Determination of the Polarization Observables C_x, C_z, and P_y for the Quasi-Free Mechanism in $\vec{\gamma} d \to K^+ \vec{\Lambda} n$

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Why Study $K\Lambda$ Photoproduction?

- The study of nucleon resonance excitation plays an important role in building a comprehensive picture of the strong interaction.

- The theoretical work on quark models in the intermediate energy range predicts a rich resonance spectrum.

- Besides have been observed reactions involving pions in the initial and/or final states, those "missing" resonances may couple strongly to other channels, such as $K\Lambda$ and $K\Sigma$ channels.

- Many experimental results for $\overrightarrow{\gamma} p \to K^+ \overrightarrow{\Lambda}$ have been published, from total cross section to polarization observables.

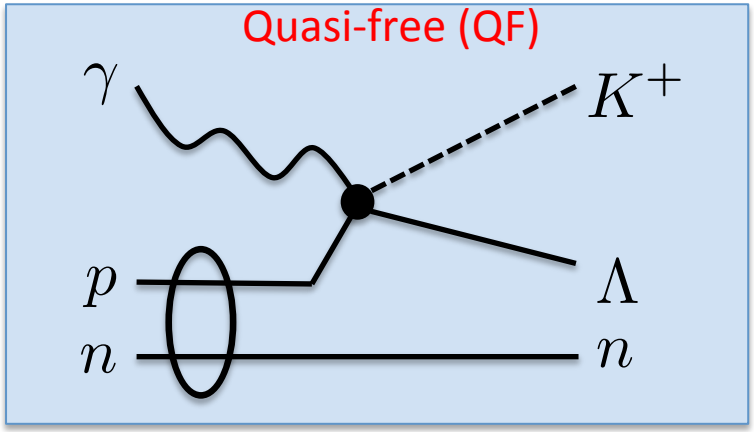
Importance of QF Study in $\overrightarrow{\gamma} d \to K^+ \overrightarrow{\Lambda} n$

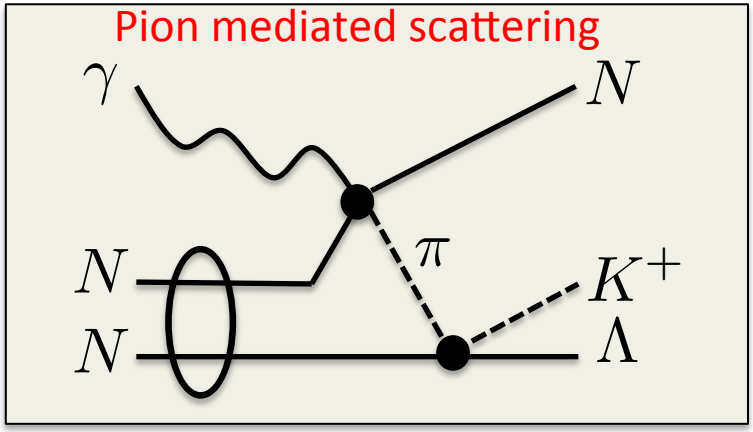
- Scattering off quasi-bound neutrons is used to extract observables for scattering off the free neutron.
  - Final-state interactions (FSI) effects.
  - Off-shell and nuclear effects.

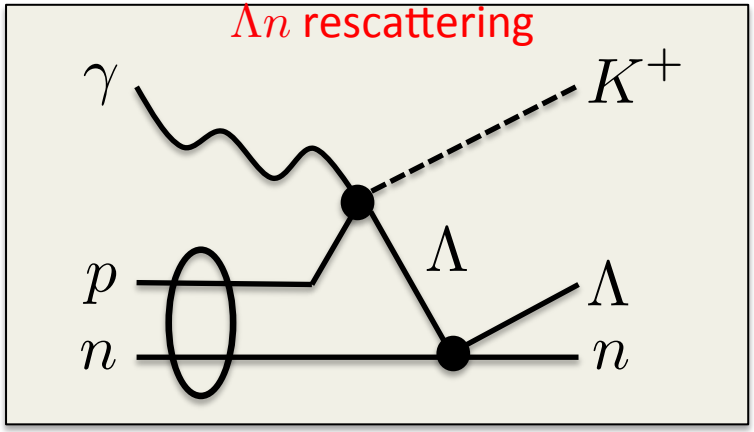
- We present an experimental study of the effect of FSI and Fermi motion on observables off the free proton obtained from data off the bound proton in $\overrightarrow{\gamma} d \to K^+ \overrightarrow{\Lambda} n$.

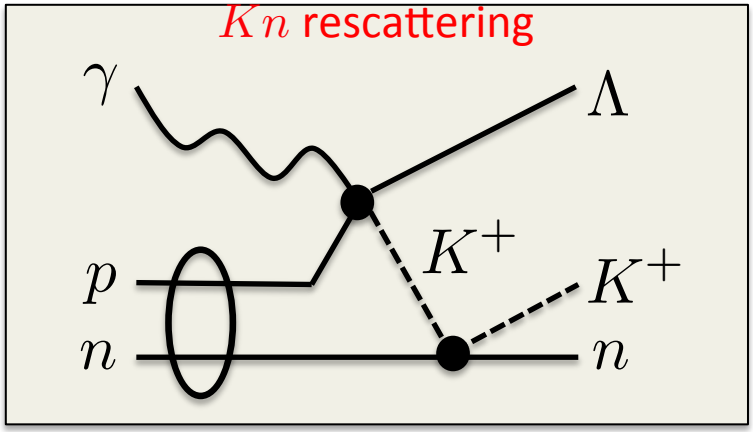
- Studied observables: induced Λ polarization and polarization transfers.

Main Mechanisms of $\vec{\gamma} d \to K^+ \vec{\Lambda} n$







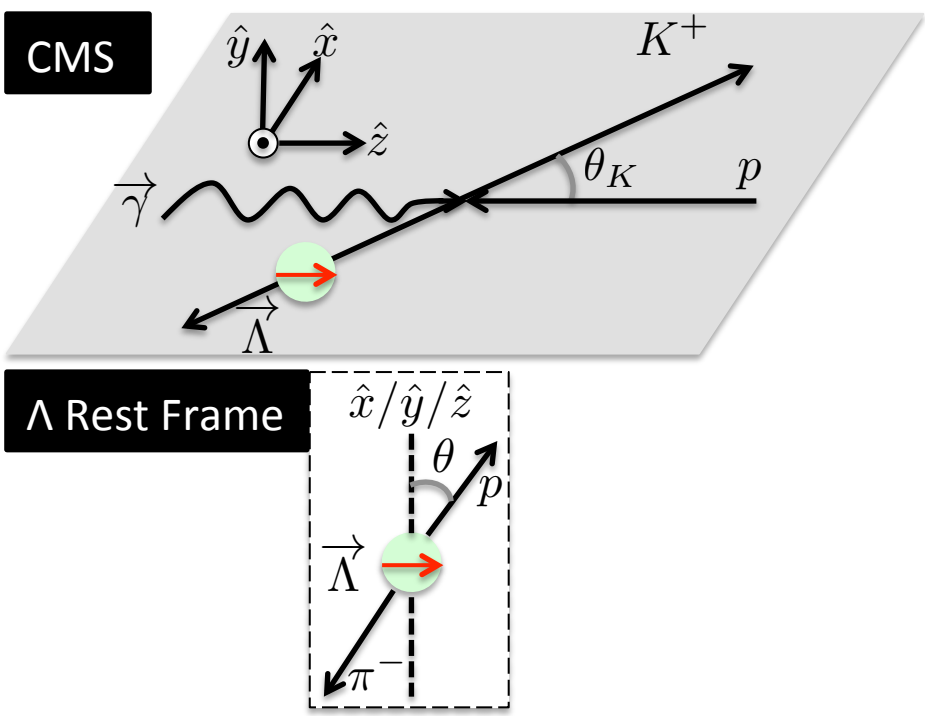


Experimental Observables

Helicity-dependent polarized differential cross section for hyperon photoproduction off the nucleon.

$$\frac{d\sigma^{\pm}}{d\Omega} = \frac{d\sigma}{d\Omega}|_{unpol}(1 \pm \alpha P_{circ} C_x \cos\theta_x \pm \alpha P_{circ} C_z \cos\theta_z + \alpha P_y \cos\theta_y)$$

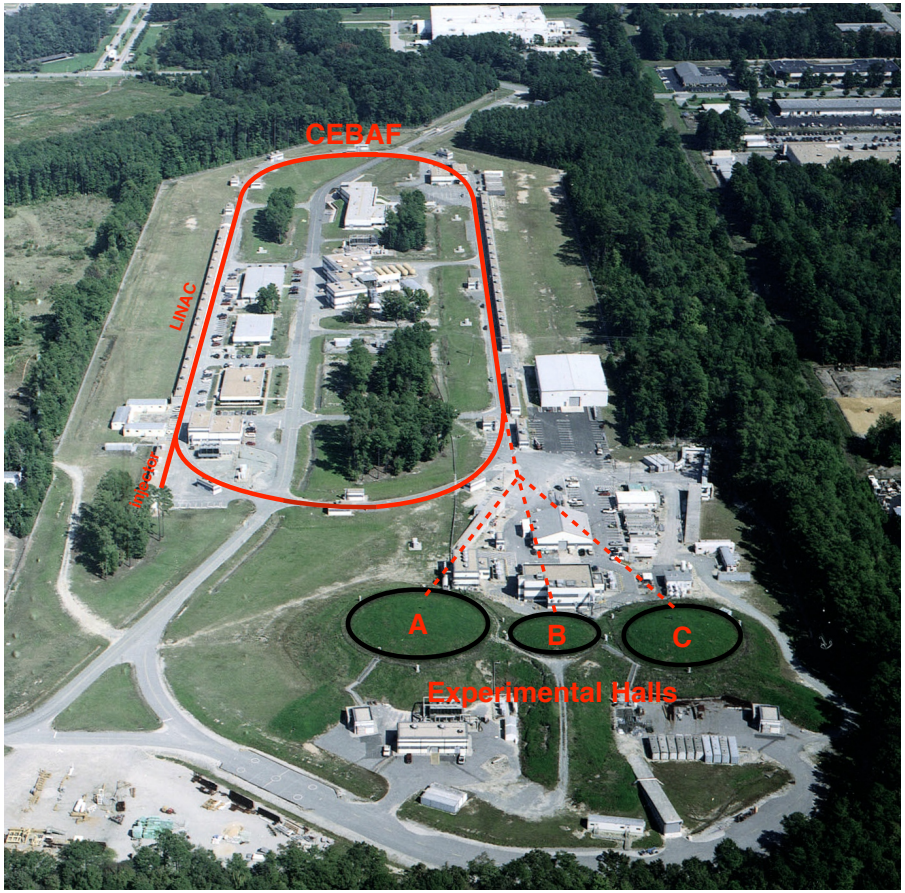
Λ self-analyzing power: $\alpha = 0.642 \pm 0.013$



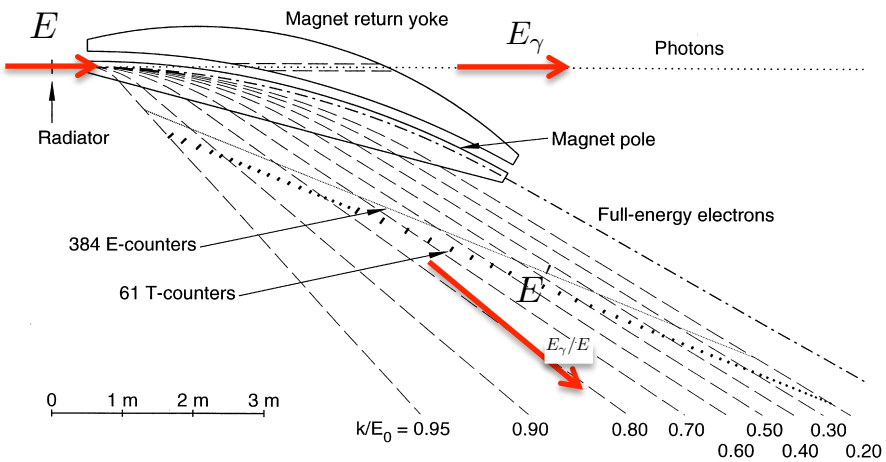
$$\begin{cases} \hat{z} = \hat{p}_\gamma \\ \hat{y} = \frac{\hat{p}_\gamma \times \hat{p}_K}{|\hat{p}_\gamma \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$

Experimental Facility: CEBAF and Photon Tagger

CEBAF



Hall-B Photon Tagger



$$E_\gamma = E - E'$$

E_e: 2 GeV, 2.65 GeV

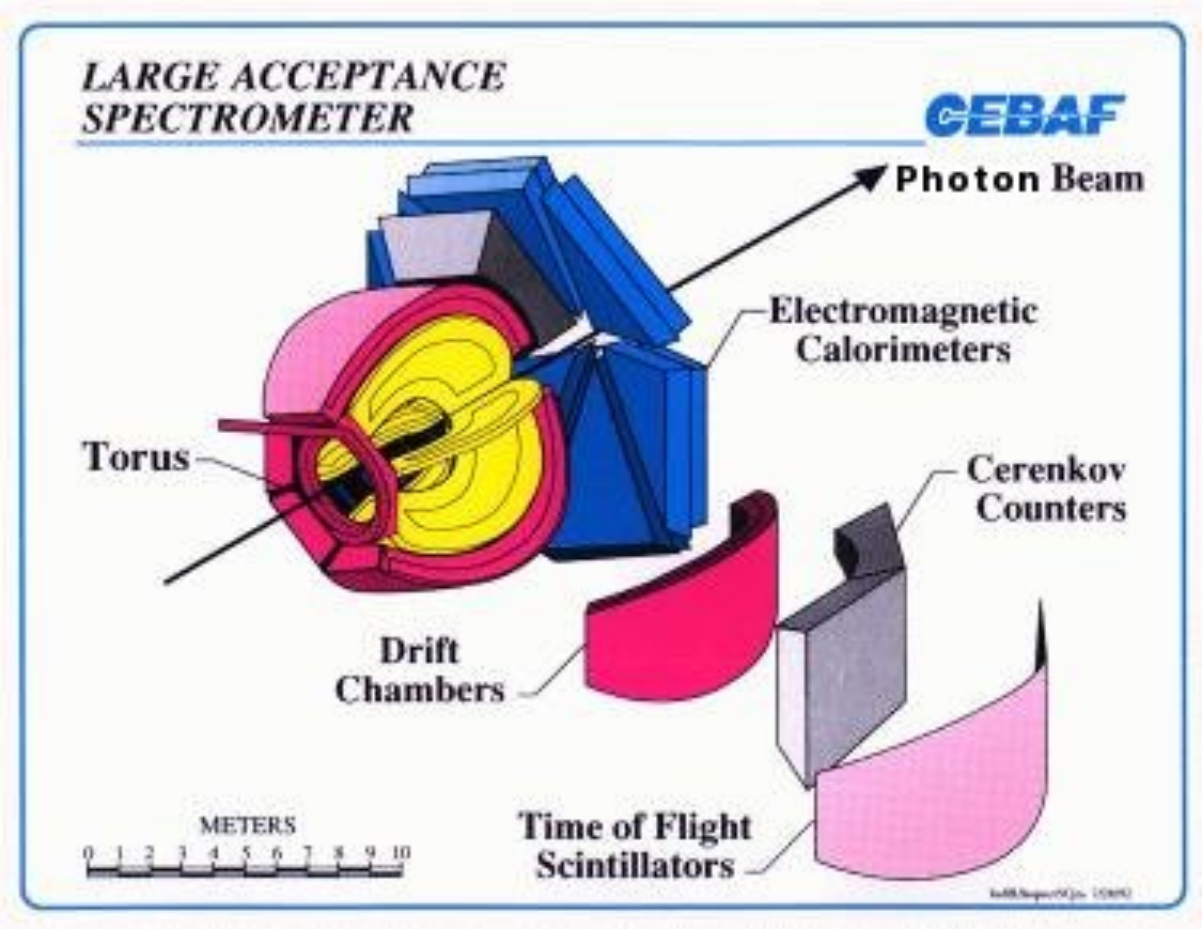
P_e: ~85%

E_γ: [0.9, 2.6] GeV

P_γ: [30%, 85%]

Currently, 12 GeV upgrade has been completed and a new hall D is in service.

Experimental Facility: CLAS



$$\vec{\gamma} d \rightarrow \boxed{K^+} \vec{\Lambda} n$$

$$\Lambda \rightarrow \boxed{p\pi^-}$$

K^+, p, π^- are detected.

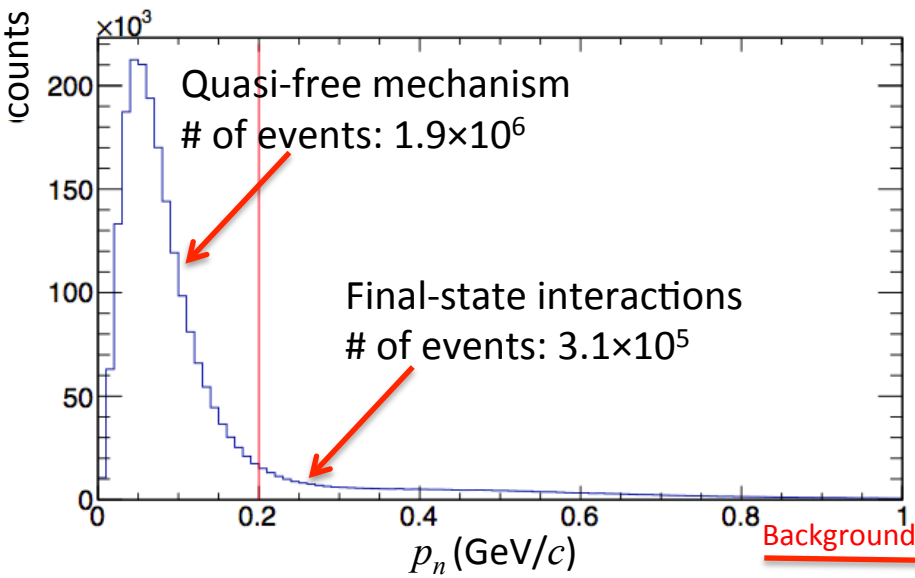
CLAS: Multi-particle charged final state
Acceptance: Almost 4π

Figure from: B. Mecking et al. Nucl. Instr. Meth. A 503, 513(2003).

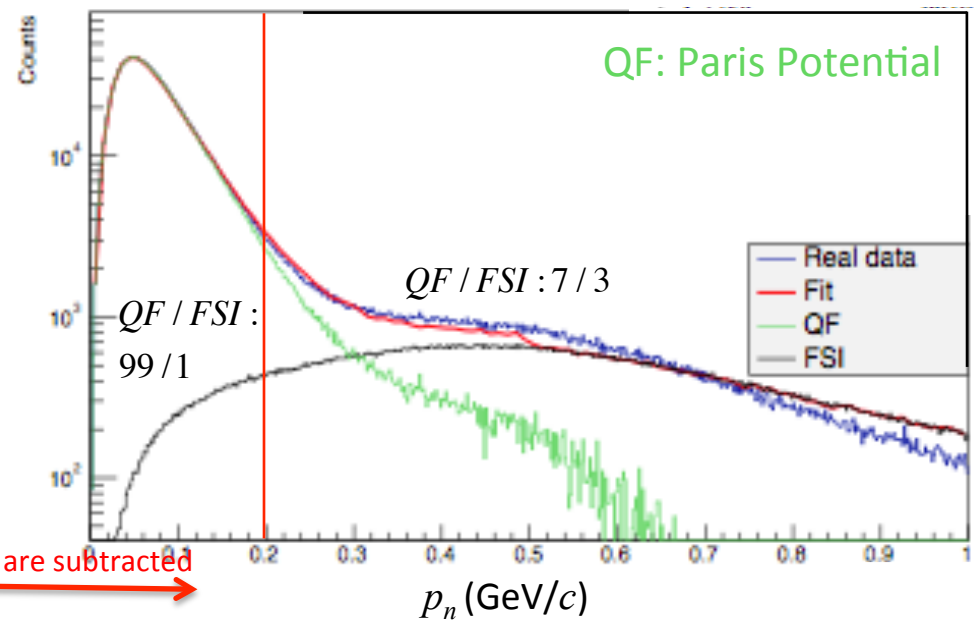
Selection of QF Mechanism

Event distribution over spectator momentum

$p_n \ (\gamma d \to K^+ \Lambda n)$



Comparison with model Distribution

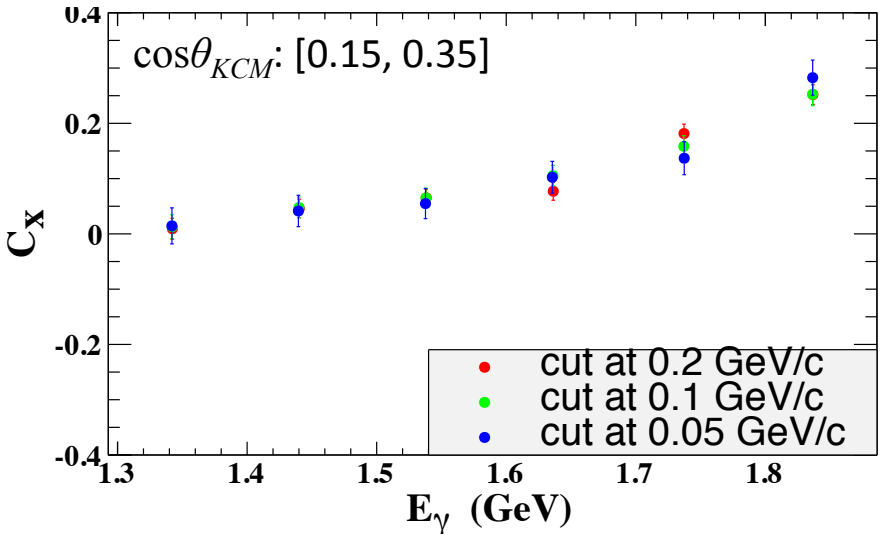


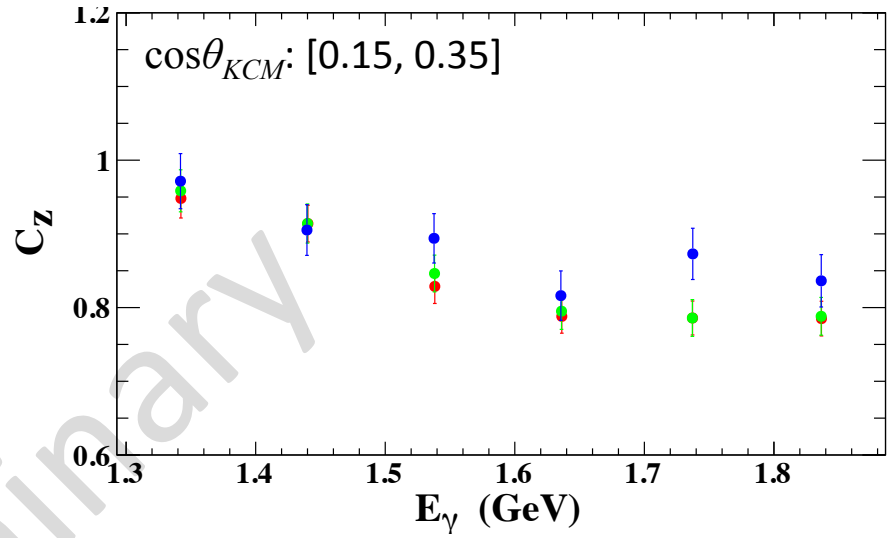
Backgrounds are subtracted

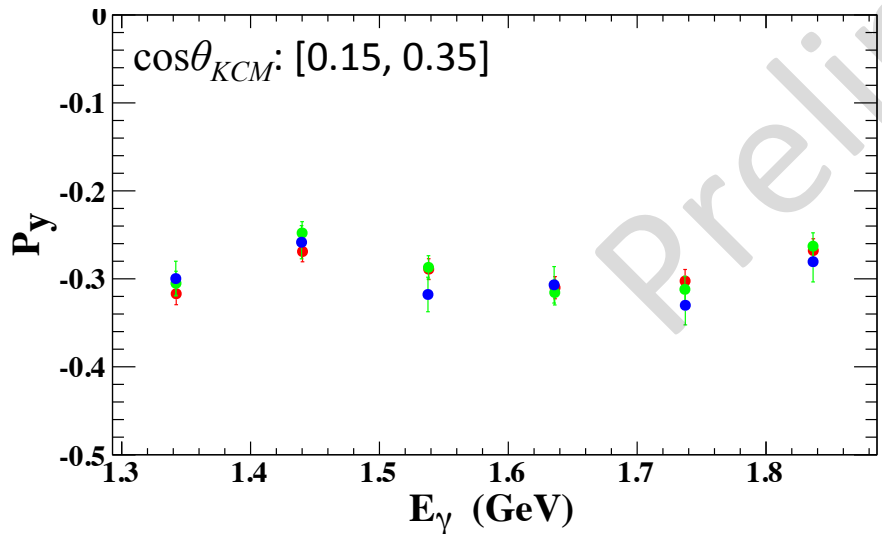
The removal of events with p_n < 0.2 GeV/c provides a sample that is by far dominated by FSI events. Standard analysis procedure.

Paris Potential describes well low p_n data. High-momentum tail drops off at ~0.6 GeV/c: effect on data interpretation.

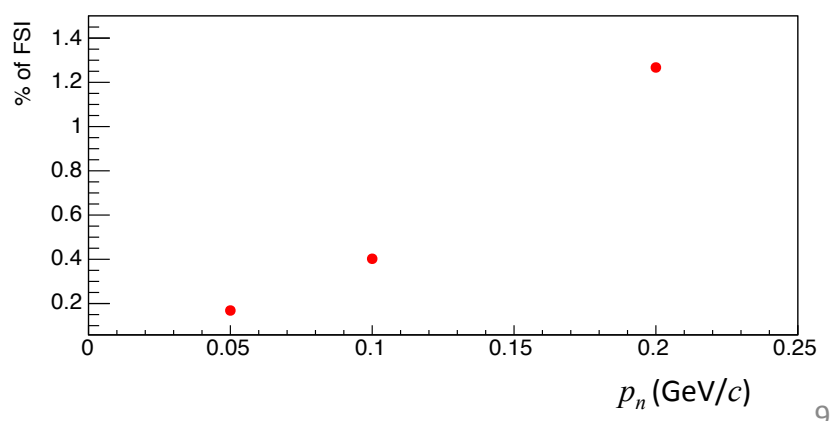
Effect of Missing Momentum Cut





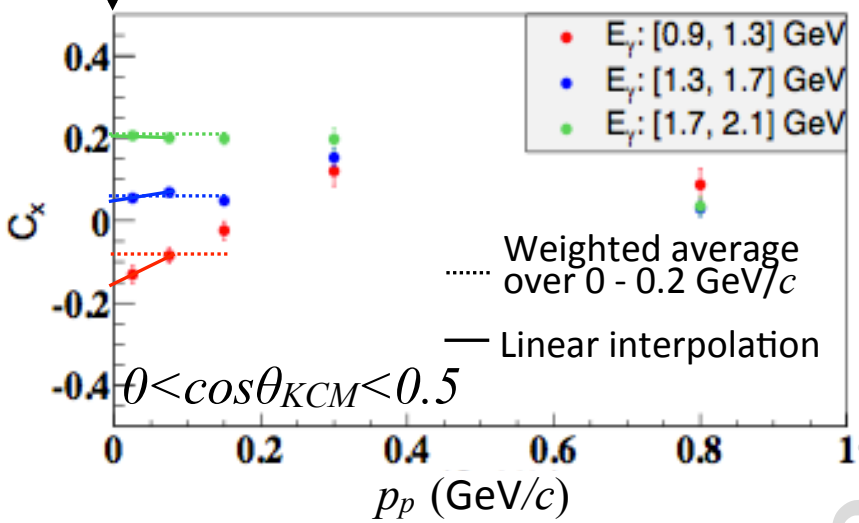


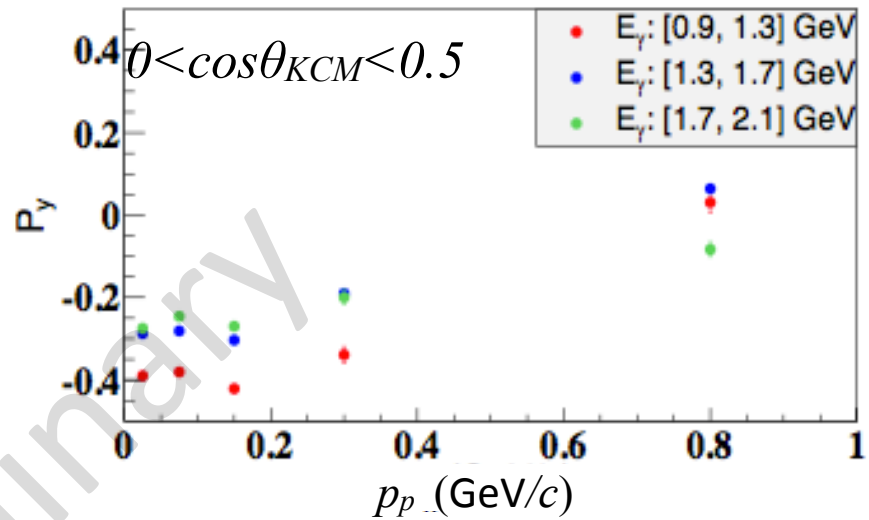
% of FSI at various missing momentum cuts

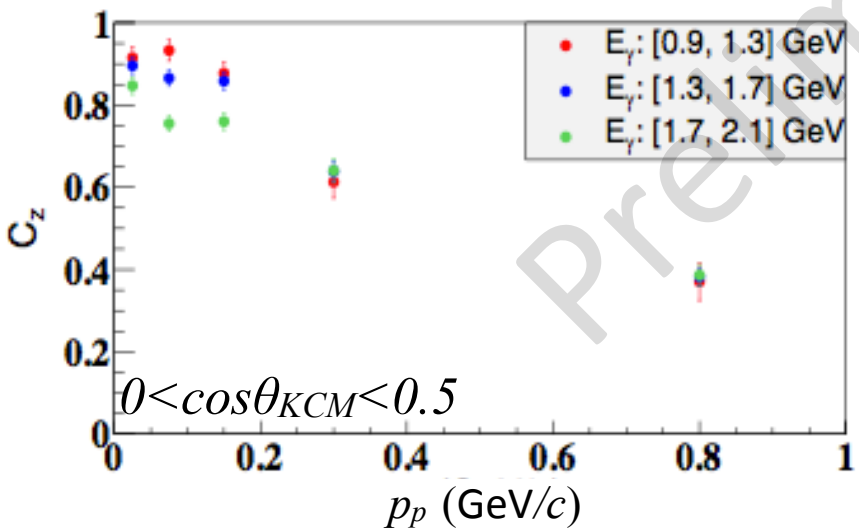


Evolution with Target Nucleon Momentum ($p_p = p_n$)

free proton, p_p= 0 GeV/c





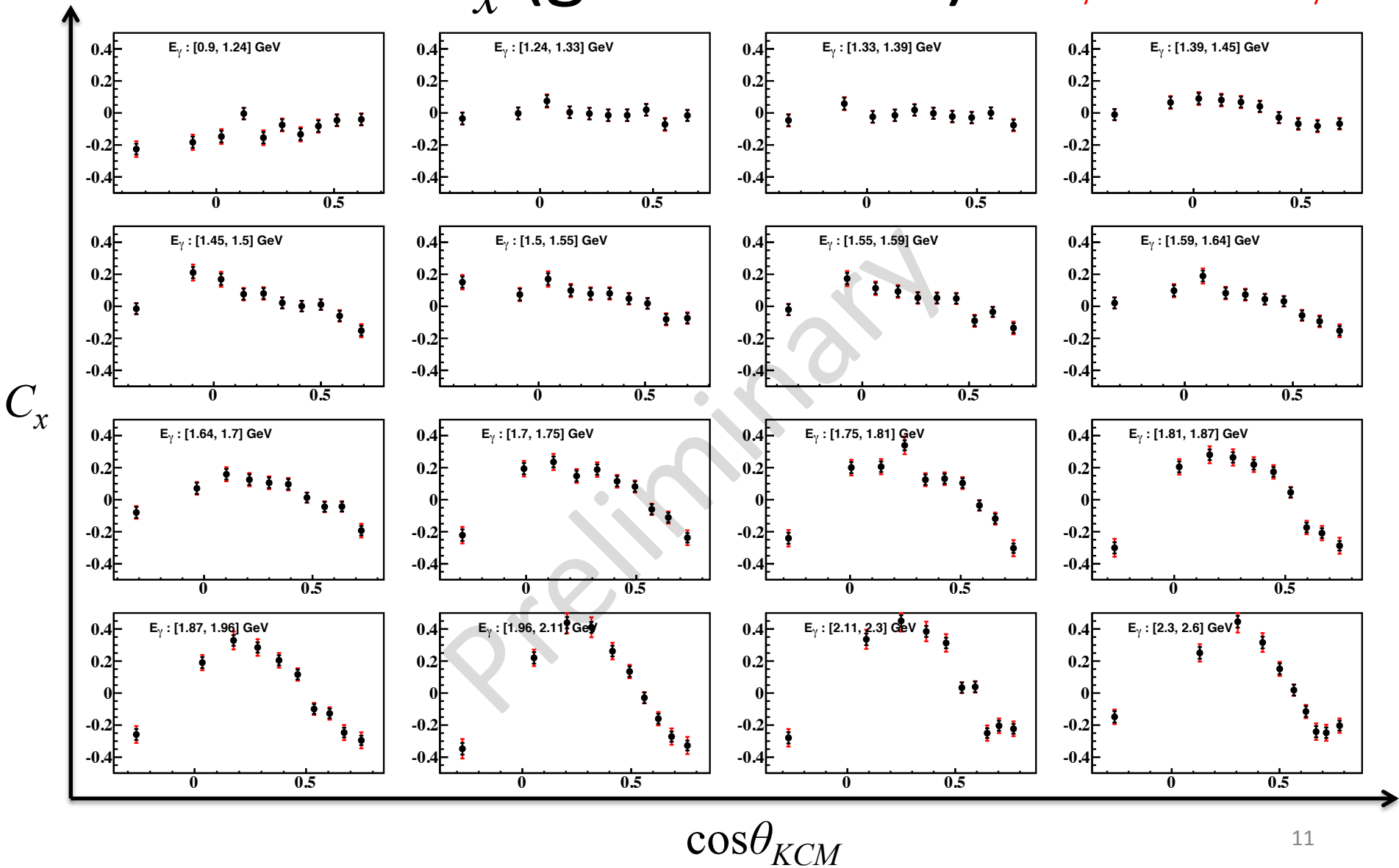


E_γ (GeV)	ΔC_x	ΔC_z	ΔP_y
0.9 - 1.3	46%	0.4%	0.3%
1.3 - 1.7	2%	4%	0.5%
1.7 - 2.1	4%	12%	10%

Δ: Difference between estimate at p_p=0 GeV/c from linear interpolation and average over 0. - 0.2 GeV/c.

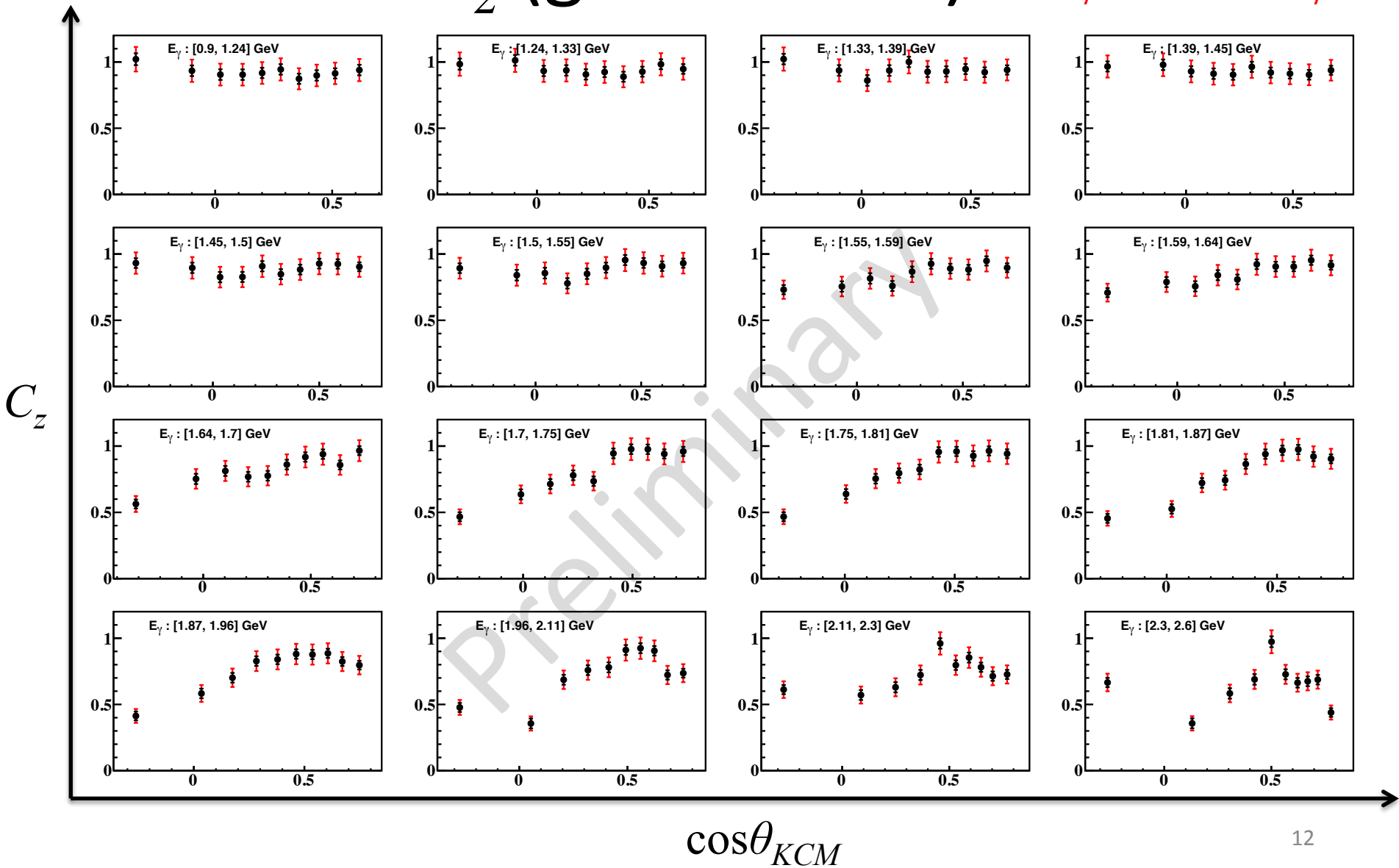
C_x (g13 Results)

Statistical uncertainty
Systematic uncertainty



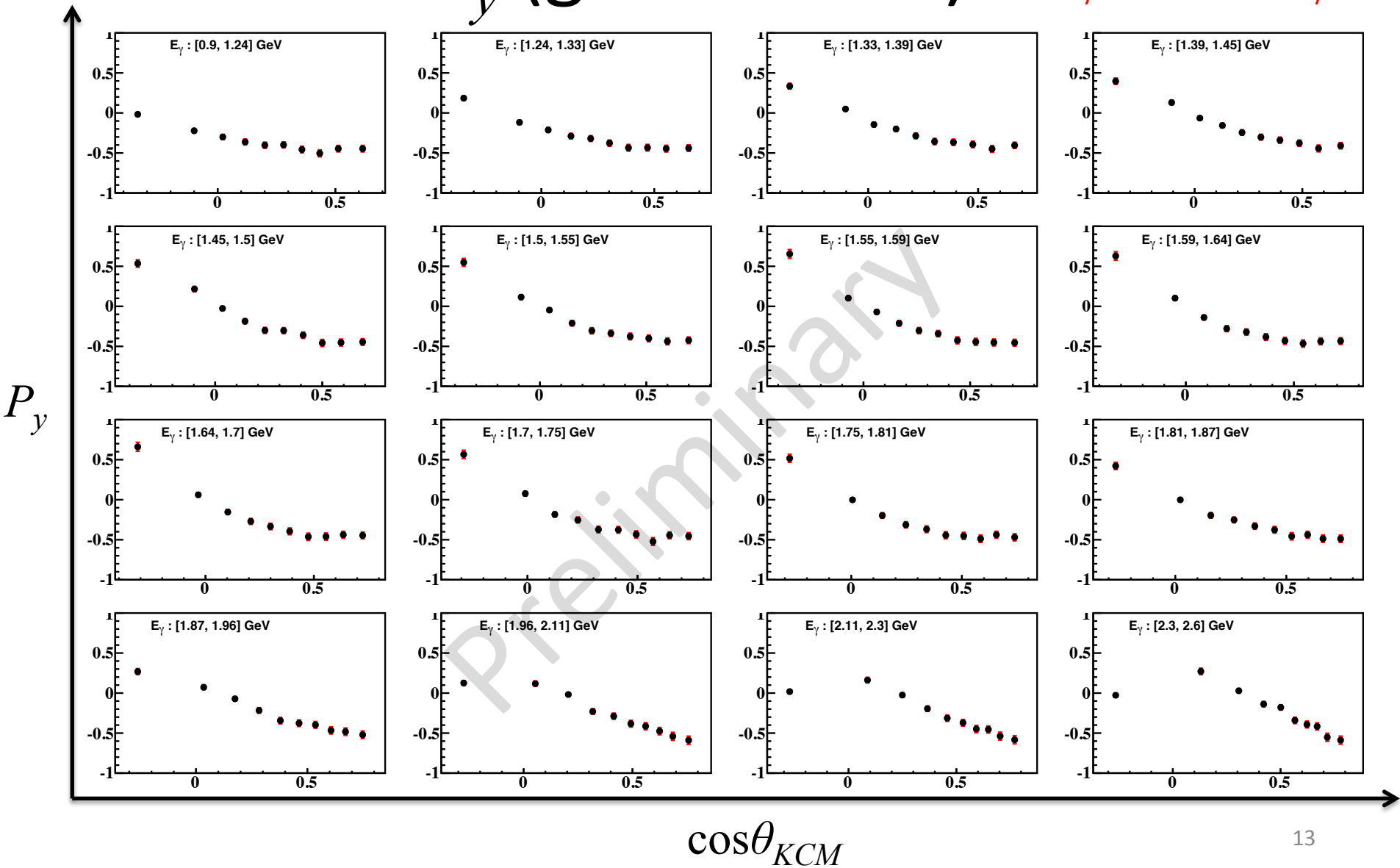
C_z (g13 Results)

Statistical uncertainty
Systematic uncertainty

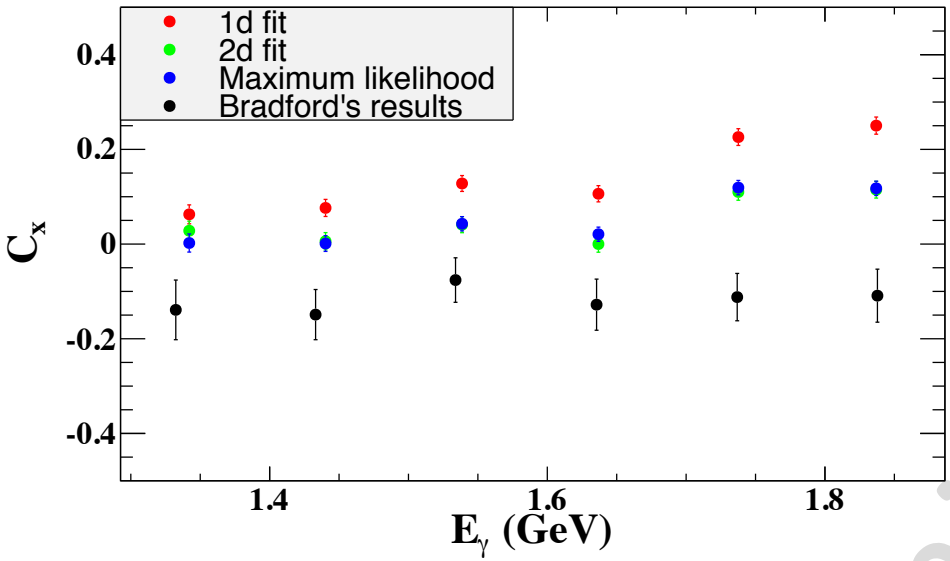


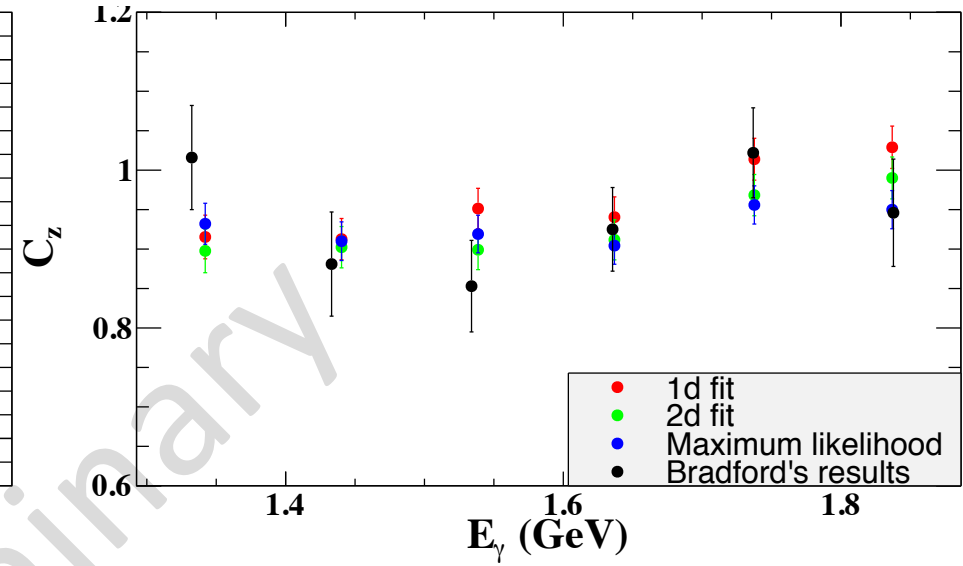
P_y (g13 Results)

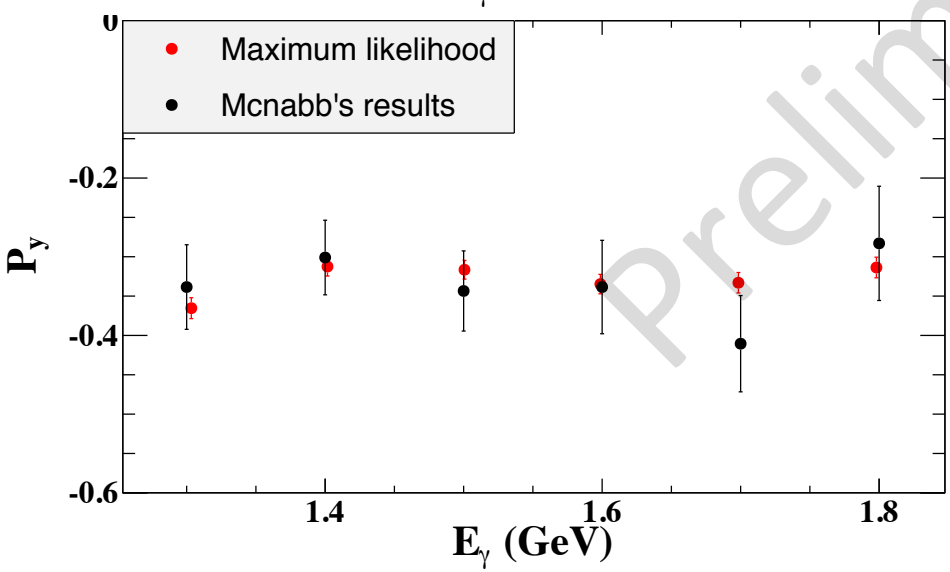
Statistical uncertainty
Systematic uncertainty



Comparison With g1c Results





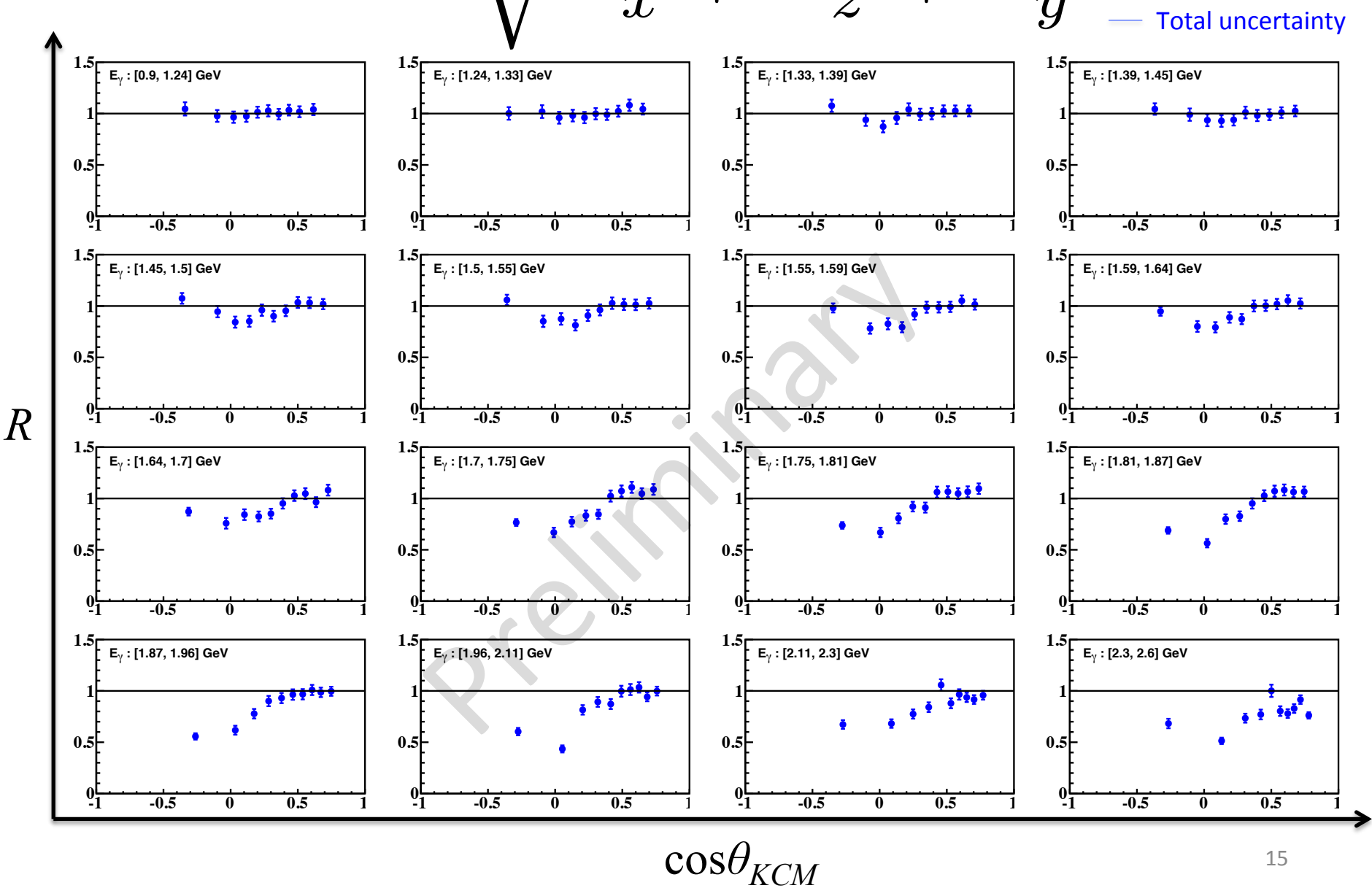


$\cos\theta_{KCM}$: [0.35, 0.55]

C_x and C_z from Robert K. Bradford
P_y from John W.C. McNabb

- Dataset: g1c
- Reaction: $\vec{\gamma} p \rightarrow K^+ \vec{\Lambda}$

$$R = \sqrt{C_x^2 + C_z^2 + P_y^2}$$



Summary

- Comprehensive results for C_x, C_z, and P_y in the kinematic bins of E_γ (0.9 – 2.6 GeV) and $\cos\theta_{KCM}$ (-1, 1) have been obtained for $K^+\Lambda$ photoproduction off the bound proton.

- The contamination of FSI events is about 1% for target nucleon momenta below 0.2 GeV/c.

- The dependence of an observable on the target nucleon momentum is different for different photon energy ranges. Polynomial extrapolation to p = 0 GeV/c could allow to obtain more accurate estimates of observables for scattering off the free nucleon than integrating over a range of p. Can be easily applied to high-statistics samples.

- Theoretical studies of quasi-free observables are very important to understand how to extract unbiased free-nucleon observables.

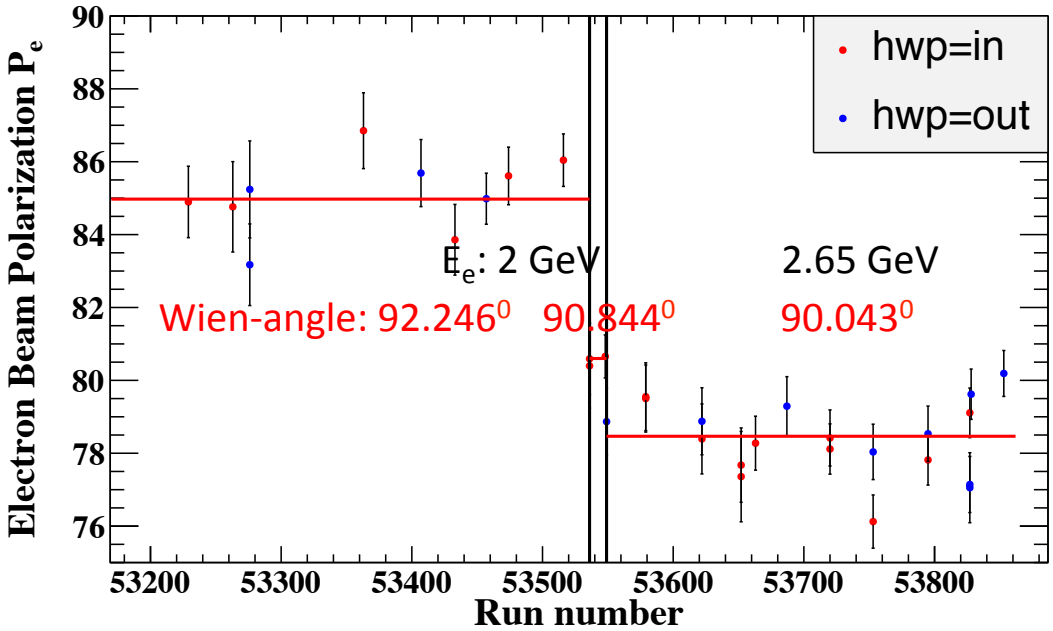
Backup Slides

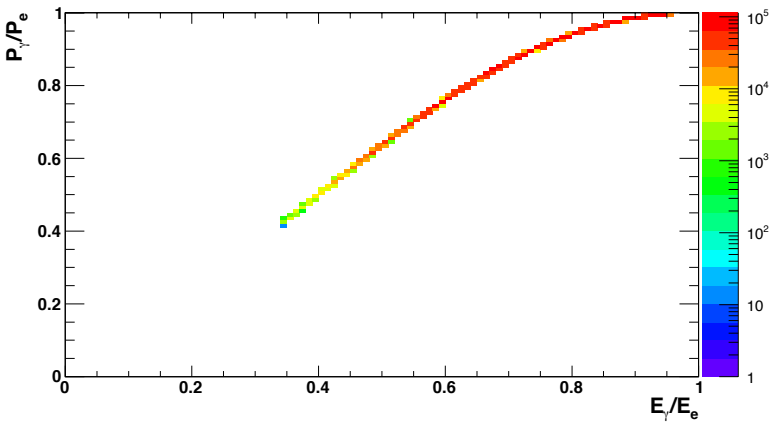
Photon Polarization

The electron polarization for some special runs were measured by the Mϕller polarimeter.

The polarization of the photon beam was calculated using the Maximon and Olson relation

$$P_{cir} = \frac{E_\gamma(E + \frac{1}{3}E')P_e}{E^2 + E'^2 - \frac{2}{3}EE'}$$

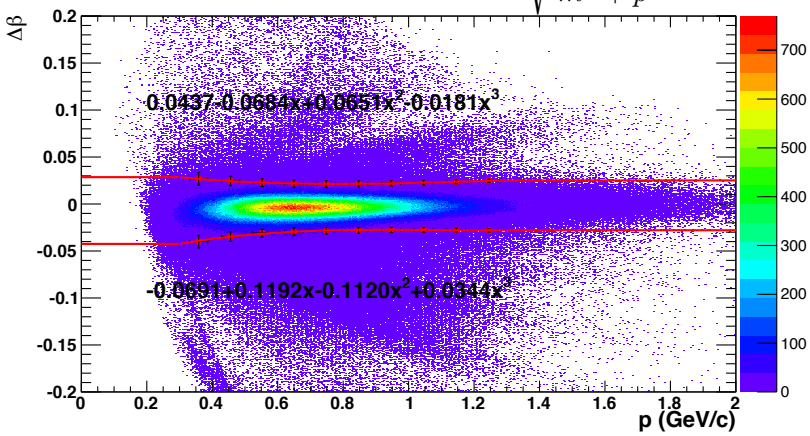




Reaction Yields

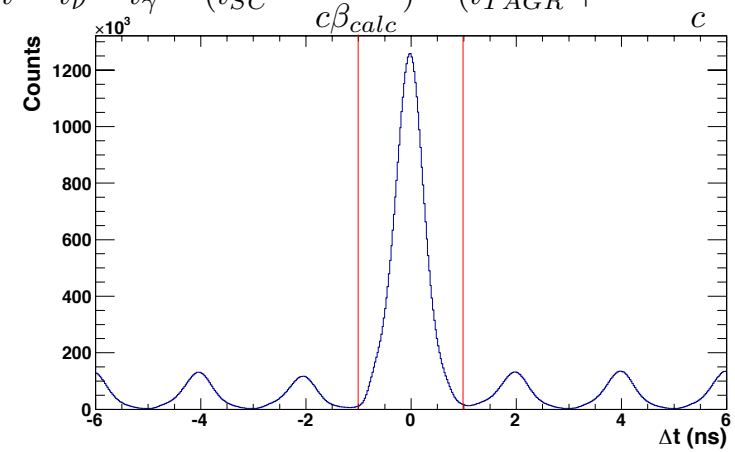
Particle Identification

$$\Delta\beta = \beta_{meas} - \beta_{calc} = \beta_{meas} - \sqrt{\frac{p^2}{m^2+p^2}}$$

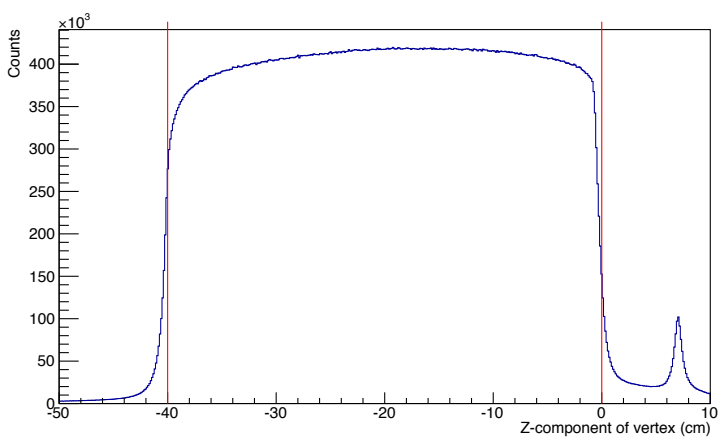


Photon Selection

$$\Delta t = t_\nu - t_\gamma = (t_{SC} - \frac{d_{SC}}{c\beta_{calc}}) - (t_{TAGR} + \frac{z+20\ (cm)}{c})$$

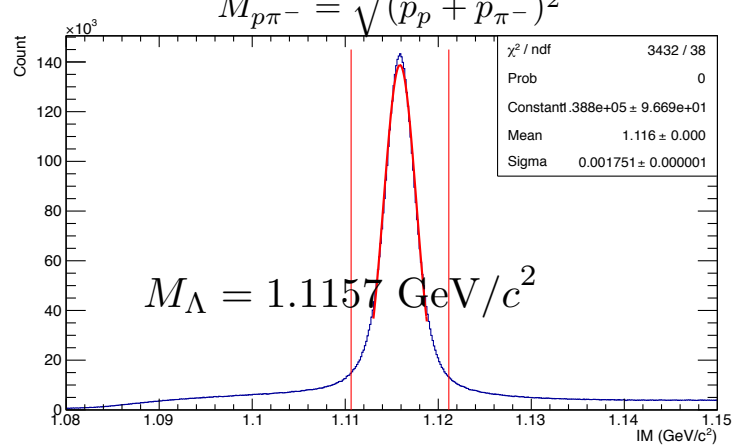


Z-Vertex Cut



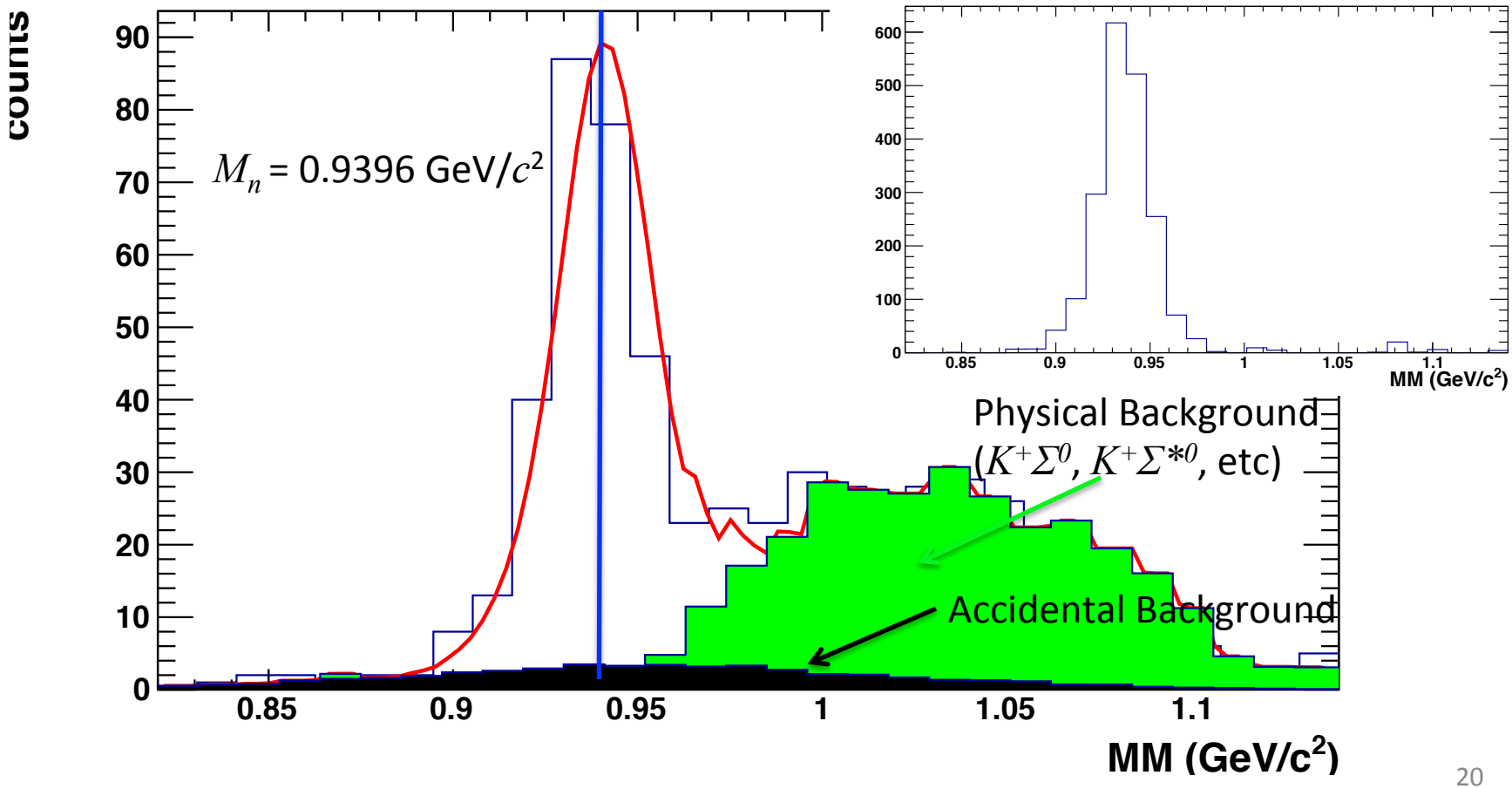
Invariant-Mass Cut

$$M_{p\pi^-} = \sqrt{(\widetilde{p}_p + \widetilde{p}_{\pi^-})^2}$$



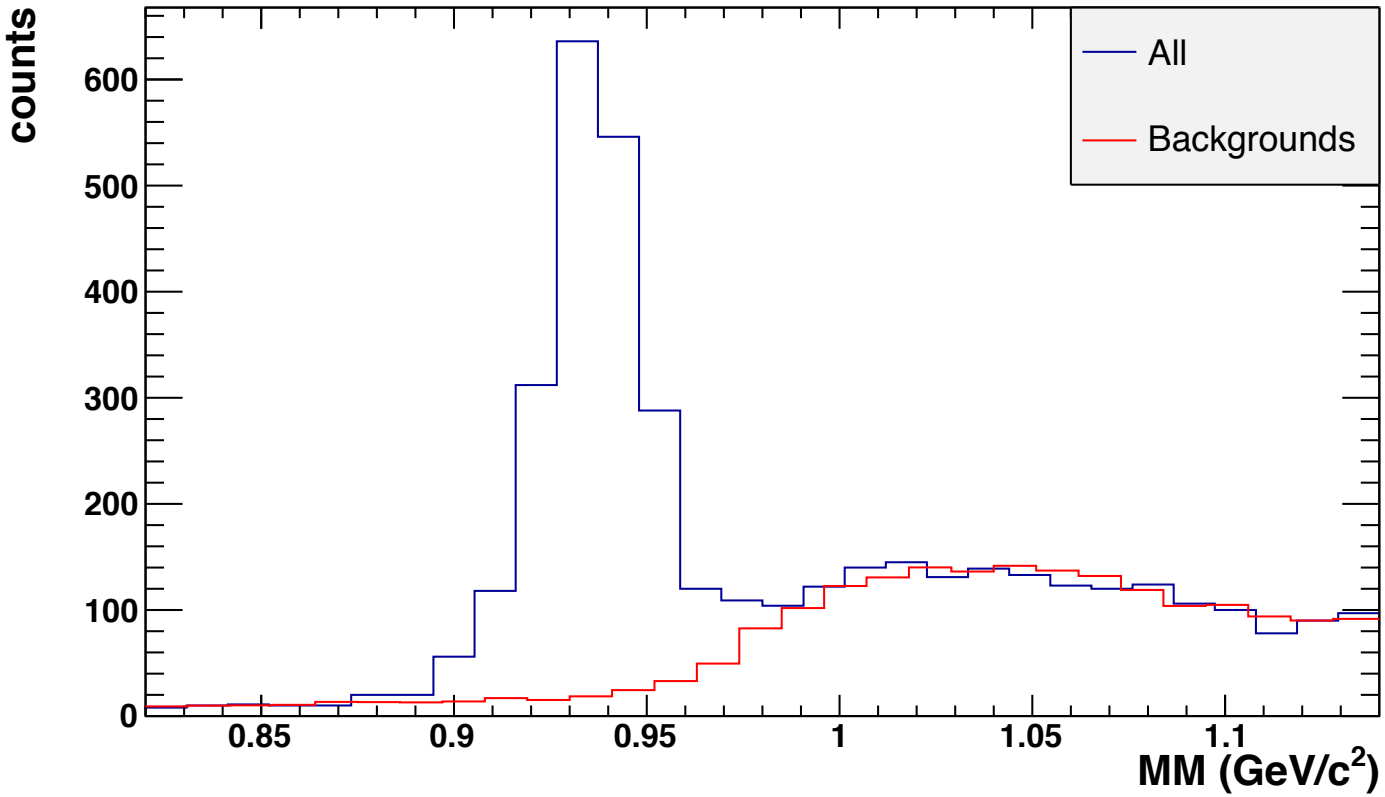
Background Subtraction

$$MM = \sqrt{(\widetilde{p}_\gamma + \widetilde{p}_d - \widetilde{p}_{K^+} - \widetilde{p}_p - \widetilde{p}_{\pi^-})^2}$$



Method 2 of Observable-Extraction: Weight for Each Event

Weight = (All – Backgrounds) / All



Method 2 of Observable-Extraction: The Maximum Likelihood Method

Non-normalized Probability Density Function (PDF) defined from the polarized differential cross section:

$$PDF = \frac{d\sigma}{d\Omega}_{|unpol}(1 \pm \alpha P_{circ} C_x \cos\theta_x \pm \alpha P_{circ} C_z \cos\theta_z + \alpha P_y \cos\theta_y)$$

Total likelihood is the product of the likelihoods for all individual events:

$$logL = b + \sum_{i=1}^{n^+} log[(1 + \alpha P^i_{circ} C_x \cos\theta^i_x + \alpha P^i_{circ} C_z \cos\theta^i_z + \alpha P_y \cos\theta^i_y) w^i]$$

$$\sum_{j=1}^{n^-} log[(1 - \alpha P^j_{circ} C_x \cos\theta^j_x - \alpha P^j_{circ} C_z \cos\theta^j_z + \alpha P_y \cos\theta^j_y) w^j]$$

Next side will introduce how to set weight w^i and w^j for each event.

Why the Maximum Likelihood Method Can Ignore Acceptance?

Non-normalized PDF with consideration of acceptance:

$$PDF = A(\theta,\phi)\sigma^{\pm}(\theta,\phi;C_x,C_z,P_y)w$$

Total likelihood: $$logL(C_x,C_z,P_y) = \sum_{i=1}^{n^+} logA(\theta_i,\phi_i) + \sum_{j=1}^{n^-} logA(\theta_j,\phi_j)$$

$$+ \sum_{i=1}^{n^+} log[\sigma^+(\theta_i,\phi_i;C_x,C_z,P_y)w_i] + \sum_{j=1}^{n^-} log[\sigma^-(\theta_j,\phi_j;C_x,C_z,P_y)w_j]$$

Equation array to obtain C_x, C_z, and P_y:

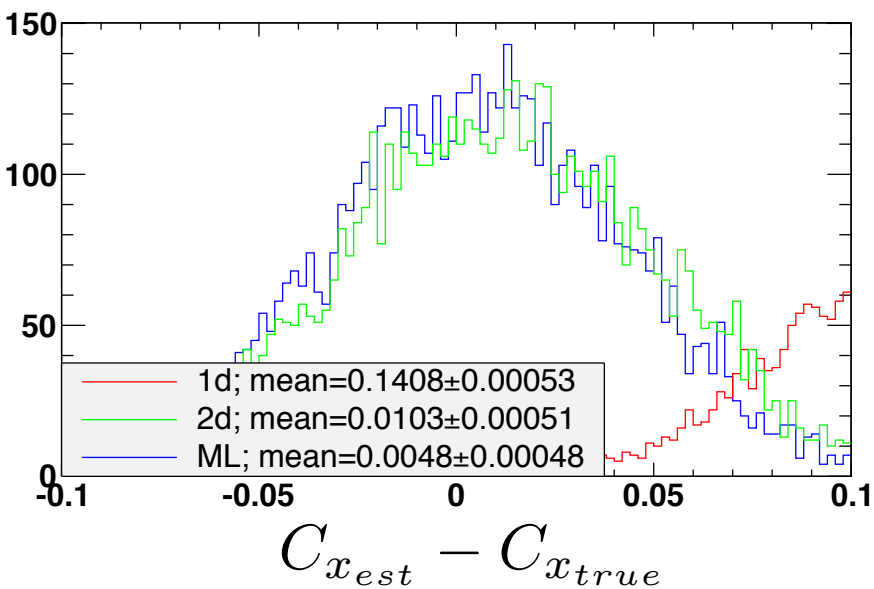
$$\begin{cases} \frac{\partial logL(C_x,C_z,P_y)}{\partial C_x} = \frac{\partial \sum_{i=1}^{n^+} log[\sigma^+(\theta_i,\phi_i;C_x,C_z,P_y)w_i]}{\partial C_x} + \frac{\partial \sum_{j=1}^{n^-} log[\sigma^-(\theta_j,\phi_j;C_x,C_z,P_y)w_j]}{\partial C_x} = 0 \\ \frac{\partial logL(C_x,C_z,P_y)}{\partial C_z} = \frac{\partial \sum_{i=1}^{n^+} log[\sigma^+(\theta_i,\phi_i;C_x,C_z,P_y)w_i]}{\partial C_z} + \frac{\partial \sum_{j=1}^{n^-} log[\sigma^-(\theta_j,\phi_j;C_x,C_z,P_y)w_j]}{\partial C_z} = 0 \\ \frac{\partial logL(C_x,C_z,P_y)}{\partial P_y} = \frac{\partial \sum_{i=1}^{n^+} log[\sigma^+(\theta_i,\phi_i;C_x,C_z,P_y)w_i]}{\partial P_y} + \frac{\partial \sum_{j=1}^{n^-} log[\sigma^-(\theta_j,\phi_j;C_x,C_z,P_y)w_j]}{\partial P_y} = 0 \end{cases}$$

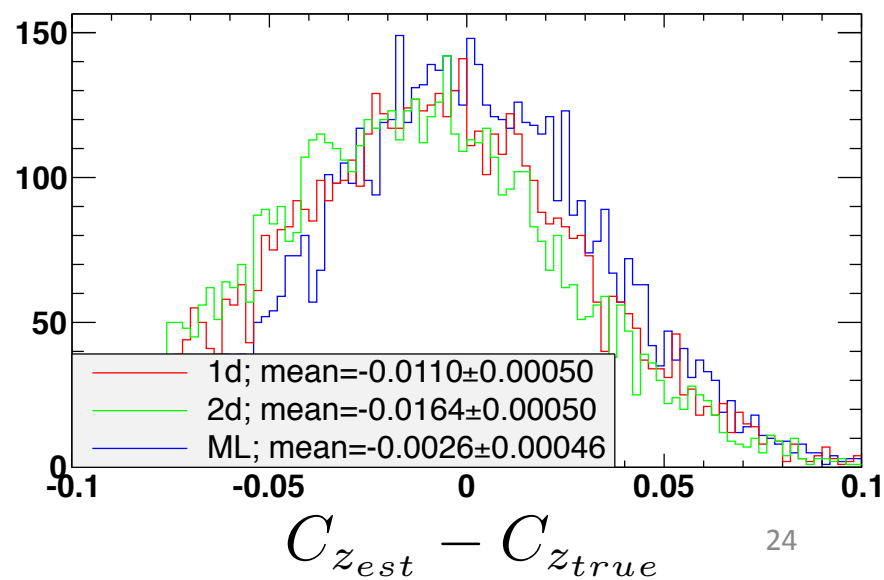
Acceptance is cancelled because it's independent of polarization observables.

Simulation Study to Understand Different Methods

A study was used to evaluate potential bias of the maximum likelihood method and the binned methods.

- 6000 different experiments, with 10^6 events in each experiment, were generated according to the differential polarized cross section with realistic values of C_x, C_z, and P_y for $\vec{\gamma} p \to K^+ \vec{\Lambda}$.
- Generated data were processed through GSIM and gpp.
- After raw data were skimmed, the observables were extracted using the maximum likelihood method and the binned methods.





Observable-Extraction Methods

- One-dimensional fit:

$$Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\int\int \frac{d\sigma^+}{d\Omega} d(cos\theta_y) d(\cos\theta_{z/x}) - \int\int \frac{d\sigma^-}{d\Omega} d(cos\theta_y) d(\cos\theta_{z/x})}{\int\int \frac{d\sigma^+}{d\Omega} d(cos\theta_y) d(\cos\theta_{z/x}) + \int\int \frac{d\sigma^-}{d\Omega} d(cos\theta_y) d(\cos\theta_{z/x})} = \alpha P_{circ} C_{x/z} \cos\theta_{x/z}$$

- Two-dimensional fit:

$$Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\int \frac{d\sigma^+}{d\Omega} d(cos\theta_y)) - \int \frac{d\sigma^-}{d\Omega} d(cos\theta_y)}{\int \frac{d\sigma^+}{d\Omega} d(cos\theta_y)) + \int \frac{d\sigma^-}{d\Omega} d(cos\theta_y)} = \alpha P_{circ} C_x \cos\theta_x + \alpha P_{circ} C_z \cos\theta_z$$

- Maximum likelihood Method:

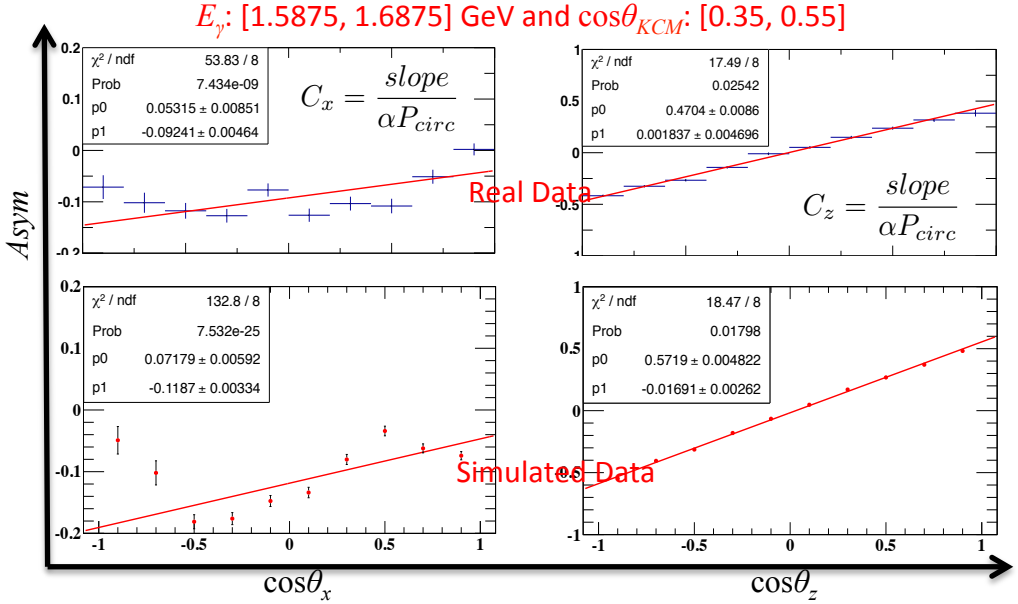
$$PDF = \frac{d\sigma}{d\Omega}|_{unpol}(1 \pm \alpha P_{circ} C_x \cos\theta_x \pm \alpha P_{circ} C_z \cos\theta_z + \alpha P_y \cos\theta_y)$$

Effect of Acceptance

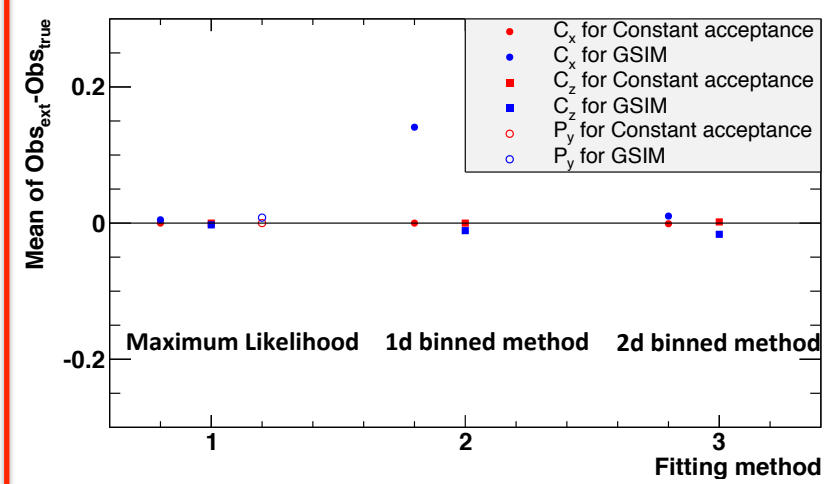
Comprehensive studies by analytical analysis and simulation tell us

- The effect of acceptance cannot be ignored in 1D fit, especially for C_x.
- 2D fitting can reduce the effect of the acceptance to some extent.
- The maximum likelihood method can reduce the effect of acceptance to the largest extent.

E_γ: [1.5875, 1.6875] GeV and $\cos\theta_{KCM}$: [0.35, 0.55]



Mean of differences between extracted and true observable values by 6000 simulated data samples



Why is the Bias Small for C_z from 1D Fit?

In the spherical coordinate system:

$$\begin{cases} \cos\theta_x = \sin\theta\cos\phi \\ \cos\theta_y = \sin\theta\sin\phi \\ \cos\theta_z = \cos\theta \end{cases}$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

θ_x, θ_y, and θ_z are not independent.

Event yield: $Y^{\pm}(\theta,\phi) = N_\gamma^{\pm} N_T \sigma^{\pm}(\theta,\phi) A(\theta,\phi)$

Integral over ϕ: $Y^{\pm}(\theta) = c(A(\theta) \pm \alpha P_{circ} C_x \sin\theta A_x(\theta) \pm \alpha P_{circ} C_z \cos\theta A(\theta) + \alpha P_y \sin\theta A_y(\theta))$

$$A(\theta) = \int_0^{2\pi} A(\theta,\phi)\,\mathrm{d}\phi; \quad A_x(\theta) = \int_0^{2\pi} A(\theta,\phi)\cos\phi\,\mathrm{d}\phi; \quad A_y(\theta) = \int_0^{2\pi} A(\theta,\phi)\sin\phi\,\mathrm{d}\phi$$

$$A_x(\theta) = \int_0^{2\pi} A(\theta,\phi)\cos\phi\,\mathrm{d}\phi < \int_0^{2\pi} A(\theta,\phi)|\cos\phi|\,\mathrm{d}\phi < |\cos\phi|_{max}\int_0^{2\pi} A(\theta,\phi)\,\mathrm{d}\phi = \int_0^{2\pi} A(\theta,\phi)\,\mathrm{d}\phi = A(\theta)$$

Asymmetry: $Asym = \dfrac{Y^+ - Y^-}{Y^+ + Y^-} = \dfrac{\alpha P_{circ} C_x \sin\theta A_x(\theta) + \alpha P_{circ} C_z \cos\theta A(\theta)}{A(\theta) + \alpha P_y \sin\theta A_y(\theta)}$

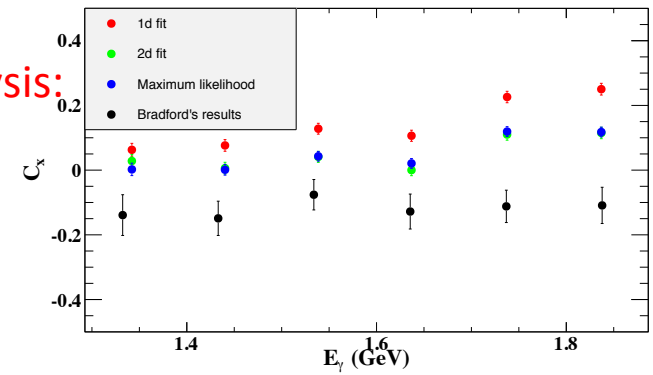
Generally, $|C_x| << |C_z|, |P_y| < |C_z|$

Therefore, $Asym \approx \alpha P_{circ} C_z \cos\theta_z$

Why is the Bias Large for C_x from 1D Fit?

Spherical coordinate system for the convenience of C_x analysis:

$$\begin{cases} \cos\theta_x = \cos\theta \\ \cos\theta_y = \sin\theta\cos\phi \\ \cos\theta_z = \sin\theta\sin\phi \end{cases}$$

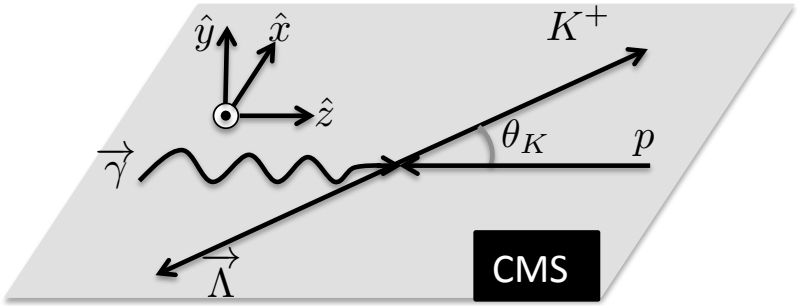


Asymmetry: $$Asym = \frac{Y^+ - Y^-}{Y^+ + Y^-} = \frac{\alpha P_{circ} C_x \cos\theta A(\theta) + \alpha P_{circ} C_z \sin\theta A_z(\theta)}{A(\theta) + \alpha P_y \sin\theta A_y(\theta)}$$

In general, C_x is small relative to C_z and P_y, so C_z and P_y terms do not cancel. Therefore, the asymmetry for C_x is not a linear function of $\cos\theta_x$.

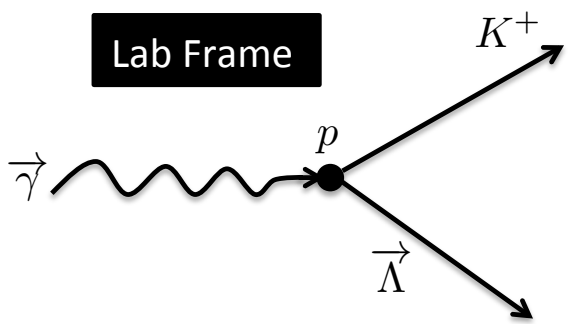
- The effect of acceptance cannot be ignored in 1D fit, especially for C_x.
- The situation with P_y is somewhat in-between C_x and C_z if it's extracted by 1D fit.
- 2D fitting can reduce the effect of the acceptance to some extent.

Effect of Axis Convention



Convention 1:

$$\begin{cases} \hat{z} = \hat{p}_\gamma \\ \hat{y} = \frac{\hat{p}_\gamma \times \hat{p}_K}{|\hat{p}_\gamma \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$



Convention 2:

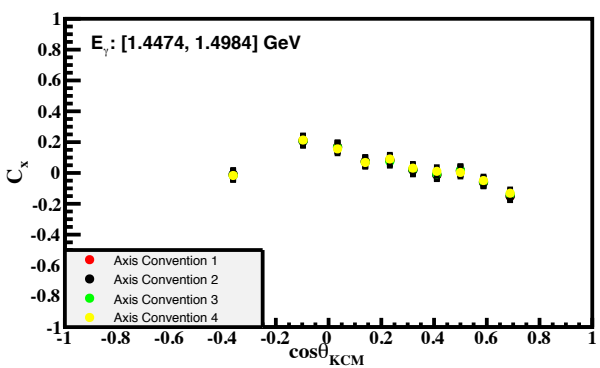
$$\begin{cases} \hat{z} = \hat{p}_\gamma \\ \hat{y} = \frac{\hat{p}_\gamma \times \hat{p}_K}{|\hat{p}_\gamma \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$

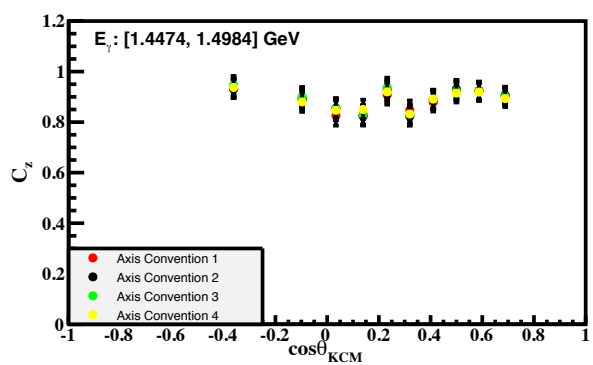
Convention 3:

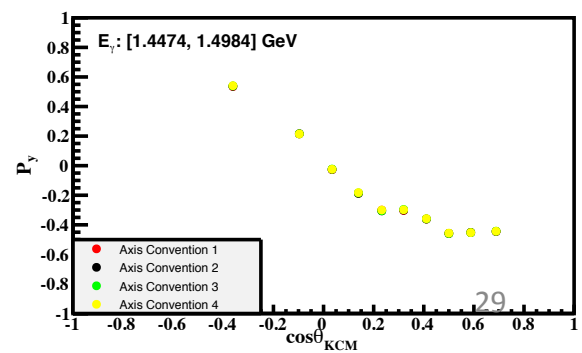
$$\begin{cases} \hat{z} = \hat{p}_\gamma \\ \hat{y} = \frac{\hat{p}_\Lambda \times \hat{p}_K}{|\hat{p}_\Lambda \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$

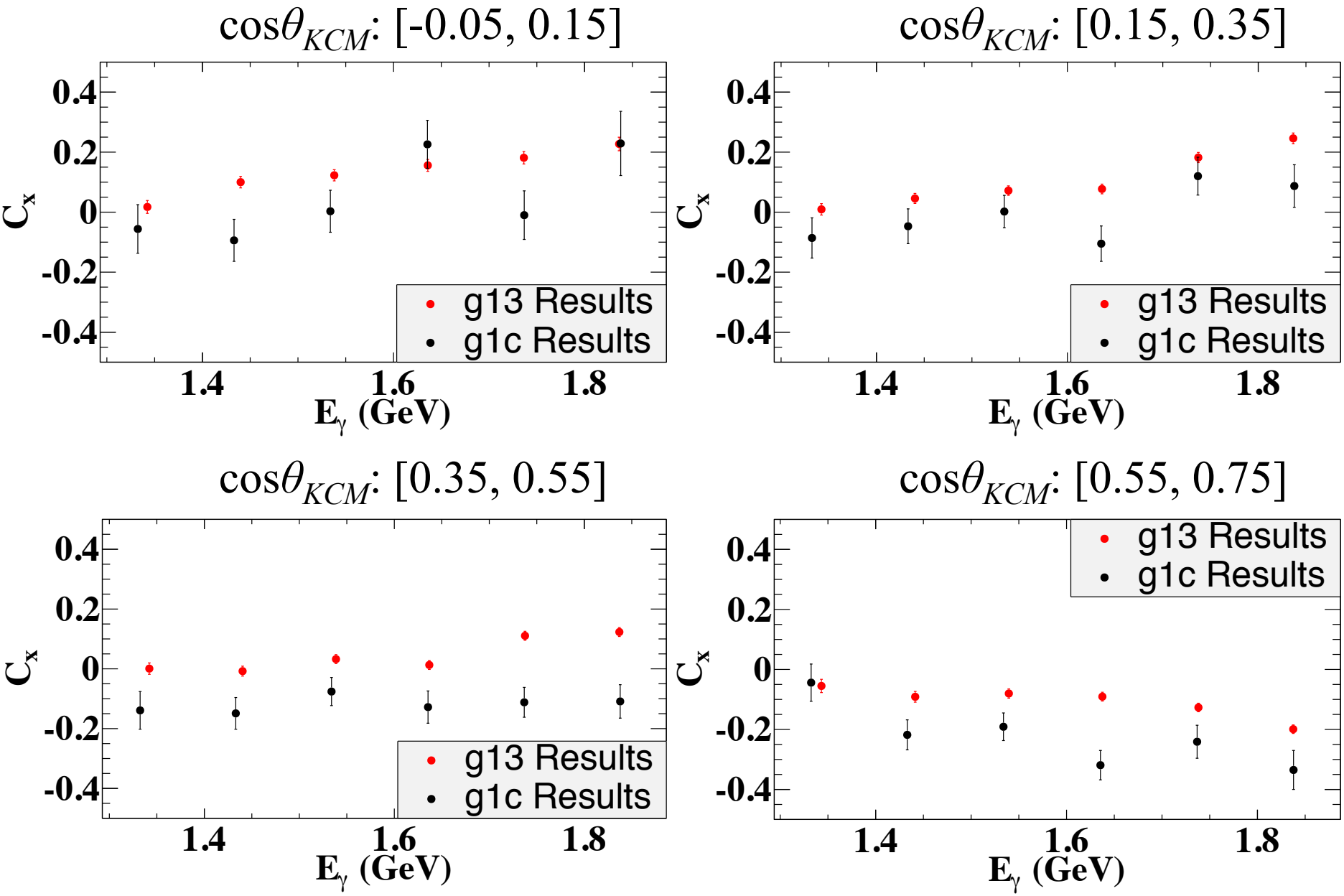
Convention 4:

$$\begin{cases} \hat{z} = \frac{\hat{p}_\Lambda + \hat{p}_K}{|\hat{p}_\Lambda + \hat{p}_K|} \\ \hat{y} = \frac{\hat{p}_\Lambda \times \hat{p}_K}{|\hat{p}_\Lambda \times \hat{p}_K|} \\ \hat{x} = \hat{y} \times \hat{z} \end{cases}$$





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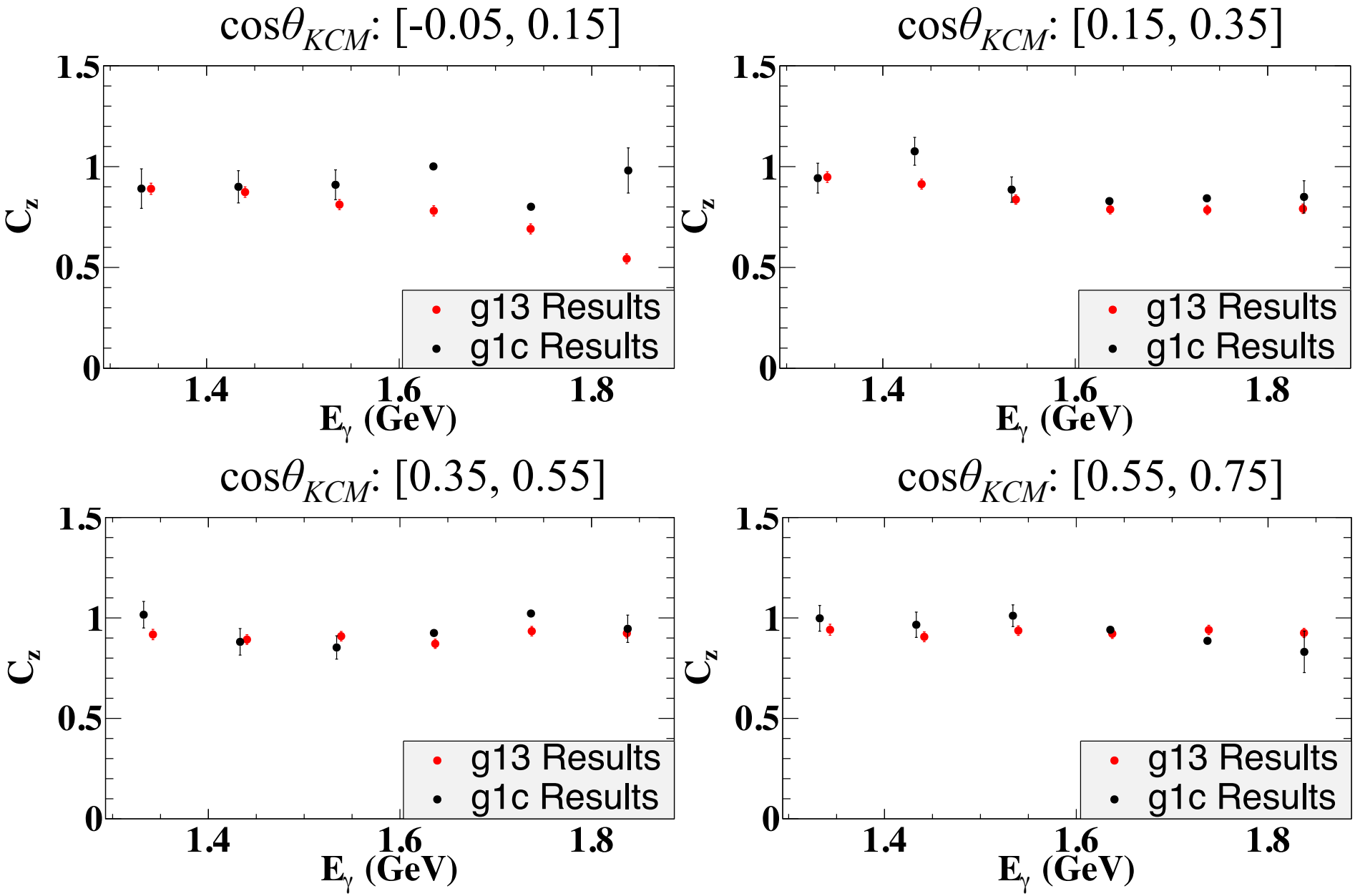
cosθ_KCM: [0.15, 0.35]
cosθ_KCM: [0.35, 0.55]
cosθ_KCM: [0.55, 0.75]
C_x
E_γ (GeV)
0.4
0.2
0
-0.2
-0.4
1.4
1.6
1.8
g13 Results
g1c Results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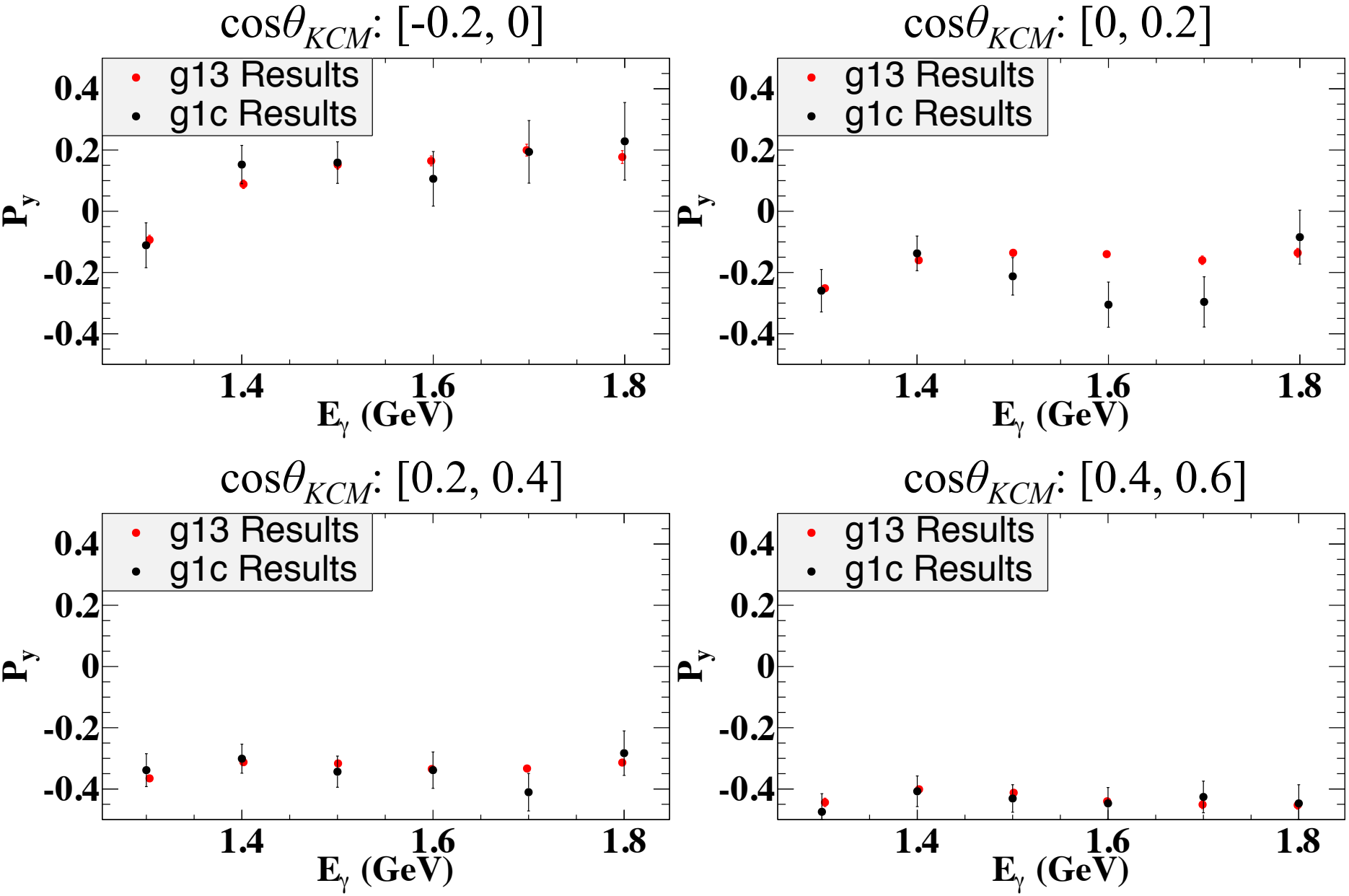
cosθ_KCM: [0.55, 0.75]
C_z
E_γ (GeV)
g13 Results
g1c Results
0
0.5
1
1.5
1.4
1.6
1.8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cosθ_KCM: [0.2, 0.4]
cosθ_KCM: [0.4, 0.6]
g13 Results
g1c Results
P_y
E_γ (GeV)
0.4
0.2
0
-0.2
-0.4
1.4
1.6
1.8