JLab Hall-C Summer Workshop
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Lambda-Sigma Coupling Effect in the Neutron-Rich Lambda Hypernuclei in a Shell-Model Calculation

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A. Umeya and T. Harada, Phys. Rev. C 79, 024315 (2009).
T. Harada, A. Umeya, and Y. Hirabayashi, Phys. Rev. C 79, 014603 (2009).

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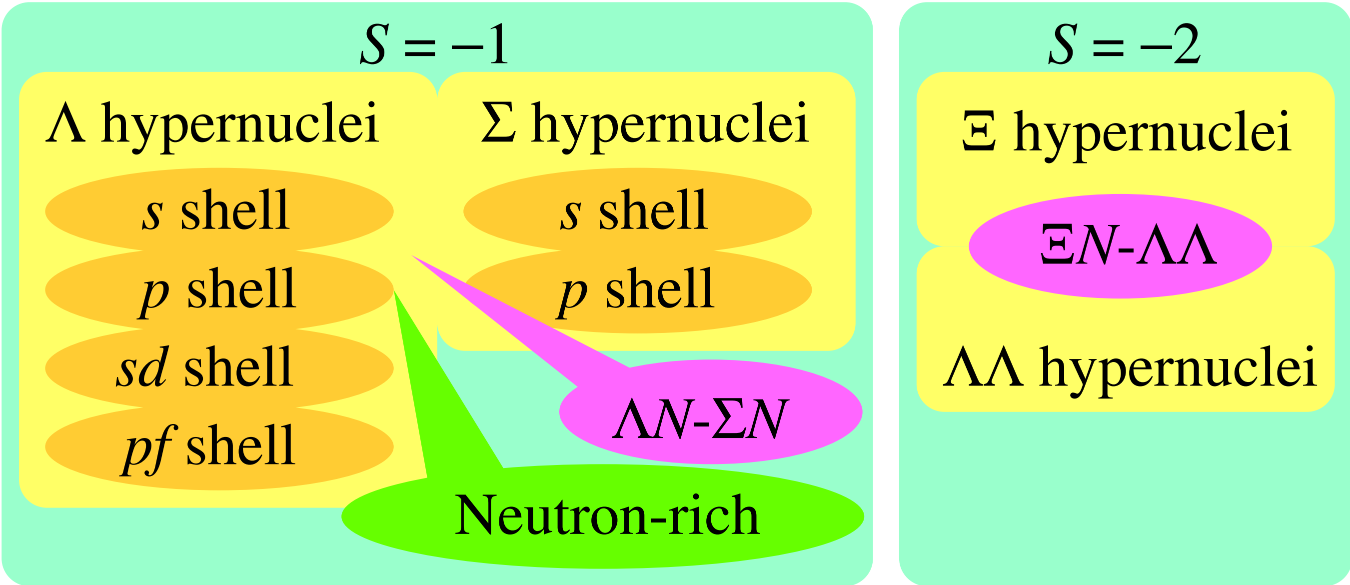
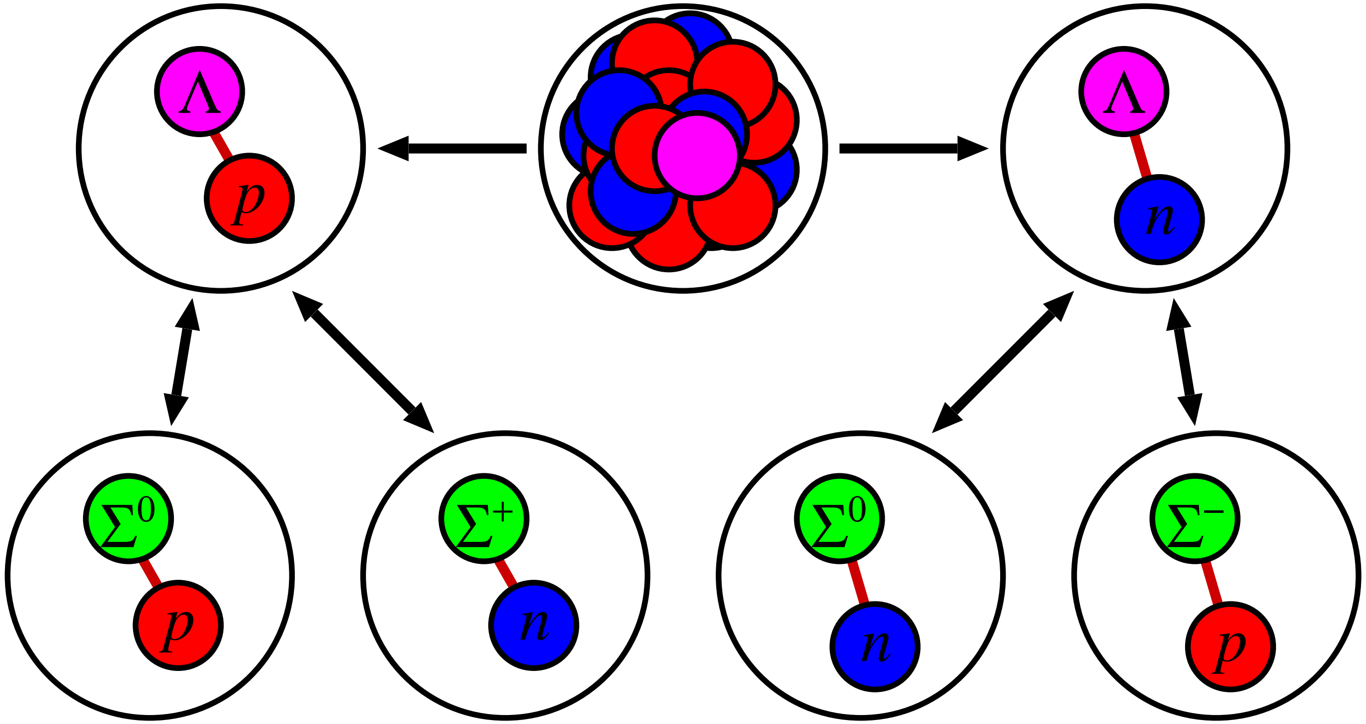
Neutron-rich hypernuclei



- A Λ hyperon plays a glue like role in nuclei beyond the neutron drip line, together with a strong ΛN-ΣN coupling, which might induce Σ-mixing in nuclei.
- The knowledge of behavior of hyperons in a neutron-excess environment significantly affects our understanding of neutron stars, because it makes the equation of state soften.

Λ-Σ coupling

A Λ-nuclear state in Λ-hypernuclei is able to convert a Σ-nuclear state by a ΛN-ΣN coupling interaction.



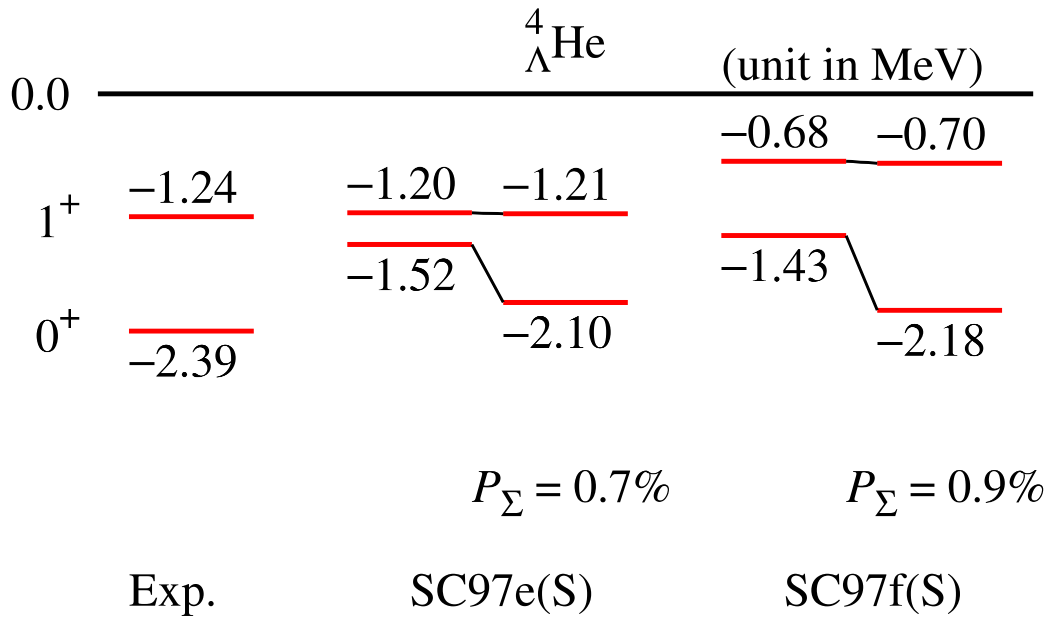
A Σ-mixing probability in a state of Λ-hypernuclei is few percent because Σ hyperon has a larger mass than a Λ hyperon by about 80 MeV.

Σ-mixing → the energy spacings of doublets

→ the production of hypernuclei by the double charge exchange reactions

Coherent Λ-Σ coupling

- The splitting of the 1^+ and 0^+ states of ${}^4_\Lambda$H and ${}^4_\Lambda$He has long been recognized as a problem to describe simultaneously the binding energies of the *s*-shell hypernuclei with a central Λ*N* interaction. R. H. Dalitz *et al.*, NPB47 (1972) 109.
- Akaishi *et al.* suggested the importance of a coherent Λ-Σ coupling in the study of He Λ-hypernuclei. Khin Swe Myint *et al.*, FBS. Suppl. 12 (2000) 383. Y. Akaishi *et al.*, PRL84 (2000) 3539.



Akaishi *et al.*, PRL 84 (2000) 3539.

Coherent Λ-Σ coupling

- The splitting of the 1^+ and 0^+ states of ${}^4_\Lambda$H and ${}^4_\Lambda$He has long been recognized as a problem to describe simultaneously the binding energies of the s-shell hypernuclei with a central ΛN interaction. R. H. Dalitz *et al.*, NPB47 (1972) 109.
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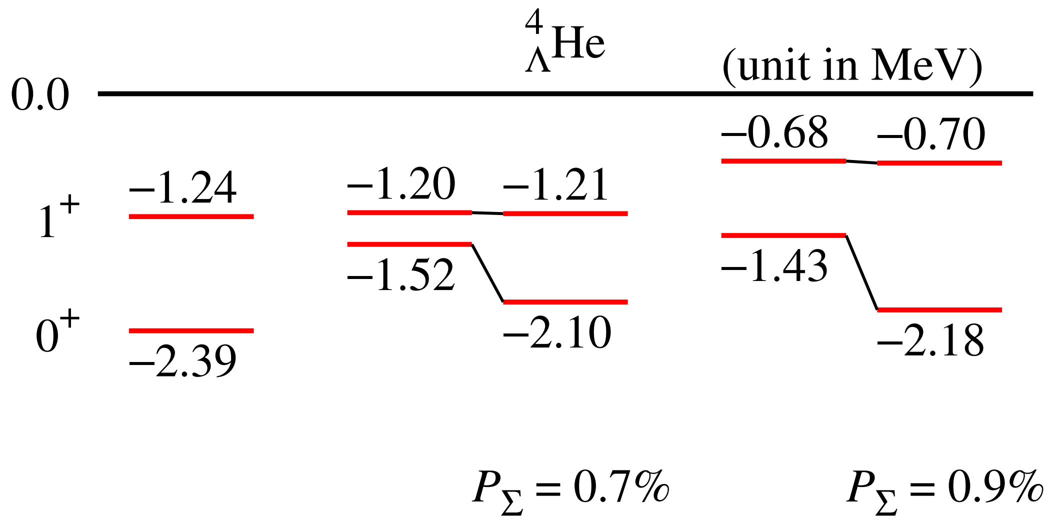
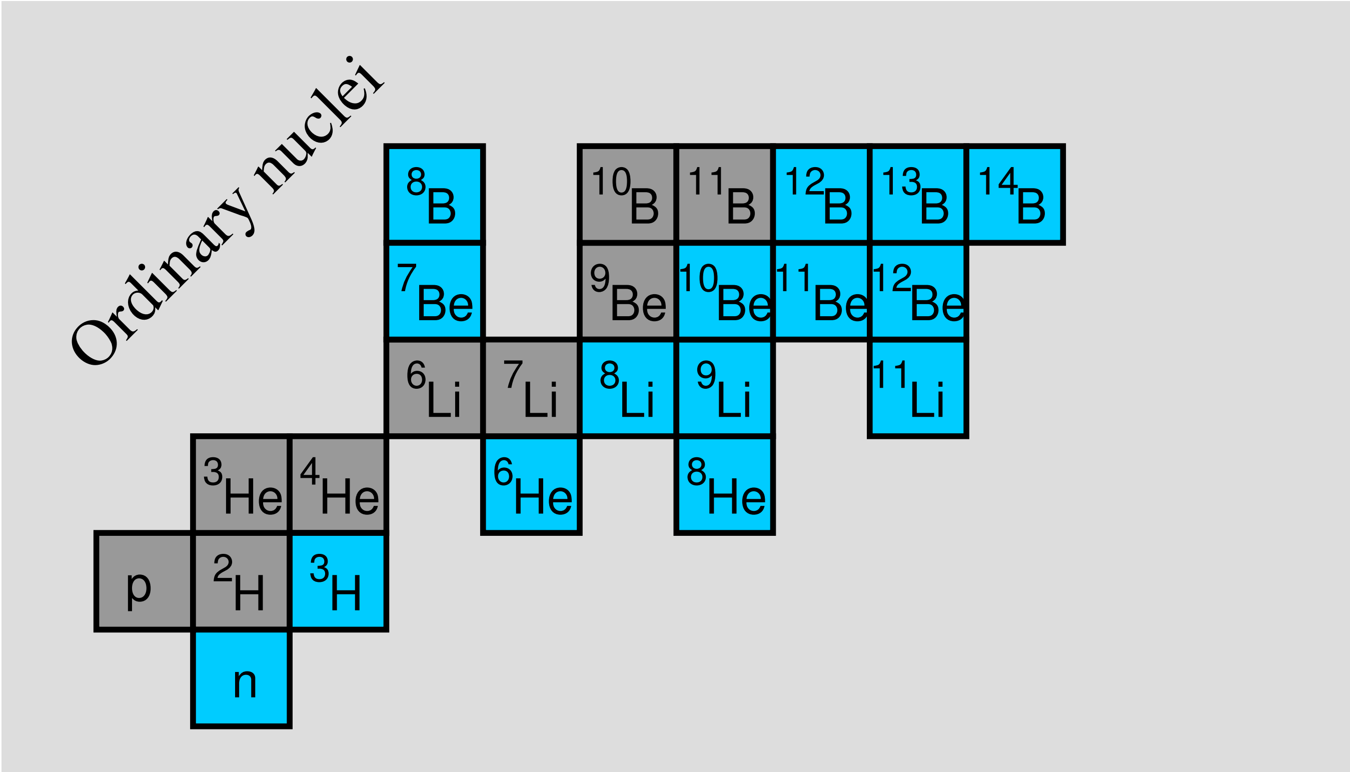


The problem might be solved by the Λ-Σ coupling which strongly affects the 0^+ states of the $A = 4$ hypernuclei.

Production of neutron-rich Λ-hypernuclei

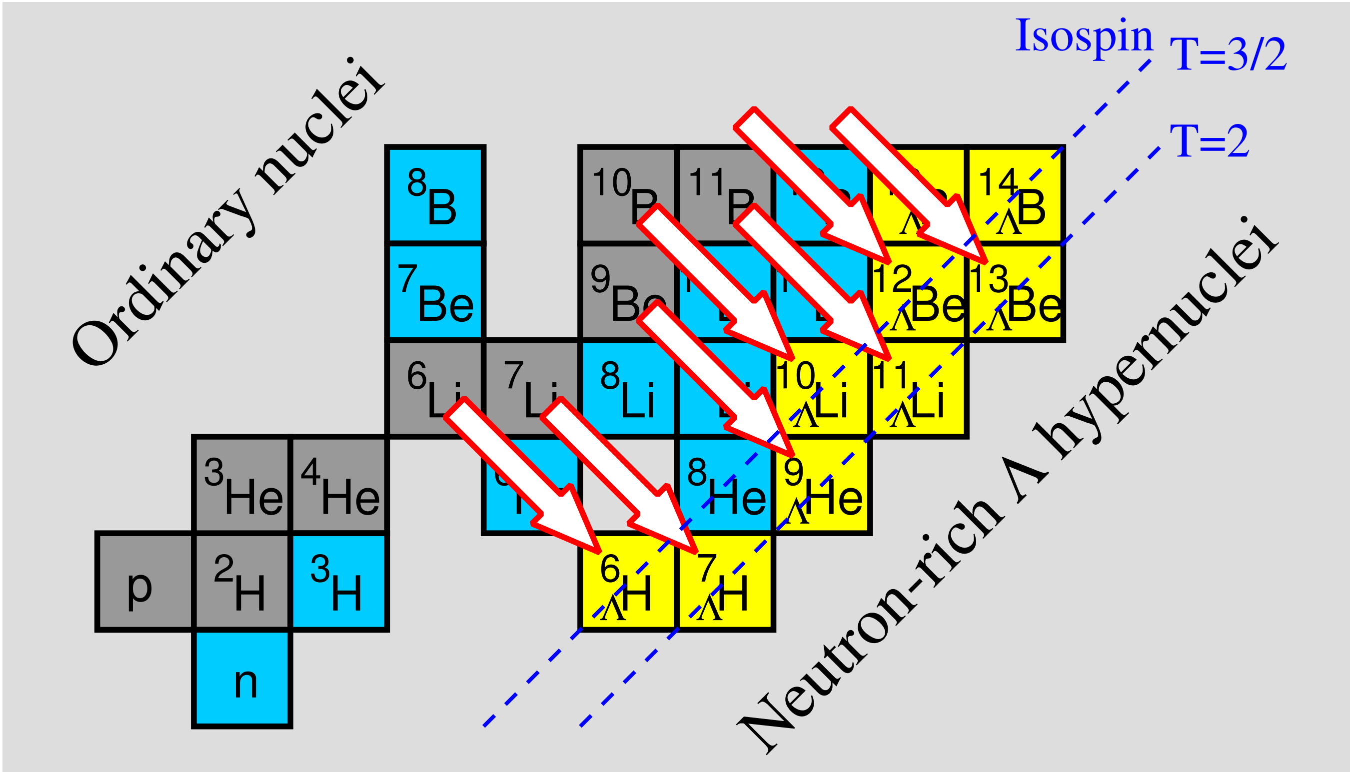
Double Charge Exchange (DCX) Reactions (K^-, π^+), (π^-, K^+)



Double charge exchange reactions can reduce two protons from target nuclei, and suitable for productions of neutron-rich Λ-hypernuclei.

Production of neutron-rich Λ-hypernuclei

Double Charge Exchange (DCX) Reactions (K^-, π^+), (π^-, K^+)



Experimental attempts to produce neutron-rich Λ-hypernuclei

KEK ^{9}Be (K^-, π^+) $^{9}_{\Lambda}$He, ^{12}C (K^-, π^+) $^{12}_{\Lambda}$Be, ^{16}O (K^-, π^+) $^{16}_{\Lambda}$C at rest. K. Kubota *et al.*, NPA602, 323 (1996).

^{10}B (π^-, K^+) $^{10}_{\Lambda}$Li at $p_\pi = 1.05, 1.20$ GeV$/c$. P. K. Saha *et al.*, PRL94, 052502 (2005).

DAΦNE ^{6}Li (K^-, π^+) $^{6}_{\Lambda}$H, ^{7}Li (K^-, π^+) $^{7}_{\Lambda}$H at rest. M. Agnello *et al.*, PLB640, 145 (2006).

J-PARC proposal P10 ^{6}Li (π^-, K^+) $^{6}_{\Lambda}$H, ^{9}Be (π^-, K^+) $^{9}_{\Lambda}$He at $p_\pi = 1.20$ GeV$/c$

Experiments in J-PARC K1.8 area Sep. 2009 –

- E03 Measurement of X-rays from Ξ-Atom (K^-, K^+)
- E05 Spectroscopic Study of Ξ-Hypernucleus ${}^{12}_{\Xi}\mathrm{Be}$, via the ${}^{12}\mathrm{C}(K^-, K^+)$ Reaction (K^-, K^+)
- E07 Systematic Study of Double Strangeness System with an Emulsion-Counter Hybrid Method (K^-, K^+)
- E10 Production of Neutron-Rich Λ-Hypernuclei with the Double Charge-Exchange Reactions (π^-, K^+)
- E13 Gamma-ray spectroscopy of light hypernuclei (K^-, π^-)
- E19 High-resolution Search for Θ^+ Pentaquark in $\pi^- p \to K^- X$ Reactions (π^-, K^-)
- E22 Exclusive Study on the ΛN Weak Interaction in $A = 4$ Λ-Hypernuclei (π^+, K^+)

Further experiments in the (π^-, K^+) reactions are also planned at J-PARC.

Experiments in J-PARC K1.8 area Sep. 2009 –

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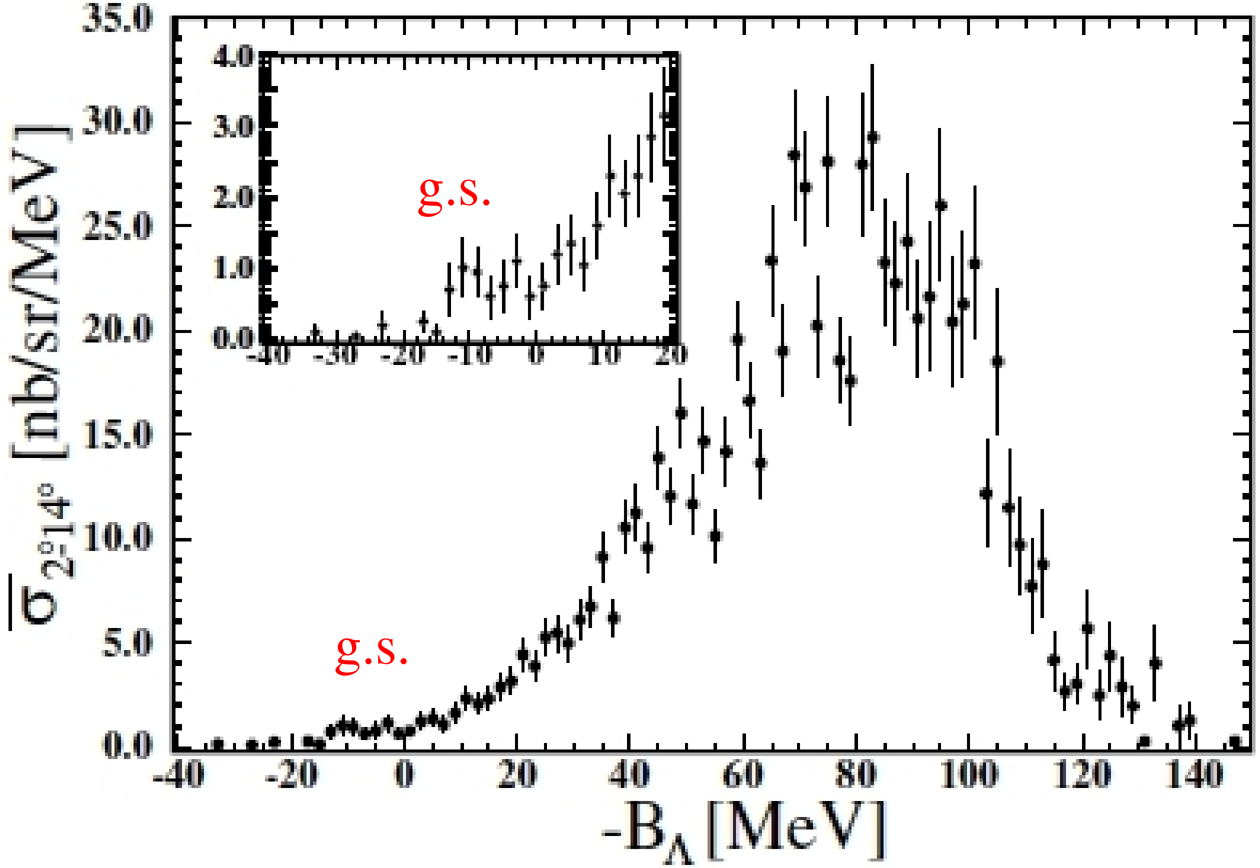
Further experiments in the (π^-, K^+) reactions are also planned at J-PARC.

Λ spectrum by (π^-, K^+) DCX reaction on a ^{10}B target at 1.2 GeV/c

$^{10}\mathrm{B}(\pi^-, K^+)\ ^{10}_{\Lambda}\mathrm{Li}$ First successful measurement

KEK-PS-E521 P. K. Saha *et al.*, PRL94 (2005) 052502.



Cross Section:

11.3 ± 1.9 nb/sr

for p_{π^-} = 1.20 GeV/c

5.8 ± 2.2 nb/sr

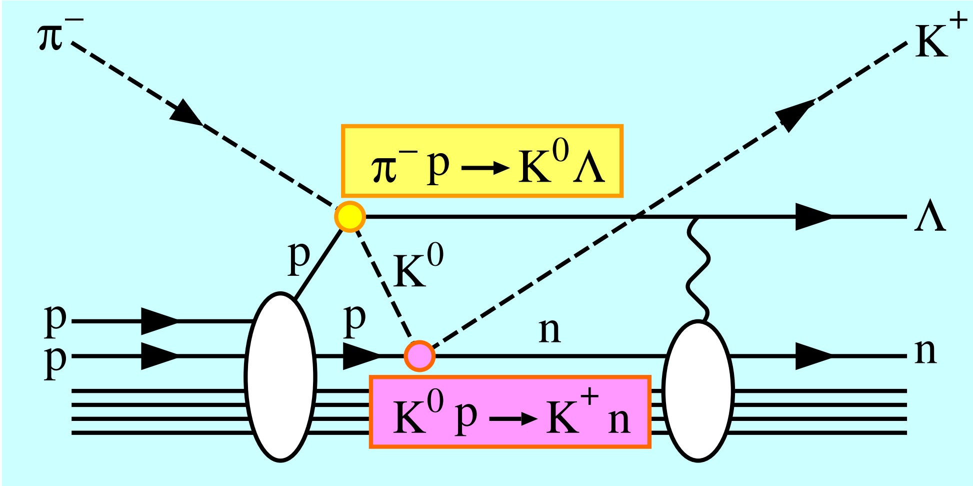
for p_{π^-} = 1.05 GeV/c

The cross sections for productions by (π^-, K^+) reactions are about 1000 times smaller than those by (π^+, K^+) reactions.

Double Charge Exchange Reaction

Two-step process

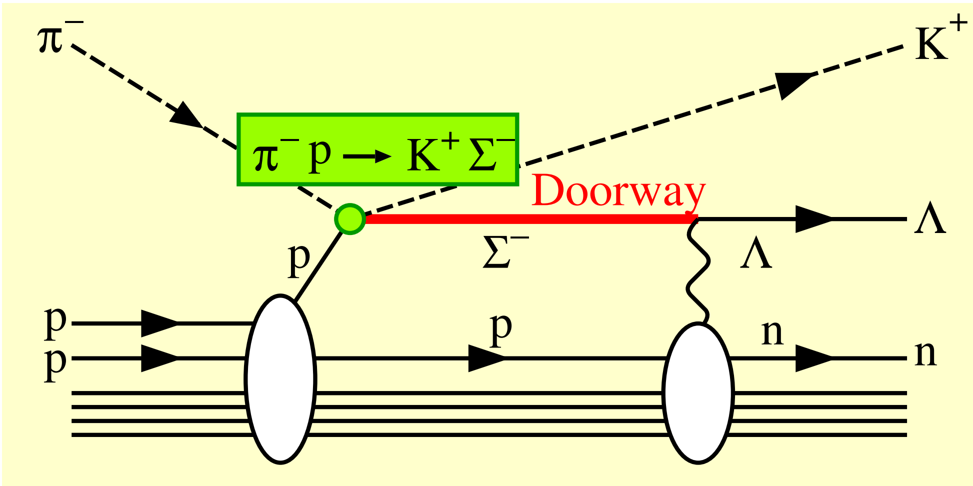
$$\pi^- p \rightarrow K^0 \Lambda$$
$$K^0 p \rightarrow K^+ n$$
$$\pi^- p \rightarrow \pi^0 n$$
$$\pi^0 p \rightarrow K^+ \Lambda$$



One-step process

$$\pi^- p \rightarrow K^+ \Sigma^-$$
$$\Sigma^- p \leftrightarrow \Lambda n$$

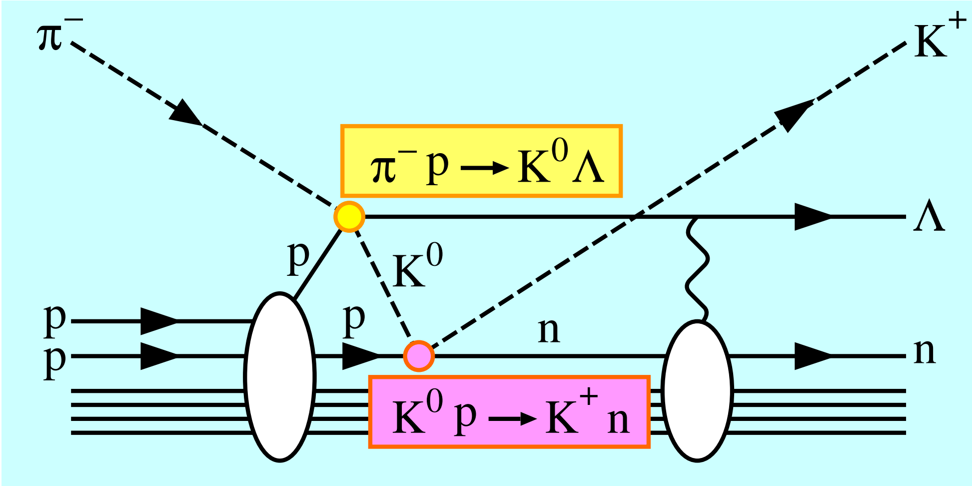
Σ-Λ coupling



Double Charge Exchange Reaction

Two-step process

$$\pi^- p \to K^0 \Lambda$$
$$K^0 p \to K^+ n$$
$$\pi^- p \to \pi^0 n$$
$$\pi^0 p \to K^+ \Lambda$$



Theoretical calculation
with the distorted-wave impulse approximation (DWIA)
T. Yu. Tretyakova, D. E. Lanskoy, Phys. At. Nucl. 66 (2003) 1651.
→ The two-step mechanism is dominant.

22 nb/sr for $p_{\pi^-} = 1.20$ GeV/c (Exp.: 11.3 ± 1.9 nb/sr)
38 nb/sr for $p_{\pi^-} = 1.05$ GeV/c (Exp.: 5.8 ± 2.2 nb/sr)
The absolute values of the cross sections and their incident-momentum dependence given by the two-step processes are different from the experimental data.

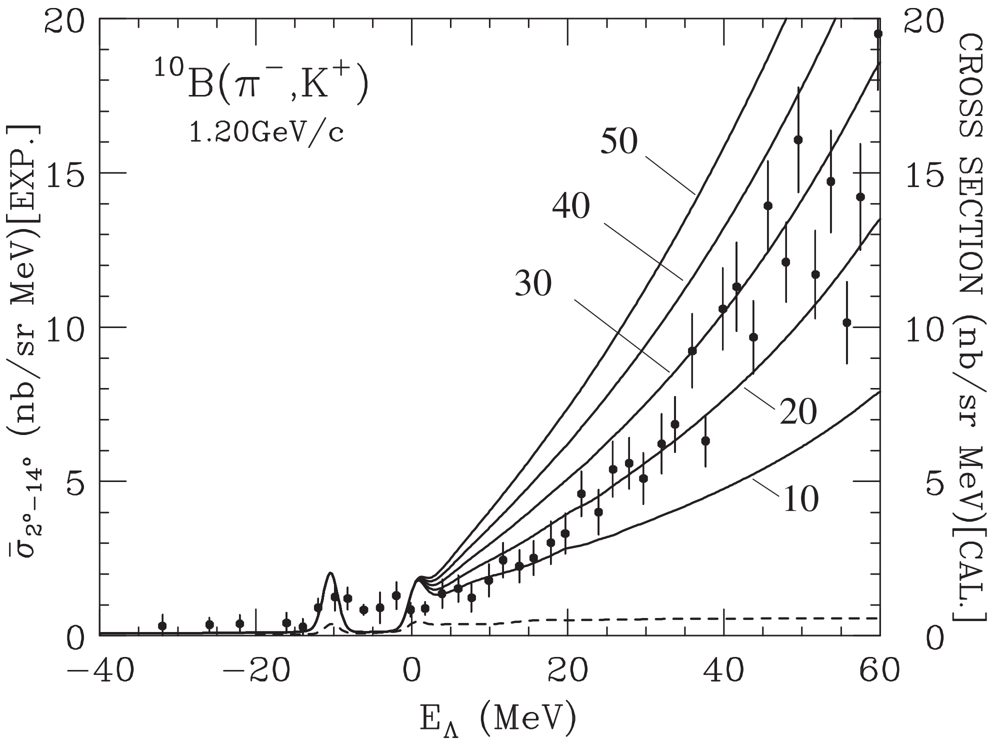
Theoretical calculation by using the coupled-channel DWIA

T. Harada, A. Umeya, Y. Hirabayashi, PRC79, 014603 (2009).

for the first successful measurement of $^{10}\mathrm{B}\ (\pi^-, K^+)\ ^{10}_{\Lambda}\mathrm{Li}$ reaction

KEK-PS-E521; P. K. Saha *et al.*, PRL94 (2005) 052502.

Λ spectrum by (π^-, K^+) DCX reaction at 1.2 GeV/c, $V_{\Sigma\Lambda}$ = 10 MeV (P_Σ = 0.57 %)



The values of W_Σ and $V_{\Sigma\Lambda}$ are phenomenologically determined by fitting to a spectral shape of the experimental data.

W_Σ : a strength of the spreading imaginary potential

→ complicated excited states for $^{10}_{\Lambda}$Li

$V_{\Sigma\Lambda}$: an effective strength of the ΛN-ΣN potential

Solid lines: One-step process with $-W_\Sigma$ = 10, 20, 30, 40, 50 MeV, Dashed line: Two-step process

The values of $-W_\Sigma$ = 20–30 MeV enable the calcurated spectrum to reproduce substantially the experimental data, and are consistent with the alalysis of Σ^- production by the (π^-, K^+) reactions.

The contribution of the two-step processes in the continuum spectrum is rather small.

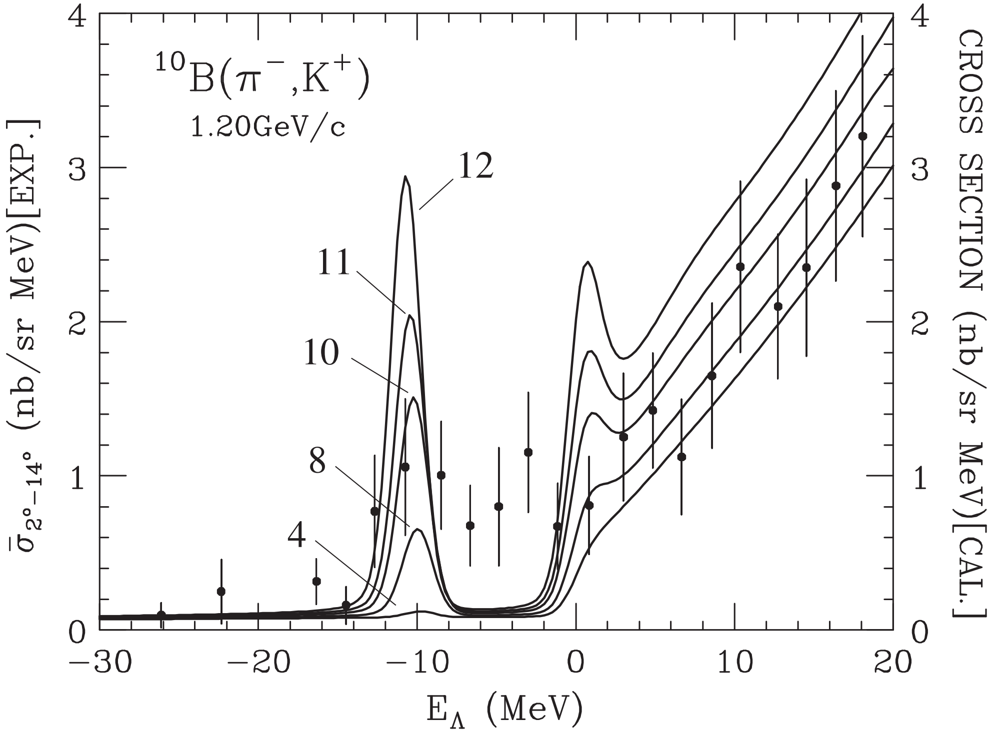
Theoretical calculation by using the coupled-channel DWIA

T. Harada, A. Umeya, Y. Hirabayashi, PRC79, 014603 (2009).

for the first successful measurement of ^{10}B (π^-, K^+) $^{10}_{\Lambda}$Li reaction

KEK-PS-E521; P. K. Saha *et al.*, PRL94 (2005) 052502.

Λ spectrum by (π^-, K^+) DCX reaction at 1.2 GeV/c, $-W_\Sigma = 20$ MeV



The values of W_Σ and $V_{\Sigma\Lambda}$ are phenomenologically determined by fitting to a spectral shape of the experimental data.

W_Σ : a strength of the spreading imaginary potential

$\rightarrow$ complicated excited states for $^{10}_{\Lambda}$Li

$V_{\Sigma\Lambda}$: an effective strength of the ΛN-ΣN potential

Solid lines: One-step process with $V_{\Sigma\Lambda}$ = 4, 8, 10, 11, 12 MeV
(P_Σ = 0.075, 0.30, 0.47, 0.57, 0.68 %)

The resultant spectrum can explain the magnitude of the recent experimental data.

Σ^--mixing probability in $^{10}_{\Lambda}$Li is $P_\Sigma \sim 0.5$ % ($V_{\Sigma\Lambda}$ = 10–12 MeV).

Approximate calculations of the integrated lab cross sections of $d\sigma/d\Omega$ for the ${}^{10}_{\Lambda}\mathrm{Li}$ 2^- bound state with two-step and one-step processes in ${}^{10}\mathrm{B}(\pi^-, K^+)$ reactions, compared with the experimental data

p_π (GeV/c)	Two-step (nb/sr)	One-step (nb/sr)	Exp. (nb/sr)
1.05	$\sim$ 1.6	2.4	5.8 ± 2.2
1.20	$\sim$ 1.2	5.4	11.3 ± 1.9

For the two-step mechanism in the DCX ${}^{10}\mathrm{B}(\pi^-, K^+)$ reaction, we roughly estimated the integrated lab cross sections of $d\sigma/d\Omega$ for the ${}^{10}_{\Lambda}\mathrm{Li}$ bound state with a harmonic oscillator model, adapting the eikonal treatment to the DWIA.

- The calculated value of $d\sigma/d\Omega$ at 1.20 GeV/c by the two-step mechanism is rather small (1-2 nb/sr) compared to that by the one-step one.
- The incident-momentum dependence of $d\sigma/d\Omega$ in the data is similar to that in the one-step mechanism.

The analysis of this reaction provides a reason to carefully examine wave functions involving Σ admixtures in ${}^{10}_{\Lambda}\mathrm{Li}$, as well as a mechanism of this reaction.

Purpose of the present shell-model study

To theoretically clarify the structure of the neutron-rich Lambda hypernuclei

We investigate the structure of the neutron-rich hypernucleus $^{10}_{\Lambda}$Li, in microscopic shell-model calculations considering the Λ-Σ coupling effect.

- Σ-mixing probabilities
- Energy shifts due to the Σ-mixing
- Effect of the ΣN interaction to the core-nuclear state
- Enhancement of the coupling strengths
- Relation to the β-transition properties of the nuclear core state

Method

A state of Λ-hypernuclei

$|({}^A_\Lambda Z)\nu T J\rangle$

A : mass number, Z : atomic number, T : isospin, J : angular momentum

ν : quantum number to distinguish states with the same T and J

Hamiltonian in the configuration space for the Λ-hypernucleus involving a Λ-Σ coupling

$H = H_\Lambda + H_\Sigma + V_{\Lambda\Sigma} + V_{\Sigma\Lambda}$

H_Λ : Hamiltonian in the Λ configuration space

H_Σ : Hamiltonian in the Σ configuration space

$V_{\Lambda\Sigma}$, $V_{\Sigma\Lambda}$: two-body Λ-Σ coupling interaction

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We can write the state of H with T, J as

$$|({}^A_\Lambda Z)\nu T J\rangle = \sum_\mu C_{\nu,\mu}|\psi^\Lambda_\mu; TJ\rangle + \sum_{\mu'} D_{\nu,\mu'}|\psi^\Sigma_{\mu'}; TJ\rangle,$$

where

$$H_\Lambda|\psi^\Lambda_\mu; TJ\rangle = E^\Lambda_\mu|\psi^\Lambda_\mu; TJ\rangle,$$
$$H_\Sigma|\psi^\Sigma_{\mu'}; TJ\rangle = E^\Sigma_{\mu'}|\psi^\Sigma_{\mu'}; TJ\rangle.$$

Method

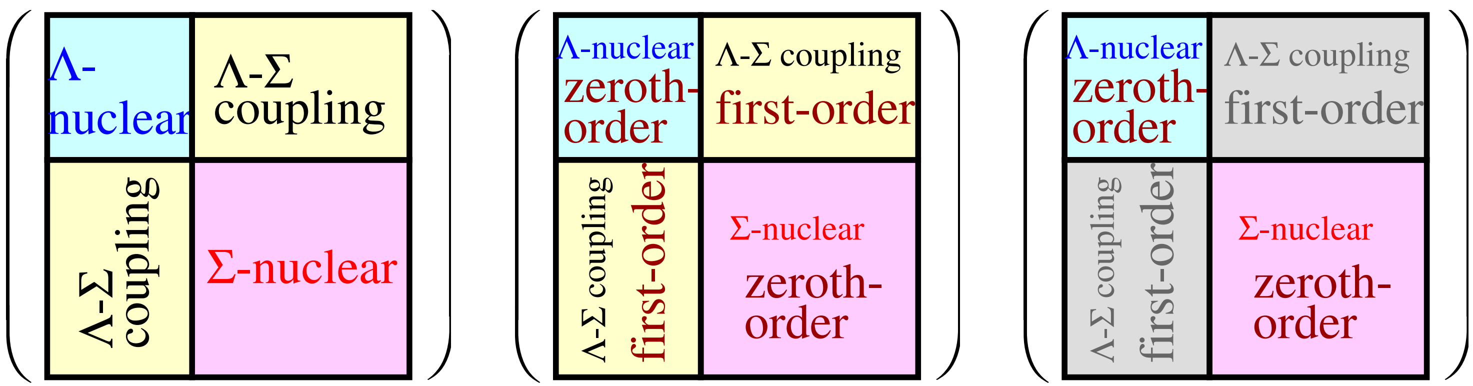
First-order perturbation

We treat $V_{\Lambda\Sigma}$ and $V_{\Sigma\Lambda}$ as perturbation because a Σ hyperon has a larger mass than a Λ hyperon by about 80 MeV.

$$|({}^{A}_{\Lambda}Z)\nu TJ\rangle = \sum_{\mu} C_{\nu,\mu}|\psi^{\Lambda}_{\mu};TJ\rangle + \sum_{\mu'} D_{\nu,\mu'}|\psi^{\Sigma}_{\mu'};TJ\rangle,$$

$$C_{\nu,\mu} = \delta_{\nu\mu}, \quad D_{\nu,\mu'} = -\frac{\langle\psi^{\Lambda}_{\nu};TJ|V_{\Lambda\Sigma}|\psi^{\Sigma}_{\mu'};TJ\rangle}{E^{\Sigma}_{\mu'} - E^{\Lambda}_{\nu}}$$

Matrix of Hamiltonian



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$$C_{\nu,\mu} = \delta_{\nu\mu}, \quad D_{\nu,\mu'} = -\frac{\langle\psi^{\Lambda}_{\nu};TJ|V_{\Lambda\Sigma}|\psi^{\Sigma}_{\mu'};TJ\rangle}{E^{\Sigma}_{\mu'} - E^{\Lambda}_{\nu}}$$

→ Λ-Σ coupling strength $|D_{\nu,\mu'}|^2$ for each Σ eigenstate $|\psi^{\Sigma}_{\mu'};TJ\rangle$

→ Σ-mixing probability $P_{\Sigma} = \sum_{\mu'} |D_{\nu,\mu'}|^2$

→ energy shift $\Delta E = \sum_{\mu'} \left(E^{\Sigma}_{\mu'} - E^{\Lambda}_{\nu}\right) |D_{\nu,\mu'}|^2$

→ binding energy $E^{\Lambda}_{\nu} - \Delta E$

Method

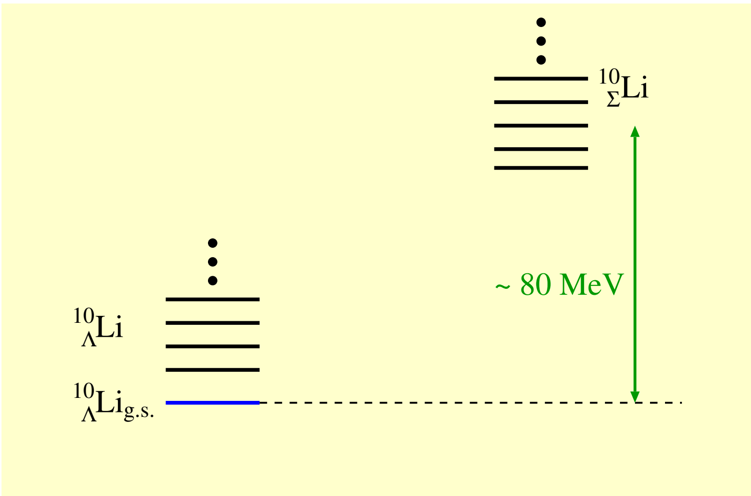
First-order perturbation

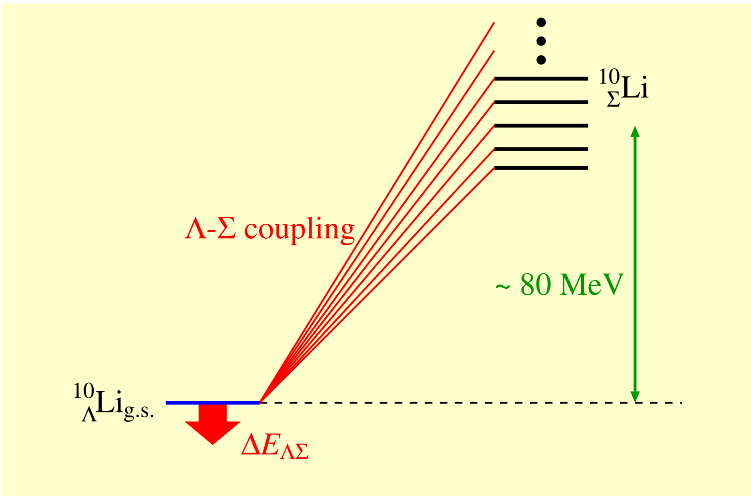
We treat $V_{\Lambda\Sigma}$ and $V_{\Sigma\Lambda}$ as perturbation because a Σ hyperon has a larger mass than a Λ hyperon by about 80 MeV.

$$|({}^{A}_{\Lambda}Z)\nu TJ\rangle = \sum_{\mu} C_{\nu,\mu}|\psi^{\Lambda}_{\mu};TJ\rangle + \sum_{\mu'} D_{\nu,\mu'}|\psi^{\Sigma}_{\mu'};TJ\rangle,$$

$$C_{\nu,\mu} = \delta_{\nu\mu}, \quad D_{\nu,\mu'} = -\frac{\langle\psi^{\Lambda}_{\nu};TJ|V_{\Lambda\Sigma}|\psi^{\Sigma}_{\mu'};TJ\rangle}{E^{\Sigma}_{\mu'} - E^{\Lambda}_{\nu}}$$

Energy shift due to the Λ-Σ coupling





Method

Spectroscopic factor

Λ : A Λ hyperon in a Λ-hypernucleus is described by the single-particle picture very well.
$\leftarrow$ The ΛN interaction is weak.

Σ : The nuclear configuration would change.
$\leftarrow$ The strong spin-isospin dependence in the ΣN interaction

Spectroscopic factor for a hyperon-pickup from the hyperon-nuclear state $|\psi^Y_\mu; TJ\rangle$

$$S_\mu(\nu_N T_N J_N, j_Y) = \frac{\left|\langle\psi^Y_\mu; TJ\|\boldsymbol{a}^\dagger_{j_Y}\|({}^{A-1}Z)\nu_N T_N J_N\rangle\right|^2}{(2T+1)(2J+1)}$$

$|({}^{A-1}Z)\nu_N T_N J_N\rangle$: an eigenstate of the core nucleus
$\boldsymbol{a}^\dagger_{j_Y}$: a creation operator of a single-particle state of the hyperon in an orbit j_Y

A hyperon in the hypernucleus provides the single-particle nature.
$\rightarrow$ The state $|\psi^Y_\mu; TJ\rangle$ is represented as a tensor product of a core-nuclear state $|({}^{A-1}Z)\nu_{\rm core} T_N J_N\rangle$ and a hyperon state $|j_Y\rangle$.
$\rightarrow$ The spectroscopic factor satisfies $S_\mu(\nu_N T_N J_N, j_Y) = \delta_{\nu_N \nu_{\rm core}}$.

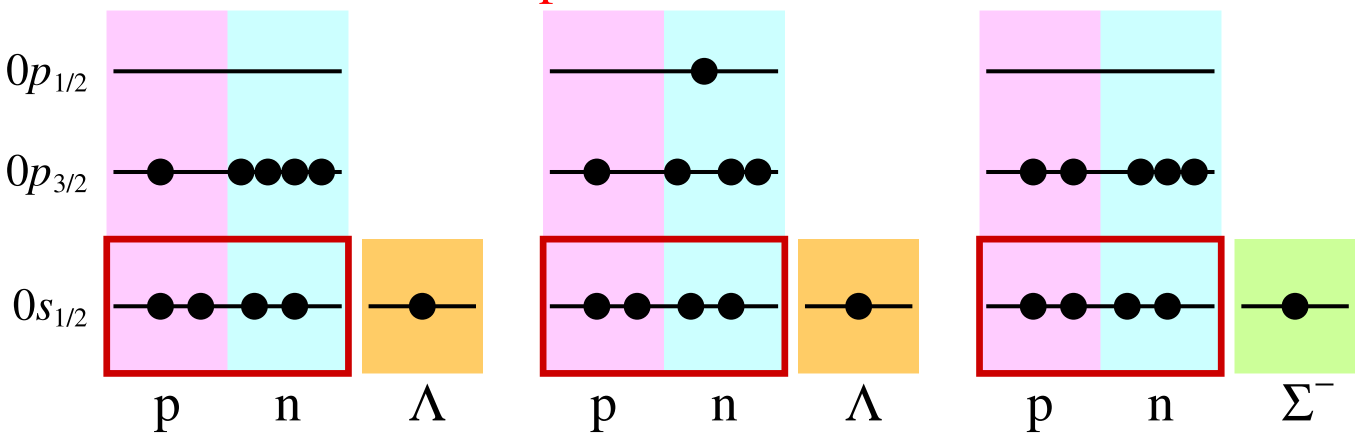
Shell-Model Setup

Model space

- Four nucleons are inert in the ^{4}He core and $(A-5)$ valence nucleons move in the p-shell orbits.
- The Λ or Σ hyperon is assumed to be in the lowest $0s_{1/2}$ orbit.

In the case of $^{10}_{\Lambda}$Li ($T=\frac{3}{2}$, $J^\pi=1^-$), 9-nucleon states have
$T_N=\frac{3}{2}$, $J_N^\pi=\frac{1}{2}^-,\frac{3}{2}^-$ for $|\psi_\mu^\Lambda;TJ\rangle$ and $T_N=\frac{1}{2},\frac{3}{2},\frac{5}{2}$, $J_N^\pi=\frac{1}{2}^-,\frac{3}{2}^-$ for $|\psi_{\mu'}^\Sigma;TJ\rangle$.

Examples of basis states



Shell-Model Setup

Shell model Hamiltonian

NN effective interaction : Cohen-Kurath (8-16) 2BME, NP73, 1 (1965).

YN effective interaction :

$\langle N\Lambda|V_\Lambda|N\Lambda\rangle_{TJ}$ $\langle N\Sigma|V_\Sigma|N\Sigma\rangle_{TJ}$ $\langle N\Lambda|V_{\Lambda\Sigma}|N\Sigma\rangle_{TJ}$ $\langle N\Sigma|V_{\Sigma\Lambda}|N\Lambda\rangle_{TJ}$

$$V_Y = \underbrace{V_0(r)}_{\bar{V}} + \underbrace{V_\sigma(r)}_{\Delta}\, \boldsymbol{s}_N \cdot \boldsymbol{s}_Y + \underbrace{V_{\mathrm{LS}}(r)}_{S_+}\, \boldsymbol{\ell} \cdot \boldsymbol{s}_+ + \underbrace{V_{\mathrm{ALS}}(r)}_{S_-}\, \boldsymbol{\ell} \cdot \boldsymbol{s}_- + \underbrace{V_{\mathrm{T}}(r)}_{T}\, S_{12}$$

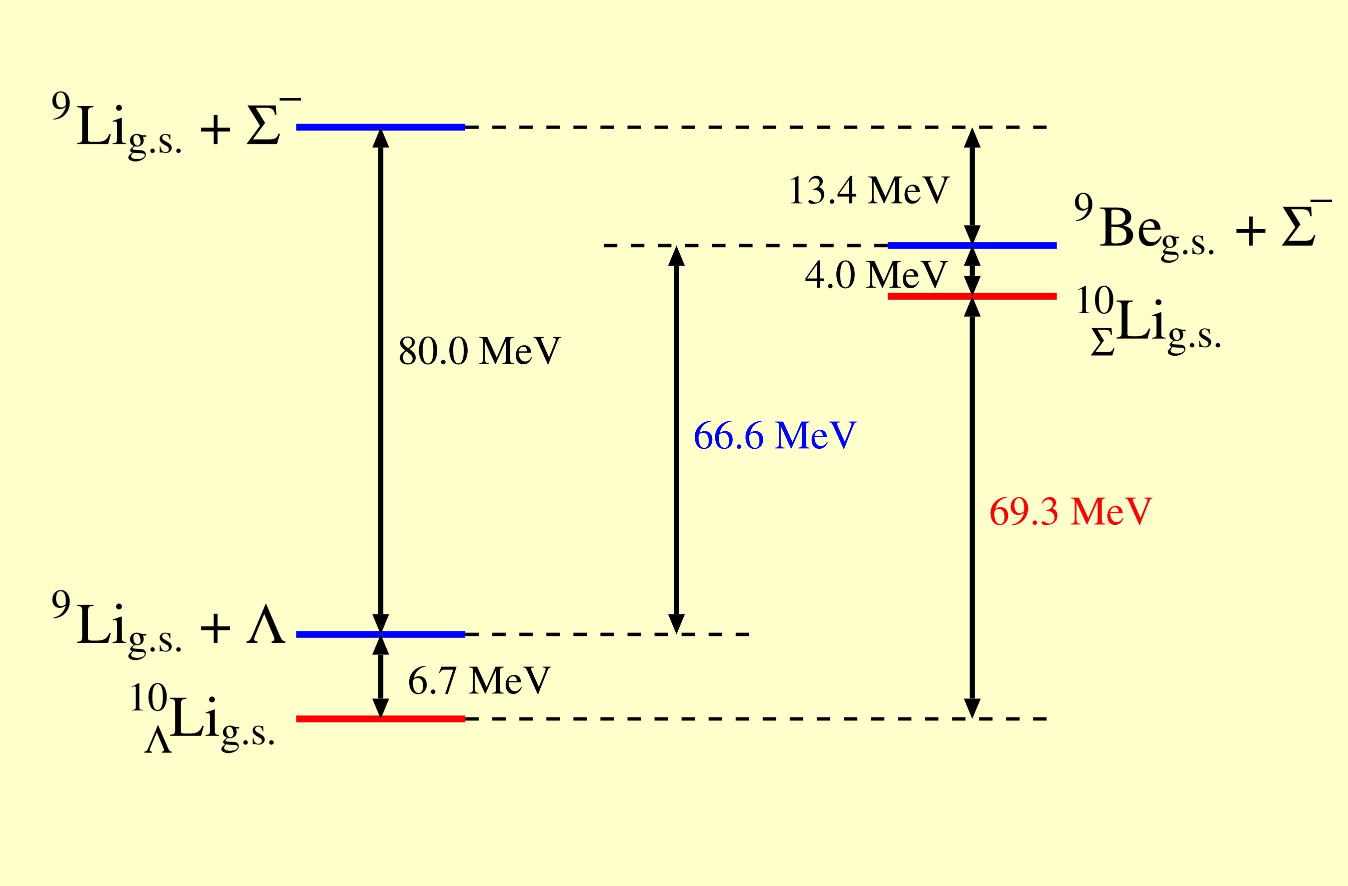
	Isospin	$\bar{V}$	Δ	S_+	S_-	T
V_Λ	$T = 1/2$	-1.2200	0.4300	-0.2025	0.1875	0.0300
V_Σ	$T = 1/2$	1.0100	-7.2150	-0.0010	0.0000	-0.3640
V_Σ	$T = 3/2$	-1.1070	2.2750	-0.2680	0.0000	0.1870
$V_{\Lambda\Sigma}, V_{\Sigma\Lambda}$	$T = 1/2$	1.4500	3.0400	-0.0850	0.0000	0.1570

← NSC97e, f(S)

$V_\Lambda, V_{\Lambda\Sigma}, V_{\Sigma\Lambda}$: D. J. Millener, Lect. Notes Phys. 724 (2007) 31.

V_Σ : D. J. Millener, private communication.

Numerical Results : Schematic energy levels



We assume that the difference between Λ and Σ threshold energies is $E(^9\mathrm{Li}_{\mathrm{g.s.}}+\Sigma) - E(^9\mathrm{Li}_{\mathrm{g.s.}}+\Lambda) = 80$ MeV.
Thus the energy of the Σ ground state $|\psi^{\Sigma}_{\mathrm{g.s.}}\rangle$ is calculated to be $E^{\Sigma}_{\mathrm{g.s.}} - E^{\Lambda}_{\mathrm{g.s.}} = 69.3$ MeV, measured from that of the Λ ground state $|\psi^{\Lambda}_{\mathrm{g.s.}}\rangle$.

Numerical Result : Σ-mixing probabilities and energy shifts

	$(J^\pi\ ;T)$	E [MeV]	P_Σ [%]	$\Delta E_{\Lambda\Sigma}$ [MeV]	$\Delta E_{\Lambda\Sigma}$ [MeV] Millener's work	
${}^{7}_{\Lambda}$Li	$(\frac{1}{2}^+;0)$	0.000	0.098	0.085	0.078	—
	$(\frac{3}{2}^+;0)$	0.612	0.017	0.015	0.004	0.070
${}^{11}_{\Lambda}$B	$(\frac{5}{2}^+;0)$	0.000	0.076	0.073	0.066	—
	$(\frac{7}{2}^+;0)$	0.328	0.015	0.015	0.011	0.058
${}^{10}_{\Lambda}$Li	$(1^-;\frac{3}{2})$	0.000	0.345	0.280		—
	$(2^-;\frac{3}{2})$	0.395	0.166	0.134		0.146

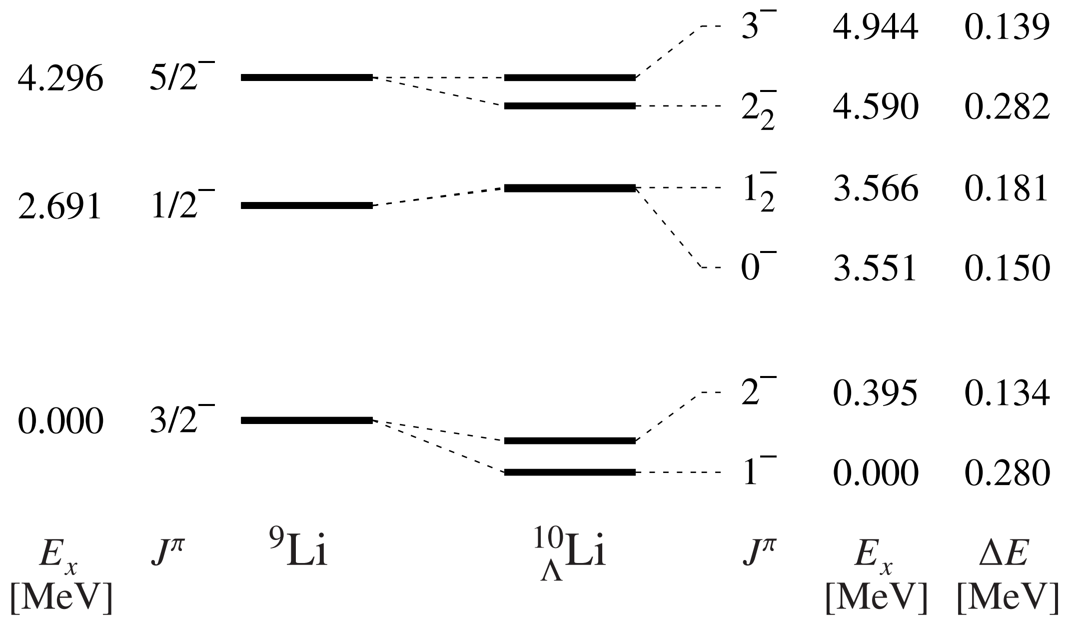
In order to check our shell-model calculation, we compare our numerical results for ${}^{7}_{\Lambda}$Li and ${}^{11}_{\Lambda}$B to the Millener's work. D. J. Millener, Lect. Notes Phys. 724, 31 (2007).

We confirm that our calculations fully reproduce the Millener's results.

The energy shift ΔE for ${}^{10}_{\Lambda}$Li is about 3 times larger than that for ${}^{7}_{\Lambda}$Li or ${}^{11}_{\Lambda}$B.

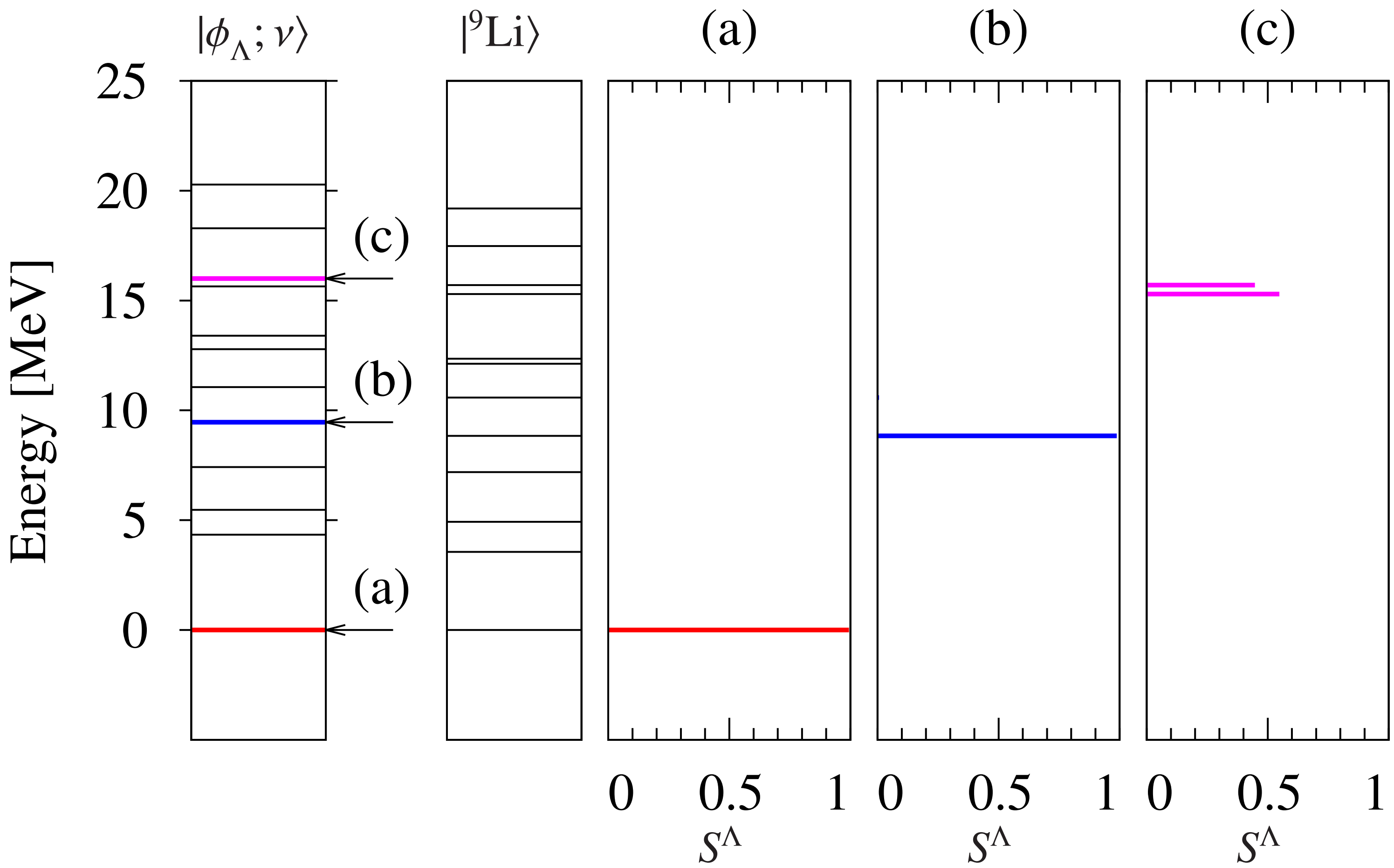
Numerical Results : Energy shifts and Σ-mixng probabilities

J^π	E_x[MeV]	ΔE[MeV]	P_Σ[%]	P_{Σ^-}[%]	P_{Σ^0}[%]	P_{Σ^+}[%]
$1^-_{\mathrm{g.s.}}$	0.000	0.280	0.345	0.183	0.159	0.002
2^-	0.395	0.134	0.166	0.096	0.070	0.000
0^-	3.551	0.150	0.185	0.098	0.086	0.001
1^-_2	3.566	0.181	0.227	0.128	0.098	0.001
2^-_2	4.590	0.282	0.350	0.187	0.162	0.001
3^-	4.944	0.139	0.175	0.104	0.071	–



Numerical Results : Energy spectra for Λ eigenstates

$({}^{10}_{\Lambda}\mathrm{Li}; T = \frac{3}{2}, J^{\pi} = 1^{-})$



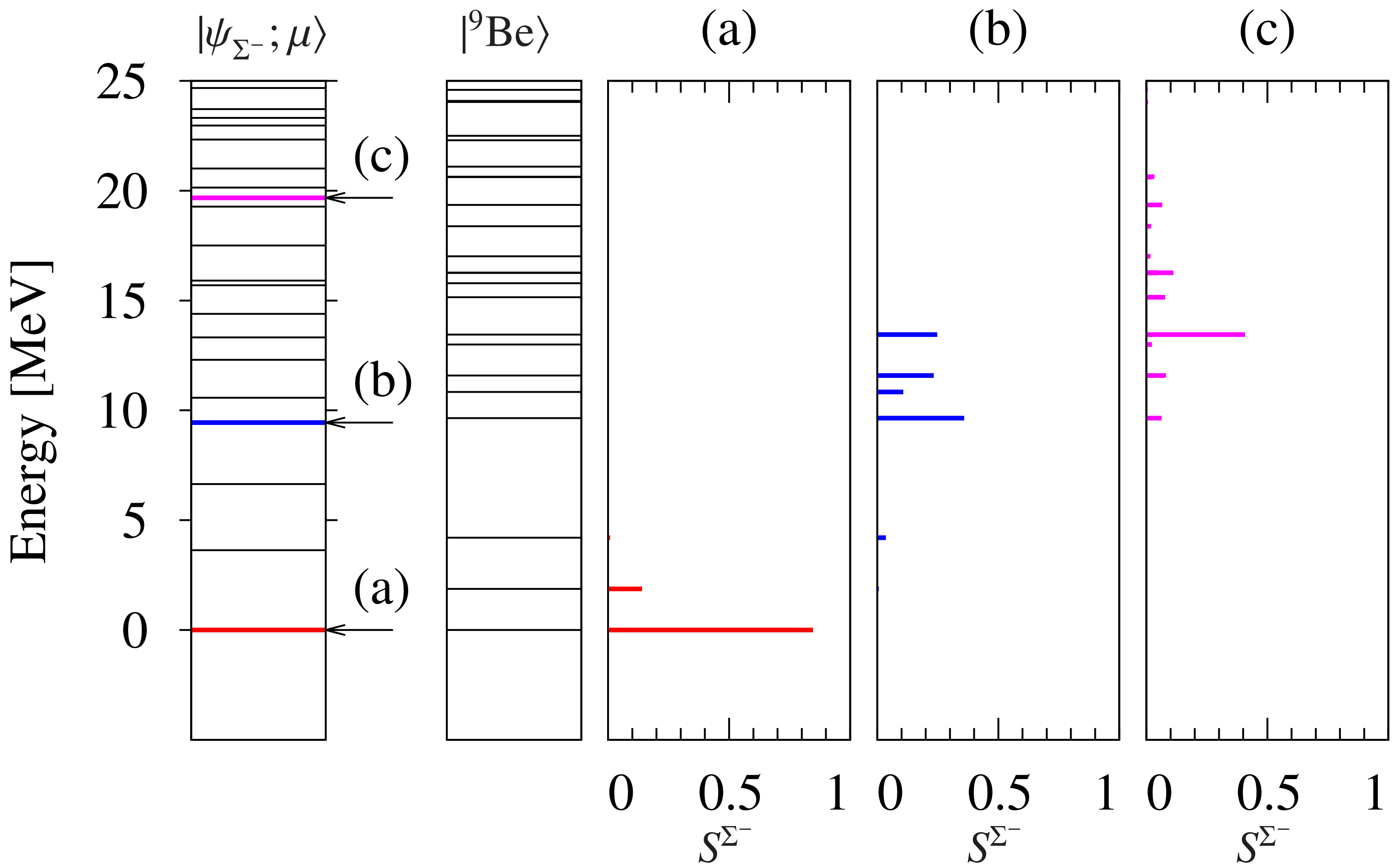
The energy spacings between the levels of ${}^{10}_{\Lambda}$Li are very similar to those of ^{9}Li.

The ^{9}Li core state is slightly changed by the addition of the Λ hyperon.

The Λ hyperon behaves as the single-particle motion in the nucleus because the ΛN interaction is rather weak.

Numerical Results : Energy spectra for Σ eigenstates

$({}^{10}_{\Lambda}\mathrm{Li}; T = \frac{3}{2}, J^{\pi} = 1^{-})$



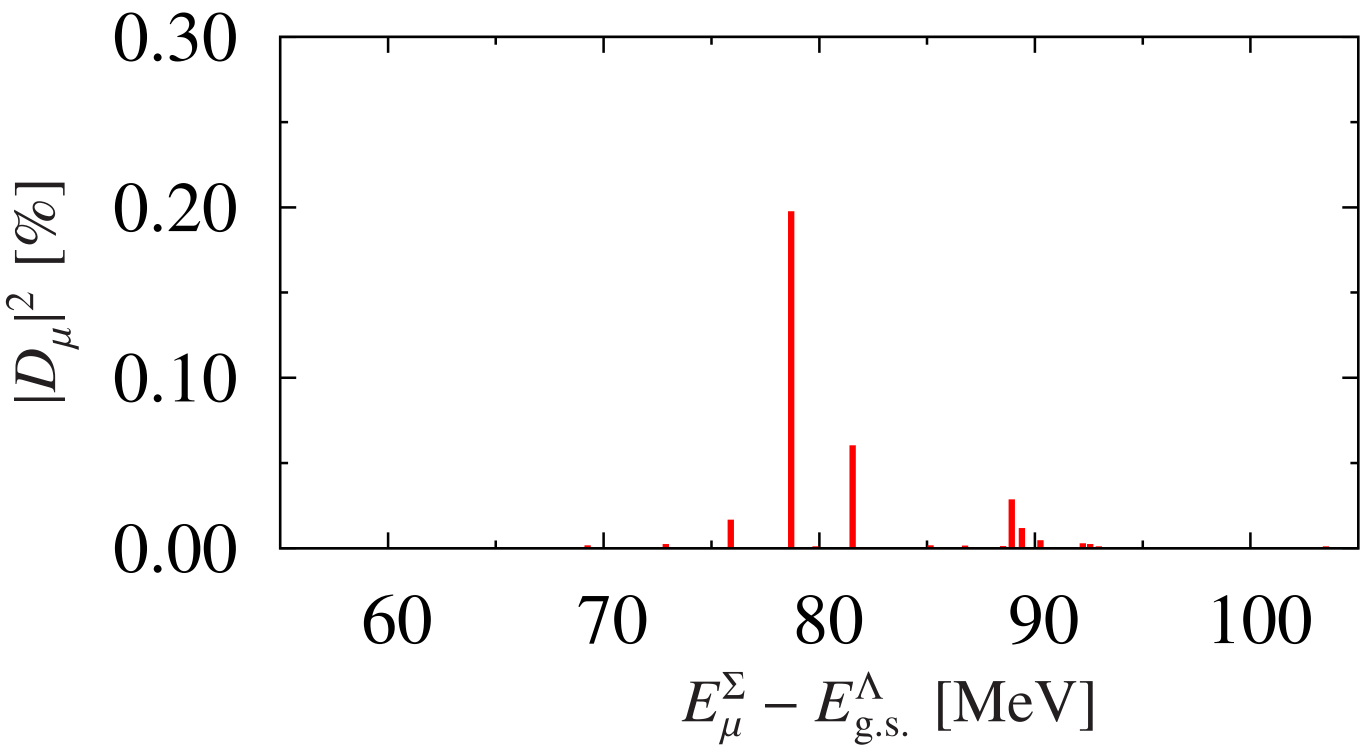
The core nucleus is ^{9}Be with $T = 1/2, 3/2, 5/2$ and $J^{\pi} = 1/2^-, 3/2^-$.

The distributions of S_{Σ^-} for excited states, (b) and (c), spread widely with the multi-configuration of ${}^9\mathrm{Be}^*$.

This implies that the Σ hyperon has the ability of largely changing the core-nuclear configuration.

Numerical Results : Λ-Σ-coupling strengths

$$|D_{\mu'}|^2 = \left| \frac{\langle \psi_\nu^\Lambda; TJ | V_{\Lambda\Sigma} | \psi_{\mu'}^\Sigma; TJ \rangle}{E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda} \right|^2$$



A contribution of the Σ ground state $|\psi_{\text{g.s.}}^\Sigma\rangle$ to the Σ-mixing of the ground state of ${}^{10}_{\Lambda}\text{Li}$ is reduced to 0.002 %.

The several Σ excited states in the $E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda \approx 80$ MeV region considerably contribute to the Σ-mixing.

These contributions are coherently enhanced by the configuration mixing. ← the Σ*N* interaction

It is shown that the nature of the Σ-nuclear states plays an important role in the Λ-Σ coupling.

Discussion

When a Λ state in $^{10}_{\Lambda}$Li converts to a Σ^- state by the Λ-Σ coupling interaction, the ^{9}Li core state changes into the ^{9}Be core state. In other words, the β^--transition, $^{9}\mathrm{Li} \to {}^{9}\mathrm{Be}$, occurs in the core-nuclear state.

It is interesting to consider the β transitions between the core-nuclear components of Λ and Σ eigenstates to investigate the strength distribution of the Λ-Σ coupling.

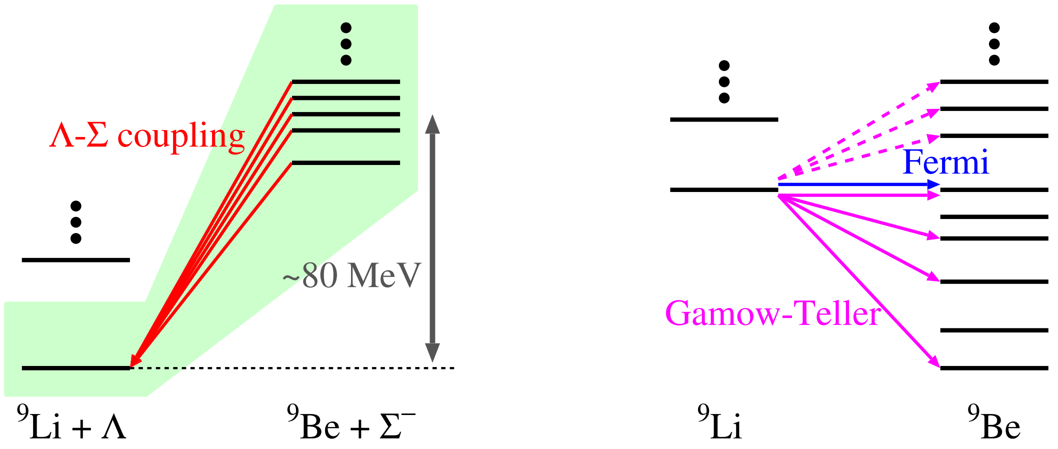
Λ-Σ coupling interaction

$$V_{\Sigma\Lambda} \simeq \underbrace{V_{\mathrm{F}}(r)\boldsymbol{t}_N \cdot \boldsymbol{\phi}_{\Sigma\Lambda}}_{V^{\mathrm{F}}_{\Sigma\Lambda}} + \underbrace{V_{\mathrm{GT}}(r)\,(\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma}_{\Sigma\Lambda})\,\boldsymbol{t}_N \cdot \boldsymbol{\phi}_{\Sigma\Lambda}}_{V^{\mathrm{GT}}_{\Sigma\Lambda}},$$

$V^{\mathrm{F}}_{\Sigma\Lambda}$: Fermi-type coupling (${\boldsymbol{t}}_N$: Fermi β-transition operator for nucleons)

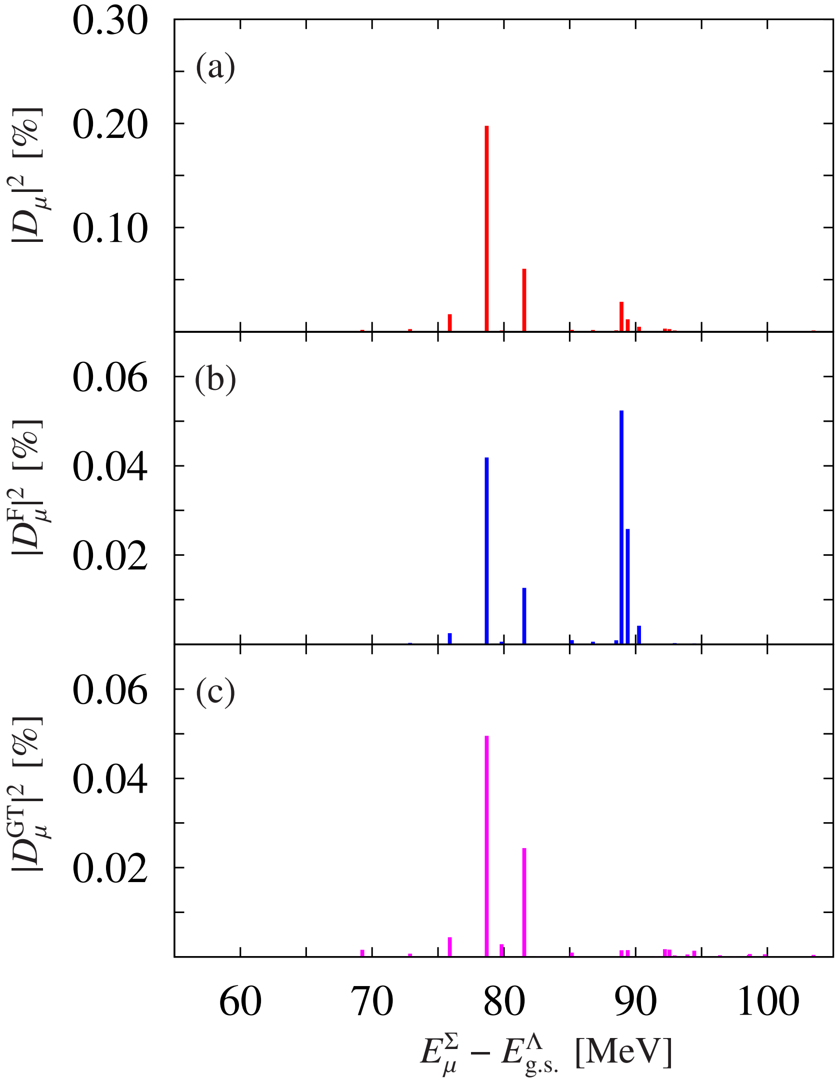
$V^{\mathrm{GT}}_{\Sigma\Lambda}$: Gamow-Teller-type coupling ($\boldsymbol{\sigma}_N \boldsymbol{t}_N$: Gamow-Teller β-transition operator for nucleons)

$|j_\Sigma\rangle = \boldsymbol{\phi}_{\Sigma\Lambda}|j_\Lambda\rangle$



Discussion

Fermi- and Gamow-Teller-type coupling strengths



$$|D_\mu|^2 = \left| \frac{\langle \psi_\Sigma;\mu | V_{\Sigma\Lambda} | \phi_\Lambda;\nu \rangle}{E_{\Lambda;\nu} - E_{\Sigma;\mu}} \right|^2 \rightarrow P_\Sigma = 0.345\ \%$$

$$|D_\mu^{\rm F+GT}|^2 = \left| \frac{\langle \psi_\Sigma;\mu | \left(V_{\Sigma\Lambda}^{\rm F} + V_{\Sigma\Lambda}^{\rm GT}\right) | \phi_\Lambda;\nu \rangle}{E_{\Lambda;\nu} - E_{\Sigma;\mu}} \right|^2 \rightarrow P_\Sigma^{\rm F+GT} = 0.350\ \%$$

$$|D_\mu^{\rm F}|^2 = \left| \frac{\langle \psi_\Sigma;\mu | V_{\Sigma\Lambda}^{\rm F} | \phi_\Lambda;\nu \rangle}{E_{\Lambda;\nu} - E_{\Sigma;\mu}} \right|^2 \rightarrow P_\Sigma^{\rm F} = 0.144\ \%$$

$$|D_\mu^{\rm GT}|^2 = \left| \frac{\langle \psi_\Sigma;\mu | V_{\Sigma\Lambda}^{\rm GT} | \phi_\Lambda;\nu \rangle}{E_{\Lambda;\nu} - E_{\Sigma;\mu}} \right|^2 \rightarrow P_\Sigma^{\rm GT} = 0.098\ \%$$

The Fermi and Gamow-Teller components coherently contribute to the Λ-Σ coupling strengths.

Discussion

Why is the energy shift for ${}^{10}_{\Lambda}$Li about 3 times larger than that for ${}^{7}_{\Lambda}$Li ?

The enhancement of the Σ-mixing probabilities in neutron-rich Λ-hypernuclei is mainly due to the Fermi-type coupling interaction $V^{\mathrm{F}}_{\Sigma\Lambda}$.

Fermi-type coupling

We use the weak-coupling limit for simplicity.

$$\rightarrow \langle V^{\mathrm{F}}_{\Sigma\Lambda} \rangle = \langle T_N{=}T\, J_N, j_\Sigma; T\, J \| V^{\mathrm{F}}_{\Sigma\Lambda} \| T_N{=}T\, J_N, j_\Lambda; T\, J \rangle$$

The Λ and Σ states must have the same core-nuclear state.

$$\Downarrow$$

$$\langle V^{\mathrm{F}}_{\Sigma\Lambda} \rangle = \bar{V} \sqrt{\frac{4T\,(T+1)}{3}},$$

$$|D^{\mathrm{F}}_{\mu'}|^2 \propto T\,(T+1)$$

	T	J^π	P^{F}_Σ [%]	P^{GT}_Σ [%]	$P^{\mathrm{F+GT}}_\Sigma$ [%]	P_Σ [%]
${}^{10}_{\Lambda}$Li	3/2	1^-	0.144	0.098	0.350	0.345
${}^{7}_{\Lambda}$Li	0	$1/2^+$	0.000	0.088	0.088	0.098

Discussion

Gamow-Teller-type coupling

In the Gamow-Teller transitions for ordinary nuclei, the Ikeda sum rule is well known as a model independent formula.

$$\sum B(\mathrm{GT}-) - \sum B(\mathrm{GT}+) = 3\,(N - Z)$$

K. Ikeda, S. Fujii, J. I. Fujita, Phys. Lett. 3, 271 (1963).

$B(\mathrm{GT}-)$: a strength of the Gamow-Teller β^--transition, $|^A Z\rangle \rightarrow |^A Z{+}1\rangle$
$B(\mathrm{GT}+)$: a strength of the Gamow-Teller β^+-transition, $|^A Z\rangle \rightarrow |^A Z{-}1\rangle$

$\boxed{\text{In general, } \sum B(\mathrm{GT}+) \text{ becomes smaller as neutron-excess grows larger.}}$

$$\begin{aligned} N \gg Z \;\rightarrow\; & \textstyle\sum B(\mathrm{GT}+) \approx 0 \\ \rightarrow\; & \textstyle\sum B(\mathrm{GT}-) \approx 3\,(N - Z) \end{aligned}$$

The Gamow-Teller-type coupling is very important in Λ-hypernuclei with large neutron excess.

Discussion

Energy shifts and Σ-mixing probabilities for ${}^{A}_{\Lambda}\mathrm{Li}$ hypernuclei

A	N	T	J	P_Σ [%]	ΔE [MeV]	ΔE^{Mil} [MeV]	P_Σ^{F} [%]	P_Σ^{GT} [%]	$P_\Sigma^{\mathrm{F+GT}}$ [%]
6	2	1/2	1^-	0.022	0.019		0.028	0.060	0.017
7	3	0	$1/2^+$	0.098	0.085	0.078	0.000	0.088	0.088
8	4	1/2	1^-	0.172	0.139		0.032	0.066	0.168
9	5	1	$3/2^+$	0.211	0.172		0.077	0.099	0.206
10	6	3/2	1^-	0.345	0.280		0.144	0.098	0.350
11	7	2	$1/2^+$	0.523	0.426		0.229	0.118	0.490
12	8	5/2	1^-	0.653	0.522		0.326	0.143	0.645

Summary and conclusion

We have investigated the structure of the neutron-rich ${}^{10}_{\Lambda}$Li hypernucleus, in shell-model calculations considering the Λ-Σ coupling in the perturbation theory. We have found that the Σ-mixing probabilities and the energy shifts of ${}^{10}_{\Lambda}$Li eigenstates are coherently enhanced by the Λ-Σ coupling configurations in the neutron-rich nucleus. We have argued the effects of the Λ-Σ coupling interaction in terms of the β-transitions for the core-nuclear states. The reasons why the Σ-mixing probabilities are enhanced are summarized as follows:

1. The multi-configuration Σ excited states can be strongly coupled with the Λ ground state with the help of the ΣN interaction.
2. These strong Λ-Σ couplings are coherently enhanced by the Fermi- and Gamow-Teller-type coupling components.
3. The Fermi-type coupling becomes more effective in the neutron-rich environment increasing as $T(T+1)$.

In conclusion, we have found that the Σ-mixing probability is about 0.35 % and the energy shift is about 0.28 MeV for the neutron-rich ${}^{10}_{\Lambda}$Li $1^{-}_{\mathrm{g.s.}}$ ground state, which is about 3 times larger than that for ${}^{7}_{\Lambda}$Li.