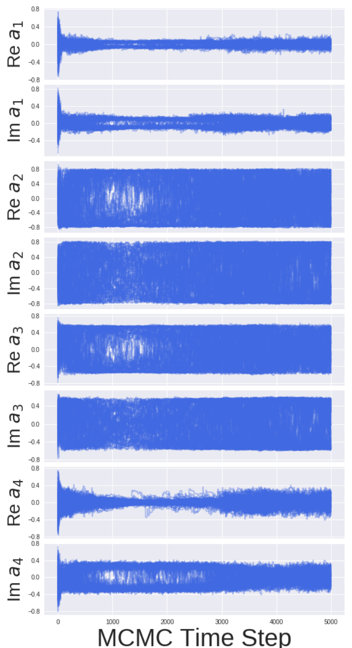



- Has detailed balance property:

$$P(\Phi' \mid \Phi)P(\Phi) = P(\Phi \mid \Phi')P(\Phi')$$

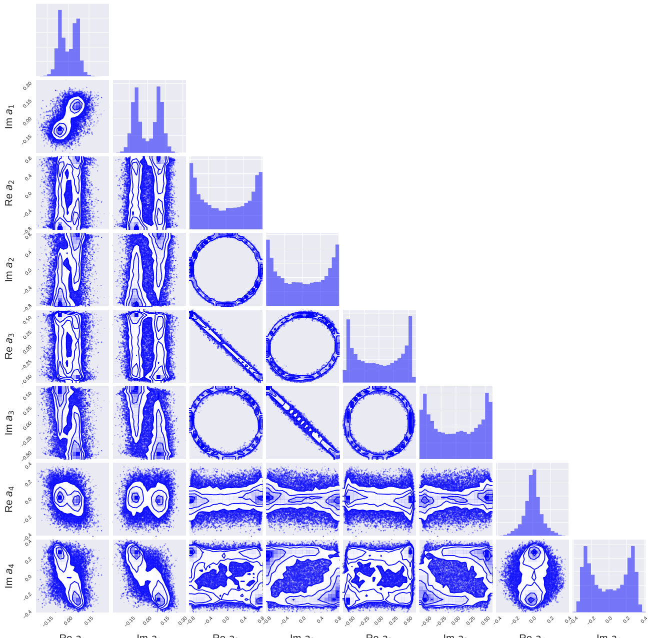
- All points in parameter space are reachable
- Sample of points can be shown to approach target distribution in the large number limit.

# MCMC Chain

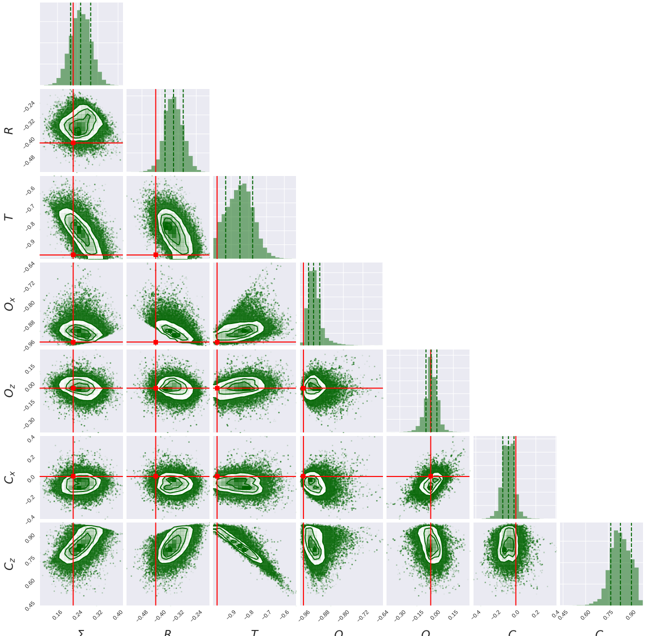


Re $a_1$
Im $a_1$
Re $a_2$
Im $a_2$
Re $a_3$
Im $a_3$
Re $a_4$
Im $a_4$
0.8
0.4
0.0
-0.4
-0.8
0
1000
2000
3000
4000
5000
MCMC Time Step

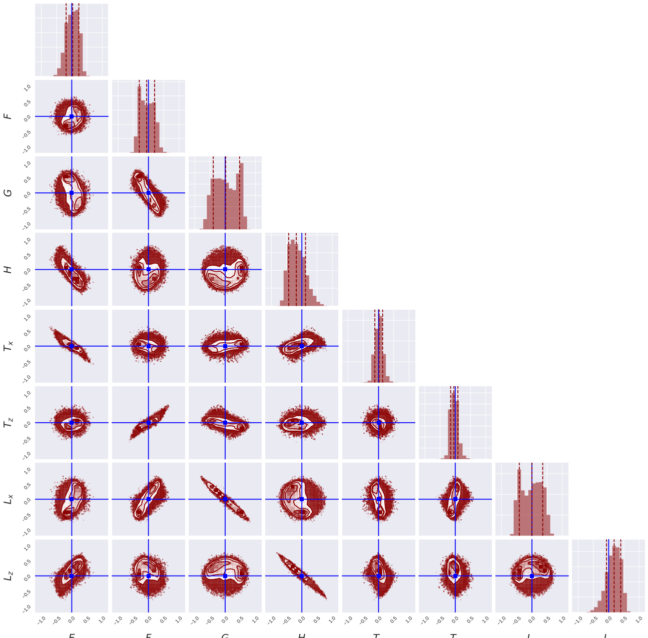
# Posterior PDF for Amplitudes



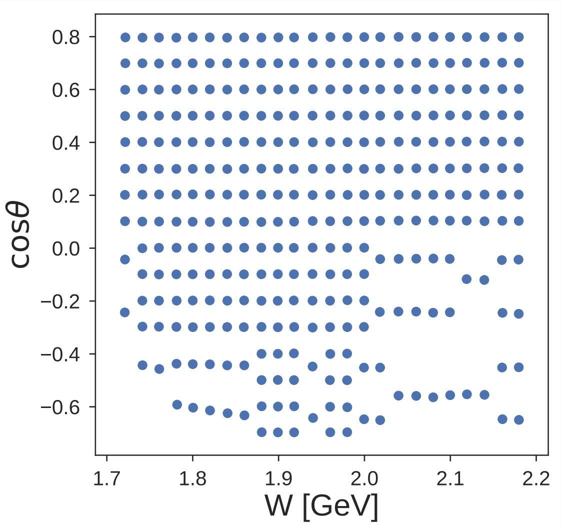
# Posterior PDF for Measured Observables



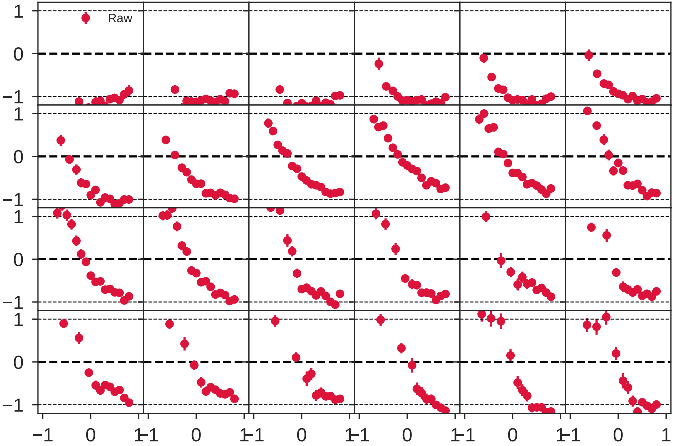
# Posterior PDF for Unmeasured Observables



# CLAS $\gamma + p \to K^+\Lambda$ Coverage

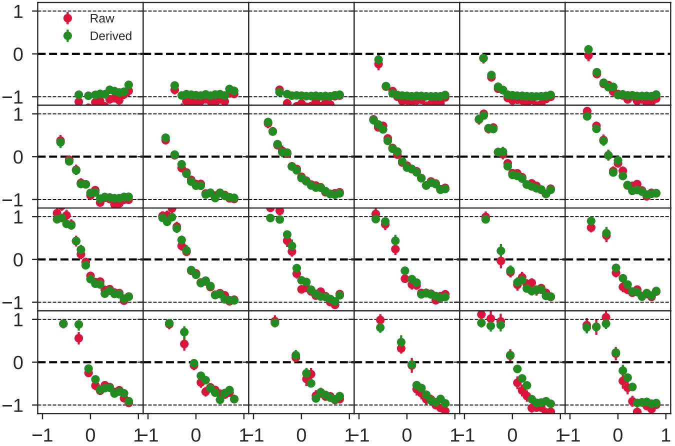


# Comparison of Raw and New (Target Asymmetry)



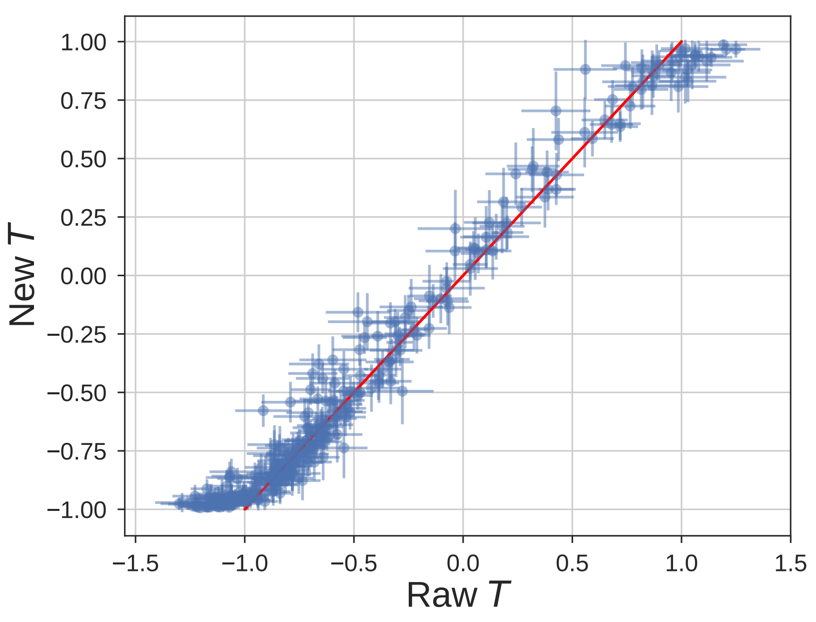

Data from unconstrained Maximum Likelihood fit.

# Comparison of Raw and New (Target Asymmetry)




Overlay newly evaluated data.

# Comparison of Raw and New



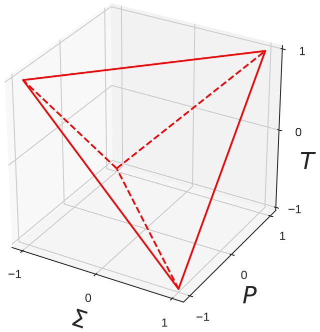
# Tetrahedral Inequality

The algebra of the transversity/helicity amplitudes leads to the inequalities:

$$|T - P| \leq 1 - \Sigma$$

and

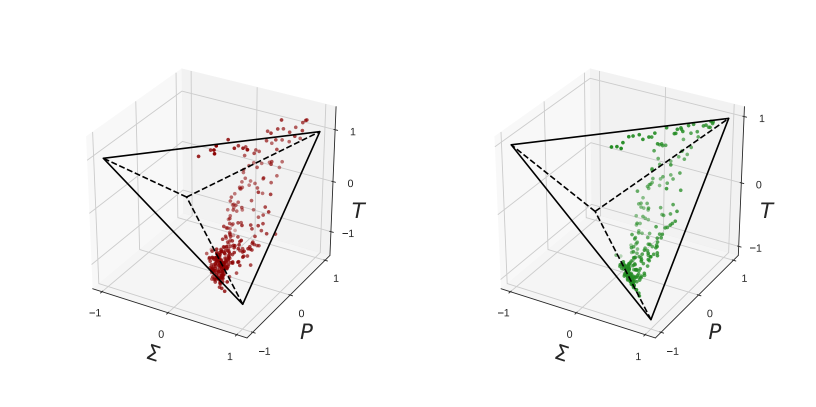
$$|T + P| \leq 1 + \Sigma$$



# Tetrahedral Inequality

Raw

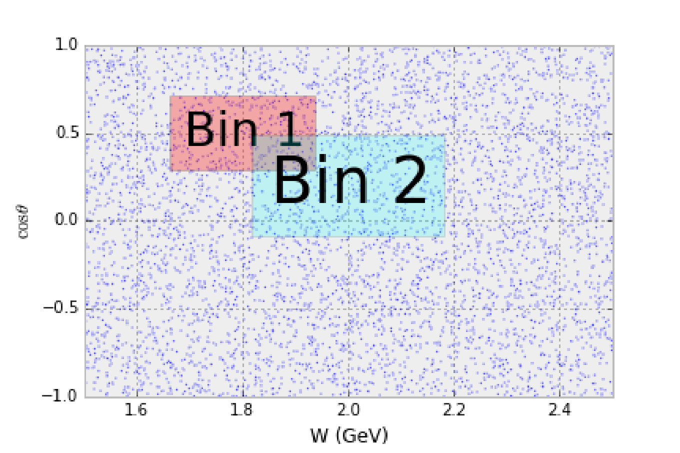
New



# Summary

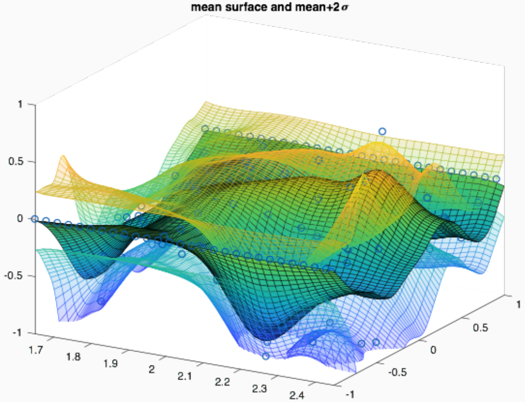
- Take data points from unconstrained fits
- Use MCMC to sample PDF in amplitude space, given measured data
- Project sampled PDF onto observable space
- Resulting PDF in observable space gives “new” consistent data

# Question: How to cope with different experiments?




Possible answer: Gaussian processes for interpolation

# Question: How to cope with different experiments?




Work in progress...

(ongoing collaboration with computer scientists)

# Take-home Messages

- Data consisting of several observables from the same channel can and should be made consistent.
- Key tools:
  - Algebra connecting observables to amplitudes
  - Evaluate full PDF (Markov Chain Monte Carlo)
- Need to be able to use independent experimental results.
- Resulting data will remove the need for arbitrary fudge factors in fits to theoretical calculations.