Semi-Inclusive Processes

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Outline

- Physics of Semi-inclusive Processes
- The QCD factotization
- Transverse Momentum Dependent Parton Distributions
- Large Logarithms resummation
- Some Phenomenology
- Conclusion

Semi-inclusive Processes

- Semi-inclusive Deep Inelastic Scattering

$$ep \rightarrow hadron(P_\perp) + X$$

  - Meng,Olness,Soper 96; Mulders,Tangelman,Boer 96&98

- Drell-Yan at low transverse momentum

$$A + B \rightarrow \mu^+\mu^-(Q_\perp) + X$$

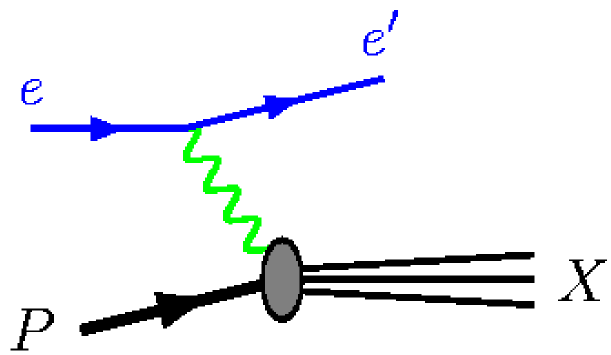
  - Dokshitzer,Diakonov,Troian 78; Paris,Petronzio 79; Collins,Soper,Sterman 85

- Back to back jet (dihadron) producti e^+e^-t coll

$$e^+e^- \rightarrow jet1 + jet2 + x$$

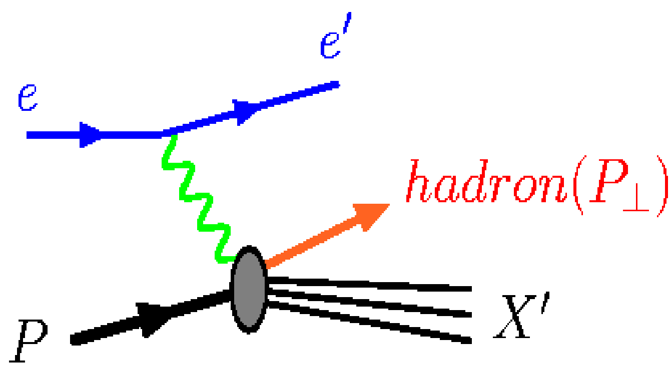
  - Collins,Soper 81; Boer,Jakob,Mulders 98

Inclusive and Semi-inclusive DIS



Inclusive DIS:

More than 30 years experimental study, many theoretical developments, e.g., NNLO splitting function 04'

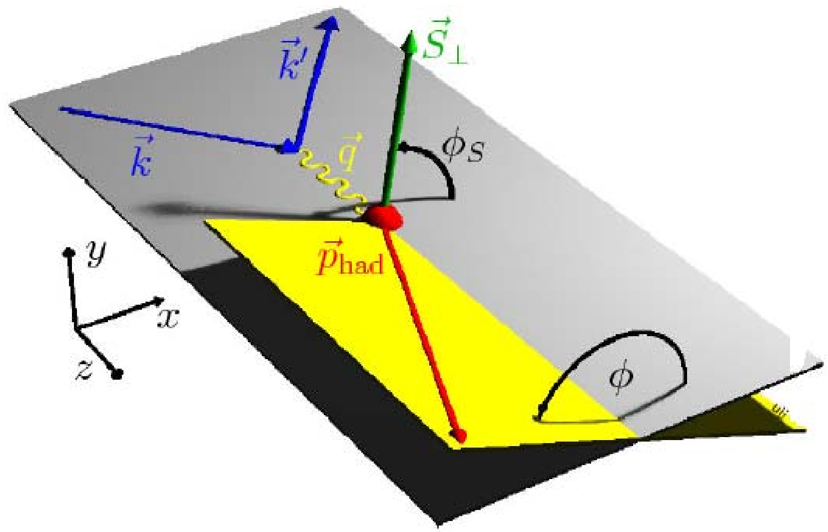


Semi-inclusive DIS:

Focus of various experiments in recent years: EMC, SMC (CERN),H1,ZEUS, HERMES(DESY),JLab,…

Provide more important information about the nucleon Structure and QCD

The Kinematics of SIDIS



Variables :

⊠$_B$ Bjorken Variable;

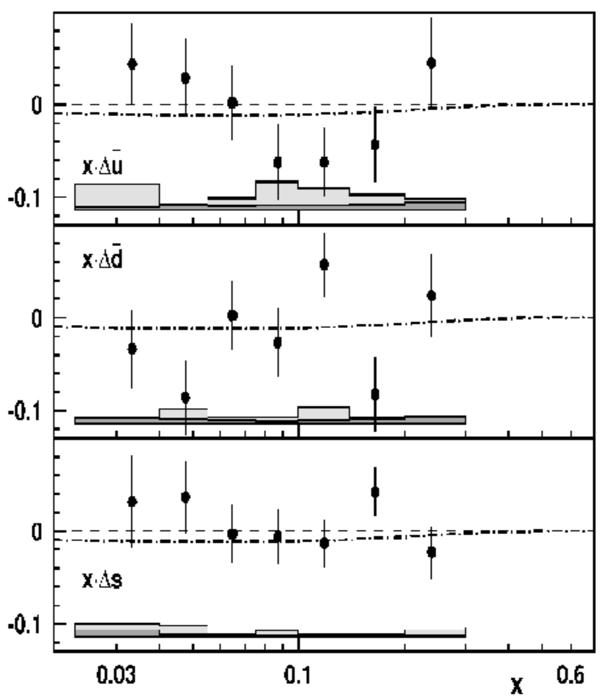
⌘ Longitudinal momentum fraction;

P_h ♈ Transverse momentum; ♐$_h$ the azimuthal angle;

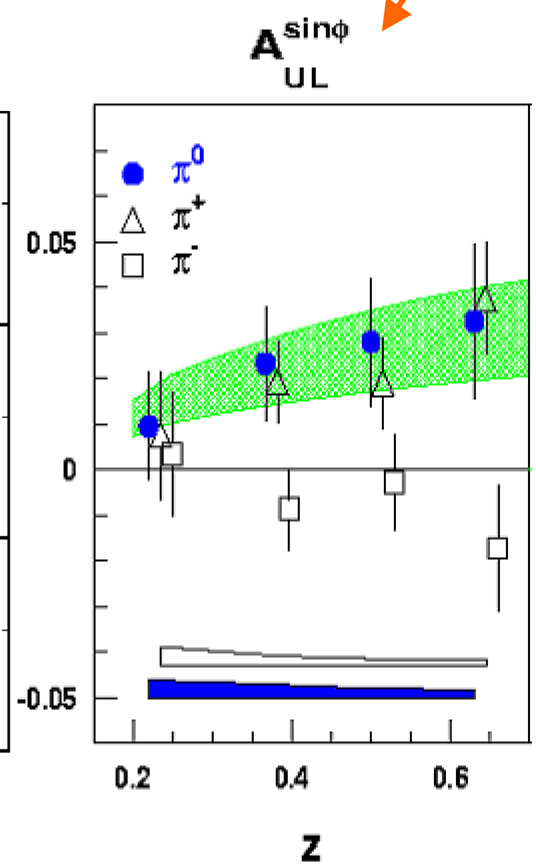
♐$_s$ azimuthal angle of the hadron polarization vector

Polarization Asymmetries in SIDIS

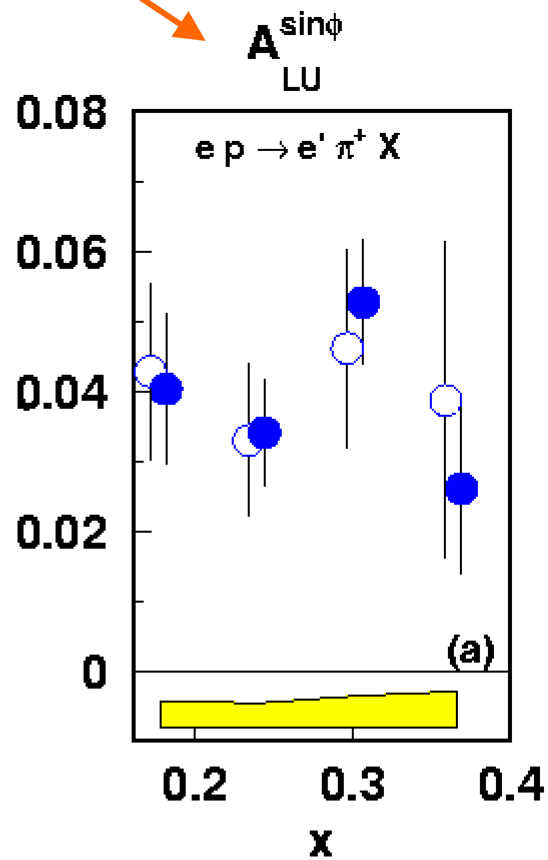
- Double Spin Asymmetry
- Single Spin Asymmetry: Target and Beam



HERMES’03



HERMES’00



CLAS’03

Naïve Factorization

- Cross section

$$\frac{d\sigma}{dx_B dy dz_h d^2 P_{h\perp}} = \frac{2\pi\alpha_{\rm em}^2}{Q^4} y \ell_{\mu\nu} W^{\mu\nu}(P,q,P_h)$$

Hadron tensor

$$W^{\mu\nu}(P,q,P_h) = \frac{1}{4z_h}\sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{iq\cdot\xi} \langle P|J_\mu(\xi)|XP_h\rangle\langle XP_h|J_\nu(0)|P\rangle$$

$$= -1/2 \;\; g_\perp^{\mu\nu} F(x_B, z_h, P_{h\perp}, Q^2) + \cdots$$

- Naïve factorization (unpolarized structure func

$$F(x_B, z_h, P_{h\perp}, Q^2) = \int q(x, k_\perp) \otimes \hat{q}(z, \vec{P}_\perp^h - z\vec{k}_\perp)$$

TMD distr. TMD frag.

Mulders, Tangelman, Boer (96 & 98)

QCD Factorization

- Factorization for the structure function

$$
\begin{aligned}
F(x_B, z_h, P_{h\perp}, Q^2) \;=\; & \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp \\
& \times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_T\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho) \\
& \times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) \;,
\end{aligned}
$$

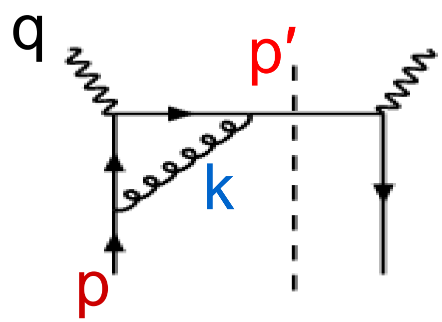
- q: TMD parton distribution
- q hat: TMD fragmentation function
- S: Soft factors
- H: hard scattering.

Impact parameter space

$$
\begin{aligned}
F(x_B, z_h, b, Q^2) \;=\; & \sum_{q=u,d,s,\ldots} e_q^2 q\left(x_B, z_h b, \mu^2, x_B\zeta, \rho\right) \hat{q}\left(z_h, b, \mu^2, \hat{\zeta}/z_h, \rho\right) \\
& \times S(b, \mu^2, \rho) H\left(Q^2, \mu^2, \rho\right) \;.
\end{aligned}
$$

One-loop Factorization (virtual gluon)

- Vertex corrections (single quark target)

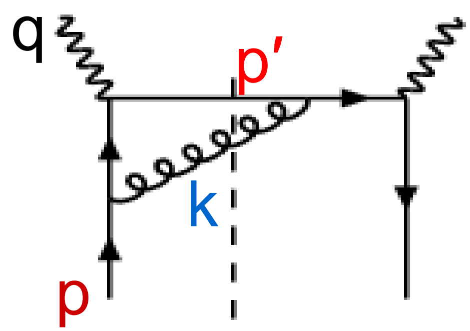


Four possible regions for the gluon momentum k:

1) k is collinear to p (parton distribution)
2) k is collinear to p′ (fragmentation)
3) k is soft (Wilson line)
4) k is hard (pQCD correction)

One-Loop Factorization (real gluon)

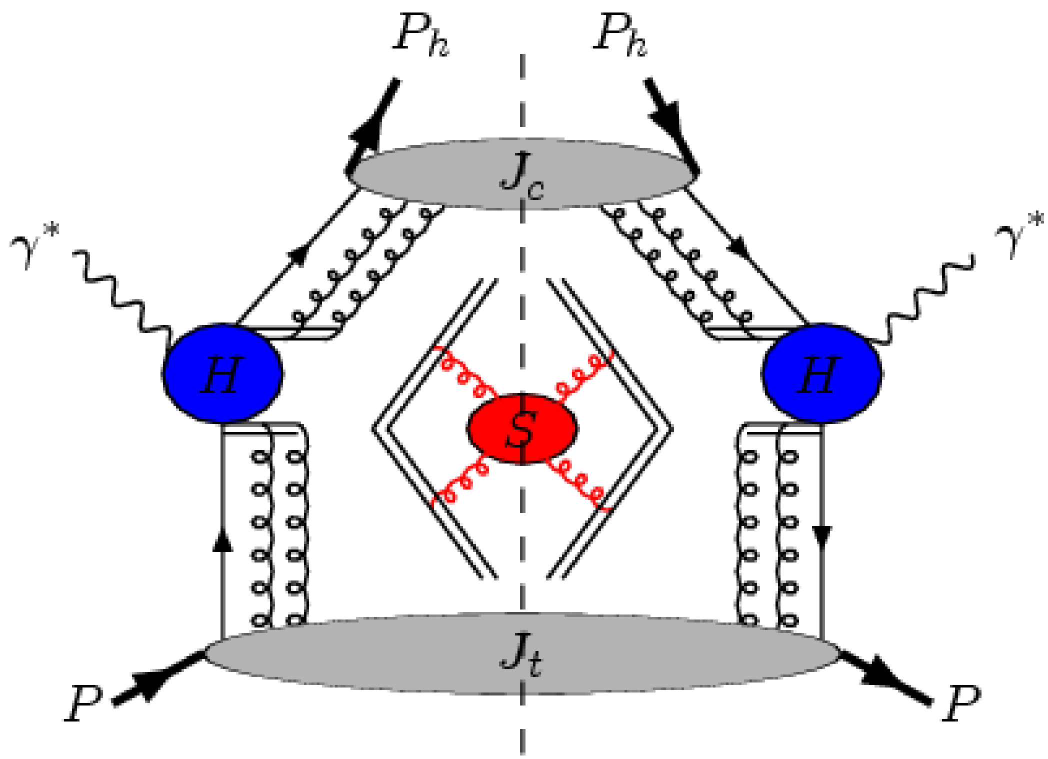
- Gluon Radiation (single quark target)



Three possible reagions for the gluon momentum k:

1) k is collinear to p (parton distribution)
2) k is collinear to p′ (fragmentation)
3) k is soft (Wilson line)

All orders Factorization



Transverse-momentum Dependent (TMD) Parton Distributions

- Generalize Feynman distribution q(x) by including the transverse momentum of the parton.

$$q(x,k_\perp)$$

- At small $k_\perp$, the transverse-momentum dependence is generated by soft non-perturbative physics.
- At large $k_\perp$, the k-dependence can be calculated in perturbative QCD and falls like powers of $1/k_\perp^2$
- Integration over transverse-momentum does not usually yield Feynman distribution

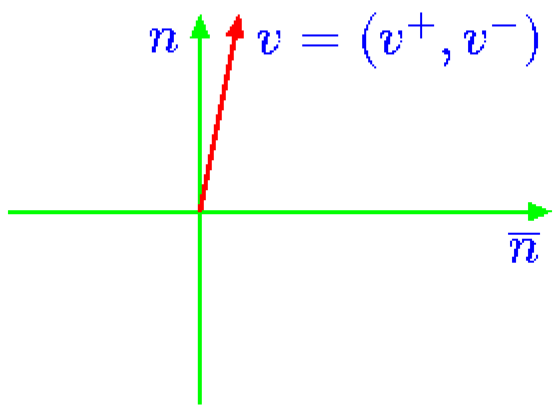
$$\int^\mu d^2k_\perp \; q(x, k_\perp) \neq q(x,\mu)$$

TMD Distributions: the definition

$$Q(x,k_\perp,\mu,x\zeta) = \frac{1}{2}\int\frac{d\xi^-}{2\pi}e^{-ix\xi^-P^+}\int\frac{d^2\vec{b}_\perp}{(2\pi)^2}e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \times \left\langle P\left|\overline{\psi}_q(\xi^-,0,\vec{b}_\perp)\mathcal{L}_v^\dagger(\infty;\xi^-,0,\vec{b}_\perp)\gamma^+\mathcal{L}_v(\infty;0)\psi_q(0)\right|P\right\rangle$$

In Feynman Gauge, the gauge link

$$\mathcal{L}_v(\infty;\xi) = \exp\left(-ig\int_0^\infty d\lambda v\cdot A(\lambda v+\xi)\right) .$$



$\overline{n} = (1,0,0_\perp)$ $n = (0,1,0_\perp)$

v is not n to avoid l.c. singularity !!

$\zeta^2 = (2v\cdot P)^2/v^2$

Polarized TMD Quark Distributions

Quark \ Nucleon	Unpol.	Long.	Trans.
Unpol.	$q(x, k_\perp)$		$q_T(x, k_\perp)$
Long.		$\Delta q_L(x, k_\perp)$	$\Delta q_T(x, k_\perp)$
Trans.	$\delta q(x, k_\perp)$	$\delta q_L(x, k_\perp)$	$\delta q_T(x, k_\perp)$ $\delta q_T'(x, k_\perp)$

Boer, Mulders, Tangerman (96&98)

SIDIS Cross Section

At leading power of 1/Q

$$
\begin{aligned}
d\sigma \quad \propto \quad & (1-y+y^2/2)x_B F_{UU}^{(1)} \\
& -(1-y)x_B\cos(2\phi_h)F_{UU}^{(2)} \\
& +\lambda_\ell \lambda y(1-y/2)x_B F_{LL} \\
& +\lambda_\ell |S_\perp| y(1-y/2)x_B\cos(\phi_h-\phi_S)F_{LT} \\
& +\lambda(1-y)x_B\sin(2\phi_h)F_{UL} \\
& +|S_\perp|(1-y+y^2/2)x_B\sin(\phi_h-\phi_S)F_{UT}^{(1)} \\
& +|\vec{S}_\perp|(1-y)x_B\sin(\phi_h+\phi_S)F_{UT}^{(2)} \\
& +|\vec{S}_\perp|(1-y)x_B\sin(3\phi_h-\phi_S)F_{UT}^{(3)}/2
\end{aligned}
$$

The structure functions depend on Q^2, ⊠$_B$, ⌘, P_h ♈

Factorization for the Structure Functions

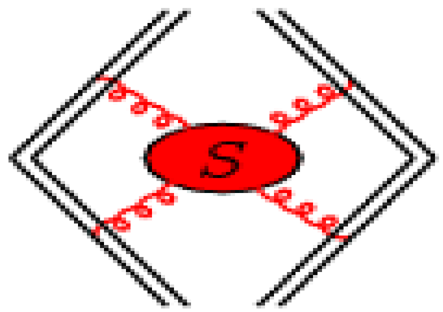
For Example,

$$
\begin{aligned}
F_{UU}^{(1)} &= \int q\left(x_B, k_\perp\right) \hat{q}\left(z_h, p_\perp\right) S^+(\vec{\ell}_\perp) H_{UU}^{(1)}\left(Q^2\right) \\
F_{LL} &= \int \Delta q_L\left(x_B, k_\perp\right) \hat{q}\left(z_h, p_\perp\right) S^+(\vec{\ell}_\perp) H_{LL}\left(Q^2\right) \\
F_{UT}^{(1)} &= \int \frac{\vec{k}_\perp \cdot \hat{\vec{P}}_{h\perp}}{M} q_T\left(x_B, k_\perp\right) \hat{q}\left(z_h, p_\perp\right) S^+(\vec{\ell}_\perp) H_{UT}^{(1)}\left(Q^2\right) \\
F_{UT}^{(2)} &= \int \frac{\vec{p}_\perp \cdot \hat{\vec{P}}_{h\perp}}{M} \delta q_T\left(x_B, k_\perp\right) \delta\hat{q}\left(z_h, p_\perp\right) S^+(\vec{\ell}_\perp) H_{UT}^{(2)}\left(Q^2\right)
\end{aligned}
$$

$$
\int = \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp \delta^2(z\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp})
$$

The Soft Factor

- The soft part can be factorized from the jet using Grammer-Yennie approximation
- The result of the soft factorization is a soft factor in the cross section, in which the target current jets appear as the eikonal lines.

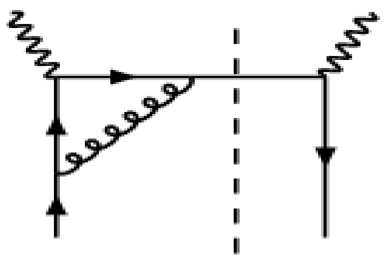


- The Operator definition

$$S(\vec{b}_\perp, \mu^2, \rho) = \frac{1}{N_c} \langle 0 | \mathcal{L}^\dagger_{\tilde{v}il}(\vec{b}_\perp, -\infty) \mathcal{L}^\dagger_{vlj}(\infty; \vec{b}_\perp) \mathcal{L}_{vjk}(\infty; 0) \mathcal{L}_{\tilde{v}ki}(0; -\infty) | 0 \rangle$$

The Hard Part

- The hard part can be calculated order by order, using subtraction method
- One-loop results for the hard parts in SIDIS



$$
\begin{aligned}
H^{(1)}_{UU}(Q^2,\mu^2,\rho) &= H_{LL}(Q^2,\mu^2,\rho) \\
&= \frac{\alpha_s}{2\pi} C_F \left[\left(1+\ln\rho^2\right)\ln\frac{Q^2}{\mu^2} - \ln\rho^2 + \frac{1}{4}\ln^2\rho^2 + \pi^2 - 4 \right]
\end{aligned}
$$

Factorization for Drell-Yan

- The cross section

$$\frac{d\sigma}{d^4Q d\Omega} = \frac{\alpha^2}{2sQ^2}\Big\{(1+\cos^2\theta)W_0(x_1,x_2,Q^2,Q_\perp)$$
$$-(1+\cos^2\theta)\lambda_1\lambda_2 W_{LL}(x_1,x_2,Q^2,Q_\perp)$$
$$+\sin^2\theta\cos(\phi_1+\phi_2)|\vec{S}_{1\perp}||\vec{S}_{2\perp}|W_{TT}(x_1,x_2,Q^2,Q_\perp)\Big\}$$

- The structure functions

$$W_0 = \int q(x_1,k_{1\perp})\,\overline{q}(x_2,k_{2\perp})\,S^-(\vec{\ell}_\perp)H_0\left(Q^2\right)$$
$$W_{LL} = \int \Delta q_L(x_1,k_{1\perp})\,\Delta\overline{q}_L(x_2,k_{2\perp})\,S^-(\vec{\ell}_\perp)H_{LL}\left(Q^2\right)$$
$$W_{TT} = \int \delta q_T(x_1,k_{1\perp})\,\delta\overline{q}_T(x_2,k_{2\perp})\,S^-(\vec{\ell}_\perp)H_{TT}\left(Q^2\right)$$

Large Logarithms Resummation

- At low transverse momentum, $P_T \ll Q$, we must resum the large logarithms $\alpha_s^n \ln^{2n}(Q^2/P_T^2)$

  -- Dokshitzer, Diakonov, Troian, 1978

  -- Parisi, Petronzio, 1979

- These large logarithms can be resummed by solving the energy evolution equation for the TMD parton dis.

  -- Collins-Soper 1981

TMD Parton Dis.: the Energy Dependence

- The TMD distributions depend on the energy of the hadron! (or Q in DIS)
- Introduce the impact parameter representation

$$\mathcal{Q}(x, b_\perp, \mu, x\zeta) = \int d^2k_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \mathcal{Q}(x, k_\perp, \mu, x\zeta)$$

One can write down an evolution equation in $\zeta=2(P^+)^2 v^-/v^+$

$$\boxed{\zeta \frac{\partial}{\partial \zeta} \mathcal{Q}(x, b, \mu, x\zeta) = \left(K(\mu, b) + G(\mu, x\zeta)\right) \mathcal{Q}(x, b, \mu, x\zeta)}$$

Collins and Soper (1981)

μ independent!

- K and G obey $\mu \frac{d}{d\mu} K = -\gamma_K = -\mu \frac{d}{d\mu} G,$

Spin-dependent Collins-Soper Equation

- The evolution kennel is the same for all the leading-twist TMD quark distributions
- K_t-even ones: $q(x, k_\perp)$, $\Delta q_L(x, k_\perp)$, $\delta q_T(x, k_\perp)$

$$\zeta\frac{\partial}{\partial\zeta}f(x,b,\mu,x\zeta,\rho)=\left(K(b,\mu,\rho)+G(x\zeta,\mu,\rho)\right)f(x,b,\mu,x\zeta,\rho)$$

- K_t-odd ones: $q_T(x, k_\perp)$, $\delta q(x, k_\perp)$, $\Delta q_T(x, k_\perp)$, $\delta q_L(x, k_\perp)$

$$\zeta\frac{\partial}{\partial\zeta}\partial_b^i f(x,b,\mu,x\zeta,\rho)=\left(K(b,\mu,\rho)+G(x\zeta,\mu,\rho)\right)\partial_b^i f(x,b,\mu,x\zeta,\rho)$$

Idilbi, Ji, Ma, Yuan: hep-ph/0406302

Large Logarithms Resummation(II)

- After resummation, large logarithms associated with Q^2 can be factorized into the Sudakov form factors. e.g.

$$\tilde{F}_{UU}^{(1)}(b,Q^2) = F_{UU}^{(1)}(x_B, z_h, b, \mu_L^2/C_2^2)\, e^{-S(Q^2,\mu_L^2,b)}$$

$$\tilde{F}_{UT}^{(1)}(b,Q^2) = \frac{-1}{M z_h}\partial_b^i\left[\left(\partial_b^i q_T(x_B, z_h b)\right)\hat{q}(z_h,b)S^+(b)H_{UT}^{(1)}(\mu_L^2/C_2^2)\, e^{-S(Q^2,\mu_L^2,b)}\right]$$

- And the Sudakov form factor

$$S(Q^2,\mu_L^2,b,C_2) = \int_{\mu_L}^{C_2 Q}\frac{d\bar{\mu}}{\bar{\mu}}\left[\ln\left(\frac{C_2 Q^2}{\bar{\mu}^2}\right)A(b\mu_L,\bar{\mu},\rho) + B(C_2, b\mu_L, \bar{\mu}, \rho)\right]$$

-- A and B are series in α_s

Double Logarithmic (DL) Approximation

- If Q^2 is not too large, DL approx. applies. The Sudakov suppression form factor then only depends on Q^2

$$\mathcal{S}^{(DL)}(Q^2, \mu_L^2, C_2) = \frac{4}{3}\int_{\mu_L}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \ln\left(\frac{C_2 Q^2}{\bar{\mu}^2}\right)$$

- The Q^2 dependence of the structure functions can b

$$\begin{aligned} F_{UU}(x_B, z_h, P_{h\perp}, Q^2) &= F_{UU}(x_B, z_h, P_{h\perp}, \mu_L^2) e^{-\mathcal{S}^{(DL)}(Q^2,\mu_L^2)} \\ F_{UT}^{(1)}(x_B, z_h, P_{h\perp}, Q^2) &= F_{UT}^{(1)}(x_B, z_h, P_{h\perp}, \mu_L^2) e^{-\mathcal{S}^{(DL)}(Q^2,\mu_L^2)} \end{aligned}$$

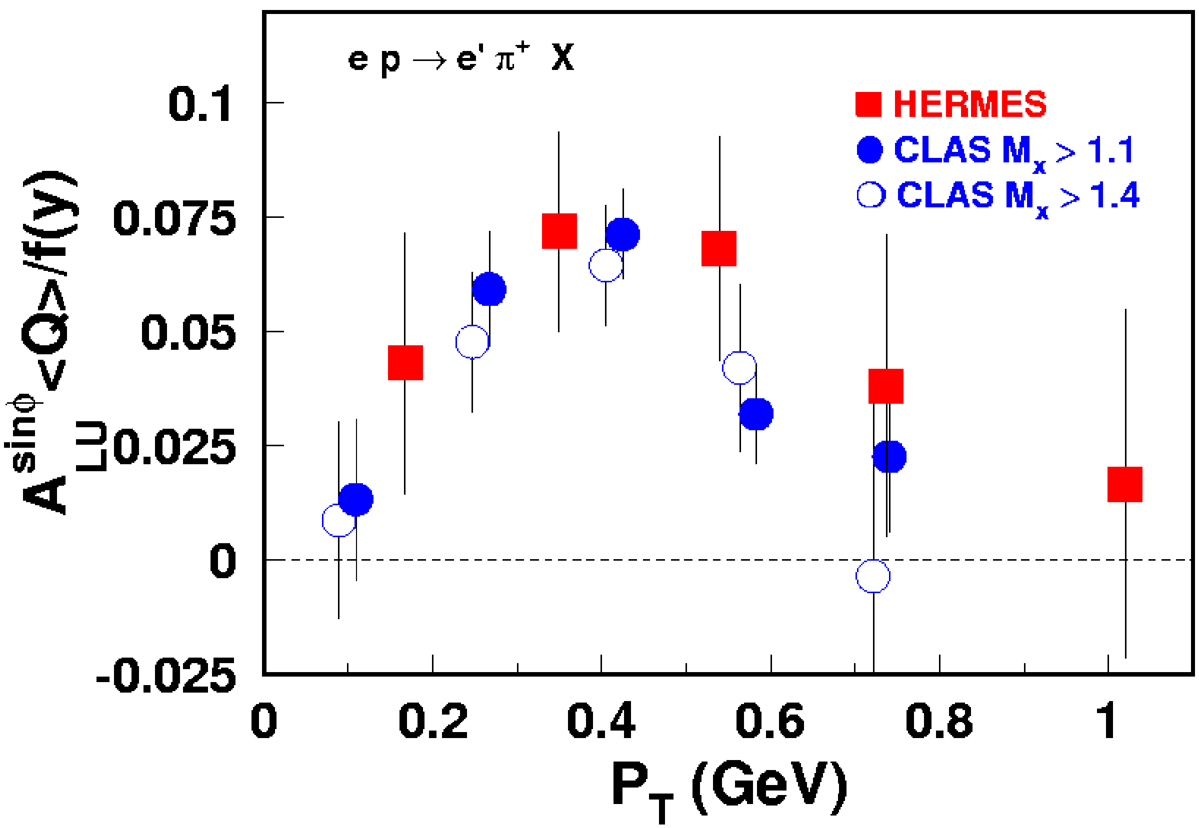
- We can predict the P_T distribution at higher Q^2 from that of lower Q^2

The P_T spectrum of the polarization asymmetry will be the same for different Q^2 at fixed ⌧$_B$ and ⌘

Exp. Evidence supporting DLA

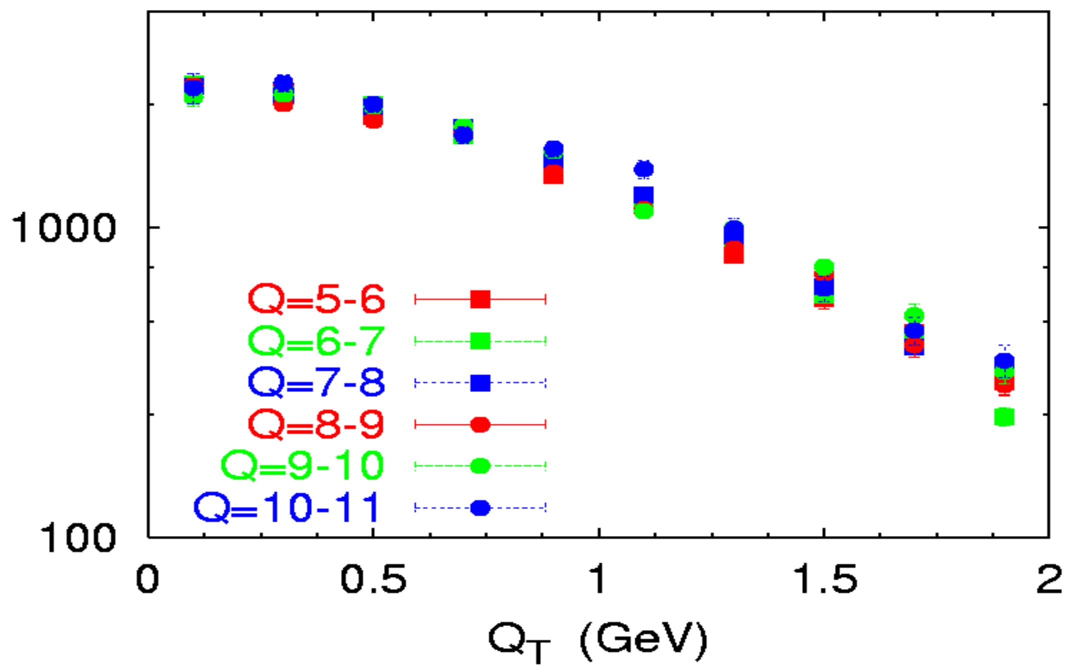
Compare the asymmetries at HERMES and JLAB

Q^2 vary a factor of 3 (H. Avakian, GDH2004)



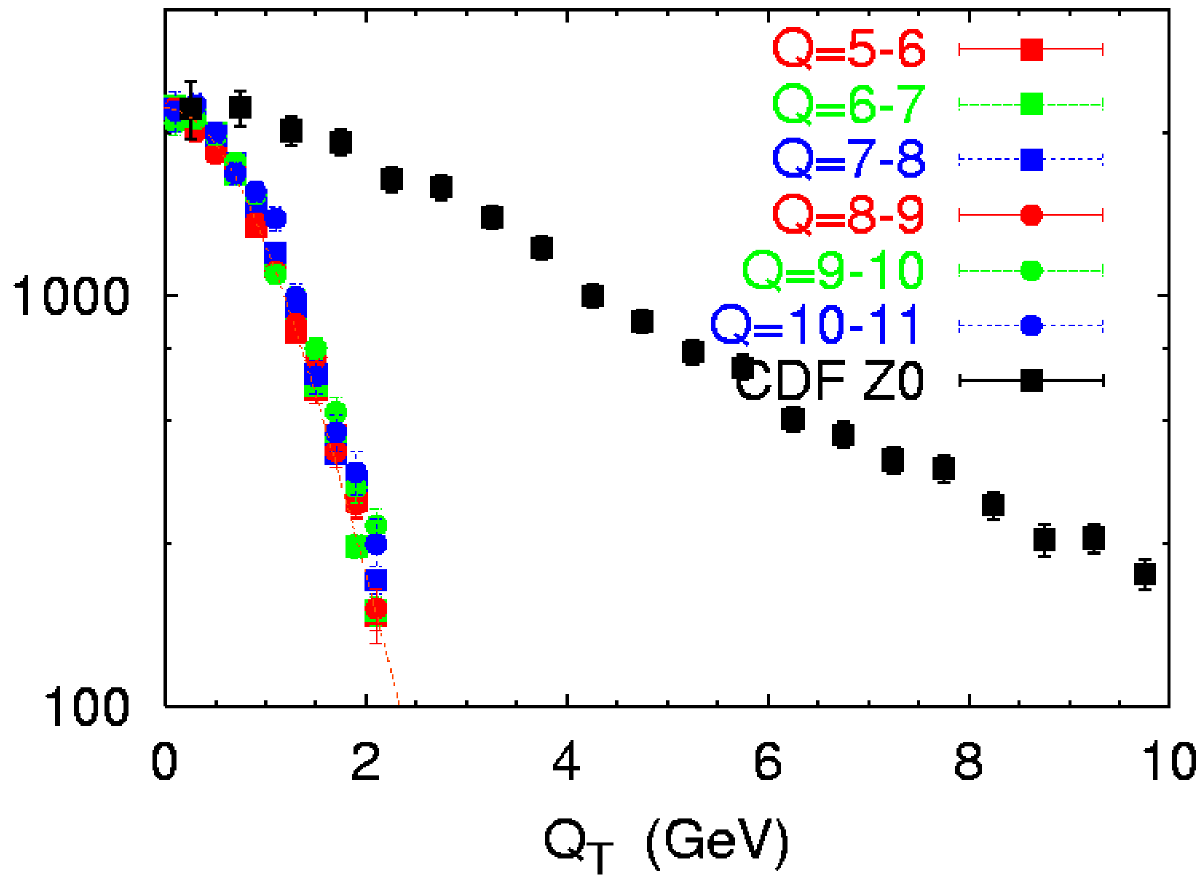
Drell-Yan Production at Fixed Target

Q_T spectrum from E288, PRD23,604(81)

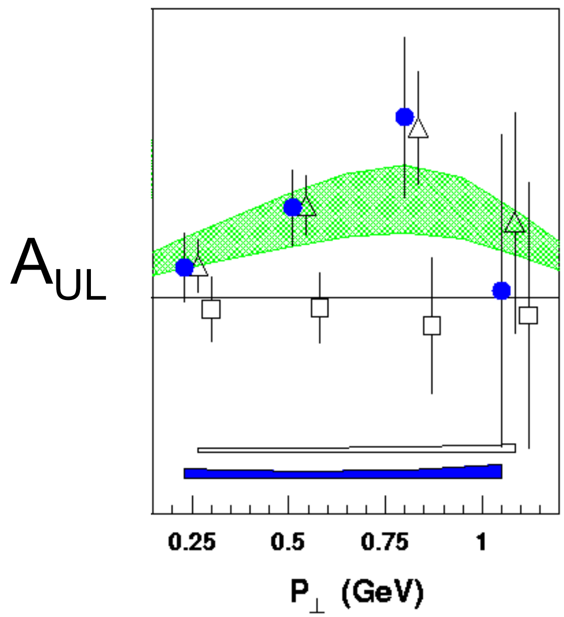


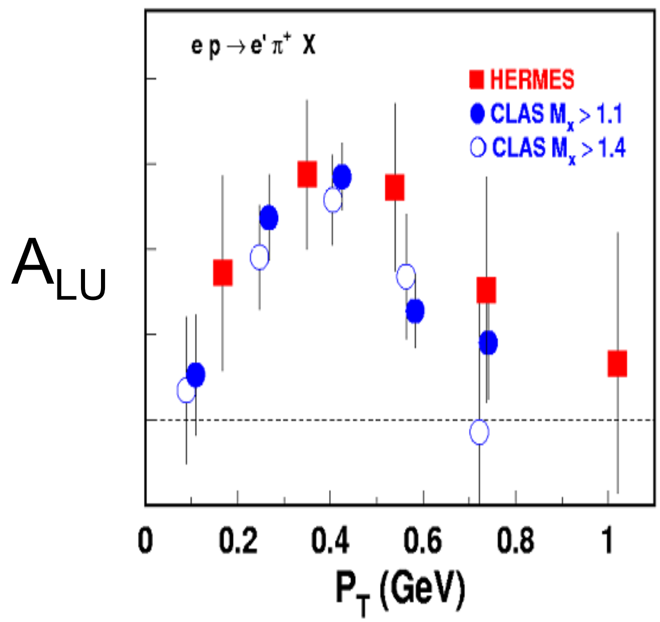
The shape of the Q_T distributions could be a hint that the x and k_t dependence in the TMD parton distributions are factorized at this kinematical range

At very large Q^2 (e.g., Z^0 and W boson), DL Approx. breaks down



P_T Dependence of the Asymmetries





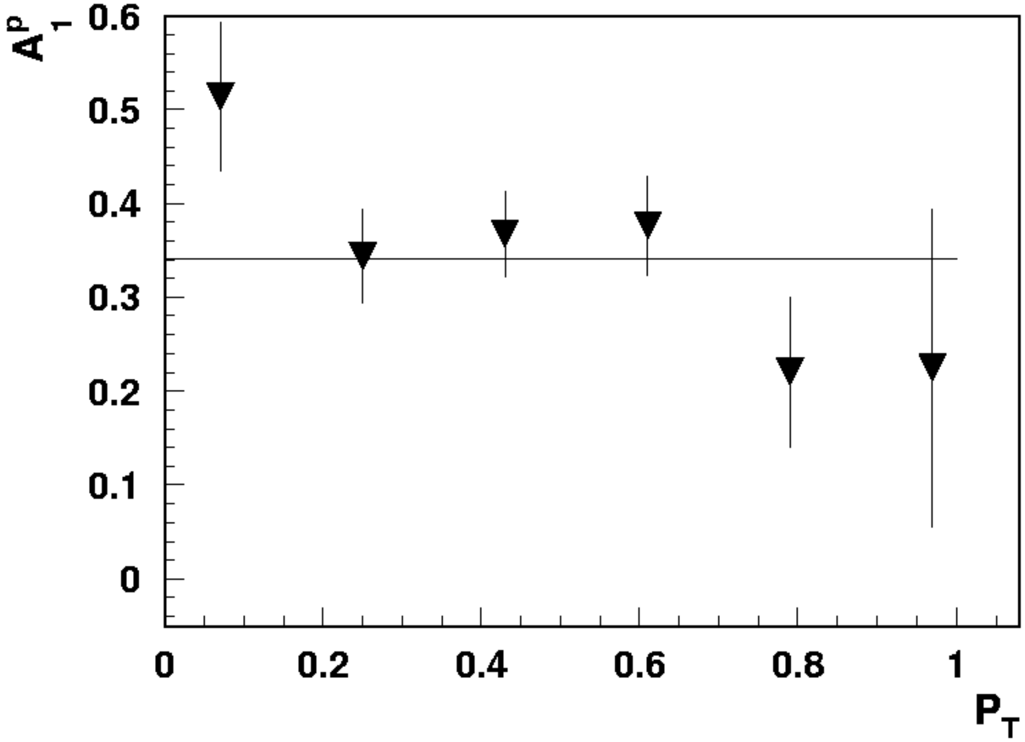
They are both k_t-odd effects, and approach 0 at large P_T

A_{UL}: Sivers dis. q_T and Collins frag. δq

A_{LU}: Sivers-like dis. δq and Collins frag. δq

Double Spin Asymmetry

k_t-even effect, A_1: $\Delta q_L(x, k_\perp)/ q(x, k_\perp)$



(H. Avakian, CLAS)

- P_T distribution of the asymmetries provides qualitative

  information for the TMD quark distributions
- k_t-even distributions have the same dependence on k_t
- k_t-odd distributions are suppressed at large k_t
  - Consistent with the power counting

    k_t-even: $1/k_t^2$

    k_t-odd: $1/k_t^3$

Conclusion

- Factorization theorem has been proved for Semi-inclusive processes
- Large logarithms can be resummed by solving the Collins-Soper equations for TMD parton dis., and the DL Approx. works for not very large Q^2
- Qualitative behavior for the TMD quark distributions have been informed from the experiments