Measurement of the Charged Pion Form Factor to High $Q^2$

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Outline

1. Motivation and techniques for measuring $F_\pi(Q^2)$
2. Data
   - Pre-JLab data
   - JLab $F_\pi$ program at 6 GeV
3. $F_\pi$ at 12 GeV: E12-06-101
   - Kinematics
   - Spectrometer requirements
   - Runplan
4. $F_\pi$ at larger $Q^2$?
The Pion Form Factor and the Interplay of Soft and Hard Physics

The pion form factor is unique in that its asymptotic form can be calculated exactly in pQCD.

However, it is unclear at what $Q^2$ the pQCD expression is relevant – soft processes play an important role at moderate $Q^2$.

Recent calculations suggest that the most significant soft physics is found in the pion distribution amplitude.

→ Calculations using pion DA from lattice give form factors similar to those using pion DA from state of the art DSE calculations.

$F_\pi(Q^2) \rightarrow \frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2} \quad Q^2 \rightarrow \infty$

$f_\pi$ = $93$ MeV is the $\pi^+ \rightarrow \mu^+\nu$ decay constant.

pQCD and the Pion Form Factor

Is it possible to apply pQCD at experimentally accessible $Q^2$?
→ Use pion DA derived using DSE formalism
→ DSE-based result consistent with DA derived using constraints from lattice

Measurement of $\pi^+$ Form Factor – Low $Q^2$

- At low $Q^2$, $F_\pi$ can be measured directly via high energy elastic $\pi^-$ scattering from atomic electrons
  - CERN SPS used 300 GeV pions to measure form factor up to $Q^2 = 0.25$ GeV$^2$
    [Amendolia et al, NPB277, 168 (1986)]
  - These data used to extract the pion charge radius
    $$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

- Maximum accessible $Q^2$ roughly proportional to pion beam energy
  - $Q^2=1$ GeV$^2$ requires 1000 GeV pion beam
Measurement of $\pi^+$ Form Factor – Larger $Q^2$

- At larger $Q^2$, $F_\pi$ must be measured indirectly using the “pion cloud” of the proton via $p(e,e^{'\pi^+})n$
  - At small $-t$, the pion pole process dominates the longitudinal cross section, $\sigma_L$
  - In Born term model, $F_\pi^2$ appears as,

$$
\frac{d\sigma_L}{dt} \propto \frac{\alpha}{(t-m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2,t)
$$

(In practice more sophisticated model is used)

- Drawbacks of the this technique
  - Isolating $\sigma_L$ experimentally challenging
  - Theoretical uncertainty in form factor extraction
Two $F_\pi$ experiments have been carried out in Hall C

$F_{\pi} - 1$: $Q^2$=0.6-1.6 GeV$^2$
$F_{\pi} - 2$: $Q^2$=1.6, 2.45 GeV$^2$

→ Second experiment took advantage of higher beam energy to access larger $W$, smaller $-t$
→ Full deconvolution of $L/T/TT/LT$ terms in cross section
→ Ancillary measurement of $\pi/\pi^+$ (separated) ratios to test reaction mechanism
→ $F_{\pi} - 1$ ran in 1997 and $F_{\pi} - 2$ in 2003

| Expt  | $Q^2$ (GeV$^2$) | $W$ (GeV) | $|t_{\min}|$ (GeV$^2$) | $E_e$ (GeV) |
|-------|-----------------|-----------|------------------------|------------|
| $F_{\pi} - 1$ | 0.6-1.6 | 1.95 | 0.03-0.150 | 2.45-4.05 |
| $F_{\pi} - 2$ | 1.6,2.45 | 2.22 | 0.093,0.189 | 3.78-5.25 |
**$F_\pi$ Extraction from JLab data**

**VGL Regge Model**

- Feynman propagator replaced by $\pi$ and $\rho$ Regge propagators.
  - Represents the exchange of a series of particles, compared to a single particle.
- Model parameters fixed from pion photoproduction.
- Free parameters: $\Lambda_\pi$, $\Lambda_\rho$ (trajectory cutoff).

\[
F_\pi(Q^2) = \frac{1}{1 + Q^2 / \Lambda^2_\pi}
\]

$\Lambda^2_\pi = 0.513, 0.491 \text{ GeV}^2$, $\Lambda^2_\rho = 1.7 \text{ GeV}^2$

**Horn et al, PRL97, 192001, 2006**

\[Q^2 = 1.60\]
\[Q^2 = 2.45\]
$F_\pi(Q^2)$ Results and Models

\rightarrow relativistic treatment of bound quarks (Bethe-Salpether equation + Dyson-Schwinger expansion)

\rightarrow Green’s function analyticity used to extract form factor

Brodsky and de Teramond
\rightarrow Anti-de Sitter/Conformal Field Theory approach

\( F_\pi(Q^2) \) at 12 GeV: E12-06-101

- E12-06-101 (G. Huber and D.G.) approved for 52 PAC days with “A” rating
  - Selected as “High impact” experiment at PAC 41
- Rosenbluth (L-T) separation experiment – helped drive some of the requirements for SHMS
  - Small angle capability
  - Good \( \pi \) ID at over large momentum range
- Extend \( F_\pi(Q^2) \) measurements to \( Q^2=6 \) GeV^2
- In addition pion form factor measurements, will perform key systematic checks of electroproduction technique for extracting form factor
\( F_\pi \) in Experimental Hall C

Spectrometer requirements

**HMS:** Electron arm
Nominal capabilities:
\[ d\Omega \sim 6 \text{ msr}, \ P_0 = 0.5 \text{ } - \text{ } 7 \text{ GeV/c} \]
\[ \theta_0 = 10.5 \text{ to } 80 \text{ degrees} \]

\( F_\pi \) requirements:
\[ P_0 = 0.5 \text{ } - \text{ } 4.7 \text{ GeV/c} \]
\[ \theta_0 = 11.0 \text{ to } 47.2 \text{ degrees} \]

**SHMS:** Pion arm
Nominal capabilities:
\[ d\Omega \sim 4 \text{ msr}, \ P_0 = 1 \text{ } - \text{ } 11 \text{ GeV/c} \]
\[ \theta_0 = 5.5 \text{ to } 40 \text{ degrees} \]

\( F_\pi \) requirements:
\[ P_0 = 2.3 \text{ } - \text{ } 8.1 \text{ GeV/c} \]
\[ \Theta_0 = 5.5 \text{ to } 10.0 \text{ degrees} \]

→ Need a fully commissioned SHMS (and HMS) before running
→ Optics, acceptance, kinematic offsets need to be understood
$F_{\pi}(Q^2)$ Kinematic Reach

JLab 12 GeV upgrade + HMS/SHMS will allow measurement up to $Q^2=6$ GeV$^2$

Require $\Delta \varepsilon > 0.2$ to minimize error amplification in L-T separation

$-t_{\text{min}}$ must not be too large $\rightarrow 0.2$ GeV$^2$
Check of Pion Electroproduction Technique

• Does electroproduction really measure the physical form-factor?
• Test by making $p(e,e'\pi^+)$ measurements at same kinematics as $\pi+e$ elastics
• Looks good so far
  
  – **Electroproduction data at $Q^2 = 0.35 \text{ GeV}^2$ consistent with extrapolation of SPS elastic data**

An improved test will be carried out as part of E12-06-101

→ smaller $Q^2 (=0.30 \text{ GeV}^2)$
→ -t closer to pole ($=0.005 \text{ GeV}^2$)
Is the model used to extract the form factor sensitive to the distance from the pion pole?

→ Test by extracting FF at different distances from $-t_{\text{min}}$

→ Ex: $F_{\pi}^{-2}$, $-t_{\text{min}}=0.093 \text{ GeV}^2$
  $F_{\pi}^{-1}$, $-t_{\text{min}}=0.15 \text{ GeV}^2$

Additional data from E12-06-101 will provide further tests:

$Q^2=1.6 \text{ GeV}^2$, $-t_{\text{min}}=0.029 \text{ GeV}^2$

$Q^2=2.45 \text{ GeV}^2$, $-t_{\text{min}}=0.048 \text{ GeV}^2$
Pole Dominance Tests

Extraction of $F_\pi$ relies on dominance of pole diagram

→ $t$-channel diagram pure isovector

→ Other Born diagrams both isovector and isoscalar

Measure (separated) $\pi^-/\pi^+$ ratios to test pole dominance

\[
\frac{\sigma_L(\pi^-)}{\sigma_L(\pi^+)} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}
\]

Ratio = 1 suggests no isoscalar backgrounds
π⁻/π⁺ Ratios from $F_{\pi^-}-1$ and $F_{\pi^-}-2$

Longitudinal ratios in general < 1: approach 0.8 at $-t_{\text{min}}$

Consistent with VGL prediction for all $-t$ at $Q^2=2.45$ GeV²

Assuming $A_V$ and $A_S$ are real: $R_L=0.8$ implies $A_S/A_V = 0.06$

Huber et al, Phys.Rev.Lett. 112 (2014) 18, 182501
Longitudinal ratios in general < 1: approach 0.8 at \(-t_{\text{min}}\)

Consistent with VGL prediction for all \(-t\) at \(Q^2 = 2.45 \text{ GeV}^2\)

Assuming \(A_V\) and \(A_S\) are real: \(R_L = 0.8\) implies \(A_S/A_V = 0.06\)

Some evidence for isoscalar backgrounds at \(Q^2 = 0.6 \text{ GeV}^2\)

\(\rightarrow\) Consistent with form factor extraction issues found during \(F_{\pi^- 1}\) analysis
π⁻/π⁺ Ratios from E12-06-101

E12-06-101 will take additional π⁻/π⁺ data at \( Q^2 = 1.6 \) and 3.5 GeV²

π⁻ detection in SHMS at forward angles especially challenging
→ High electron singles rates limits maximum current at some settings
Related experiment: E12-07-105
“Scaling Study of the L-T Separated Pion Electroproduction Cross Section at 11 GeV” (T. Horn and G. Huber)

→ Study the $Q^2$ dependence of $\sigma_L$ at fixed $x$ and $-t$
→ Factorization theorem predicts $\sigma_L \sim 1/Q^6$
→ Derivation of this theorem assumes asymptotic form of pion DA - synergy with pion form factor measurements

Common experimental technique and systematic challenges → makes sense to try and run together as a mini “run-group”
E12-06-101 \((F_\pi)\) has joined with E12-07-105 (Pion Scaling) to make a combined runplan.

→ Both are pion electroproduction, L-T separation experiments with many common experimental issues (PID, systematics, etc.)

→ Kinematic overlap is not large, but some time is saved (on the order of a few days) by running together.

→ Likely, more significant time savings comes from shared calibration and supplemental data (elastic ep runs, target boiling, etc.).
### Fπ/Pion Scaling Runplan

<table>
<thead>
<tr>
<th>Run period</th>
<th>Settings</th>
<th>Energy</th>
<th>Run Time (Days)</th>
<th>Target Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pion Scaling: $x=0.39$</td>
<td>2.2 GeV/pass</td>
<td>16</td>
<td>2018</td>
</tr>
<tr>
<td>2</td>
<td>$F_π$: $Q^2=1.6, 3.5, 4.46$</td>
<td>2.2 GeV/pass 1.96 GeV/pass</td>
<td>39</td>
<td>2019</td>
</tr>
<tr>
<td>3</td>
<td>Pion Scaling: $x=0.55$ (partial)</td>
<td>2.2 GeV/pass 1.96 GeV/pass</td>
<td>38</td>
<td>2020</td>
</tr>
<tr>
<td>4</td>
<td>$F_π$: $Q^2=2.45, 5.25$ Pion Scaling: $x=0.31,0.55$ (partial)</td>
<td>2.2 GeV/pass 1.96 GeV/pass 1.82 GeV/pass</td>
<td>38</td>
<td>2020/2021</td>
</tr>
<tr>
<td>5</td>
<td>$F_π$: $Q^2=0.3, 6.0$ Pion Scaling: $x=0.31,0.39$ (partial)</td>
<td>2.2 GeV/pass 1.96 GeV/pass 1.82 GeV/pass</td>
<td>35</td>
<td>2021</td>
</tr>
<tr>
<td>6</td>
<td>$F_π$: $Q^2=0.3$</td>
<td>1.37 GeV/pass</td>
<td>4</td>
<td>2020?</td>
</tr>
</tbody>
</table>

Ideal runplan, if we were free to choose dates/schedule
→ Would like 6-9 month gap between period 1 and 2 to make sure data understood – other periods could be combined as needed
**Beyond \( Q^2 = 6 \, \text{GeV}^2 \)**

There is great interest in extending measurements of the pion form factor to even larger \( Q^2 \)

- A measurement near \( Q^2 = 8-9 \, \text{GeV}^2 \) would access region where lattice-based pion DA calculations are valid
- \( Q^2 = 8-9 \, \text{GeV}^2 \) ultimate limit dictated by beam energy and \( \Delta \varepsilon \)
- At such large \( Q^2 \), \(-t_{\text{min}} \sim 0.5 \, \text{GeV}^2 \)

**Figure 2.2:** Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.

**$F_\pi$ Beyond $Q^2=6$ GeV$^2$**

Possible measurement at larger $-t_{min}=0.54$ GeV$^2$

JLab 12 GeV + SHMS/HMS allows measurements of $\sigma_L$ up to $Q^2 \sim 8-9$ GeV$^2$

$-t_{min}$ grows to 0.5 GeV$^2$ – non-pole backgrounds need to be addressed

Projected precision using $R$ from VR model and assuming pole dominance
$F_\pi$ Beyond $Q^2=6$ GeV$^2$

- Investigations underway to optimize a measurement at larger $Q^2$
- Possible scenario: modify one of the Pion Scaling settings ($Q^2=9.1$ GeV$^2$)
  - Shift to lower $Q^2=8.3$ GeV$^2$ to improve rates, increase $\Delta\epsilon$
  - Need to make sure this change doesn’t negatively impact scaling tests
- $-t_{\text{min}}$ is still large $\rightarrow$ 0.54 GeV$^2$
  - Need some kind of constraint on non-pole backgrounds/pole dominance
  - No recent theoretical work in this area – some future 12 GeV data may be useful (exclusive $\pi^0$)
  - Most straightforward test comes from $\pi^-/\pi^+$ ratios $\rightarrow$ extra time/data required
- Looking at rates to see if $Q^2=8.3$ GeV$^2 + \pi^-/\pi^+$ ratio measurement can be accomplished with some (but not large) amount of extra beam time (G. Huber, T. Horn, D.G.)
Summary

• The Jefferson Lab pion form factor has provided precise, reliable measurements up to $Q^2=2.45\text{ GeV}^2$
• The JLab 12 GeV Upgrade + SHMS will extend our precision measurements up to $Q^2=6\text{ GeV}^2$
• Combined running with Pion Scaling experiment will maximize efficiency and experimental output
  – Initial partial run with some time to evaluate data would be helpful
• Pion form factor at even larger $Q^2$ may be accessible – understanding the reaction mechanism is crucial.
  – Supplemental measurements will be required