The Nucleon Structure

Bogdan Wojtsekhowski, Hall A

- Nucleon Constituents
- Electromagnetic Form Factors
- Neutron GEN experiment
- Flavor Form Factors
- IMF densities
- Large $Q^2$ program
Experimental study of structure

The beam and the target are required for study
e.g. an electron beam and atomic electrons

How one can study the quark-quark "potential"?
Create "an internal beam" inside the nucleon
Virtual photon absorption used to accelerate quark

Very productive investigations are DIS, FFs, DVCS, ...

At large $Q^2$ FF is a measure of q–q correlations at short distances
3-d picture of the nucleon in IMF

Proton form factors, transverse charge & current densities

Correlated quark momentum and helicity distributions in transverse space - GPDs

Structure functions, quark longitudinal momentum & helicity distributions
GPDs of the nucleon

Ji, Muller, Radyushkin (1994-1997)

\[ \gamma^* Q^2 \quad \gamma \]
\[ k \quad k' \]
\[ x - \xi \quad x + \xi \]

\[ \begin{array}{c}
\text{GPDs} \\
p \quad \text{N} \\
p' \quad \text{N}
\end{array} \]

where \( \xi = \frac{(p_q^+ - p_q'^+)}{(p_q^+ + p_q'^+)} \)

Quark dynamics of the nucleon are encoded in GPDs functions
\[ H(x, \xi, t), \tilde{H}(x, \xi, t) \] hadron helicity-conserving;
\[ E(x, \xi, t), \tilde{E}(x, \xi, t) \] helicity-flipping distributions.
GPDs information

Reduction formulas at $\xi = t = 0$
for DIS and $\xi = 0$ for FFs

$$H^q(x, \xi = 0, t = 0) = q(x)$$

$$\tilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x)$$

$$\int_{-1}^{+1} dx \ H^q(x, 0, Q^2) = F_1^q(Q^2)$$

$$\int_{-1}^{+1} dx \ E^q(x, 0, Q^2) = F_2^q(Q^2)$$

Ji’s sum rule for quark orbital momentum

$$\langle L_v^q \rangle = \frac{1}{2} \int_0^1 dx [x E_v^q(x, \xi = 0, t = 0) + x q_v(x) - \Delta q_v(x)]$$

DVCS will access low $t$, large $Q^2$ kinematics

FFs presently are the main source for $E_v^q$
Unpolarized and Polarized Structure functions

Figure 7: The polarized structure function $g_1^p$ as function of $Q^2$ in intervals of $x$. The error bars shown are the statistical and systematic uncertainties added in quadrature. The data are well described by our QCD NLO curves (solid lines), ISET=3, and its fully correlated 1σ error bands calculated by Gaussian error propagation (shaded area). The values of $C(x)$ are given in parentheses. Also shown are the QCD NLO curves obtained by AAC (dashed lines) [15] and GRSV (dashed-dotted lines) [16] for comparison.
Now the focus is Form Factors
Lepton-Nucleon scattering

\[ l(k, h) + N(p, \lambda_N) \rightarrow l(k', h') + N(p' \lambda'_N) \]

\( h, h', \lambda_N, \) and \( \lambda'_N \) are helicities

\[ P = \frac{p + p'}{2}, \quad K = \frac{k + k'}{2}, \quad q = k - k' = p' - p \]

\[ s = (p + k)^2, \quad t = q^2 = -Q^2, \quad u = (p - k')^2 \]

\[ T_{\lambda'_N, \lambda_N}^{h', h} \equiv \langle k', h'; p', \lambda'_N | T | k, h; p, \lambda_N \rangle \]

Total 16 amplitudes.
Parity invariance \( \rightarrow \) number of independent helicity amplitudes from 16 to 8.
Time reversal invariance \( \rightarrow \) to 6.
When neglect the lepton mass \( \rightarrow \) to 3.

\[ T_{+, +}^{+, +}; \quad T_{-, -}^{+, +}; \quad T_{-, +}^{+, +} = T_{+, -}^{+, +} \]

which are functions of \( (s - u) \) and \( t \).
Dirac, Pauli and Sachs Form Factors

Hadron current, one-photon approximation, $\alpha_{em} = 1/137$, Rosenbluth, 1950

$$\mathcal{J}_{\text{hadron}}^\mu = ie\bar{N}(p_f) \left[ \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2M} F_2(Q^2) \right] N(p_i)$$

Cross section and asymmetry for electron-nucleon scattering

$$d\sigma = d\sigma_{NS} \left\{ \varepsilon(G_E)^2 + \tau(G_M)^2 \right\} \cdot [1 + h_e A(G_E, G_M)]$$

$$A = A_\perp + A_\parallel = \frac{a \cdot G_E G_M \sin \theta^* \cos \phi^*}{G_E^2 + c \cdot G_M^2} + \frac{b \cdot G_M^2 \cos \theta^*}{G_E^2 + c \cdot G_M^2}$$

Sachs, 1962

Does a nucleon have a core?

$$G_E = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \quad G_M = F_1(Q^2) + F_2(Q^2)$$

$$J_{fi} = 2E \cdot F(-\vec{q}^2), \quad \vec{J} = 0 \quad \rho(r) = \frac{1}{(2\pi)^3} \int F(-\vec{q}^2) e^{i\vec{q}\vec{r}} d^3\vec{q}$$
The mechanism of electron-nucleon scattering

Generalized Parton Distributions

\[ F_1(Q^2) = \sum_q \int H_q(x, Q^2)dx \]

\[ F_2(Q^2) = \sum_q \int E_q(x, Q^2)dx \]

Vector Meson Dominance

\[ V = \rho, \rho', \omega... \]

Two-gluon exchange (with OAM)

\[ F_2/F_1 \propto \frac{1}{Q^2} \ln^2(Q^2/\Lambda^2) \]
Kelly’s Parameterization

\[ G(Q^2) = \sum_{k=0}^{n=1} a_k \tau^k / (1 + \sum_{k=1}^{n+2=3} b_k \tau^k) \]

scaling constraint: \( Q \to \infty, G \sim Q^{-4} \)
Duality constrained parameterization

\[ \xi^{p,n} = \frac{2}{1 + \sqrt{1 + \tau^{-1}_{p,n}}} \]

constrains: at \( \xi \to 1 \)

\[
G_{BABB} = A(\xi) \times G_{Kelly}(Q^2)
\]

1) \[
\frac{G_{Mn}^2}{G_{Mp}^2} = \frac{1+4(d/u)}{4+(d/u)}
\]

2) \[
\frac{G_{En}^2}{G_{Mn}^2} = \frac{G_{Ep}^2}{G_{Mp}^2}
\]
Now the focus is GEn
Why we study the neutron Charge Form Factor?

- Test of the QCD motivated FF models is a powerful approach to the understanding of confinement
- Charge density is a fundamental property of the neutron
- Flavor separated FFs are a productive test of lattice QCD
- Unique constraint on the model of GPDs $E_u$ and $E_d$
- Dirac/Pauli density for up and down quarks and its connection to the Siver’s effect
- Applications e.g. for the neutrino-nuclei cross section
Concept of High $Q^2 \, G_E^n$ experiment

◆ **Optimization** of the large-acceptance high-luminosity $G_E^n$ experiment:
  - a polarized $^3\text{He}$ target (re-use E94-010 target)
  - a dipole magnet for electron arm (re-use BigBite from NIKHEF)
  - a matching neutron detector (re-use UVa and CMU bars)
  - a trigger with a calorimeter (re-use E99-114 electronics)

◆ **A key idea**: focus on higher $Q^2$
  at 2-3 GeV$^2$ there is $G_E^p/G_M^p$ effect, 3q-state dominance at high $Q^2$
  also $Q^2 > 2$ GeV$^2$ Glauber method becomes sufficiently accurate

◆ **Target Figure-of-Merit**: $J_{\text{beam}}^{\text{max}} \, t_{\text{target}} \, P_{\text{target}}^2$
  Electron-polarized neutron luminosity and high polarization of $^3\text{He}$
  target make the measurement about **10 times** more effective than
  with ND$_3$ polarized target.

◆ **Productivity of experiment** – target FoM in combination with a large
  acceptance electron spectrometer: the total enhancement is more
  than **100**, which is a key to reaching $Q^2$=3.5 GeV$^2$
Conceptual setup of E02-013

\[ A_{phys} = A_\perp + A_\parallel = \frac{a \cdot G_E G_M \sin \theta^* \cos \phi^*}{G_E^2 + c \cdot G_M^2} + \frac{b \cdot G_M^2 \cos \theta^*}{G_E^2 + c \cdot G_M^2} \]

\[ ^3\text{He}(\vec{e}, e'n) \]
Conceptual setup of EO2-013

\[ A_{\text{phys}} = A_\perp + A_\| = \frac{a \cdot G_E G_M \sin \theta^* \cos \phi^*}{G_E^2 + c \cdot G_M^2} + \frac{b \cdot G_M^2 \cos \theta^*}{G_E^2 + c \cdot G_M^2} \]

\[ \theta^* \sim 90^\circ \]

\[ A_{\text{phys}} \propto G_E^n \]

\[ p_\perp = |(p_n^* - \vec{q}) \perp \vec{q}| \]

Selection of Quasi-Elastic by cut \( p_\perp < 150 \text{ MeV} \)
Hall A $G_E^n$ experiment

Beam

Electron arm

Target

Neutron arm

$^3\vec{He}(\vec{e}, e'\vec{n})$
Hall A $G_E^n$ experiment

Beam

Laser light

Polarized beam

Target chamber

Recoiled neutron

Scattered electron

Rb + K

Pumping chamber

Polarization pumping

$J_{\text{polarized nuclei}} \times P^2_{\text{nuclei}}$

$G_E^n$

$^3\text{He}(\bar{e}, e' n)$
Hall A $G_E^n$ experiment

- Solid angle of 76 msr (12 times higher than HRS)
- 40 cm long target
- Momentum resolution of 1%
Electron Spectrometer

Optimization of acceptance: for $\Delta Q^2/Q^2 \sim 0.1$ with max $\Omega$ leads to a large aspect ratio, limited by $\cos(\phi^*)$ for asymmetry. BigBite was designed at NIKHEF for aspect ratio $\Delta \theta/\Delta \phi = 1/5!$. Spectrometer has solid angle (for 40 cm long target) 76 msr.

Detector:

15 planes of MWDCs ($X,U,V$)
Two-layer lead-glass calorimeter
Segmented Scint. Plane
"world’s largest" dedicated Neutron Detector
- Match BigBite solid angle for Quasi Elastic kinematics
- Flight distance ~ 11 m
- Operation at $3 \cdot 10^{37}$ cm$^2$/s
- Neutron efficiency 35%
- Time resolution 0.35 ns
Considerations/Optimization

\[ ^3He(e, e'n) \]

- Range of the momentum transfer should be < 10%
  => angular acceptance < 10°
- Asymmetry vs. polarization direction, q-vector, and the e,e’ plane
  => azimuth coverage < 60°
- Electron arm resolution requirement => 1%
- Neutron arm rate limitations => luminosity
- MWDC rate limitations => luminosity
- Trigger and DAQ capabilities => calorimeter
- Field gradient at the target < 1 mT/cm => “sheet-metal” dipole

Configuration of E02·13 is close to ideal for a Form Factor experiment
Data analysis: step 1 - Time-of-Flight

Raw events (BLACK lines) have significant accidental level and large tail for slower protons

RED lines present events after cut on e’-n angular correlation: accidentals and tails almost gone
Analysis: step 2 - $q_\perp$ vs W; 1.7 GeV$^2$

perpendicular “q” = q $\times$ sin($\theta_{qh}$); $W^2 = M^2 + 2M(E-E') - Q^2$

Quasi elastic events dominates after Full Cuts applied

max value of used perp. q
Analysis: step 3 – W distribution

for 3.5 GeV$^2$ the quasi-elastic signal is very small in e,e$'$ spectrum. However, after angular correlation cut applied the peak is just as supposed to be for quasi-elastic process.
The pQCD log-scaling provided a good fit to the $G_E^n/G_M^n$ data.

- Is pQCD log-scaling good fit for the $G_E^n/G_M^n$ from 1.5 GeV$^2$?
- How large $Q^2$ will be a limit for Constituent Quark Model?
The semi-final results E02-013

- The pQCD log-scaling should wait much larger $Q^2$
- Constituent Quark Model doesn’t work above 2 GeV$^2$
- The $q(qq)$, ANL model is only one in agreement with the data
Now return to structure
(e,e′) ⇒ Nuclear Charge Distributions

Model-independent analysis -> accurate nuclear charge distributions
Study of nucleon structure requires IMF GPDs in the impact parameter representation

\[ F_1(t) = \sum_q e_q \int dx H_q(x, t) \]

Muller, Ji, Radyushkin

\[ q(x, b) = \int \frac{d^2q}{(2\pi)^2} e^{i \mathbf{q} \cdot \mathbf{b}} H_q(x, t = -q^2) \]

M.Burkardt

\[ \rho(b) \equiv \sum_q e_q \int dx \ q(x, b) = \int d^2q \ F_1(q^2) e^{i \mathbf{q} \cdot \mathbf{b}} \]

P.Kroll: u/d segregation

\[ \rho(b) = \int_0^\infty \frac{Q \cdot dQ}{2\pi} J_0(Qb) \frac{G_E(Q^2) + \tau G_M(Q^2)}{1 + \tau} \]

G.Miller

center of momentum \( R_\perp = \sum_i x_i \cdot r_\perp,i \)

Transverse center of the quarks longitudinal momentum fractions

\( b \) is defined relative to \( R_\perp \)
QCD: Dirac and Pauli densities

The impact parameter $b$ is defined relatively to the transverse center of the quarks longitudinal momentum fractions $R = \sum x_i r_i$. The transverse density $\rho_T(b)$ can be expressed as:

$$\rho_T(b) = \rho_{\text{Dirac}} - \sin(\phi_b - \phi_S) \rho_{\text{Pauli}}$$

The Dirac and Pauli densities are given by:

$$\rho_{\text{Dirac}}(b) = \int_0^{\infty} \frac{QdQ}{2\pi} J_0(bQ) F_1(Q^2)$$

$$\rho_{\text{Pauli}}(b) = \int_0^{\infty} \frac{Q^2dQ}{4\pi M} J_1(bQ) F_2(Q^2)$$

Direct implications for physics of the Semi Inclusive Deep Inelastic Scattering $(e,e'\pi)$:

- Separation of $u/d$ in transversely polarized neutron (along $X$) in high $Q^2$ events.
Polarized neutron

SIDIS should have many effects due to this u/d separation
Rotation of u/d quarks in neutron

Let see how quark rotation leads to u/d separation:

amplitude is small

virtual photon  quark

amplitude is large

motion inside nucleon

M.Burkardt (2003)
Rotation of u/d quarks in neutron

Let see how quark rotation leads to u/d separation:

M. Burkardt (2003)

amplitude is small

virtual photon \quad \text{quark}

amplitude is large

Interaction selects one side because of rotation
Rotation of u/d quarks in neutron
The u/d separation, observed in Form Factor data, is possibly a result of the collective rotation of the u-quark and the d-quark, which is going in opposite directions.
Flavor view with EMFFs
F_1^{d(2)}/F_1^{u(2)} \text{ with proton and neutron FFs}

F_1 = \frac{G_E + \tau G_M}{1 + \tau}
F_2 = -\frac{G_E - G_M}{1 + \tau}
F_1^u = 2 F_{1p} + F_{1n}
F_1^d = 2 F_{1n} + F_{1p}

Lattice calculation => very good agreement with the trend, need accuracy
q(qq) ANL => good, possibly a signature of dominant degrees of freedom
Our data will require a new fit of E_d and E_u GPDs
$F_{1d}^{(2)}, F_{1u}^{(2)}$ with proton and neutron FFs
Why need higher $Q^2$?
The semi-final results E02-013

pQCD with orbital angular momentum

What is happening at higher $Q^2$?
Additional advance in polarized target and the Super BigBite components (the magnet and a high-resolution high-rate capability GEM tracker) are required to extend experiment to 10 GeV².
Beam energy of 8.8 GeV, 60 $\mu$A. Target: He-3, polarization 60%, 36 days

$G_E^n$ at 10 GeV$^2$ with uncertainty of 20% * $G_{\text{Galster}}$ (or 0.07 $G_{\text{Dipole}}$).
12 GeV approved GEn experiments
Super BigBite apparatus
Proton $G_E^p/G_M^p$ with SBS

Proton form factors ratio, $E_{12} - 07 - 109$

Proton Arm
Electron Arm

Beam
Target

BNL
BigBen

INFN
GEM

48D48

NSF
SciFi

FNAL

DOE / Dubna

HCalo

Lead–glass Calorimeter
Single settings of SBS will provide full coverage for all $P_\perp < \frac{1}{6} P_\parallel$
SBS physics program

- **GEP**: reach unique high 15 (GeV/c)^2 - productivity!
- **GMN**: reach absolute max 18 (GeV/c)^2
- **GEN**: reach fantastic value 10 (GeV/c)^2
- **SSA in nSIDIS**: 30,000 gain vs HERMES

=================================

- **A1n/d2n** – productivity gain ~ 20-30 compare with SHMS
- **F_π**: new approach with a polarimeter for the forward going neutron will allow to perform L/T separation without Rosenbluth
- **H(e,e’ϕ)p** – access to gluon at JLab

=================================

- **A1p/d2p** – as for A1n has a very large gain of productivity
- **D(e,e’d)** – elastic A, event rate gain ~ 50 at 6 (GeV/c)^2
- **T/³He(e,e’)** – u/d at high-x
- **SRC**: e’(HRS) + p(SBS) + N(BB)
- **PVDIS** – gain 10-15 compare with two HRSs
- **A(e,e’p), A(e,e’π⁺⁻)** - each item is a big program