Outline

• Introduction to SANE-HMS
• Experimental Setup and SANE-HMS
• Dilution Factor
• Radiative Corrections
• HMS Asymmetries
• Spin Structure Functions
• $d_2$, Twist-3 Matrix Element
• Summary
Purpose of SANE-HMS

SANE-HMS resonance data explores high Bjorken x region at intermediate $Q^2$:  
• Resonances and $Q^2$ dependence of $A_1$ and $A_2$  
• SSF $g_2(x, Q^2)$  
• Higher twist effects  
• Twist-3 $d_2$ matrix element
Experimental Setup

Polarized Electron Beam: 4.7, 5.9 GeV

Polarized Proton Target: \( \perp, \parallel \)

Ammonia (NH\(_3\)) Polarized via DNP in 5T Magnetic Field

\( \Theta_B = 180^\circ \)

\( \Theta_B = 80^\circ \)

"BETA" Electron Arm

BigCal

Čerenkov

Chicane

Magnet

Coils

Helium Bag

HMS
HMS Coverage for SANE

- **Q^2 dependence**
- **RSS**
- **Extension of x_{Bj} range**
Dilution Factor from Packing Fraction

The target and beam are not completely polarized. It contains also un-polarizable materials.

\[ A = \frac{1}{P_b P_t f} \frac{d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\downarrow\uparrow} + d\sigma^{\uparrow\uparrow}} \]

Beam Polarization ~80%
Proton(target) Polarization ~70%
Dilution Factor

Dilution factor \( f \) is the ratio of free polarizable nucleons to the total amount of nucleons in the sample.

\( A = A_{180} \) or \( A_{80} \) is the measured asymmetry without radiative corrections.
Packing fraction is the relative volume ratio of ammonia to the target cell, or the fraction of the cell’s length that would be filled with ammonia by cylindrical symmetry.

\[ f = \frac{3H}{(3H + N)\text{pf} + He(1 - \text{pf}) + Others} \]
Dilution Factor from Packing Fraction

Total yield has linear relation with packing fraction:

\[ Y_T = m \, pf + b \]

Using MC (P. E. Bosted and M. E. Christy, PRC81 (2010) 055213) assuming two different \( pf \), the slope(\( m \)) and intercept(\( b \)) can be calculated and then the yield of real data produces \( pf \) of real target.

SANE packing fractions are 56% - 62% with ~4.5% error.
Packing Fraction

Data and MC comparison (Red is MC)
Packing fraction

![Graph showing packing fraction (pf) against PF number. The graph includes error bars and indicates unique values with blue squares and run average with red diamonds.](image)
Dilution Factor

Dilution factor is calculated using MC, comparing cross sections of each materials in target cell. And packing fraction is the only necessary input for each target cell.

Dilution factor for resonance with PF of 61.9%
Dilution factor is calculated using MC, comparing cross sections of each materials in target cell. And packing fraction is the only necessary input for each target cell.

Dilution factor for DIS with PF of 58.8%
Radiative Corrections

Following S. Stein et al., PRD12 (1975) 1884, corrections were mainly done by POLRAD 2.0. Initial fit parameters came from RSS, basically Breit-Wigner resonance and polynomial (with some correction) deep inelastic tail. Newly corrected data was refitted to iterate.
HMS Asymmetries

![Graph showing HMS electron data with various Q^2 and W values for different energies and momenta.]

(1) 5.9 GeV 80 3.1 GeV/c 15.4 deg
(2) 3.57 GeV/c 22 deg
(3) 4.4 GeV/c 15.4 deg
(4) 5.9 GeV para 2.2 GeV/c 16 deg
(5) 3.1 GeV/c 15.4 deg
(6) 4.7 GeV para 2.2 GeV/c 16 deg
(7) 3.2 GeV/c 20.2 deg
(8) 4.7 GeV 80 part 2.2 GeV/c 16 deg
(9) 4.7 GeV 80 part 11.6 GeV/c 18 deg
(10) 1.65 GeV/c 18 deg
(11) 2.2 GeV/c 16 deg
(12) 2.7 GeV/c 16 deg
### HMS Asymmetries

<table>
<thead>
<tr>
<th>Setting</th>
<th>Beam energy (GeV)</th>
<th>HMS central momentum (GeV)</th>
<th>HMS angle from beamline (degree)</th>
<th>$&lt; Q^2 &gt;$ (GeV$^2$)</th>
<th>$&lt; W &gt;$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>4.7 (par) / 5.9 (per)</td>
<td>3.2 (par) / 4.4 (per)</td>
<td>20.2 (par) / 15.4 (per)</td>
<td>1.863</td>
<td>1.353</td>
</tr>
<tr>
<td>(2)</td>
<td>5.9</td>
<td>3.1</td>
<td>15.4</td>
<td>1.313</td>
<td>2.196</td>
</tr>
<tr>
<td>(3)</td>
<td>4.7</td>
<td>2.2</td>
<td>16</td>
<td>0.806</td>
<td>2.196</td>
</tr>
</tbody>
</table>
SANE-HMS Region 1

\[ Q^2 = 1.86 \text{ GeV}^2 \]

Resonance region
Parallel and Perpendicular Asymmetries

\[ Q^2 = 1.86 \text{ GeV}^2 \]
Asymmetries $A_1$ and $A_2$

$A_1$ and $A_2$ are virtual photoabsorption asymmetries.

$$A_1 = \frac{\sigma_{1/2}^T - \sigma_{3/2}^T}{\sigma_{1/2}^T + \sigma_{3/2}^T} = \frac{\sigma_{TT}}{\sigma_T} = \frac{g_1 - \gamma^2 g_2}{F_1}$$

$$A_2 = \frac{2\sigma_{LT}}{\sigma_{1/2}^T + \sigma_{3/2}^T} = \frac{\sigma_{LT}}{\sigma_T} = \frac{\gamma(g_1 + g_2)}{F_1}$$

$\sigma_{1/2}^T$ and $\sigma_{3/2}^T$ are the virtual photon absorption transverse cross sections when total helicity of photon and nucleon is 1/2 and 3/2 respectively. $\sigma_{LT}$ is the interference term between the transverse and longitudinal photon-nucleon amplitude.

Radiative correction done by iterating the fits of $A_1$ and $A_2$ until they converged.
Asymmetry $A_1 = \frac{\sigma_{TT}}{\sigma_T}$

$Q^2 = 1.86 \text{ GeV}^2$
Asymmetry $A_1 = \frac{\sigma_{TT}}{\sigma_T}$

$Q^2 = 1.86 \text{ GeV}^2$

$Q^2 \sim 1.44 \text{ GeV}^2$

$Q^2 \sim 1.44 \text{ GeV}^2$

$Q^2 \sim 1.71 \text{ GeV}^2$

$Q^2 \sim 2.05 \text{ GeV}^2$
Asymmetry $A_1 = \frac{\sigma_{TT}}{\sigma_T}$

$Q^2 = 1.86 \text{ GeV}^2$

$Q^2 \sim 1.71 \text{ GeV}^2$
Asymmetry $A_1 = \frac{\sigma_{TT}}{\sigma_T}$

$Q^2 = 1.86 \text{ GeV}^2$

$Q^2 \sim 2.05 \text{ GeV}^2$
Asymmetry $A_1 = \frac{\sigma_{TT}}{\sigma_T}$

$Q^2 = 1.86 \, GeV^2$
Asymmetry $A_2 = \frac{\sigma_{LT}}{\sigma_T}$

$Q^2 = 1.86 \text{ GeV}^2$
Fitting Function

\[ \text{fit} = \sum_{i=1}^{4} BW_i + x^\alpha \sum_{n=0}^{3} \beta_n x^n \times \frac{1}{\sqrt{Q^2}} \]

Resonances \hspace{2cm} DIS \hspace{2cm} A_2 \text{ only}

where

\[ BW_i = \frac{a_i \kappa_i^2 \omega_i^2 \Gamma_i \Gamma_i^\gamma}{\kappa_{cm}^2 \left[ (w_i^2 - W^2)^2 + w_i^2 \Gamma_i^2 \right]} \]

\[ = g_i \left( \frac{q_{cm}}{q_i} \right)^{(2l_i+1)} \left( \frac{q_i^2 + X_i^2}{q_{cm}^2 + X_i^2} \right)^{l_i} \]

\[ = g_i \left( \frac{\kappa_{cm}}{\kappa_i} \right)^{(2j_i)} \left( \frac{\kappa_i^2 + X_i^2}{\kappa_{cm}^2 + X_i^2} \right)^{j_i} \]

\[ \kappa_i = \sqrt{(w_i^2 + M^2 + Q^2)^2 - 4w_i^2} \]
\[ q_i = \sqrt{(w_i^2 + M^2 - m_i^2)^2 - 4w_i^2} \]
\[ \kappa_{cm} = \sqrt{(W^2 + M^2 + Q^2)^2 - 4W^2} \]
\[ q_{cm} = \sqrt{(W^2 + M^2 - m_{\pi}^2)^2 - 4W^2} \]
Table 3.2: The fitting parameters of $A_1$ and $A_2$. $a_i$ is the amplitude, $\omega_i$ is the centroid, and $g_i$ is the width of the i-th BW peak.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$A_1$ Fit</th>
<th>$A_2$ Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$-0.553 \pm 0.204$</td>
<td>$-0.306 \pm 0.152$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$0.724 \pm 0.267$</td>
<td>$-0.474 \pm 0.210$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$0.615 \pm 0.071$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>$1.186 \pm 0.016$</td>
<td>$1.232$ (fixed)</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>$1.381 \pm 0.006$</td>
<td>$1.323 \pm 0.010$</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>$1.547 \pm 0.012$</td>
<td>$-$</td>
</tr>
<tr>
<td>$g_1$</td>
<td>$0.031 \pm 0.025$</td>
<td>$0.070 \pm 0.057$</td>
</tr>
<tr>
<td>$g_2$</td>
<td>$0.053 \pm 0.036$</td>
<td>$0.058 \pm 0.035$</td>
</tr>
<tr>
<td>$g_3$</td>
<td>$0.197 \pm 0.068$</td>
<td>$-$</td>
</tr>
</tbody>
</table>
Spin Structure Function $g_1$

$Q^2 = 1.86 \text{ GeV}^2$

$$g_1(Q^2) = 1.86 \text{ GeV}^2$$
Spin Structure Function $g_1$

$Q^2 = 1.86 \text{ GeV}^2$
Spin Structure Function $g_1$

$Q^2 = 1.86 \text{ GeV}^2$
Spin Structure Function $g_2$

$Q^2 = 1.86 \text{ GeV}^2$
Preliminary Twist-3 $d_2$ for the Region 3

\[ d_2 = 3 \int_0^1 x^2 (g_2 - g_2^{WW}) \, dx = \int_0^1 x^2 (2g_1 + 3g_2) \, dx \]

OPE valid to

\[ N = 2 < Q^2/M_0^2 \sim 1.8/0.5^2 \]

per DIS – resonances duality

Ji & Unrau, PRD52 (1995) 72
$d_2^{\text{Matrix Element}}$

$$d_2(x \geq x_{\text{min}}) = x_{\text{min}}^2 g_1 + 3 g_2 dx$$

$$\int_{0.5}^{0.87} x^2 (2g_1 + 3g_2) dx$$
$d_2$ Matrix Element

$$ d_2 = -0.0087 \pm 0.0014 $$

$$ d_2 = \int_{0.47}^{0.87} x^2 (2g_1 + 3g_2) \, dx $$
$d_2$ Matrix Element

Bag Model

Lattice QCD

Chiral Soliton Model

RSS

HERMES

SLAC

QCD sum rule

SANE-HMS
SANE-HMS Region 2

\[ Q^2 = 1.31 \text{ GeV}^2 \]

Extension of RSS data into DIS
Parallel and Perpendicular Asymmetries

\[ Q^2 = 1.31 \text{ GeV}^2 \]
Asymmetry $A_1 = \frac{\sigma_{TT}}{\sigma_T}$

$Q^2 = 1.31 \text{ GeV}^2$
Asymmetry $A_2 = \frac{\sigma_{LT}}{\sigma_T}$

$Q^2 = 1.31 \text{ GeV}^2$
Spin Structure Function $g_1$

$Q^2 = 1.31 \, GeV^2$
Spin Structure Function $g_1$

$Q^2 = 1.31 \text{ GeV}^2$

 EG1b A2 $Q^2=1.44$
 quadratic polynomial fit

 SANE-HMS $<Q^2> = 1.31 \text{ GeV}^2$
 CLAS eg1b $<Q^2> = 1.2 \text{ GeV}^2$
 CLAS eg1b $<Q^2> = 1.44 \text{ GeV}^2$
 RSS $<Q^2> = 1.3 \text{ GeV}^2$
Spin Structure Function $g_1$

$Q^2 = 1.31 \, \text{GeV}^2$

$$g_1(1, Q^2) = 1.31 \, \text{GeV}^2$$
Spin Structure Function $g_2$

$$Q^2 = 1.31 \, GeV^2$$
SANE-HMS Region 3

\[ Q^2 = 0.81 \text{ GeV}^2 \]

DIS region
Parallel and Perpendicular Asymmetries

\[ Q^2 = 0.81 \text{ GeV}^2 \]
Asymmetry \( A_1 = \frac{\sigma_{TT}}{\sigma_T} \)

\[ Q^2 = 0.81 \text{ GeV}^2 \]
Asymmetry $A_2 = \frac{\sigma_{LT}}{\sigma_T}$

$Q^2 = 0.81 \text{ GeV}^2$
Spin Structure Function $g_1$

$Q^2 = 0.81 \text{ GeV}^2$
Spin Structure Function $g_2$

$Q^2 = 0.81 \text{ GeV}^2$
SANE collaboration


Spokespersons: S. Choi (Seoul), M. Jones(Jlab), Z-E. Meziani (Temple), O. A. Rondon (U. of Virginia)
Summary

- SANE-HMS is a measurement of spin structure functions in high Bjorken $x$ and intermediate $Q^2$.
- Parallel and perpendicular asymmetries and structure functions show good agreement with previous experiments.
- $A_2$ and $g_2$ show significant $Q^2$ evolution. Negative $A_2$ at $W=1.3$ GeV is shown. And negative $A_2$ at DIS region can affect $g_1$ deduced from parallel asymmetry only (e.g. Hall B results).
- $\bar{d}_2 = -0.0087 \pm 0.0014$ is the first negative result of $d_2$ matrix element, although its integration range is limited.
Backup Slides
## Systematic Errors

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Polarization</td>
<td>4.0%</td>
</tr>
<tr>
<td>Beam polarization</td>
<td>1.5%</td>
</tr>
<tr>
<td>Dilution Factor</td>
<td>3.3%</td>
</tr>
<tr>
<td>Nitrogen Correction</td>
<td>0.4%</td>
</tr>
<tr>
<td>Radiative Corrections</td>
<td>10% ($A_1$ and $A_2$)</td>
</tr>
<tr>
<td>Kinematic Reconstruction</td>
<td>0.5%</td>
</tr>
</tbody>
</table>
Twist-3 $d_2$
Useful relations

\[ g_1 = \frac{F_1}{1 + \gamma^2} \left( A_1 + \gamma A_2 \right), \]
\[ g_2 = \frac{F_1}{1 + \gamma^2} \left( -A_1 + \frac{A_2}{\gamma} \right) \]
\[ A_{\parallel} = D \left( A_1 + \eta A_2 \right), \]
\[ A_{\perp} = d \left( A_2 - \zeta A_1 \right), \]
\[ A_1 = \frac{g_1 - \gamma^2 g_2}{F_1} \]
\[ A_2 = \gamma \frac{g_1 + g_2}{F_1} , \]
Spin Structure Functions

Inclusive DIS cross section depends on four structure functions, two unpolarized \( (F_1, F_2) \) and two polarized \( (g_1, g_2) \). The spin structure functions \( g_1 \) and \( g_2 \) can be experimentally determined by measuring spin asymmetries:

\[
A_{\parallel} = \frac{\sigma_{\downarrow \uparrow} - \sigma_{\uparrow \uparrow}}{\sigma_{\downarrow \uparrow} + \sigma_{\uparrow \uparrow}}, \quad A_{\perp} = \frac{\sigma_{\downarrow \rightarrow} - \sigma_{\uparrow \rightarrow}}{\sigma_{\downarrow \rightarrow} + \sigma_{\uparrow \rightarrow}}.
\]

\[
g_1(x, Q^2) = \frac{F_1(x, Q^2)}{d'} \left[ A_{\parallel} + \tan(\theta/2)A_{\perp} \right],
\]

\[
g_2(x, Q^2) = \frac{yF_1(x, Q^2)}{2d'} \left[ \frac{E + E' \cos(\theta)}{E' \sin(\theta)} A_{\perp} - A_{\parallel} \right].
\]
ep deep inelastic scattering

High-energy electron-nucleon scattering (Deep Inelastic Scattering) \( ep \rightarrow e'X \)

\( k \) and \( k' \) are the four-momenta of the incoming and outgoing electrons, \( P \) is the four-momentum of a proton with mass \( M \), and \( W \) is the mass of the recoiling system \( X \).

\( q \) is the four-momentum of the virtual photon (the exchanged particle). \( (Q^2 = -q^2) \)
Spin structure functions

When the spins of electron and nucleon are all polarized, we can see the dependence of scattering cross section on the spin structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$.

$$g_1 = \frac{1}{2} \sum_i e_i^2 [q_i^+ - q_i^-]$$

$$g_2 = g_{2W} + \overline{g_2}$$

$$g_{2W}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(x', Q^2)}{x'} dx'$$
High Momentum Spectrometer

High Momentum Spectrometer

2/25/2009 9:29:15 AM

Momentum
Input (GeV): 4.40000

Running
Stop

Input OK

Q1 at Current
Q2 at Current
Q3 at Current
Dipole at Field

Magnets ON / OFF

Q1
Q2
Q3
Dipole

Spectrometer is On

Using Standard Tune

ESR

4K Return

4K Supply

Ln2 Level

LHe
LN2
PSU

RB: 573.4 A
Set: 573.3 A

7572 Gauss

-9226 Gauss

4749 Gauss

LN2 Level: 59%

Ln2 Level: 61%

Ln2 Level: 61%

4K Supply

4K Return

RB: 456.0 A
Set: 455.8 A

RB: 221.2 A
Set: 221.6 A

RB 1.2093810 T
Set 1.2093765 T

1360.4 A

SOS Angle 146.88

HMS Angle 15.39

Print
World data lacks big region, especially in the perpendicular asymmetry. SANE covers broad region of

$$0.3 \leq x \leq 0.8, \quad 2.5 GeV^2 \leq Q^2 \leq 6.5 GeV^2$$
Comparing data with Monte Carlo results assuming 50% and 60% packing fraction of target, 60.9% packing fraction is determined for the target material #9 6-28-07 14NH3.
Unpolarized Structure Functions
ep deep inelastic scattering

Invariant quantities:
\[ \nu = \frac{q \cdot P}{M} = E - E' \] is the lepton’s energy loss in the nucleon rest frame (in earlier literature sometimes \( \nu = q \cdot P \)). Here, \( E \) and \( E' \) are the initial and final lepton energies in the nucleon rest frame.

\[ Q^2 = -q^2 = 2(EE' - \vec{k} \cdot \vec{k}') - m_{\ell}^2 - m'_{\ell'}^2 \] where \( m_{\ell}(m_{\ell'}) \) is the initial (final) lepton mass. If \( EE' \sin^2(\theta/2) \gg m_{\ell}^2, m_{\ell'}^2 \), then

\[ \approx 4EE' \sin^2(\theta/2), \] where \( \theta \) is the lepton’s scattering angle with respect to the lepton beam direction.

\[ x = \frac{Q^2}{2M\nu} \] where, in the parton model, \( x \) is the fraction of the nucleon’s momentum carried by the struck quark.

\[ y = \frac{q \cdot P}{k \cdot P} = \frac{\nu}{E} \] is the fraction of the lepton’s energy lost in the nucleon rest frame.

\[ W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2 \] is the mass squared of the system \( X \) recoiling against the scattered lepton.
Radiative Correction

\[ A = \frac{1}{f C_N P_b P_t f_{RC}} \frac{d\sigma^{\downarrow \uparrow} - d\sigma^{\uparrow \uparrow}}{d\sigma^{\downarrow \uparrow} + d\sigma^{\uparrow \uparrow}} + A_{RC} \]

1. Incoming and outgoing electron lose energy before and after scattering.
2. Elastic tail should be subtracted.
3. QED processes other than Born contributes to data.
CLAS eg1b data