New Muon Lifetime and the Fermi Coupling Constant $G_F$

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HallC meeting, JLAB, Aug 10, 2007
Precision Muon Physics

“Of all the recent muon experiments, the measurement of the muon anomalous magnetic moment, $a_\mu = (g_\mu - 2)/2$ by experiment E821 at BNL stands out both experimentally and theoretically. “

“That measurement was statistics limited and could be further improved. Indeed, an upgrade proposal by the collaboration outlines a plan to reduce the error in $a_\mu$ by a about factor of 2.”
Precision Muon Physics

MUONS

\( \mu \equiv \frac{g_\mu - 2}{2} \)

Michel parm \( \rho, \delta, \eta, P_\mu \xi \)

Lifetime \( G_F \) (1 ppm)

\( \mu^- \rho \) capture \( \Lambda_S \rightarrow g_P \)

\( \mu^- d \) capture \( L1A \)

\( g-2 \) SUSY, BSM

\( \mu A \rightarrow e A \) LFV, BSM

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MUON Physics

- Physical Review Letters, 20 July 2007, Volume 99, Number 3
  - Improved Measurement of the Positive-Muon Lifetime and Determination of the Fermi Constant
    D. B. Chitwood et al. (MuLan Collaboration)
    Published 16 July 2007
  - Measurement of the Muon Capture Rate in Hydrogen Gas and Determination of the Proton's Pseudoscalar Coupling $g_P$
    V. A. Andreev et al. (MuCap Collaboration)
    Published 16 July 2007 032002
  - Electroweak Radiative Corrections to Muon Capture
    Andrzej Czarnecki, William J. Marciano, and Alberto Sirlin
    Published 16 July 2007 032003
### Standard Model Electroweak Interaction

(4 parameters)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g'$</td>
<td>SU(1)</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>SU(2)</td>
<td></td>
</tr>
<tr>
<td>$M_H$ (Higgs)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>expectation value of the Higgs field</td>
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<td>$\alpha$</td>
<td>Fine structure constant</td>
<td>00.045 ppm</td>
</tr>
<tr>
<td>$G_F$</td>
<td>Fermi coupling constant</td>
<td>09.000 ppm</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>Mass of the Z boson</td>
<td>23.000 ppm</td>
</tr>
<tr>
<td>$M_H$</td>
<td>Higgs mass</td>
<td>constrained</td>
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$m_i$ masses of quarks and leptons (12)

CKM quark mass mixing matrix, [Cabbibo] Kobayashi-Maskawa (4)

MKS mass matrix for neutrinos, Maki-Nakagawa-Sakata matrix (4)

Cosmological Constant (1)

QCD Coupling (1)

26 dimensionless constants for standard model

- PDG, Particle Data Group
Electroweak Interaction

\[ L_F = - \frac{2 \sqrt{2} G_F}{4} \left( \bar{u}(\nu_e) \gamma^\alpha (1 - \gamma_5) u(e^+) \right) \left( \bar{u}(\bar{\nu}_\mu) \gamma^\beta (1 - \gamma_5) u(\mu^+) \right) \]

\[ L_W = - \frac{g^2}{8} \left( \bar{u}(\nu_e) \gamma^\alpha (1 - \gamma_5) u(e^+) \right) \frac{-g_{\alpha\beta} + \frac{k_\alpha k_\alpha}{M_W^2}}{k^2 - M_W^2} \left( \bar{u}(\bar{\nu}_\mu) \gamma^\beta (1 - \gamma_5) u(\mu^+) \right) \]

\[ L_W = - \frac{g^2}{8} \left( \bar{u}(\nu_e) \gamma^\alpha (1 - \gamma_5) u(e^+) \right) \frac{g_{\alpha\beta}}{M_W^2} \left( \bar{u}(\bar{\nu}_\mu) \gamma^\beta (1 - \gamma_5) u(\mu^+) \right) \]
The theoretical uncertainty on $G_F$ as extracted from the muon lifetime is $< 0.3$ ppm.
**Electroweak**

- Basic parameters [2006] (e.g. PDG)

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**Standard Model Electroweak Interaction**

\[
\frac{\delta G_F}{G_F} = -\frac{1}{2} \frac{\delta \tau}{\tau} - \frac{5}{2} \frac{\delta m_\mu}{m_\mu} - \frac{1}{2} \frac{\delta (1+q)}{(1+q)}
\]

9 ppm \quad 0.08 ppm \quad 0.3 ppm
MuLan Collaboration

University of Illinois at Urbana Champaign (UIUC)
Boston University (BU)

University of California, Berkeley (UCB and LBNL)
University of Kentucky, Lexington
TRIUMF, Vancouver
James Madison University, (JMU)
KVI Groningen, Netherlands
Istanbul Technical University, Turkey
The experimental concept

- Accumulation Period, $T_A$
- Measurement Period, $T_M$
- Background Level

- Kicker Transition

- Real data

- $-12.5 \text{ kV}$
- $12.5 \text{ kV}$

- Number (log scale)
Rates

• 1ppm requires at least $10^{12}$ observed decays

• Probability for a set of measurements $(t_1, t_2, \ldots)$

\[ dP(t_1, t_2 \ldots \mid \tau) = \rho(t_1 \mid \tau)\rho(t_2 \mid \tau) \ldots dt_1 dt_2 \ldots \]

• Probability that a parameter has a value based on $(t_1, t_2, \ldots)$

\[ L(\tau \mid t_1, t_2 \ldots) = \rho(t_1 \mid \tau)\rho(t_2 \mid \tau) \]
Rates and Uncertainty

- For a simple distribution one can find the value for \( \tau \) that make the experimental result highly probable and estimate the uncertainty. (same result from fitting)

\[
\frac{dL(\tau \mid t_1, t_2 \ldots)}{d\tau} = 0
\]

\[
\rho(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}}
\]

\[
\tau = \frac{t_1 + t_2 + \ldots}{N}
\]

\[
\Delta \tau = \left[ \int \frac{1}{\rho} \left( \frac{\partial \rho}{\partial \tau} \right)^2 dt \right]^{-\frac{1}{2}}
\]

\[
\Delta \tau = \frac{\tau}{\sqrt{N}}
\]
Uncertainty with Background

Assume measurement period of $10\tau$
Uncertainty

• Uncertainty is increased due to new terms in the distribution

• Lifetime is distorted due to an incorrect distribution
  • The average is no longer an unbiased estimator for the lifetime when there is background

• Reduce your uncertainty through statistics therefore an unbiased measurement of $t$ is critical

• Individual precise measurements are not critical

KNOW YOUR DISTRIBUTION

REMOVE SYSTEMTICS
Rates

- To accumulate the statistics necessary in a one month period
  - $4.0 \times 10^5$ muon decays/sec
  - 1 muon every 3 µs

- Need more than 1 muon in target at a given time.
  - Granularity (FAST approach)
  - Pulsed beam (µLAN Approach)
    - Duty factor $(35 \, \mu s - \text{total}) / (5 \, \mu s - \text{on})$ [factor of 7]
    - $3 \times 10^6$ muons/sec

Beam rate of a FEW MHz
MuLan Experiment at the Paul Scherrer Institut Villigen, Switzerland

Swiss Light Source Neutrons SINQ

1.8 mA cyclotron, 590 MeV protons
πe3 beamline

Surface Muon Beam 29 MeV/c

- Dedicated beam studies for several years
- A Beam Rate > 11 MHz;
- Spot at target, only few cm2.
- Separator leaves about 5–10% positron background.
- Many elements to tune. Straight beam is tricky.
- Extinction fraction looks reasonable. (1000)
Entrance Muon Counter

- 96 active wires on each plane
- 1 mm spacing
- reading every two wires
- resolution: 2 mm

- Logic pulses between 40 and 100 ns long
- FPGA for time and position hits
- Accumulation: Prescaling
- Measurement: No-prescaling
Quads
+10 kV
0
-10 kV
Kicker Plates
Kicker HV supply; 50 kHz cycle time

SSL E3 4.2 MeV
muon source

Subsystem #1

Subsystem #2

Subsystem #3

Simulations:
Tunes
GEANT 4
Systematics
Fitting

Subsystem #4

Subsystem #5

Subsystem #6

Online DAQ and Analysis Farm
500 MHz WFD Bank
LEDs
25 kV (+/- 12.5 on each plate) kick, 12 cm gap, 30 kHz: a gift from TRIUMF
1) Continuous vacuum
2) AK-3 target with rotation mechanism
3) Thin-walled vacuum tube
4) EMC - High-rate wire chamber
2-sided, soccer-ball geometry

Hex-house

Pent-house

Outer (smaller)

Inner (larger)

2 Analog Pulses

Pulse Area (outer)

Pulse Area (inner)