Possible relations between GPDs and TMDs

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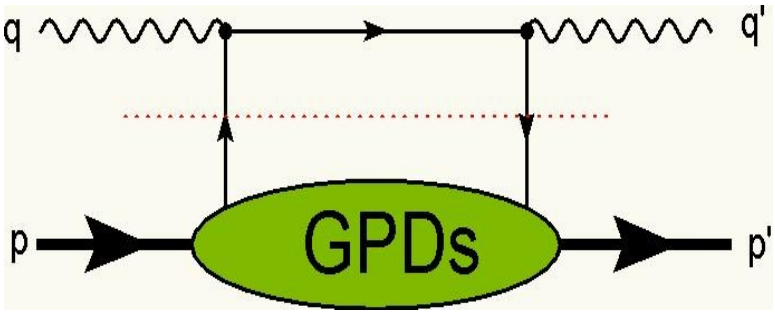
Hall C summer meeting:
Physics opportunities in Hall C at 12 GeV

Generalized Parton Distributions

- Exclusive processes (DVCS, meson production, ...):



$$P = \tfrac{1}{2}(p + p')$$

$$\Delta = p' - p$$

Skewness-parameter:

$$\Delta^+ = -2\xi P^+$$

- GPDs $\longrightarrow$ "off-diagonal" matrix elements of quark-quark operator:

$$F_{ij}(x,\xi,\vec{\Delta}_T) = \int \frac{dz^-}{2(2\pi)}\, e^{ixP^+z^-} \langle p'|\, \bar{\psi}_j(-\tfrac{z^-}{2})\, [-\tfrac{z^-}{2}\,;\tfrac{z^-}{2}]\, \psi_i(\tfrac{z^-}{2})|p\rangle$$

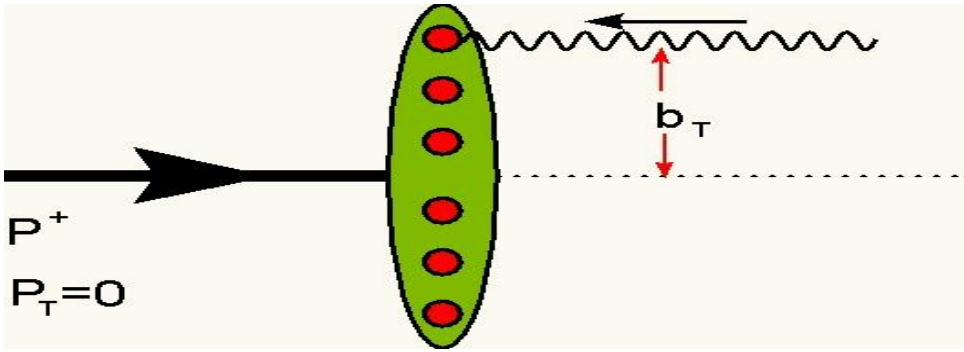
- Project out quark polarizations: 8 leading twist GPDs

unpolarized: $(H\,,\,E)(x,\xi,t)$ longitudinally: $(\tilde{H}\,,\,\tilde{E})(x,\xi,t)$

transverse [chirally-odd]: $(H_T\,,\,E_T\,,\,\tilde{H}_T\,,\,\tilde{E}_T)(x,\xi,t)$

Impact Parameter Space

- Impact Parameter Space: (ξ=0) [M. Burkardt, PRD62, 071503]



- Impact parameter b_T and transv. momentum transfer Δ_T ⟶ FT

$$\mathcal{F}_{ij}(x,\vec{b}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2}\, \mathrm{e}^{-i\vec{\Delta}_T\cdot\vec{b}_T}\, F_{ij}(x,0,\vec{\Delta}_T)$$

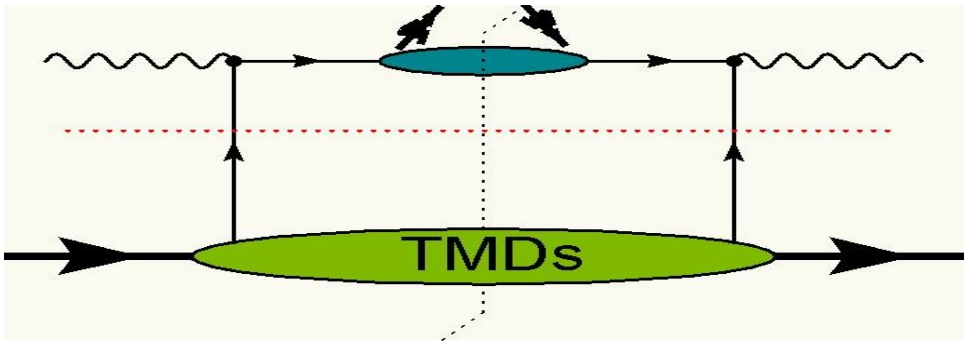
- Impact parameter space ⟶ "diagonal" matrix element $z_{1/2} = \mp\frac{z^-}{2}n_- + b_T$

$$\boxed{\mathcal{F}_{ij}(x,\vec{b}_T) = \int \frac{dz^-}{2(2\pi)}\, \mathrm{e}^{ixP^+z^-}\, \langle P^+;\vec{0}_T|\bar{\psi}_j(z_1)\,[z_1;z_2]\,\psi_i(z_2)|P^+;\vec{0}_T\rangle}$$

- (ξ=0): probability density of partons in transverse plane.

Transverse Momentum Dependence (TMD)

- Semi-inclusive processes (SIDIS, Drell-Yan, $e^+e^- \longrightarrow H_1 + H_2 + X$)



$$d\sigma \sim f \otimes D$$

f: TMD parton distribution

D: TMD fragmentation function

- TMDs $\longrightarrow$ "diagonal" matrix element, but k_T-dependence.

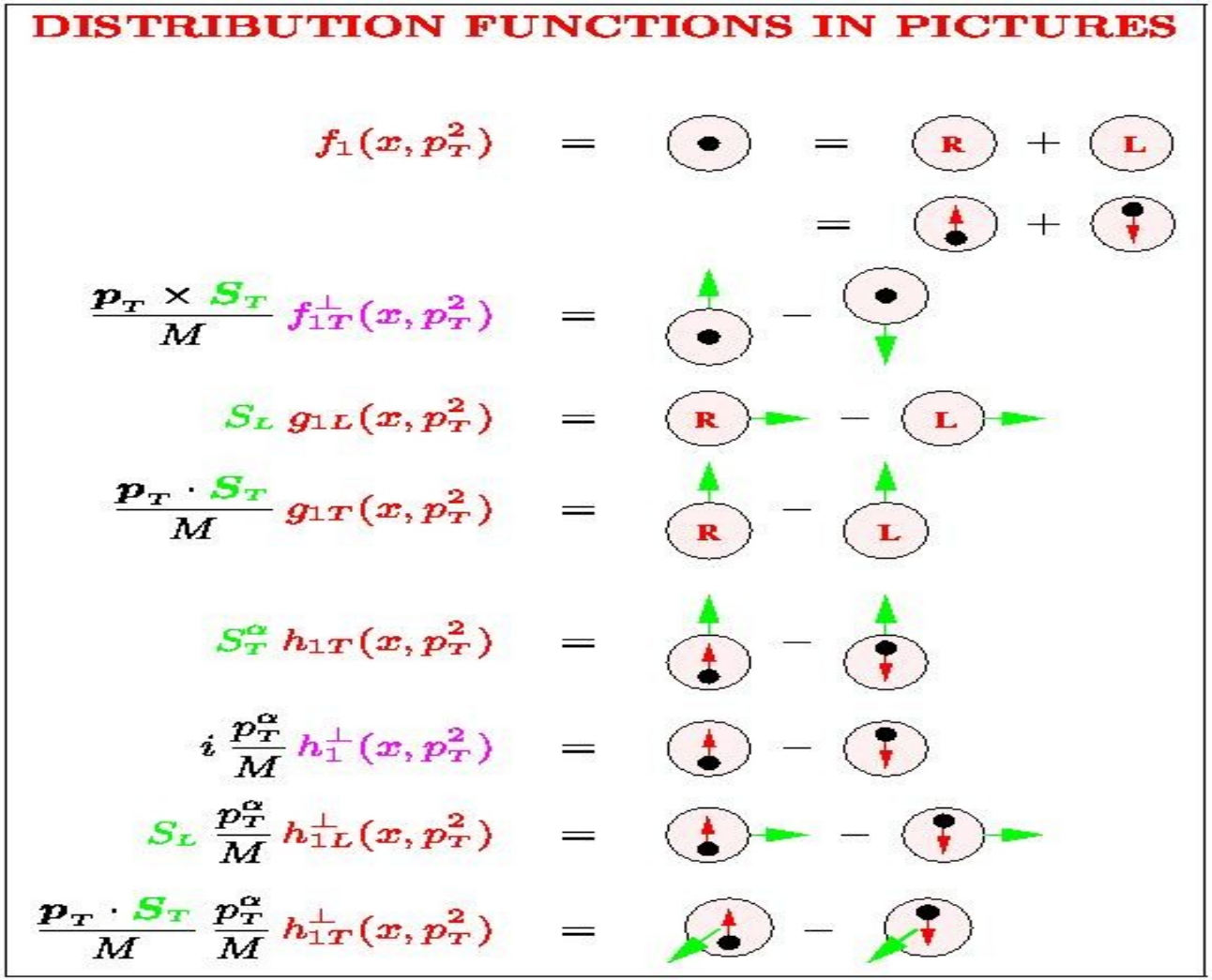
$$\Phi_{ij}(x, \vec{k}_T; S) = \int \frac{dz^- d^2 z_T}{2(2\pi)^3} \, e^{ik\cdot z} \langle P, S | \bar{\psi}_j(0) \mathcal{W}_{\mathrm{SIDIS/DY}}[0, z] \, \psi_i(z) | P, S \rangle \Big|_{z^+=0}$$

- Project out polarizations: 8 leading twist TMDs

unpolarized: $(f_1 \, , \, f_{1T}^{\perp})(x, \vec{k}_T^2)$ longitudinally: $(g_{1L} \, , \, g_{1T})$

transversely [chirally-odd]: $(h_1 \, , \, h_{1T}^{\perp} \, , \, h_{1L}^{\perp} \, , \, h_1^{\perp})$

TMDs in pictures



"unpolarized"

"Sivers"

"helicity"

"transversity"

"Boer-Mulders"

"pretzelosity"

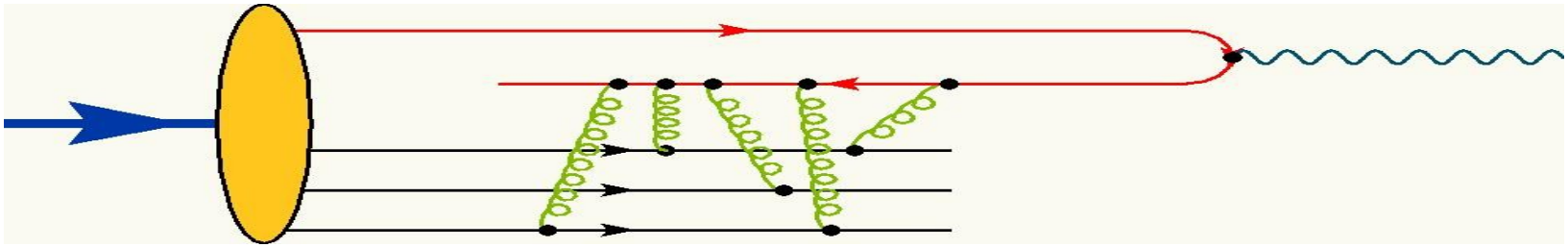
T-odd TMDs

- *Neglect* gauge link operator:

$\Longrightarrow$ Time-reversal forbids → Sivers function $f_{1T}^{\perp}$

→ Boer-Mulders function $h_1^{\perp}$

- If T-odd TMDs ≠ 0: Gauge Link not neglegible, physical effect:



Initial / Final state interactions

Time reversal → switches sign:

$$f_{1T}^{\perp}\Big|_{DIS} = -f_{1T}^{\perp}\Big|_{DY}$$

$$h_1^{\perp}\Big|_{DIS} = -h_1^{\perp}\Big|_{DY}$$

Prominent Observables in SIDIS

1.) Sivers asymmetry:

$$\sigma_{UT}^{\mathrm{SIDIS}} \propto \sin(\phi_h - \phi_S) f_{1T}^{\perp} \otimes D_1$$

2.) Collins asymmetry:

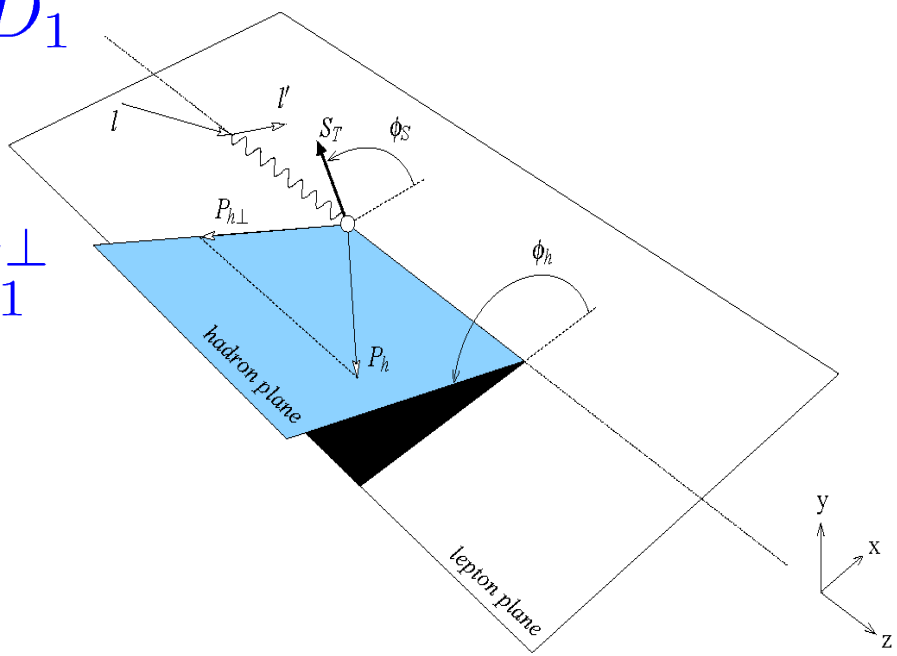
$$\sigma_{UT}^{\mathrm{SIDIS}} \propto \sin(\phi_h + \phi_S) h_1 \otimes H_1^{\perp}$$

3.) Boer-Mulders:

$$\sigma_{UU}^{\mathrm{SIDIS}} \propto \cos(2\phi) h_1^{\perp} \otimes H_1^{\perp}$$

4.) "pretzelosity":

$$\sigma_{UT}^{\mathrm{SIDIS}} \propto \sin(3\phi_h - \phi_S) h_{1T}^{\perp} \otimes H_1^{\perp}$$

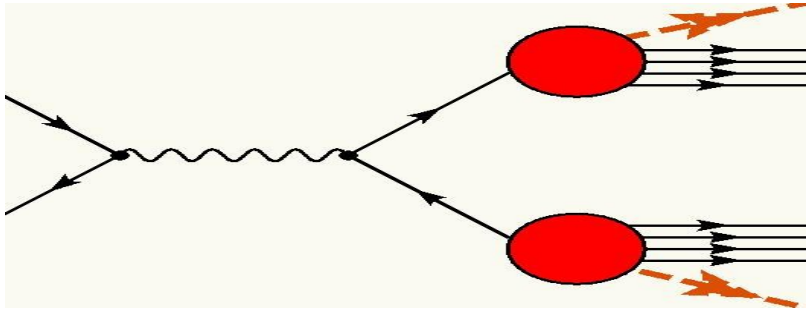


Collins-function important ⟶ BELLE data

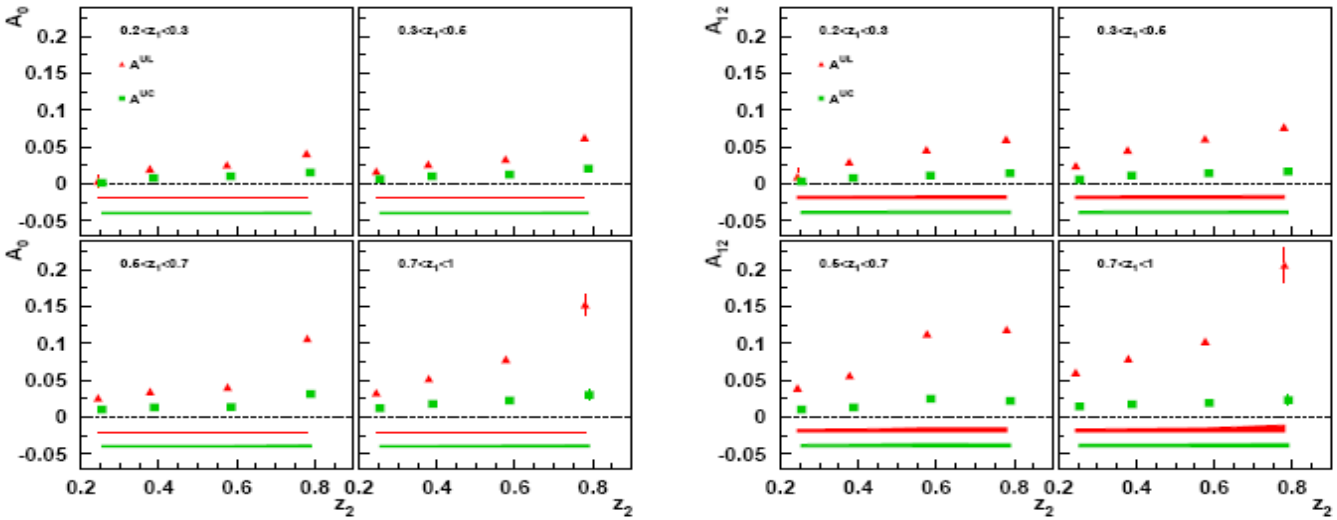
$e^+e^- \longrightarrow H_1 + H_2 + X$

- Azimuthal asymmetry:

$$\sigma^{e^+e^-} \sim H_1^{\perp,q/H_1} \otimes H_1^{\perp,\bar{q}/H_2}$$



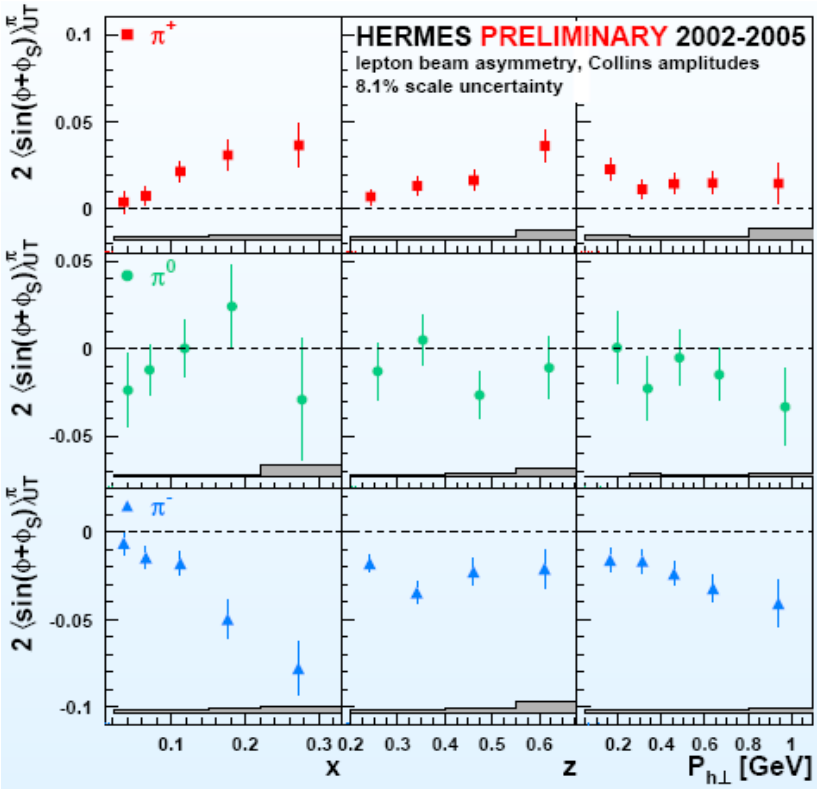
- Belle data, 2008:



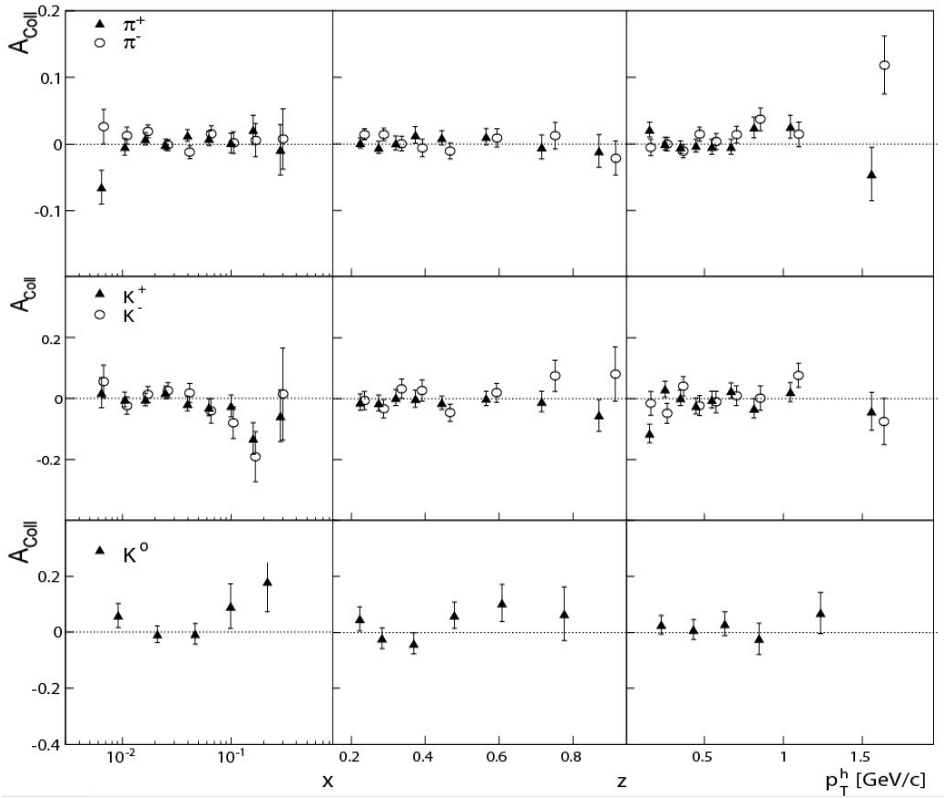
Enables extraction of Collins function.

Collins effect in SIDIS

HERMES data: proton target

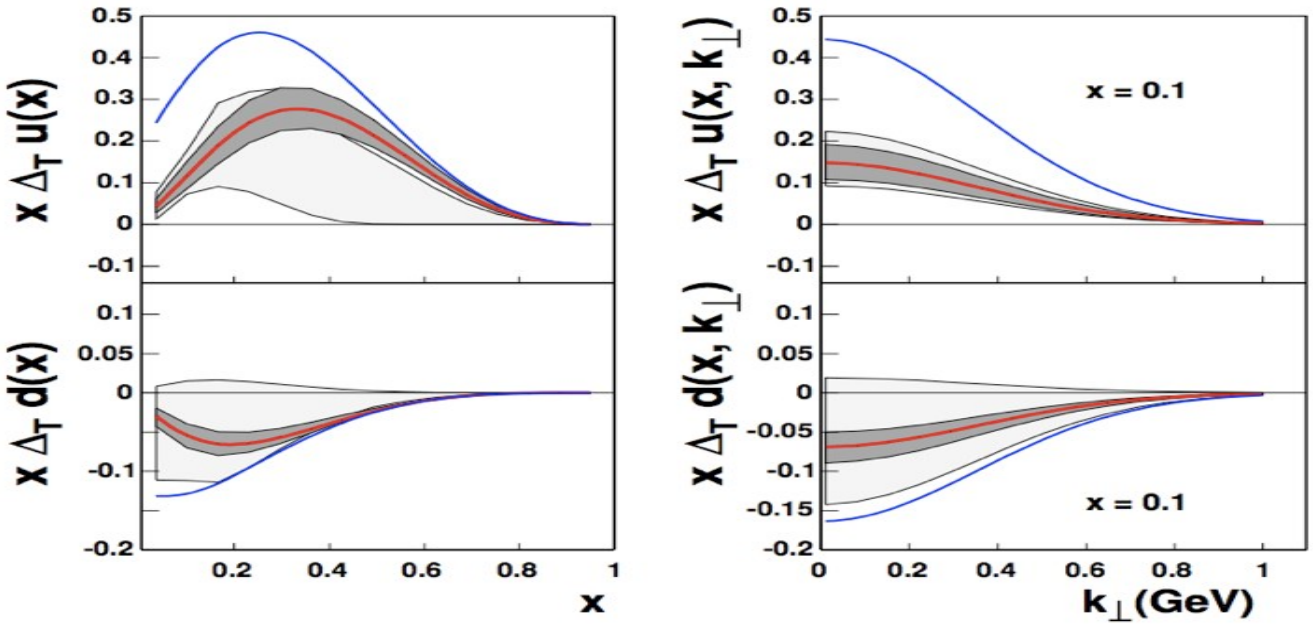


COMPASS data: deuterium target



Extraction of transversity

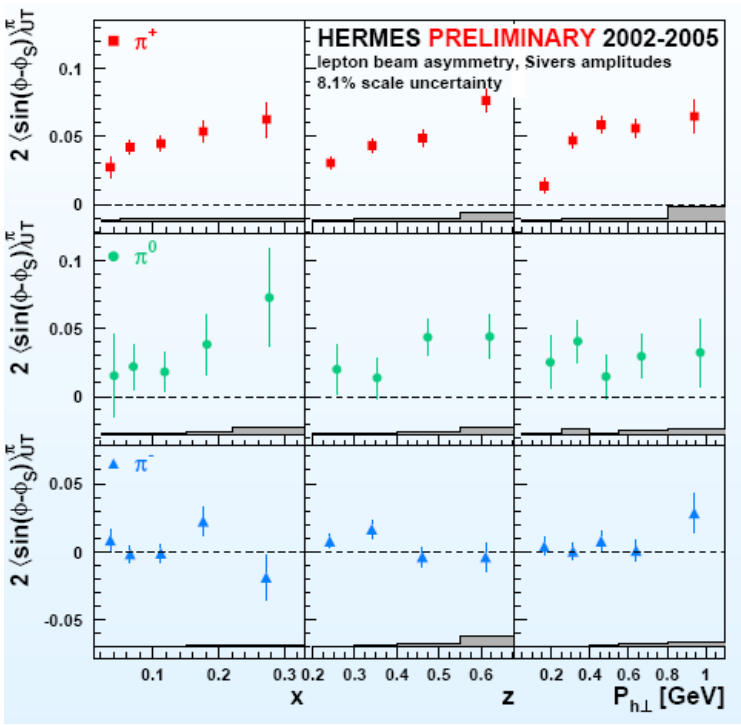
Combined data analysis: M. Anselmino et al. [Spin workshop, Beijing, 2008]

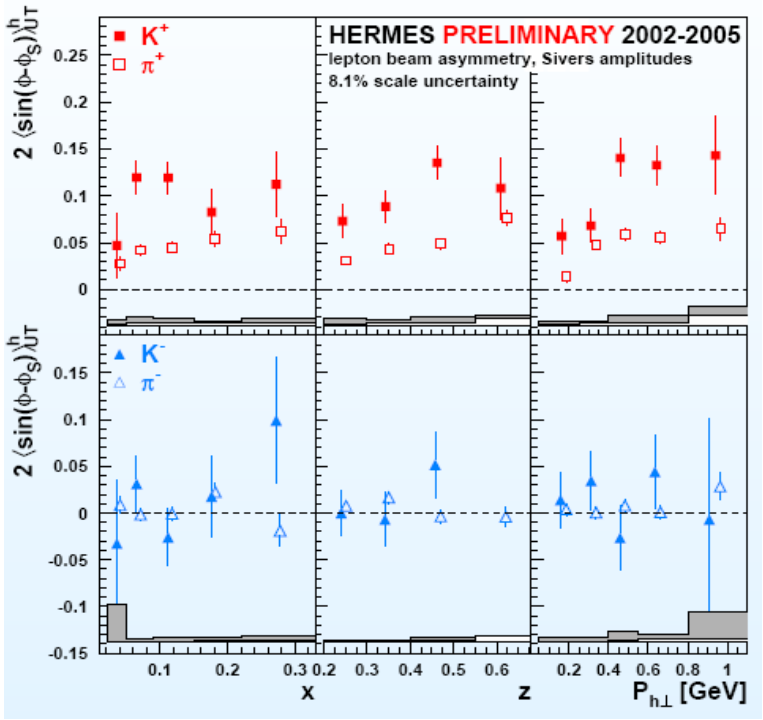


- $h_1^u > 0 \quad h_1^d < 0$ in agreement with most models
- Soffer bound not saturated
- Improved fit, statistical errors reduced
- *But:* Fits not constraint by data for x > 0.3...

Sivers effect in SIDIS

HERMES data: Pions Kaons

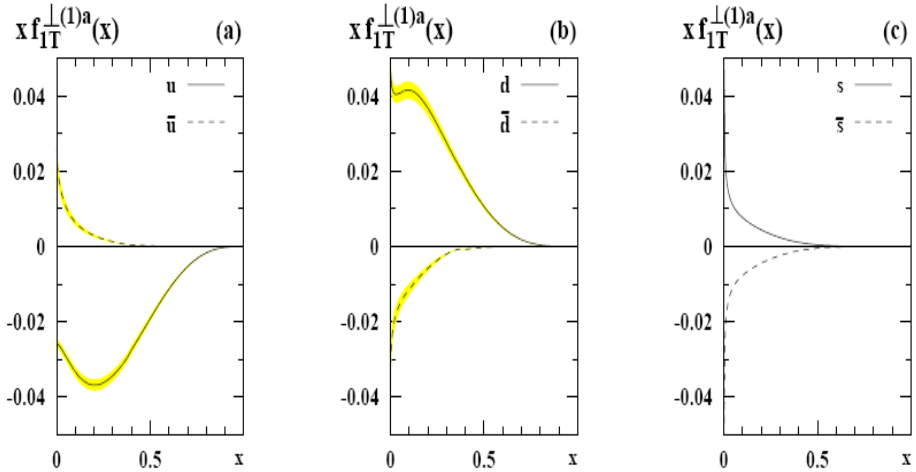




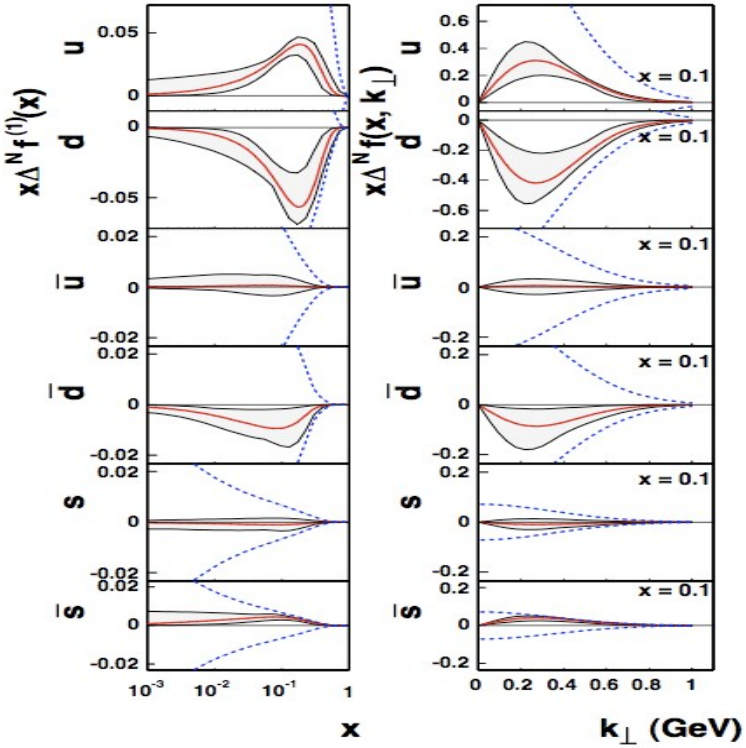
- Non-zero effects for pions and kaons ⟶ Sivers function exist.
- COMPASS data compatible with zero ⟶ flavor dependence.
- Kaon SSA > Pion SSA at x ≈ 0.2... sea-quark effect?

Extractions of Sivers functions

S. Arnold et al., 2008



M. Anselmino et al., 2008



- u- and d-Sivers roughly equal, but opposite sign
- non-zero sea-quark Sivers functions.
- both ansätze: small statistical errors, $\chi^2 \approx 1$
- again, Sivers functions not constraint for x > 0.4...

Relations between TMDs and GPDs

Trivial Relations are well-known:

$$f_1(x) = H(x,0,0) = \int d^2k_T\, f_1(x,\vec{k}_T^2) = \int d^2b_T \mathcal{H}(x,\vec{b}_T^2)$$

$$g_1(x) = \tilde{H}(x,0,0) = \int d^2k_T\, g_{1L}(x,\vec{k}_T^2)$$

$$h_1(x) = H_T(x,0,0) = \int d^2k_T\, h_1(x,\vec{k}_T^2)$$

→ model-independent, integrated relations

also for twist-3 PDFs e(x), g_T(x), ...

Non-trivial Relations

Non-trivial relations for "T-odd" parton distributions:

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]

Average transverse momentum of unpolarized partons in a transversely polarized nucleon:

$$\langle k_T^i \rangle_T(x) = \int d^2k_T\, k_T^i\, \frac{1}{2}\Big[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\Big] \propto f_{1T}^{\perp,(1)}(x)$$

Manipulation of Gauge Links + Impact parameter representation

$$\langle k_T^i \rangle(x) = \int d^2b_T \int \frac{dz^-}{2(2\pi)}\, e^{ixP^+z^-} \langle P^+;\vec{0}_T;S_T|\bar{\psi}(z_1)\gamma^+[z_1;z_2]I^i(z_2)\psi(z_2)|P^+;\vec{0}_T;S_T\rangle$$

$z_{1/2} = \mp\frac{z^-}{2}n_- + b_T$ Impact parameter representation for GPD E

$I^i(z^-) = \int dy^-\,[z^-;y^-]gF^{+i}(y^-)[y^-;z^-]$ coll. "soft gluon pole" matrix element

Non-trivial Relations

- *Conjecture:* factorization of final state interactions and spatial distortion

$$\langle k_T^i \rangle = -M \epsilon_T^{ij} S_T^j \, f_{1T}^{\perp,(1)}(x) \simeq \int d^2 b_T \, \mathcal{I}^i(x, \vec{b}_T) \, \frac{\vec{b}_T \times \vec{S}_T}{M} \, \frac{\partial}{\partial b_T^2} \mathcal{E}(x, \vec{b}_T^2)$$

$\mathcal{I}^i(x, \vec{b}_T^2)$: Lensing Function = net transverse momentum

- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

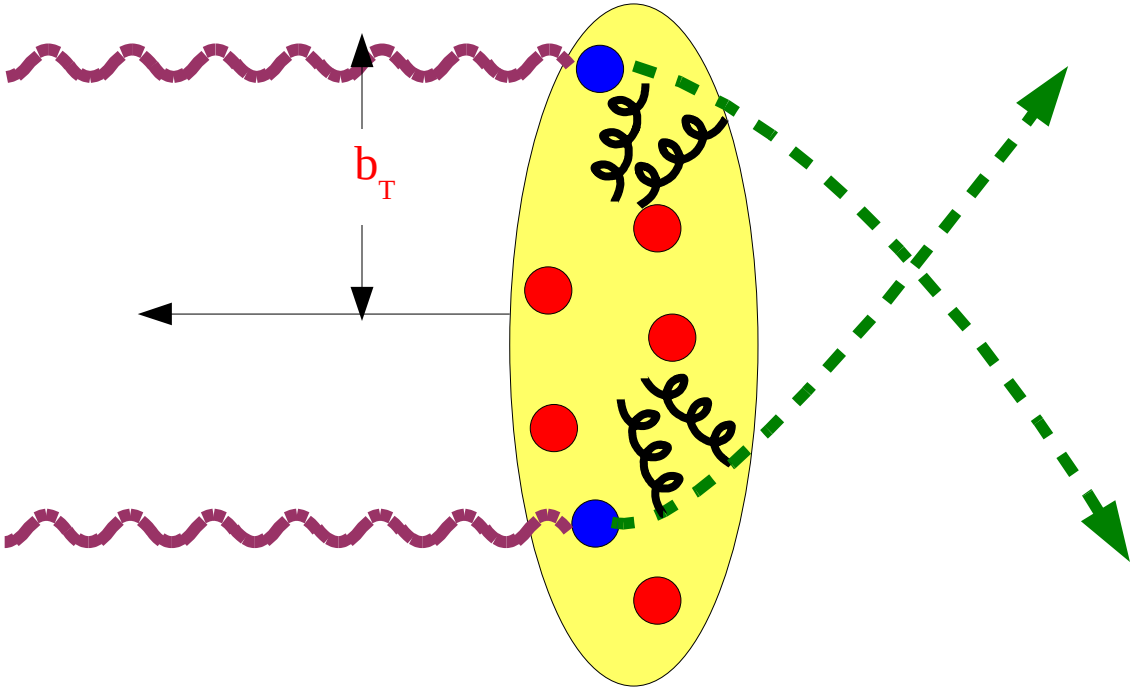
$$-2M^2 \, h_1^{\perp,(1)}(x) \simeq \int d^2 b_T \, \vec{b_T} \cdot \vec{\mathcal{I}}(x, \vec{b}_T) \, \frac{\partial}{\partial b_T^2} \left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T \right)(x, \vec{b}_T^2)$$

- Quantify distortion effect $\longleftrightarrow$ flavor anormalous magnetic moment κ^q

$\longrightarrow$ Prediction of signs for u- and d-quark T-odd TMDs.

Physical picture of the Relation

Intuitive picture of the Final State Interactions:



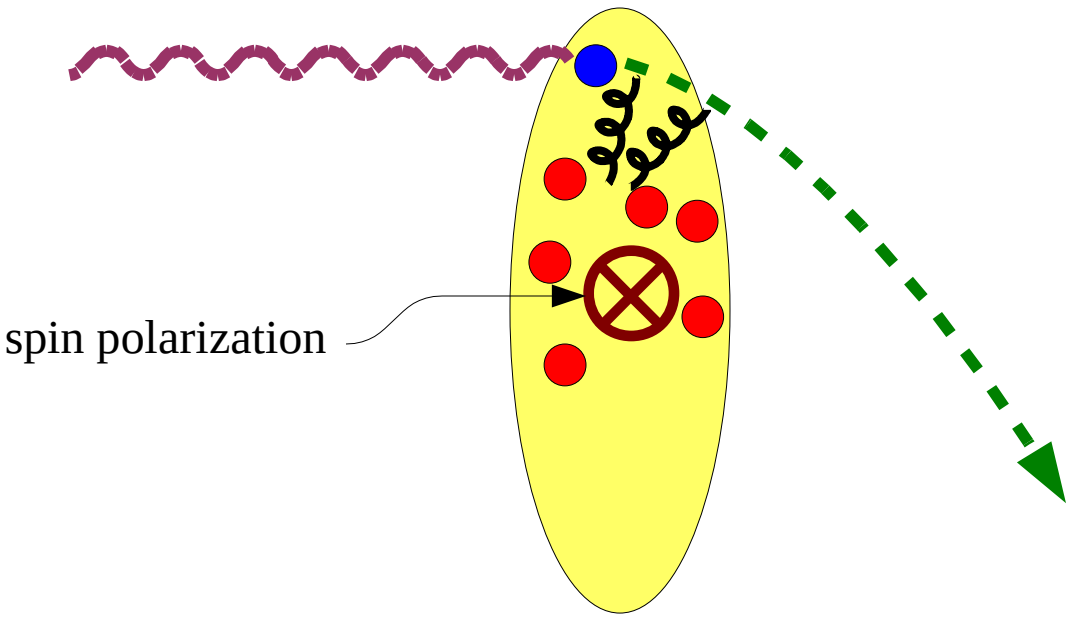
Final State interactions are assumed to be attractive

→ Lensing!

Physical picture of the Relation

Intuitive picture of the Sivers asymmetry:

Spatial distortion in the transverse plane due to polarization!



Mechanism leads to non-zero Sivers asymmetry!

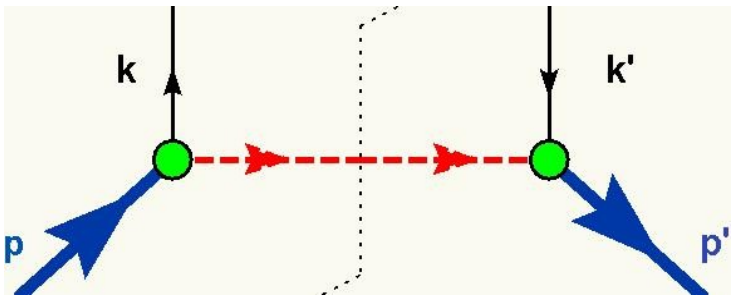
Relations in Spectator Models

Explicit checks of relations in a diquark spectator model:

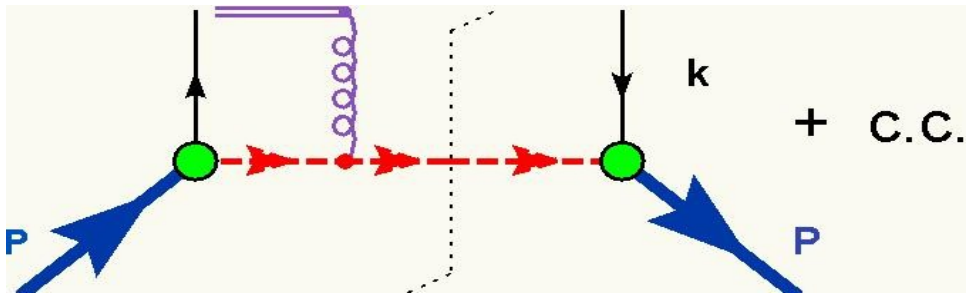
[Burkardt, Hwang, PRD69, 074032], [Meissner, Metz, Goeke, PRD76, 034002]

Lowest order calculations:

GPDs:



(T-odd) TMDs:



Non-trivial relations are *exactly* fulfilled!

$$-M\epsilon^{ij}S_T^j\, f_{1T}^{\perp,(1)} = \int d^2b_T\, \mathcal{I}^i\, \frac{\vec{b}_T\times\vec{S}_T}{M}\,\mathcal{E}'$$

$$-2M\, h_1^{\perp,(1)} = \int d^2b_T\, \frac{\vec{b}_T\cdot\vec{\mathcal{I}}}{M}\left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T\right)'$$

Relations in Spectator Models

In the diquark-spectator model:

- Relations between *arbitrary* moments:

$$f_{1T}^{\perp,(n)}(x) \propto E^{(n)}(x)\,,\ 0 \leq \mathrm{n} \leq 1$$

- TMD: $f^{(n)}(x) \sim \int d^2k_T\,(\vec{k}_T^2)^n f(x,\vec{k}_T^2)$
- GPD: $E^{(n)}(x) \sim \int d^2\Delta_T\,(\vec{\Delta}_T^2)^{n-1} E(x,0,-\frac{\vec{\Delta}_T^2}{(1-x)^2})$

- Relation between GPDs and *T-even* TMDs:

$$h_{1T}^{\perp,(n)}(x) \sim \tilde{H}_T^{(n)}(x)$$

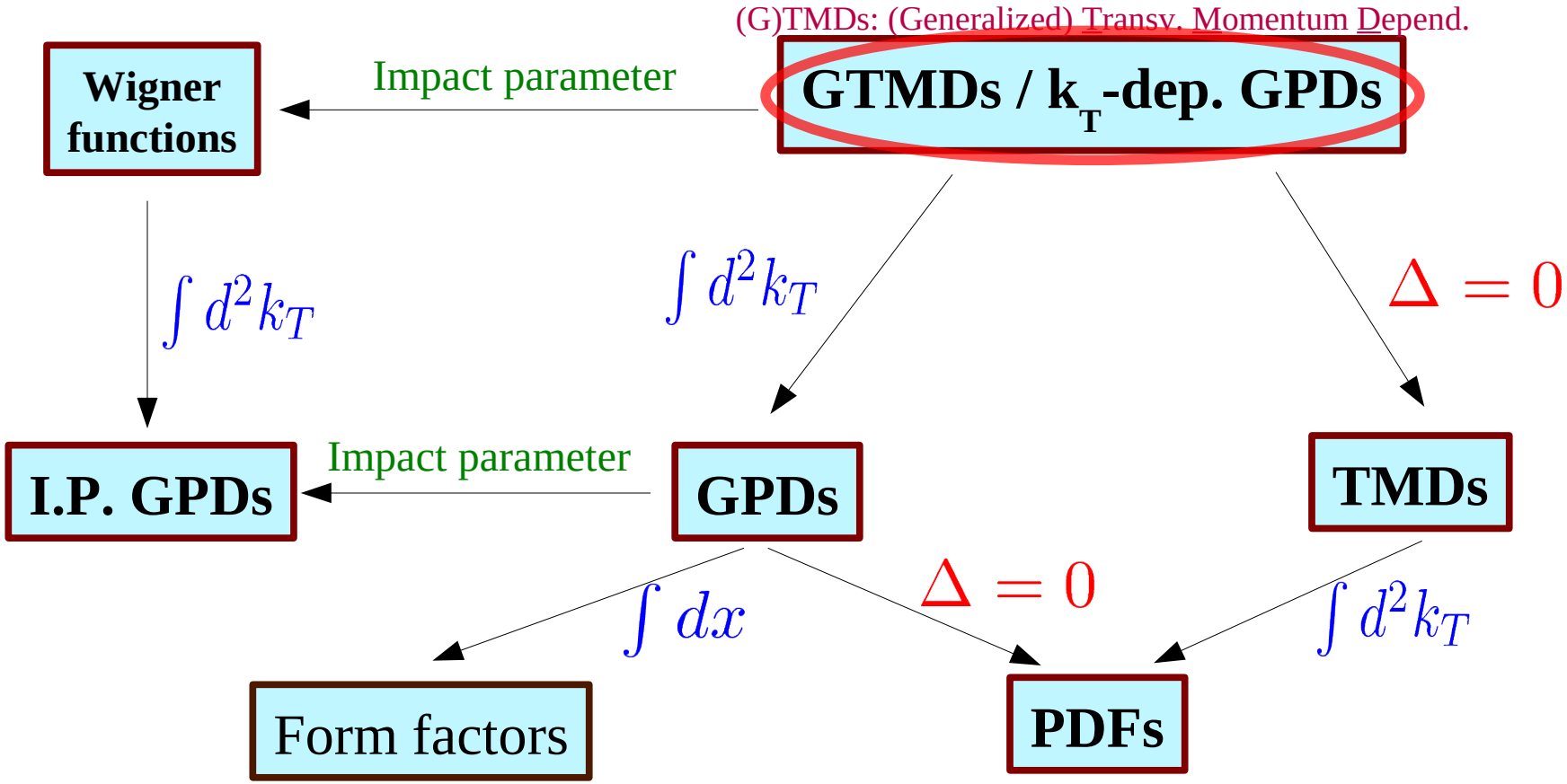
→ No FSI / Lensing function needed!

valid also in a light-cone constituent quark model (Pasquini et al.,arXiv:0806.2298)

- Relations for gluon-GPDs and gluon-TMDs.
- Relations are likely to be broken for higher order diagrams.

Mother functions

Relations between functions:



Which GPDs and TMDs have the same mother functions?

Classification of Generalized TMDs

- Define "off-diagonal" k_T-dependent matrix element:

  [Meissner, Metz, M.S., Goeke, arXiv:0805.3165]

$$W_{ij}(x,\xi;\vec{k}_T;\vec{\Delta}_T) = \int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ik\cdot z} \langle p',\lambda'| \bar{\psi}_j(-\tfrac{z}{2}) \mathcal{W}[-\tfrac{z}{2},\tfrac{z}{2}] \psi_i(\tfrac{z}{2}) |p,\lambda\rangle \Big|_{z^+=0}$$

For spin-0: Classification 4 leading twist GTMDs

chiral-odd:

$$\mathrm{Tr}[W\, i\sigma^{i+}\gamma_5] = -\frac{2i}{M}\left(\epsilon_T^{ij} k_T^j\, H_1^k + \epsilon_T^{ij}\Delta_T^j\, H_1^\Delta\right)$$

- GTMDs are complex-valued functions (Time-reversal)

$$X(\eta) = \Re[X] + i\eta\Im[X]$$

Limits: TMD

$$h_1^\perp(x,\vec{k}_T^2) = \eta\Im[H_1^k](x,0,0,0,\vec{k}_T^2)$$

GPD

$$H_1^\pi(x,\xi,t) = \int d^2k_T \left[\frac{\vec{\Delta}_T\cdot\vec{k}_T}{\vec{\Delta}_T^2}\Re[H_1^k] + \Re[H_1^\Delta]\right]$$

No exact, model-independent relation!

Approx. relation?

Limiting cases: Spin 1/2

Example: Sivers function:

GTMDs: [Meissner, Metz, M.S., Goeke, in preparation]

$$\mathrm{Tr}\left[\tilde{W}_{\lambda',\lambda}\gamma^+\right] = \frac{1}{P^+}\bar{u}(p',\lambda')\left[\frac{P^+}{M}\mathcal{F}_1 + \frac{i\sigma^{i+}k_T^i}{M}\mathcal{F}_1^\perp + \frac{i\sigma^{i+}\Delta_T^i}{M}\mathcal{F}_1^\Delta + \frac{P^+}{M}\frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\mathcal{F}_1^{\perp\Delta}\right]u(p,\lambda)$$

TMD-limit: $\Delta = 0$

$$f_{1T}^\perp(x,\vec{k}_T^2) = \Im[\mathcal{F}_1^\perp](x,0;\vec{k}_T^2,0,0)$$

GPD-limit: $\int d^2k_T\,,\ \xi = 0$

$$(E+H)(x,0,\vec{\Delta}_T^2) = 2\int d^2k_T\left[\Re[\mathcal{F}_1^\Delta] + \frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\Re[\mathcal{F}_1^\perp]\right]$$

No model-independent *exact* relation! Approximate relations?

- Relation $h_{1T}^\perp \leftrightarrow \tilde{H}_T$ *not* supported by GTMD-analysis!

Summary

- Non-trivial relations between GPDs and (T-odd) TMDs were suggested on the basis of a separation between:

  distortion of parton distribution in the *transverse plane*

  + *Final state interactions*

- Relations seems to work "numerically"

  $\longrightarrow$ predicts signs of u- and d-quark *Sivers*- and *BM*-function.

- Relations are exact in lowest-order spectator models.

  Also relations between T-even TMDs and GPDs.

- GTMD-investigation: does not support *model-independent, exact relations*. Does not rule out "approximate" relations...