Moments of Structure Functions and Glue

Cynthia Keppel

Hampton University / Jefferson Lab

Hall C Summer Meeting 2005

Moments of the Structure Function

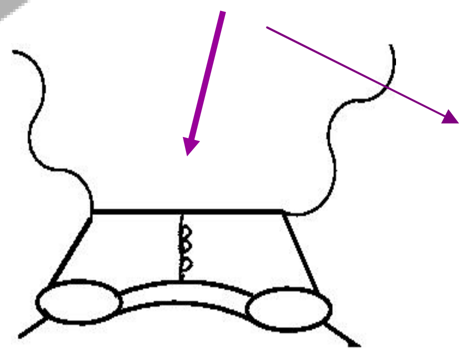
$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F(x,Q^2)$$

where $x = Q^2/2M\nu$

- Operator Product Expansion

$$M_n(Q^2) = \sum_{k=1}^{\infty} (nM_0^2/ Q^2)^{k-1} B_{nk}(Q^2)$$

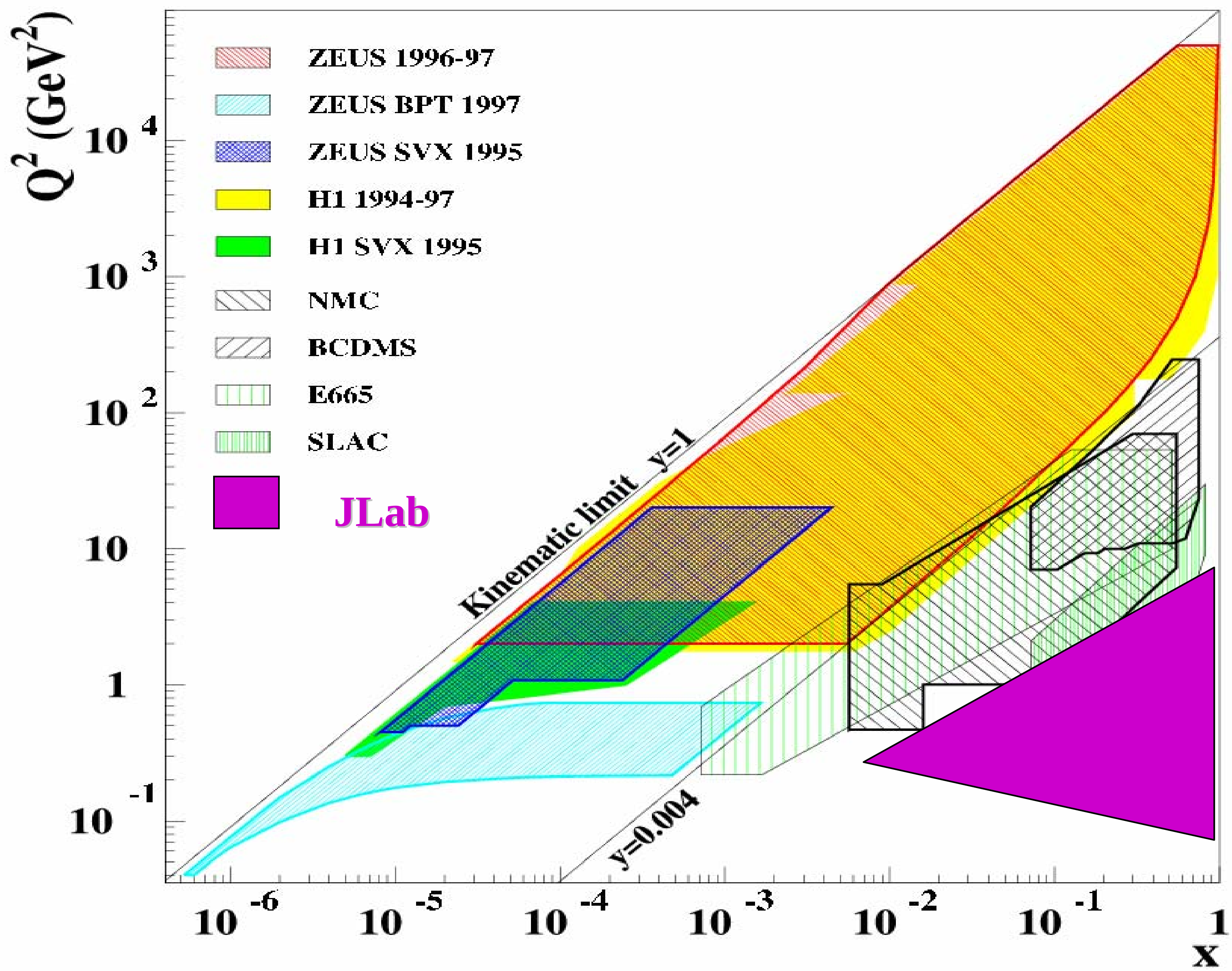
higher twist logarithmic (pQCD)



interactions between struck quark and other quarks in the target nucleon

- Large x dominates particularly the higher order moments
- Higher twist expected to become important at low Q^2

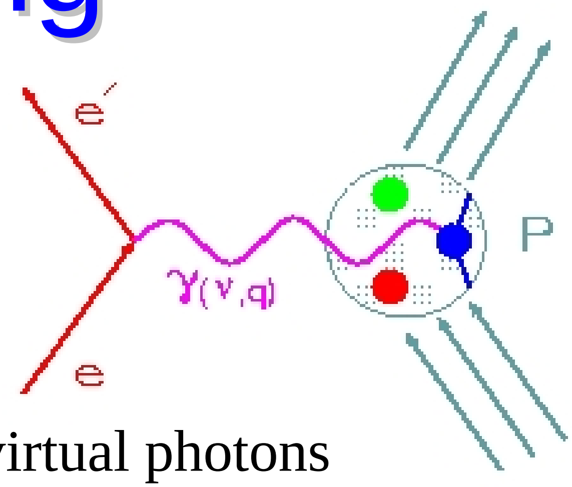
The Jefferson Lab Regime



Inclusive e + p → e + X Scattering

Rosenbluth:

$$\frac{d\sigma}{d\Omega dE'} = \Gamma(\sigma_T + \varepsilon\sigma_L)$$



Where: Γ = flux of transversely polarized virtual photons
ε = relative longitudinal polarization

Alternatively:

$$\frac{d\sigma}{d\Omega dE'} = \sigma_{mott}(F_2/\nu + 2F_1\tan^2(\theta/2)/M)$$

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{2xF_1}$$

$$F_L = (1 + \frac{4M^2x^2}{Q^2})F_2 - 2xF_1$$

longitudinal

TRANSVERSE

mIxEd

A Program of L/T Separated Structure Function Measurements in Hall C at Jefferson Lab

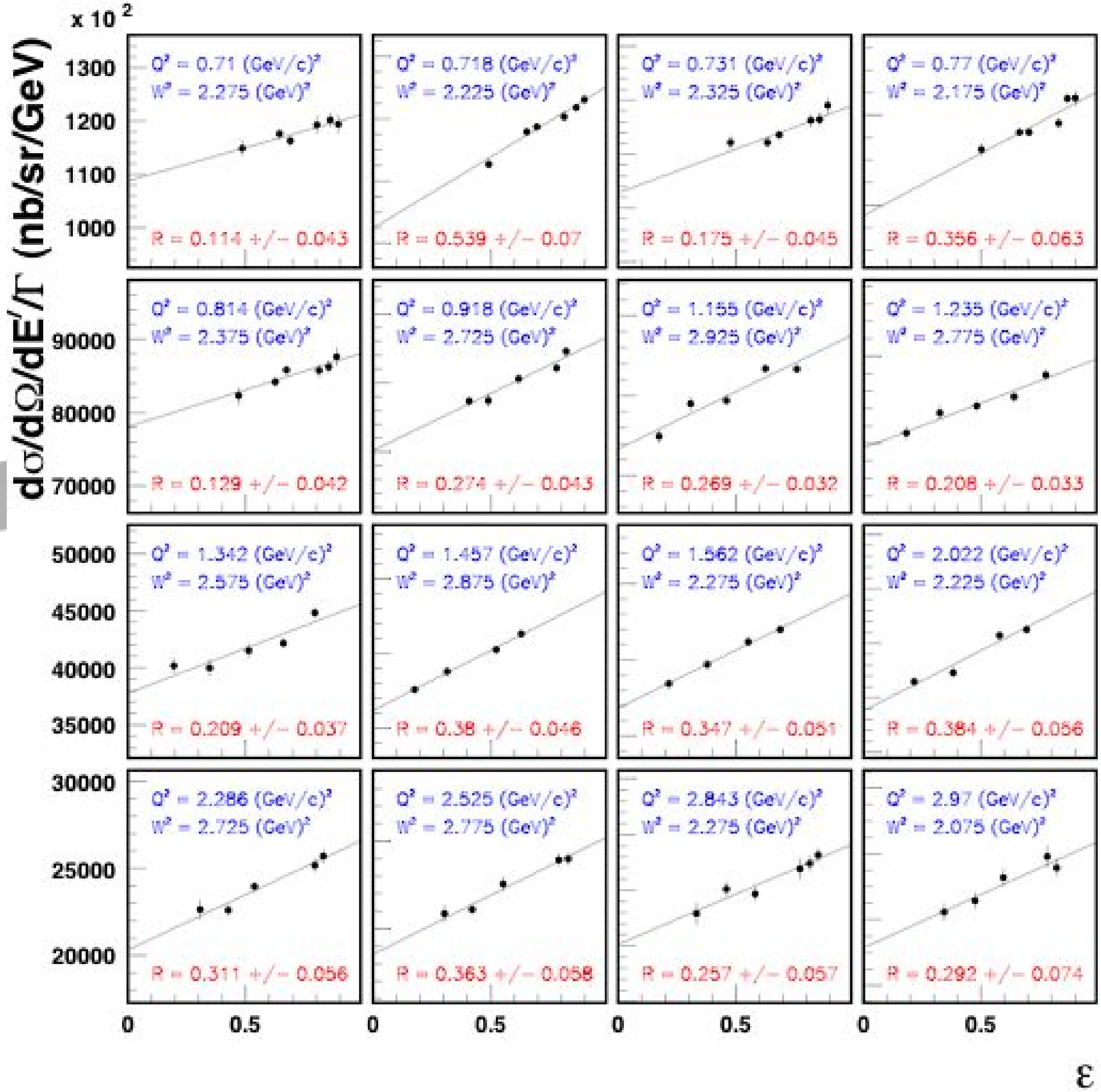
- E94-110: Hydrogen Resonance Region
- E99-118: Low x, Q^2 A-Dependence
- E00-002: Low Q^2 Deep Inelastic H, D
- E04-001: Nuclear Dependence, Neutrino Modeling
- E02-109: Deuterium Resonance Region, Neutron Structure Function Moments

Rosenbluth Separations (195 from E94-110 alone)

Need:

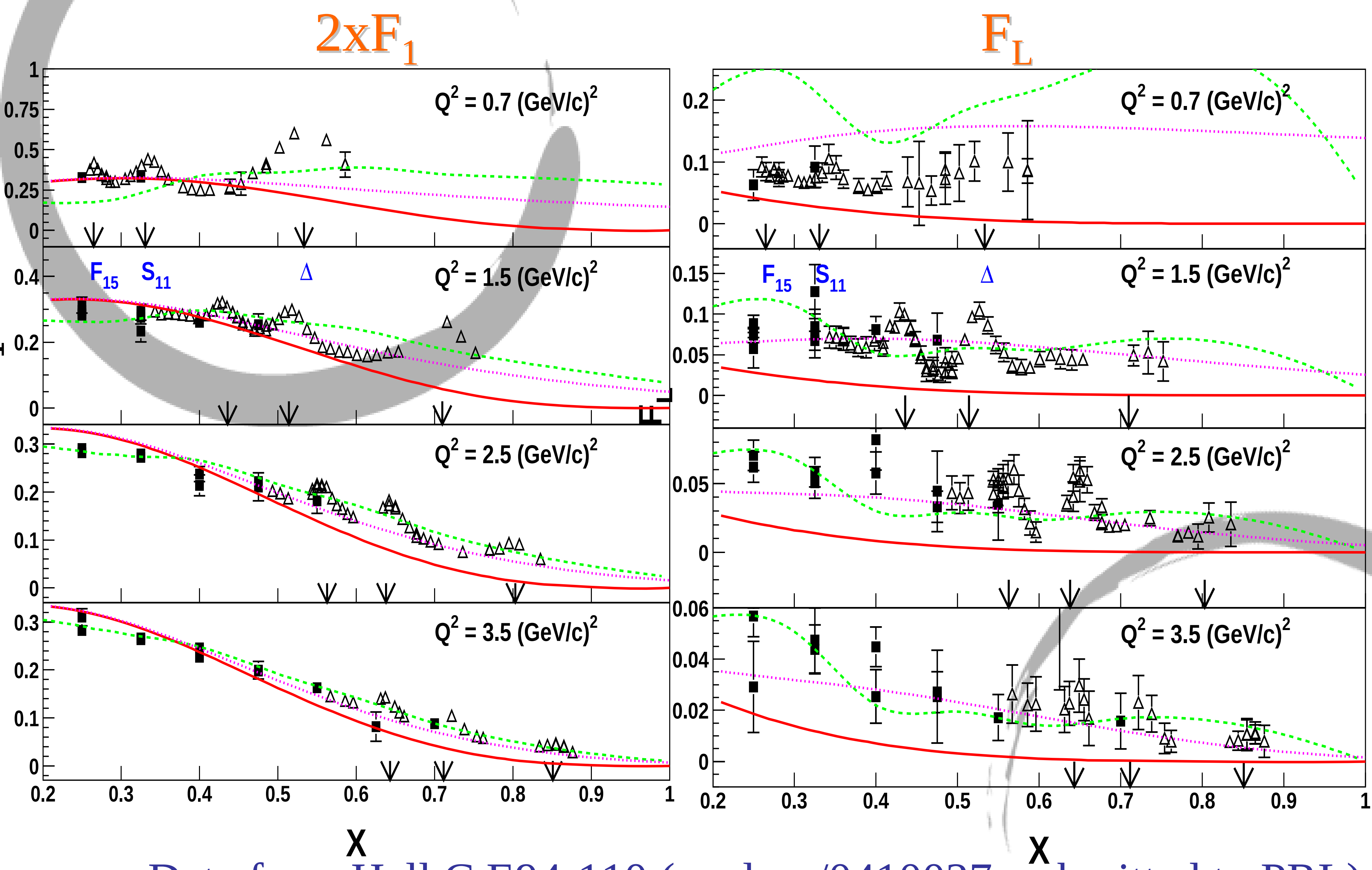
range in ε

small point-to-point systematic uncertainty!



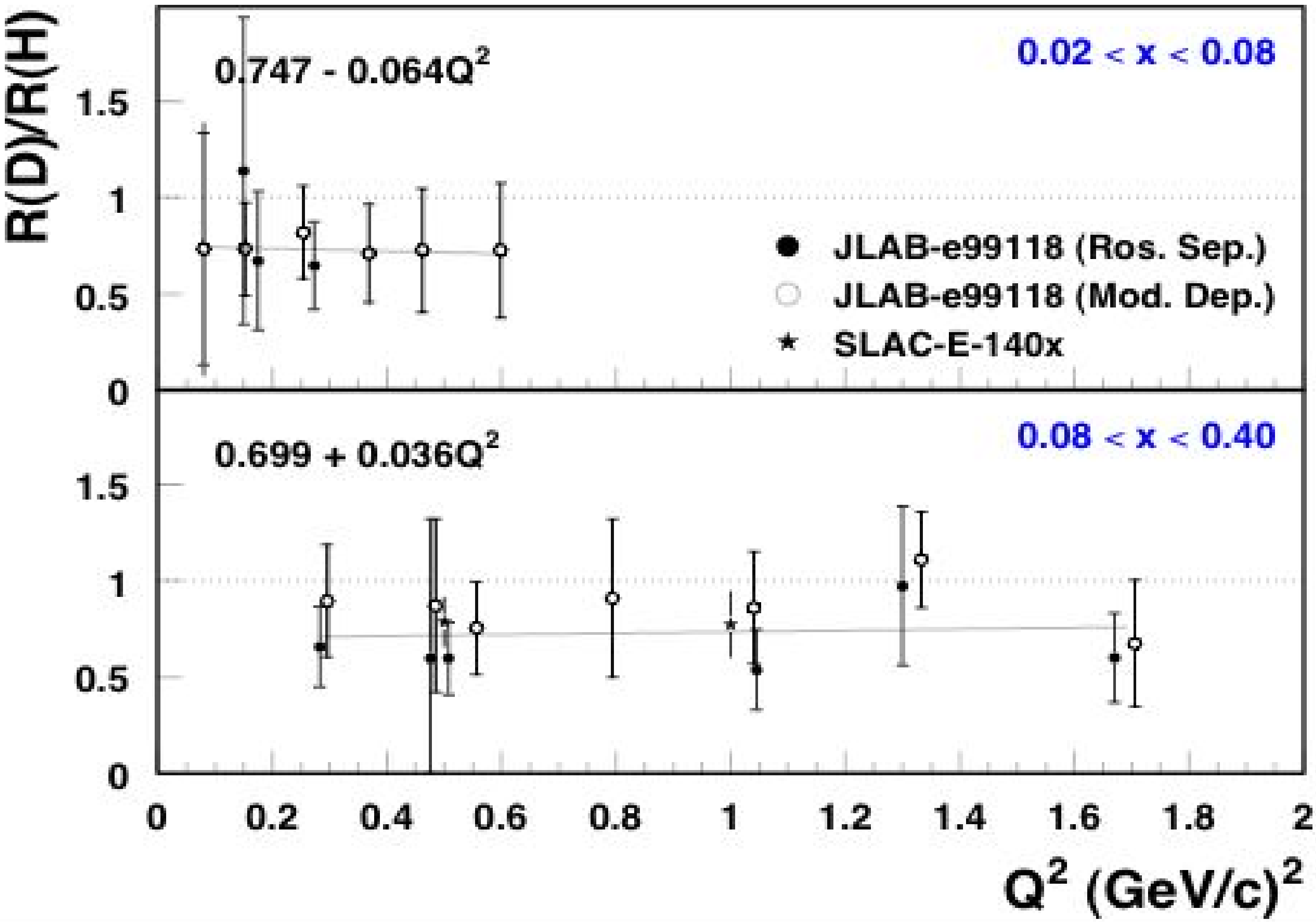
Precision Separated Structure Function Measurements

Alekhin NNLO MRST NNLO MRST NNLO + Target Mass



Data from Hall C E94-110 (nucl-ex/0410027, submitted to PRL)

Ratio R_D / R_H



Very limited previous data

$R_D \sim 0.7\, R_H$ (within *large* uncertainty)

The new data buys us, in particular:

- Ability to extract structure function moments
  - -Evaluate higher twist
  - -Compare to lattice QCD
  - -F_L yields G(x)
- Unique access to large x
  - -Reduce large pdf uncertainties

$$M^n(Q^2) = \int_0^1 dx\, x^{n-2} F(x,Q^2)$$

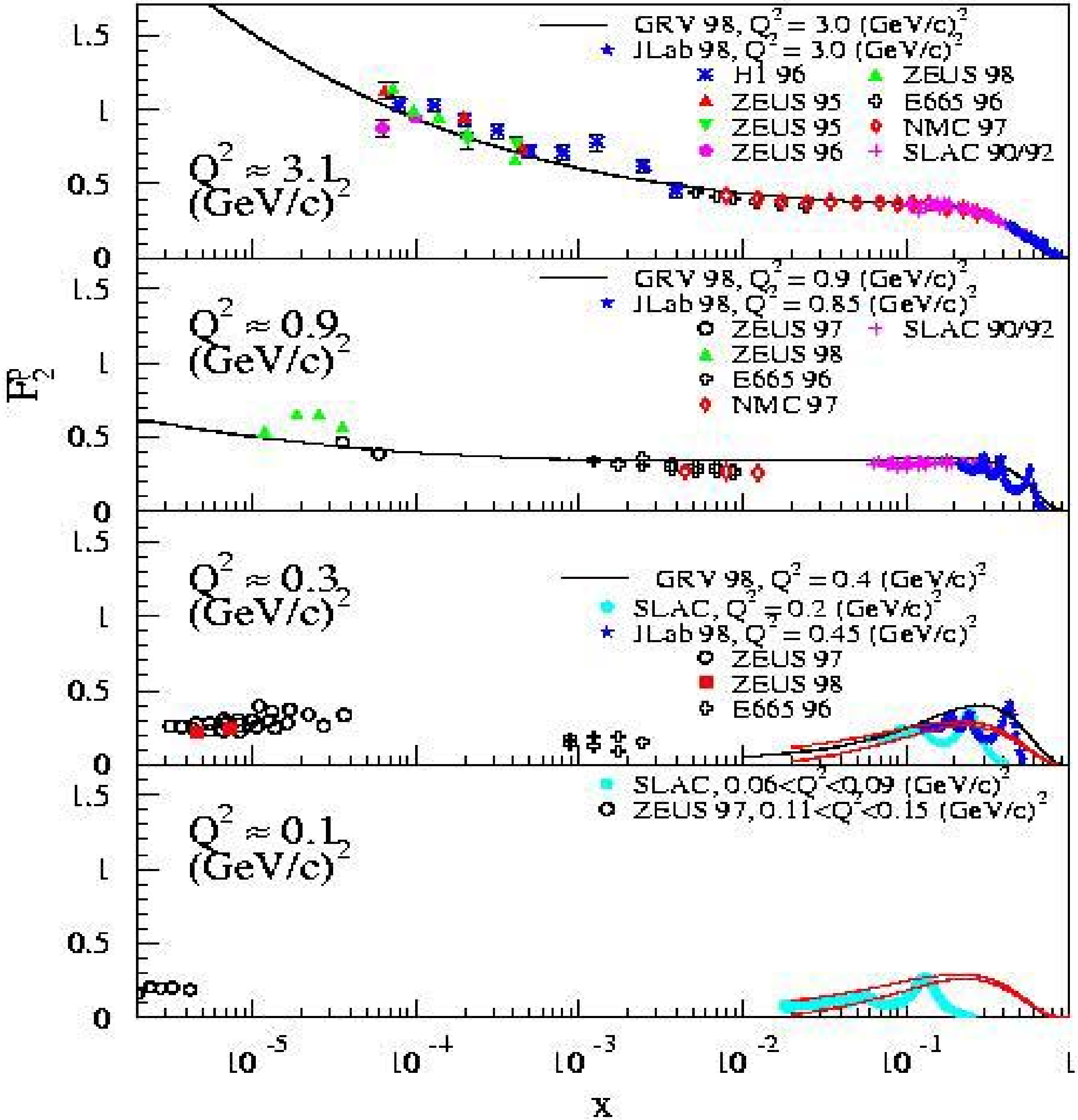
F_2

For moment extractions:

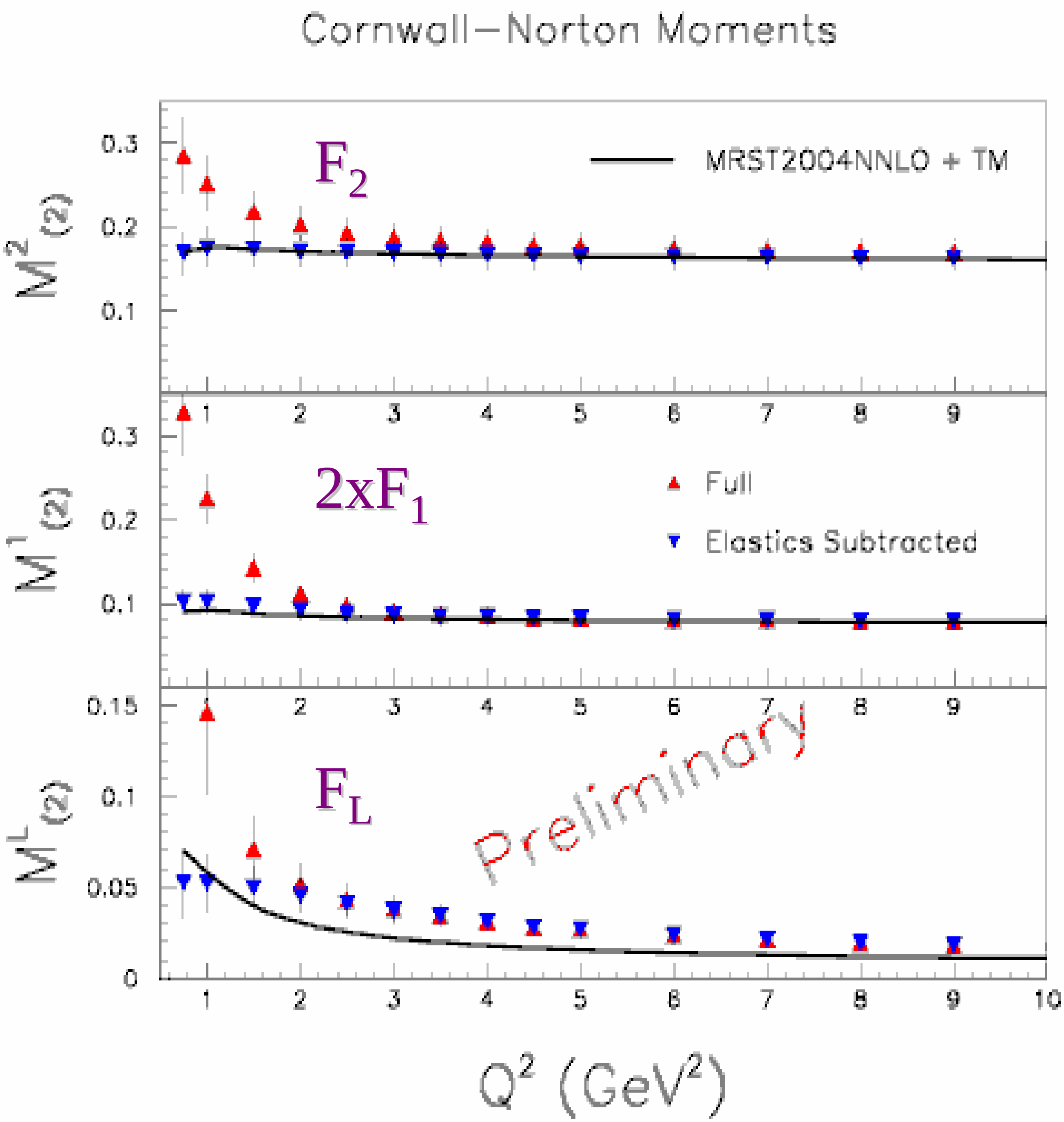
Need data covering *wide range* in x, at fixed Q^2

Large x increasingly important at large n

+ elastics……



n = 2 Cornwall-Norton Moments



F_2, F_1 in excellent agreement with NNLO + TM above $Q^2 = 2$ GeV^2

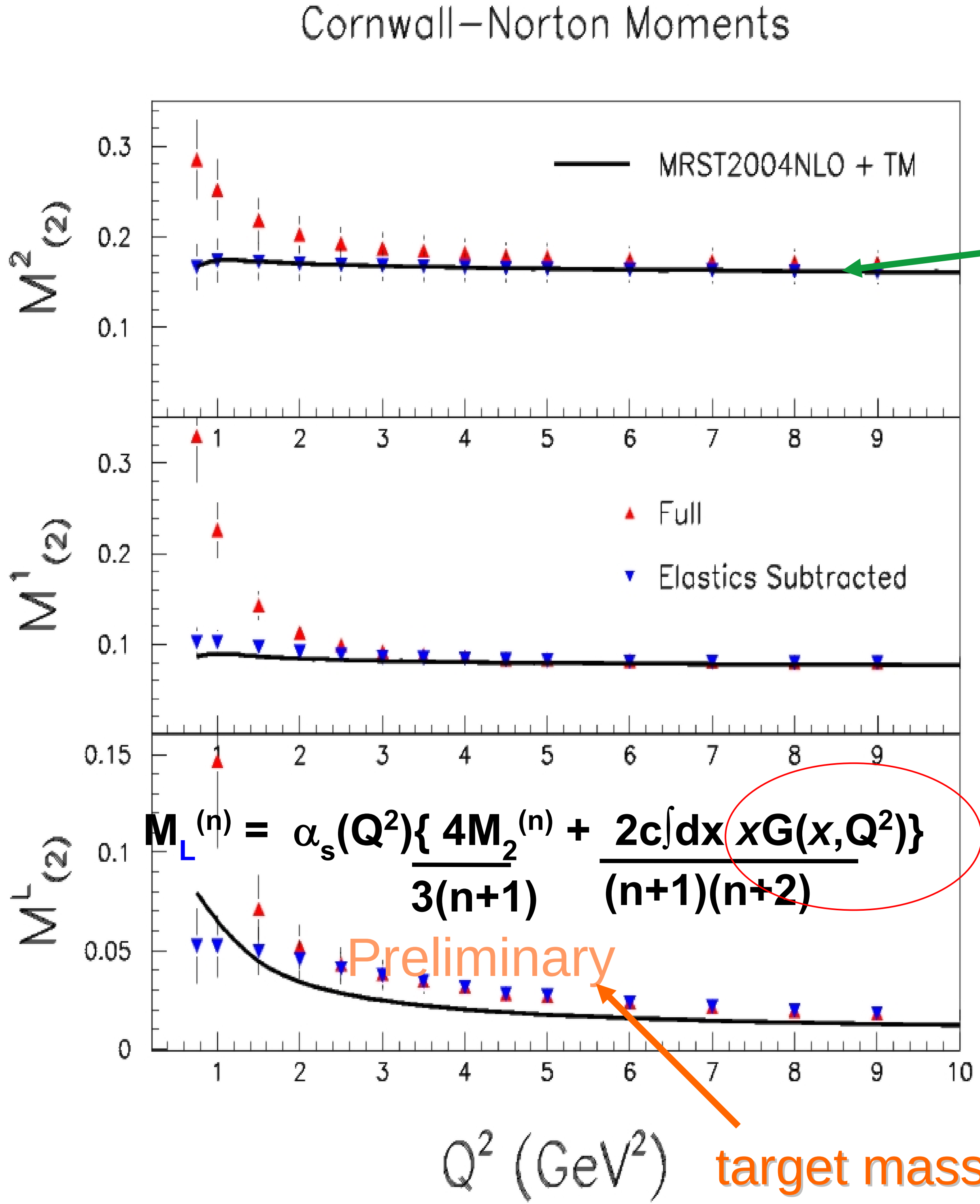
No (or canceling) higher twists

Yet, dominated by large x and resonance region

Remove known HT (a bit novel), the elastic, and there is no more down to $Q^2 = 0.5$ GeV2

The case looks different for F_L (data or curve?)

Momentum Sum Rule



For F_2, QPM gives: $(1/3)^2(0.17) + (2/3)^2(0.34) = 0.17$

~50% of momentum carried by quarks - the rest, assumably, by the glue

F_L gives a direct measurement of the glue

But, we get 0.70 (preliminary) - also different from pdfs

target mass, higher twist, NNLO,…..

Application of TMC to Unpolarized Structure Functions

Formalism (original form due to Georgi and Politzer, PRD 14, 7 (1976) 1829:

$$F_2(x,Q^2) = x^2k^3F(\xi) + 6M^2x^3k^4/\, Q^2)\int_{\xi}^{1} dx'F(x') + 12M^4x^4k^5/\, Q^4)\int_{\xi}^{1} dx''F(x'')$$

$$F_1(x,Q^2) = xk/2F(\xi) + M^2x^2k^3/\, Q^2)\int_{\xi}^{1} dx'F(x') + 2M^4x^3k^3/\, Q^4)\int_{\xi}^{1} dx''F(x'')$$

where $k = (1 + 4M^2x^2/Q^2)^{-1/2}$

In the limit $M^2/Q^2 \Rightarrow 0$, $F_1(x,Q^2) \Rightarrow x/2\, F_1^{\text{"bg"}}(x)$

$F_2(x,Q^2) \Rightarrow x^2F_2^{\text{"bg"}}(x)$

and $kF_2 - 2xF_1 = F_L \Rightarrow x^2(F_2^{\text{"bg"}} - F_1^{\text{"bg"}})$

$\Rightarrow$ Difference due to F_L, or Callan-Gross at all orders if no TM

"background function" (appears 3 times in each equation) - G&P call it a "quark distribution function", so pdf-based structure functions often utilized

In *original* form due to Georgi and Politzer:

$F_1^{bg} = F_2^{bg} \quad \Rightarrow \quad F_L = 0$

FL only due to target mass?
At all orders in α_s?

Clearly, not the case…..this is because they considered only twist-2 operators at LO.

However, the formalism works at *all orders* for twist-2

Assuming the background functions are the same is equivalent to applying target mass corrections at leading order only - possibly responsible for portion of large higher twist reported in literature

(HT operators can have different TMC)

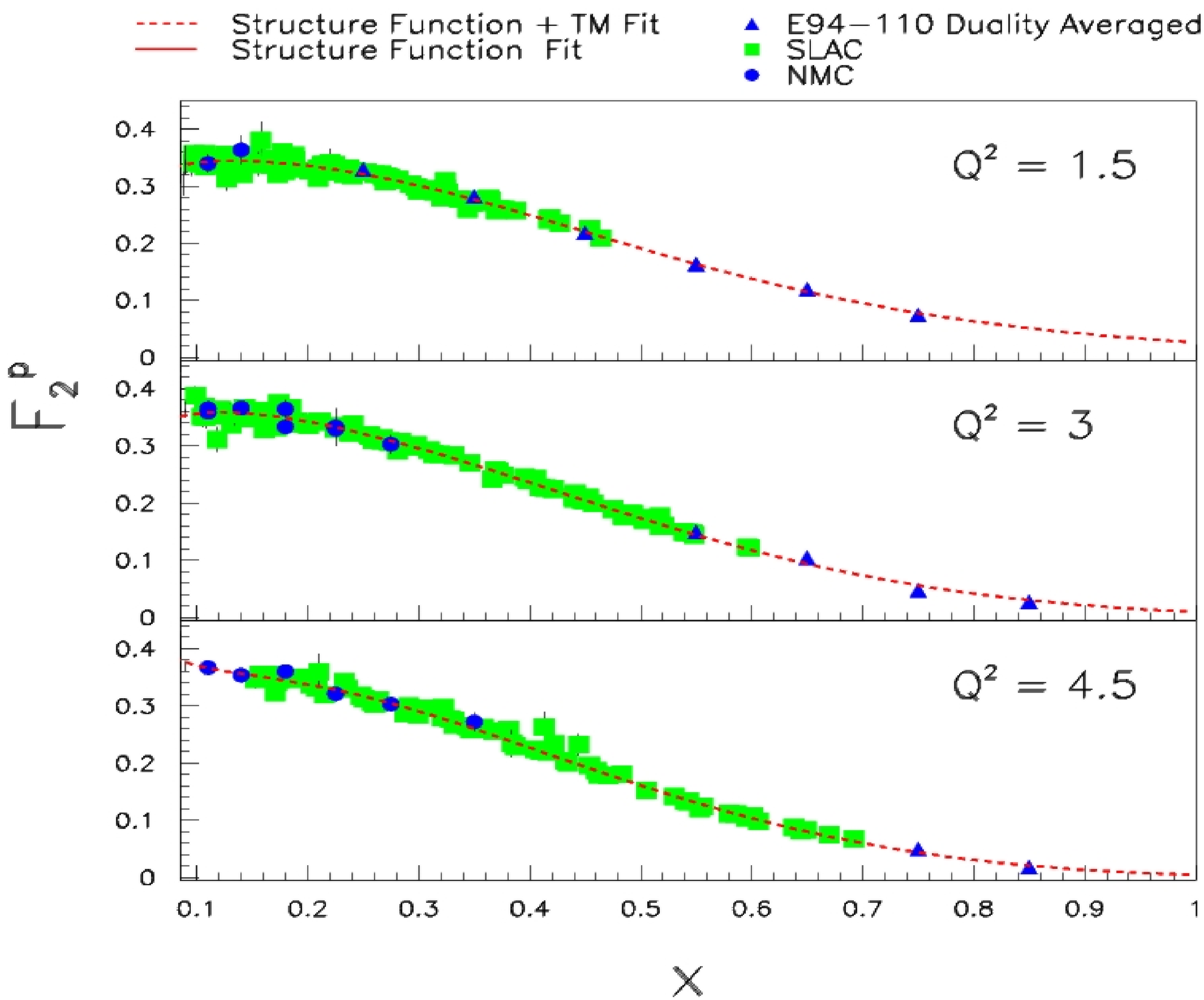
Q: How to determine $F_{1,2}^{bg}$ from data (to all orders!)?

A: Use paramaterization of $F_{1,2}^{bg}$ inserted into Eqs on previous slide to calculate $F_{1,2}$ and minimize with respect to data to determine parameters.

Current fit form is $F_{1,2}^{bg} = A\, x^{\alpha}(1-x)^{\beta}\,(1+Cx^{1/2} + Dx)$

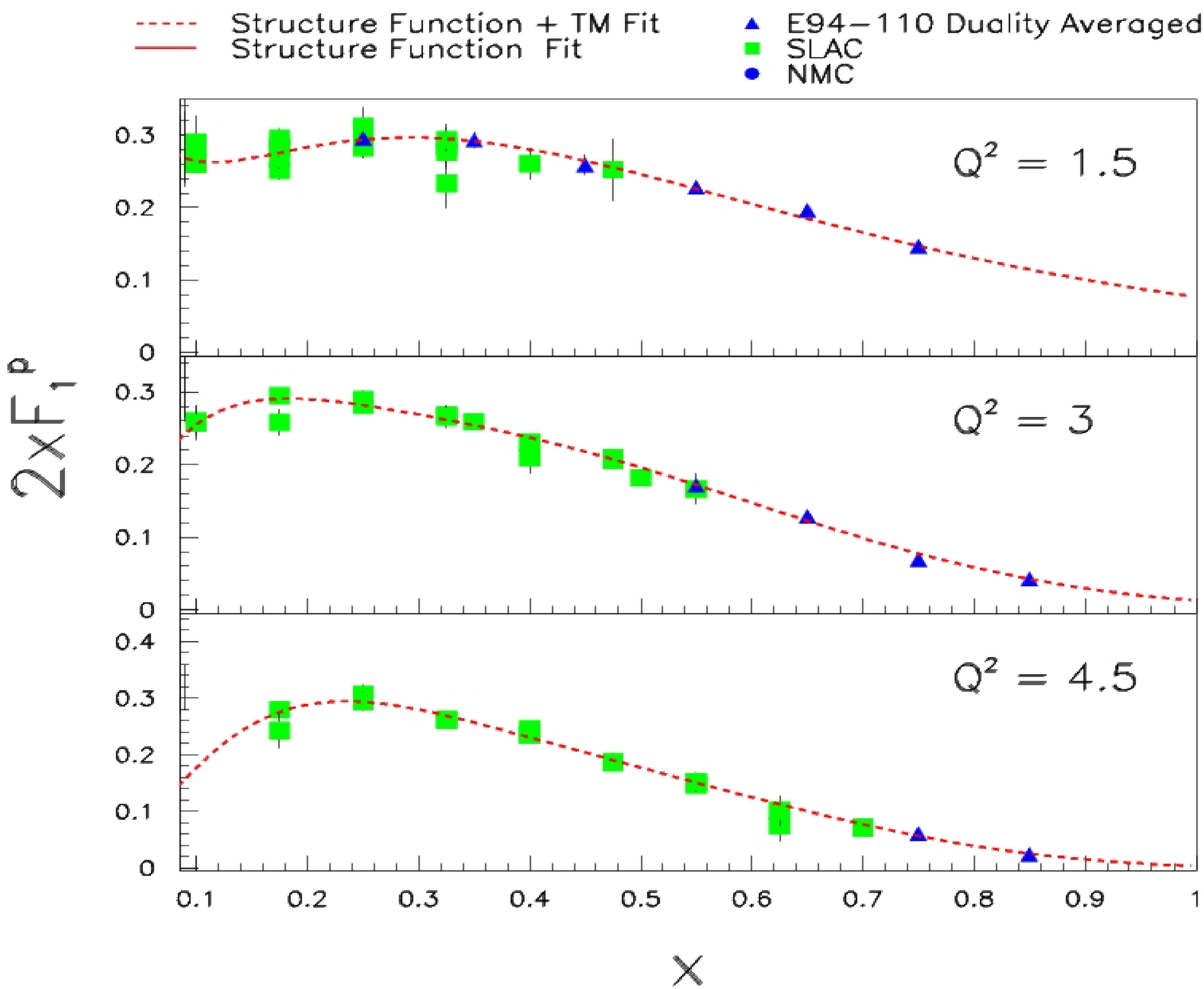
depends crucially on shape of large x data!

Fit Results



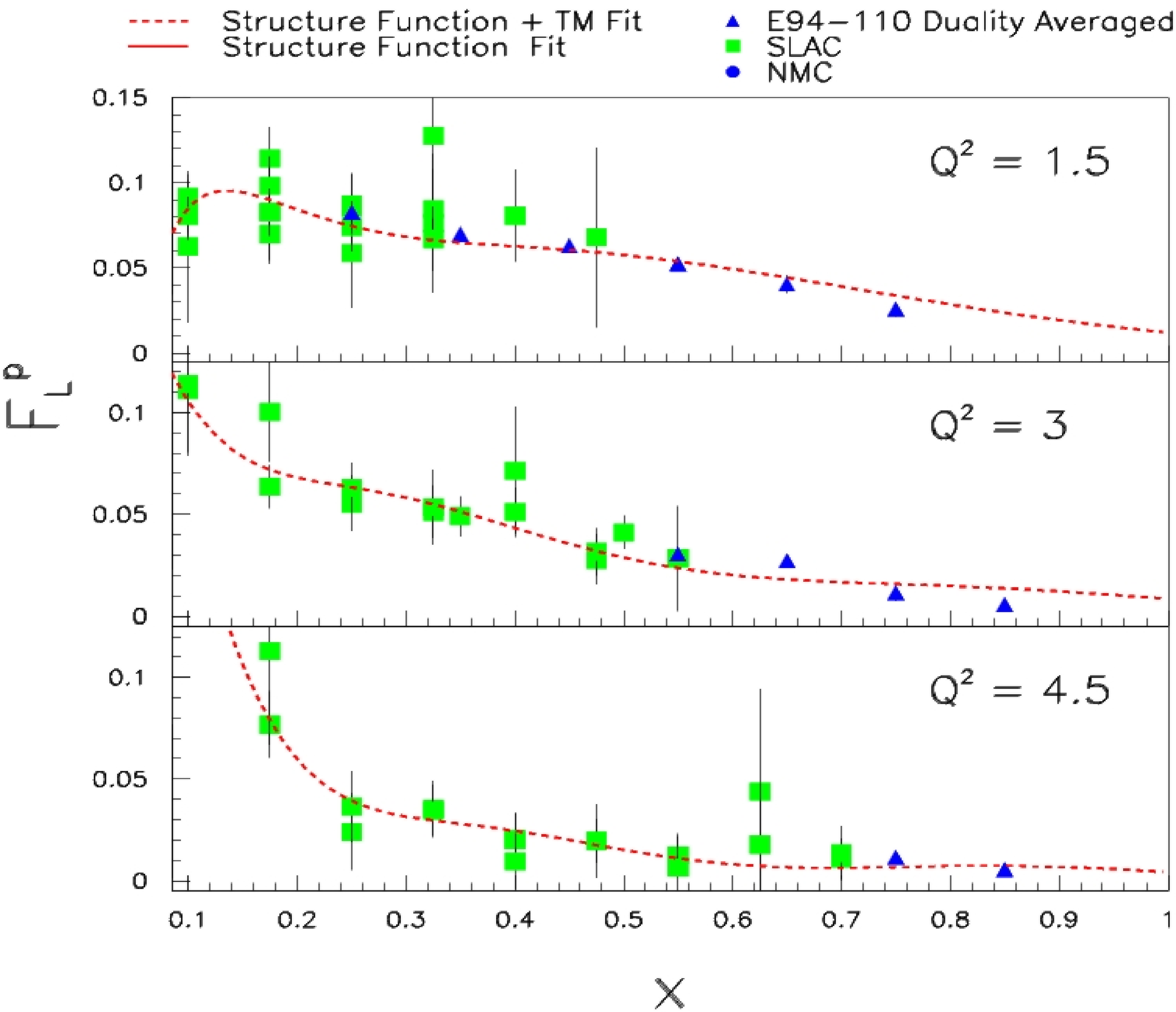
Structure Function + TM Fit
Structure Function Fit
E94−110 Duality Averaged
SLAC
NMC
$Q^2 = 1.5$
$Q^2 = 3$
$Q^2 = 4.5$
F_2^p
X
0 0.1 0.2 0.3 0.4
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Fit Results



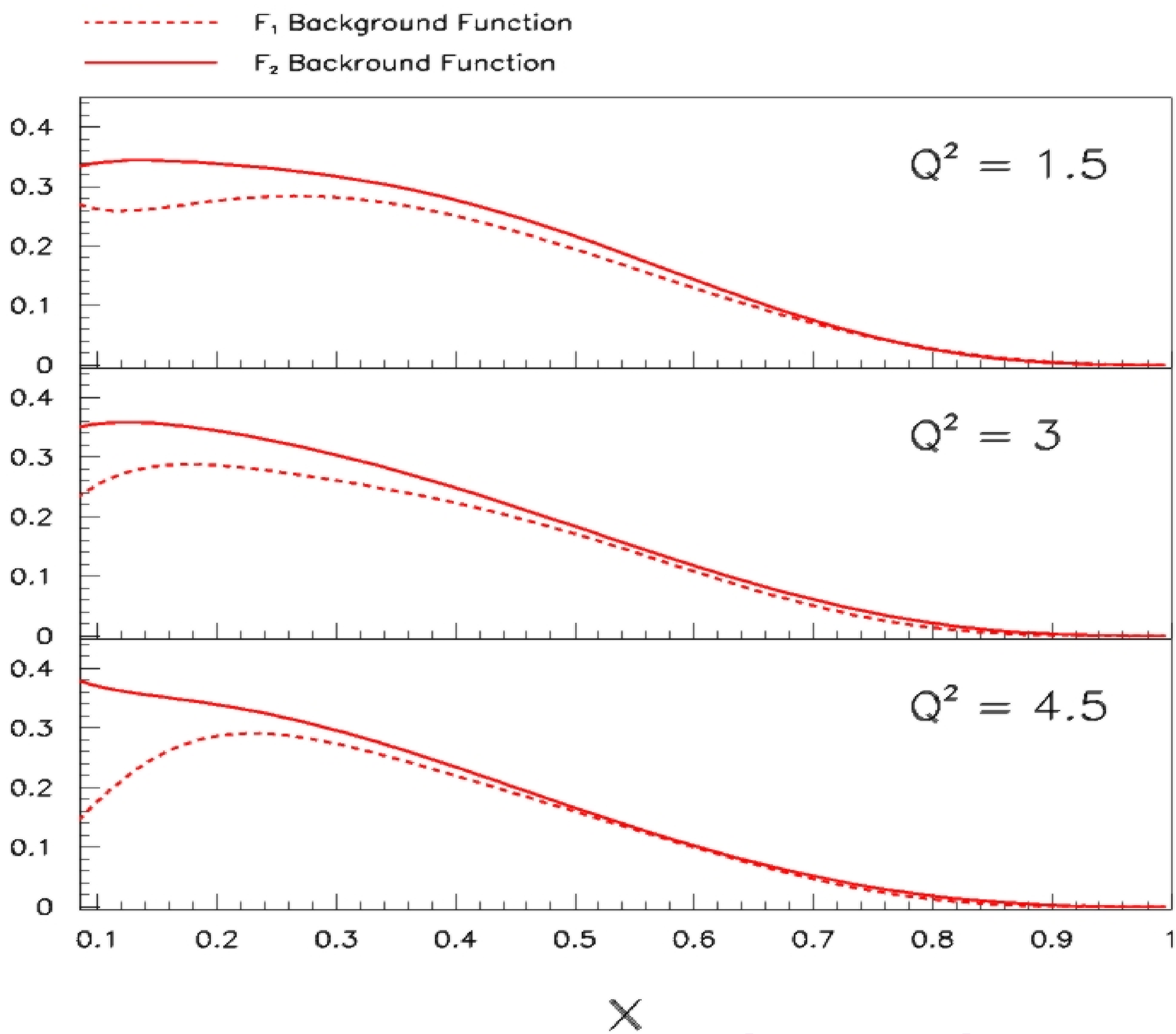
Structure Function + TM Fit
Structure Function Fit
E94−110 Duality Averaged
SLAC
NMC
$Q^2 = 1.5$
$Q^2 = 3$
$Q^2 = 4.5$
$2xF_1^p$
x
0 0.1 0.2 0.3 0.4
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Fit Results



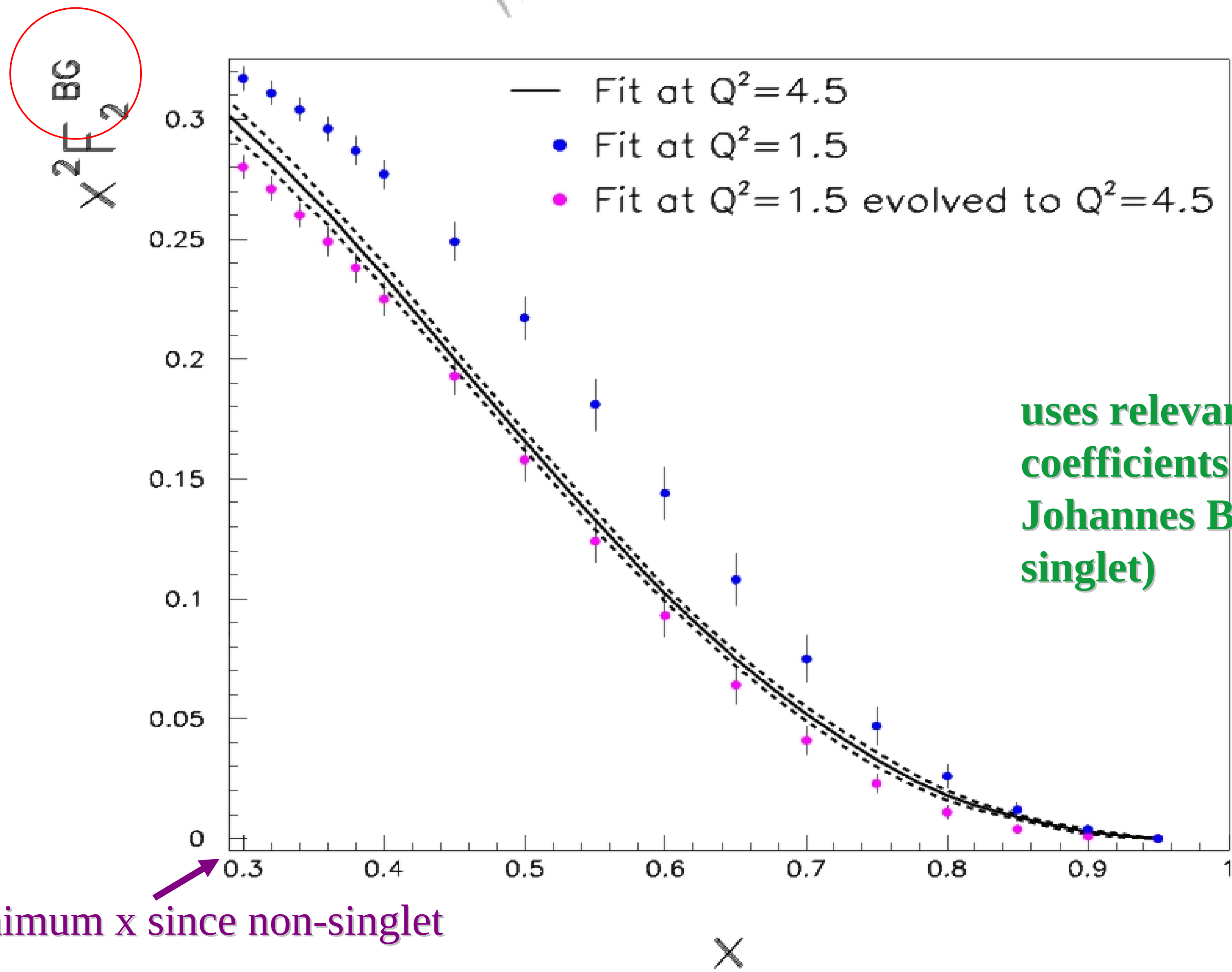
Structure Function + TM Fit
Structure Function Fit
E94−110 Duality Averaged
SLAC
NMC
F_L^p
$Q^2 = 1.5$
$Q^2 = 3$
$Q^2 = 4.5$
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
X

F_2^{bg}, F_1^{bg} difference



There must be a non-zero F_L!

Preliminary pQCD (nonsinglet) Evolution Tests



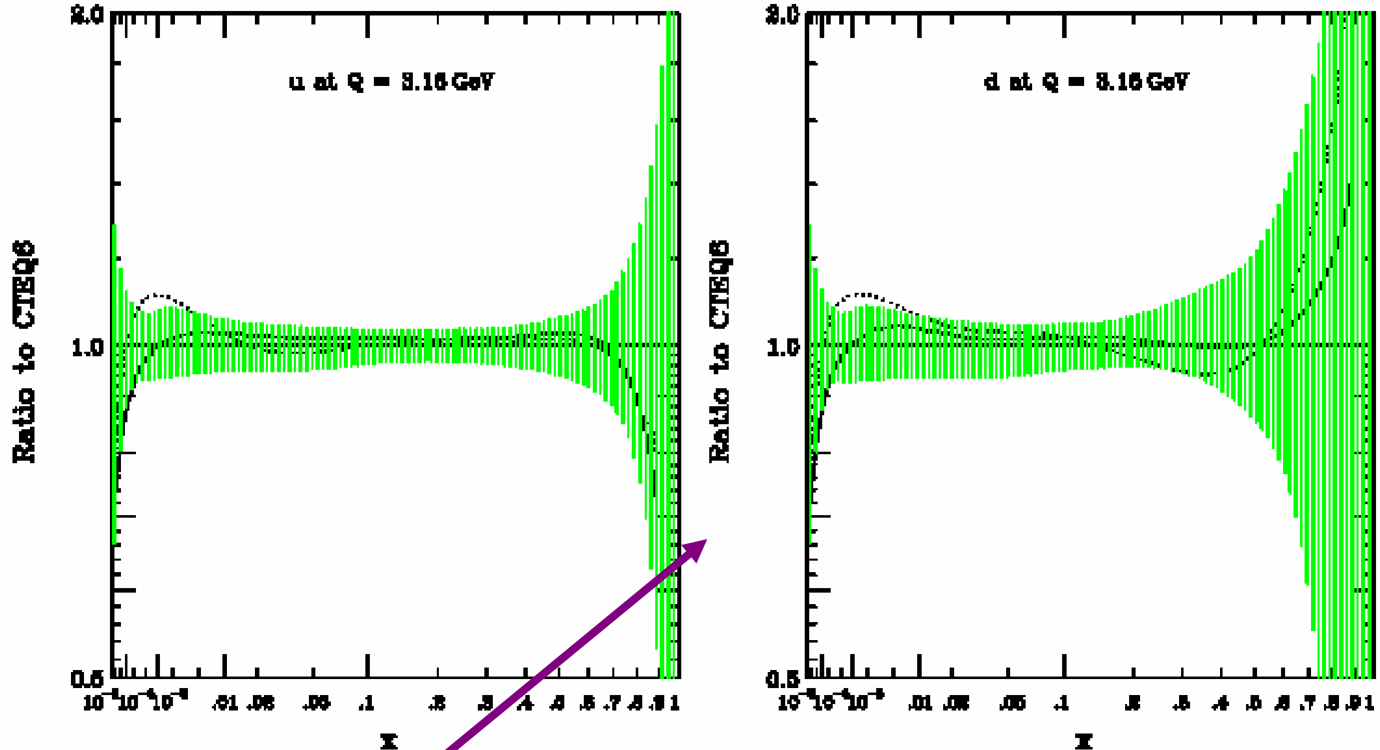
uses relevant Wilson coefficients from Johannes Blumlein (non-singlet)

minimum x since non-singlet

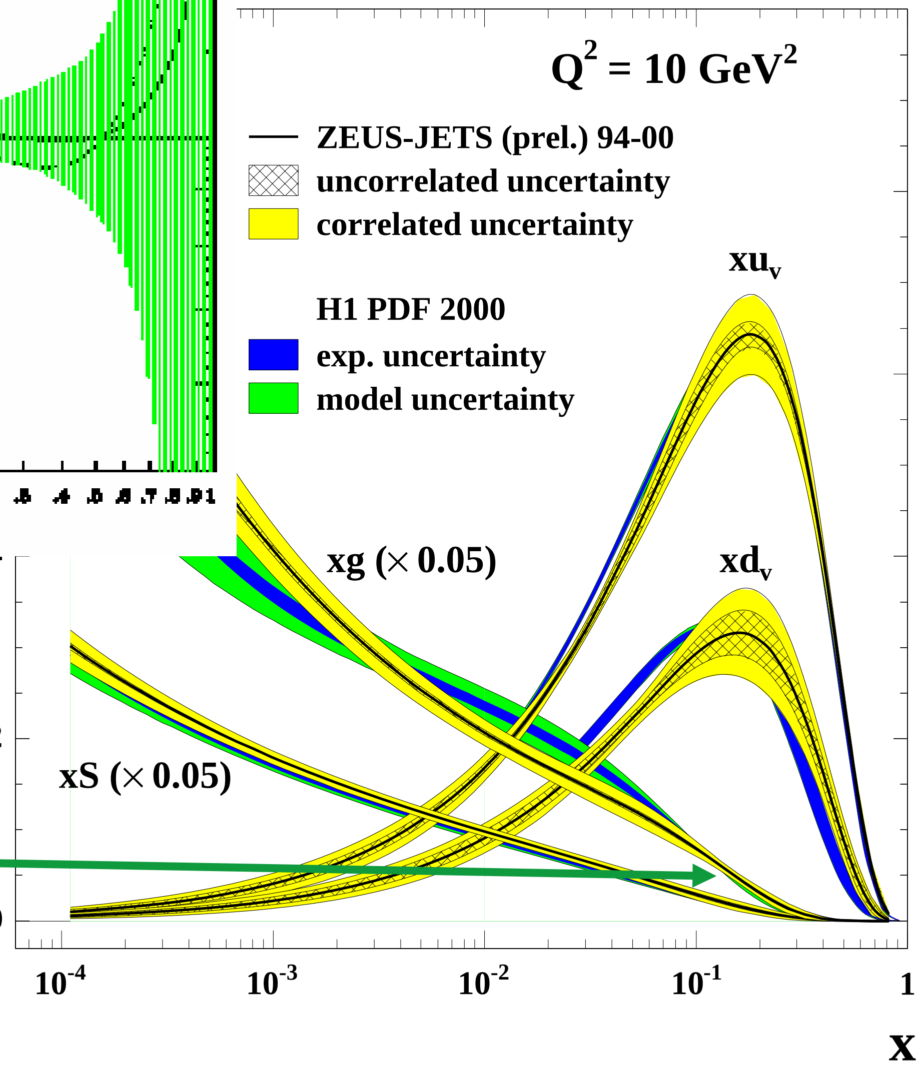
A work in progress (Eric still in Zeuthen with Johannes)…

- Extract Λ_{QCD}
- Quantify Q^2 dependence of higher twist (as separate from target mass)
- Move to singlet analysis (glue)
- Preliminary results show F_2 following pQCD in Blumlein's scheme-independent evolution
- To pin down singlet fully, necessary to extracting non-singlet, need deuterium data

Parton Distribution Functions - not well known at large x

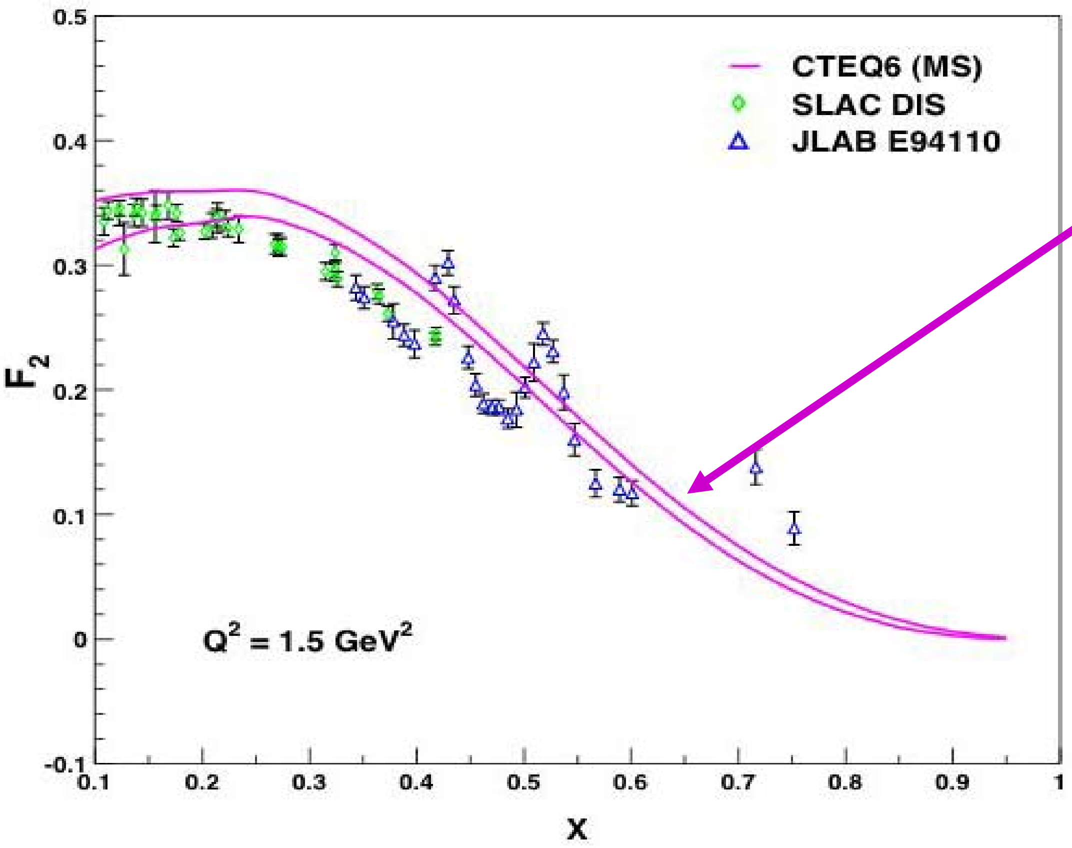


CTEQ u,d(x) uncertainty bands



Similar for HERA, note also glue important still at large x (scaled by factor of 20!)

Can the JLab data help reduce the uncertainties?



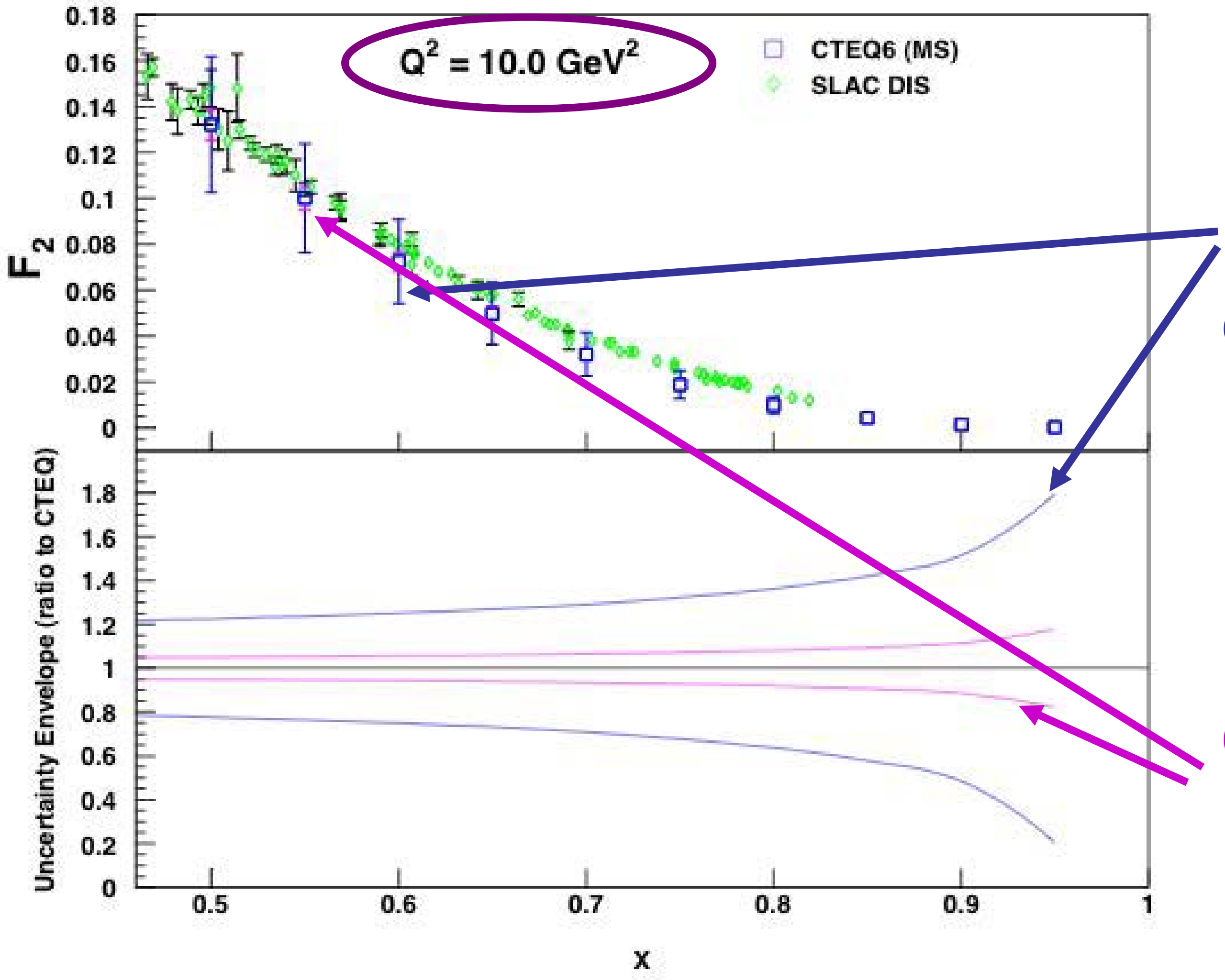
CTEQ6 Hessian error envelope

Errors smaller at low x?

PDF envelope constrained by fit form

…and by choice of d/u (fixed)

Can the JLab (SLAC) data help reduce the uncertainties? - another approach



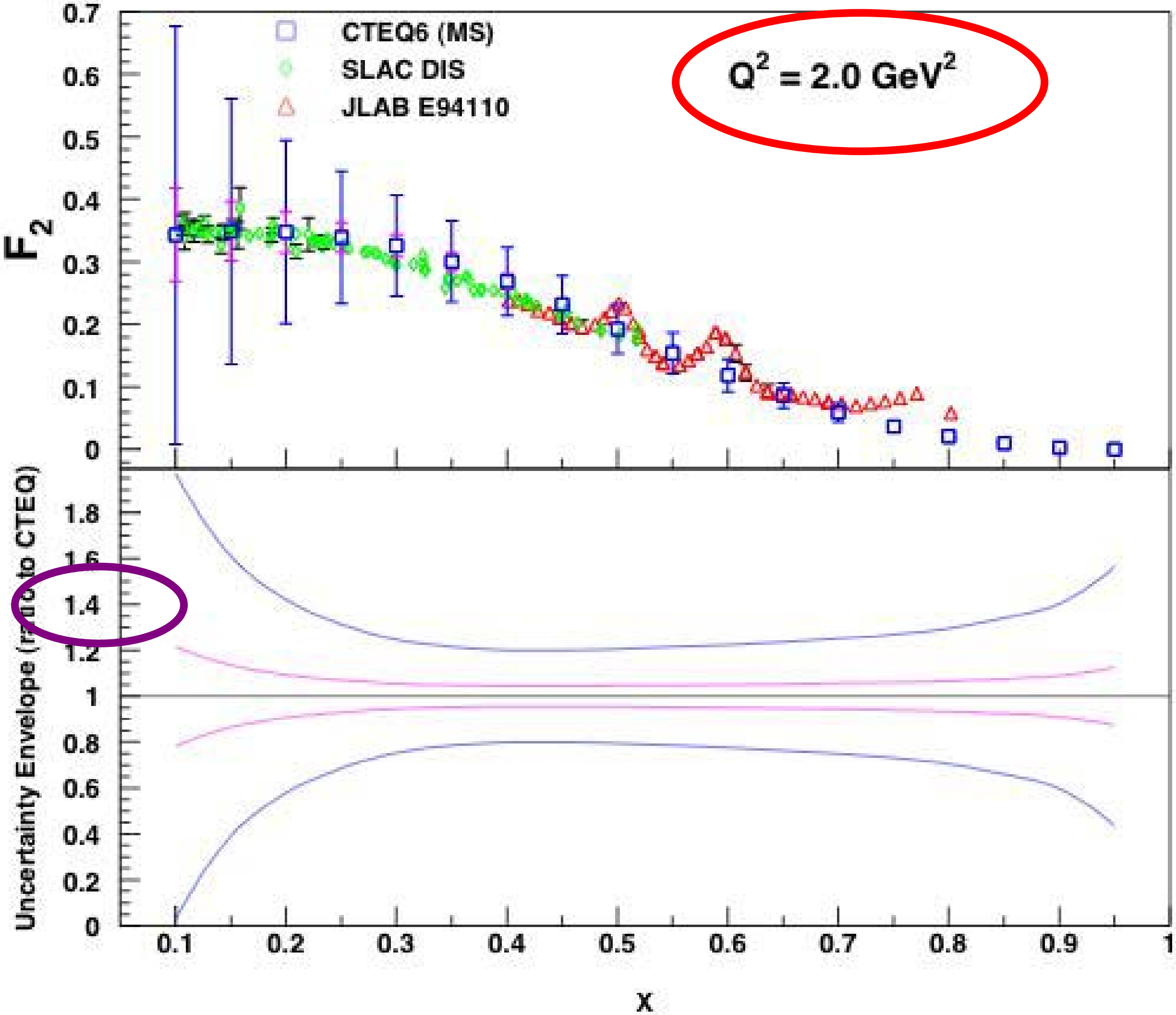
Current cut is $W^2 = 10$ GeV, $Q^2 = 10$ GeV^2

Blue is SF error due to pdf errors (likely an overestimate)

Pink is SF error due to pdf errors, but normalized to x = 0.5 data

Bottom plot better shows error at large x

Can the Jlab (SLAC) data help reduce the uncertainties? - another approach



Uncertainties associated with higher twist, large x evolution

But, it looks maybe useful even if the "data uncertainty" were increased 20%

May help LHC….

A work in progress

Summary

- Precision measurements of $R = \sigma_L/\sigma_T$, and therefore separated structure functions, now available from Jefferson Lab, more to come in the future
- Significant longitudinal strength and structure at large x
- Dictates L/T separations necessary even for F_2
- New data allows for high precision moment extractions - very minimal higher twist observed
- Need to better quantify target mass contribution, data can help
- F_L moments probe the glue
- Perhaps we use this large x data to better constrain large x pdfs