Theoretical overview of current status and future directions of spin studies

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I will concentrate on two features of spin physics:

- Spin sum rule: the proton’s spin budget?
- Spin as a tool to probe the hadron’s partonic structure and QCD dynamics
Hadron properties in terms of dynamics of quarks and gluons:

- **Hadron properties**
  - Charge, Mass, Spin, Magnetic moment, ...

- **QCD**
  - Quarks Color, Flavor, Charge, Mass, Spin, ...
  - Gluons Color, Spin, ...

**Lattice QCD:**

Could calculate all hadron properties in principle!
Has done an excellent job in reproducing the hadron mass spectrum

**But,**

It does not reveal the space-time distribution of partons inside a hadron, details of interactions, reasons of confinement, ...
Hadron properties – parton dynamics

- How color is distributed inside a hadron?
  (possible clue for color confinement, …)
  ✷ Electric form factor ➔ charge distribution
    But, no color form factor!
  ✷ Hadron is a color singlet, but, gluon carries color

- How partons and their interaction build up the hadron mass?
  ✷ Atom mass – heavy nucleus + light electrons
    – concentrated mass and “localized” charge source
    No “localized” color source for light hadrons!
  ✷ Hadron mass < Energy scale to “see” localized partons (live long enough) - hard for pQCD approach
Something special about spin

- **Spin of an elementary particle:**
  An intrinsic quantum property of the particle

- **Spin of a composite particle – like a proton:**
  Angular momentum when the particle is at rest
  \[ = \text{Spin of elementary partons} \]
  \[ \text{(intrinsic quantum effect)} \]
  \[ + \]
  \[ \text{Motion of the partons} \]
  \[ \text{(dynamical – fundamental interaction)} \]

- **Proton’s spin budget in QCD:**
  Jaffe-Manohar, Ji, Chen et al, Wakamatsu, …

  \[ S(\mu) = \sum_f \langle P, S | \hat{J}^z_f(\mu) | P, S \rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu) \rightarrow \frac{1}{2} \Sigma_q + L_q + (\Delta G + L_g) \]

  The decomposition is not unique! **Only the total sum is physical!**
Spin as a hadron property

- Complexity of the proton state – scale dependence:

\[ S(\mu) = \frac{1}{2} \]

- Asymptotic limit:

\[ J_q(\mu \to \infty) \Rightarrow \frac{1}{2} \frac{3N_f}{16 + 3N_f} \sim \frac{1}{4} \quad J_g(\mu \to \infty) \Rightarrow \frac{1}{2} \frac{16}{16 + 3N_f} \sim \frac{1}{4} \]

- Spin sum rule – not unique!

\[ S(\mu) = \frac{1}{2} \Sigma(\mu) + L_q(\mu) + \Delta G(\mu) + [J_g(\mu) - \Delta G(\mu)] \]

Intrinsic parton’s spin:
\[ \Sigma(Q^2) = \sum [\Delta q(Q^2) + \Delta \bar{q}(Q^2)] \], \quad \Delta G(Q^2)

dynamical parton motion:
\[ L_q(Q^2), \quad L_g(Q^2) \]

- Spin decomposition – at different distance scales:

Learn QCD dynamics, not much details in partonic structure!
Role of quark’s spin – twenty years’ effort

The EMC’s “Plot” is improved:

But, the puzzle remains!

\[ \Delta \Sigma(Q^2) = \sum_q \left[ \Delta q(Q^2) + \Delta \bar{q}(Q^2) \right] \ll 1 \]

Quark spin ~ 25%
Role of gluon’s spin – RHIC’s spin program

- **Definition – in terms of an non-local operator! – small x?**
  \[
  \Delta G = \int_0^1 dx \Delta G(x) = \int_0^1 \frac{dx}{xp^+} \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p, s|F^{+\mu}(0)F^{+\nu}(y^-)|p, s\rangle (-i\epsilon_{\mu\nu})
  \]

- **NLO QCD global fit - DSSV:**
  [Graph showing the distribution of x*Δg over a range of x values]
  \[
  \Delta G \approx \int_{0.001}^1 dx \Delta G(x) = -0.084
  \]
  Strong constraint on ΔG from
  \[
  0.05 \lesssim x \lesssim 0.2
  \]

- **Proton’s spin budget:**
  Quark spin ~ 25%, Gluon spin ~ 0%

PRL101,072001(2008)
New PHENIX data – Run 9

- Combine data from Run 5, 6, and 9:
Generalized parton distributions (GPDs) - quark:

\[ F_q(x, \xi, t, \mu^2) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P'| \bar{\psi}_q(\lambda/2) \frac{\gamma \cdot n}{2P \cdot n} \psi_q(-\lambda/2) | P \rangle \]

\[ = H_q(x, \xi, t, \mu^2) \left[ \bar{U}(P') \gamma^\mu U(P) \right] \frac{n_\mu}{2P \cdot n} \]

\[ + E_q(x, \xi, t, \mu^2) \left[ \bar{U}(P') \frac{i \sigma^{\mu\nu}(P' - P)_{\nu}}{2M} U(P) \right] \frac{n_\mu}{2P \cdot n} \]

with \( \xi = (P' - P) \cdot n/2 \) and \( t = (P' - P)^2 \Rightarrow -\Delta_1^2 \) if \( \xi \to 0 \)

Net quark’s orbital motion:

\[ J_q = \frac{1}{2} \lim_{t \to 0} \int dx \, [H_q(x, \xi, t) + E_q(x, \xi, t)] \]

\[ = \frac{1}{2} \Delta_q + L_q \]

Similarly, for gluon GPDs

C. Weiss’ talk on exclusive process tomorrow

Need to measure the exclusive processes – DIS is ideal
(no factorization for hadronic diffractive processes!)
Contribution from parton’s orbital motion

- Moments of GPDs on lattice:
  \[ \langle J_q^i \rangle = S^i \int dx \left[ H_q(x, 0, 0) + E_q(x, 0, 0) \right] x \]

- Ji’s relation:
  \[ L^z_q = J^z_q - \frac{1}{2} \Delta q \]

- Both \( L_u \) and \( L_d \) large:
  But, \( L_u + L_d \sim 0 \)

- Role of disconnected diagram – cloud?

  EIC is an ideal place to measure GPDs – DVCS – energy and luminosity
Challenges in determining the sum rule

“Proper” definition:

\[
S(\mu) = \frac{1}{2} \Sigma(\mu) + L_q(\mu) + \Delta G(\mu) + [J_g(\mu) - \Delta G(\mu)]
\]

◇ Confinement – No free quarks and gluons – only the “sum” is physical

◇ A good or meaningful decomposition:

Every term is connected to the physically measurable quantities with controllable approximation

Exclusive processes – GPDs:

◇ Exchange a “vacuum” quantum number

Any partonic combination with a “vacuum” quantum number can contribute

\[q\bar{q}, \ q(g)^n\bar{q}, \ gg, \ g(g)^ng, \ldots\]

◇ Both real and imaginary part of amplitude contribute

Require a large \(Q^2\) – “localized” probe, and a large range of \(Q^2\), ...
Spin as a powerful tool

- The sum rule, such as the proton’s spin budget, is interesting and important

- BUT, the $x$- and $k_T$-dependence of the distributions (the motion of a parton), and the correlation of multiple partons inside a hadron are even more interesting, more rich in dynamics

  - Transverse momentum dependent (TMD) distributions
    - 3-D motion of partons
  
  - Multi-parton correlation functions
    - Measurement of quantum interference

Spin helps to separate and extract these information

- Power of various spin asymmetries
Parton’s transverse motion

- Transverse single spin asymmetry (SSA):

\[ A(p_A, s^\uparrow) + B(p_B) \rightarrow \pi(p) + X \]

\[ A(l, \vec{s}) \equiv \frac{\Delta \sigma(l, \vec{s})}{\sigma(l)} = \frac{\sigma(l, \vec{s}) - \sigma(l, -\vec{s})}{\sigma(l, \vec{s}) + \sigma(l, -\vec{s})} \]

- Vanish without parton’s transverse motion:

A direct probe for parton’s transverse motion, Spin-orbital correlation, QCD quantum interference
SSA in collinear parton model

- SSA corresponds to a naively T-odd triple product:

\[ A_N \propto i \bar{s}_p \cdot (\vec{p} \times \vec{l}) \Rightarrow i \epsilon^{\mu\nu\alpha\beta} p_\mu s_\nu \ell_\alpha p'_\beta \]

Novanish \( A_N \) requires a phase, enough vectors to fix a scattering plan, and a spin flip at the partonic scattering

- Leading power in QCD:

\[ \sigma_{AB}(p_T, \vec{s}) \propto \begin{array}{c}
\text{diagram 1} + \text{diagram 2} + \ldots \end{array} \Rightarrow \propto \alpha_s \frac{m_q}{p_T} \]

\( A_N \) connects to parton’s transverse motion!

Kane, Pumplin, Repko, PRL, 1978
TMD parton distributions

- **Quark TMD distributions:**
  \[ \hat{k}_\mu = x p_\mu + \frac{k^2_T}{2x p^+} n_\mu + k^\mu_T \]
  \[ dk^2 dk^+ \delta(x - k^+/P^+) \]

\[ \Phi(x, k_\perp) = \frac{1}{2} \left[ f_1 h_+ + f_{1T} \right] \frac{\epsilon_{\mu\nu\rho\sigma} \gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left( S_L g_{1L} + \frac{k_\perp \cdot S_T}{M} g_{1T} \right) \gamma^5 \gamma^\mu \]
  \[ + \left( h_{1T} i \sigma_{\mu\nu} \gamma^5 n_+^\nu S_T^\mu + \left( S_L h_{1L} + \frac{k_\perp \cdot S_T}{M} h_{1T} \right) i \sigma_{\mu\nu} \right) \frac{\gamma^5 n_+^\mu k_\perp^\nu}{M} \]
  \[ + \frac{h_{1T} \sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \]

Total 8 TMD quark distributions

- **Gluon TMD distributions,** …

  Production of quarkonium, two-photon, …
Measure TMD’s

- **Sivers’ effect – Sivers’ function:**
  - Hadron spin influences parton’s transverse motion
  - Diagram showing di-jet, photon-jet not exactly back to back
  - Photons have asymmetry
  - Jet vs. Photon sign flip predicted

- **Collin’s effect – Collin’s function:**
  - Transversity
  - No asymmetry for the jet axis
  - Parton’s transverse spin affects its hadronization

- **Need TMD factorization to quantify parton transverse motion!**

  **Two-scale problem in QCD:**
  \[ Q_1 \gg Q_2 \sim \Lambda_{QCD} \]
SIDIS is ideal for studying TMDs

- SIDIS has the natural kinematics for TMD factorization:

\[ \ell(s_e) + p(s_p) \rightarrow \ell + h(s_h) + X \]

Natural event structure:
high Q and low \( p_T \) jet (or hadron)

- Separation of various TMD contribution by angular projection:

\[
A_{UT}^{\text{Collins}}(\varphi_h, \varphi_S) = \frac{1}{P} \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} \\
= A_{UT}^{\text{Collins}} \sin(\phi_h + \phi_S) + A_{UT}^{\text{Sivers}} \sin(\phi_h - \phi_S) + A_{UT}^{\text{Pretzelosity}} \sin(3\phi_h - \phi_S)
\]

\[
A_{UT}^{\text{Collins}} \propto \left\langle \sin(\phi_h + \phi_S) \right\rangle_{UT} \propto h_1 \otimes H_1^\perp \\
A_{UT}^{\text{Sivers}} \propto \left\langle \sin(\phi_h - \phi_S) \right\rangle_{UT} \propto f_{1T}^\perp \otimes D_1 \\
A_{UT}^{\text{Pretzelosity}} \propto \left\langle \sin(3\phi_h - \phi_S) \right\rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp
\]
Our knowledge of TMDs

- Sivers function from SIDIS:

  EIC can do much better job in extracting TMDs

- NO TMD factorization for hadron production in p+p collisions!

  Collins and Qiu, 2007, Vogelsang and Yuan, 2007, Mulders and Rogers, 2010, …
Critical test of TMD factorization

- TMD distributions with non-local gauge links:

\[ f_{q/h^\uparrow}(x, k_\perp, \vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{i x y^+ - i k_\perp \cdot y_\perp} \langle p, \vec{S} | \bar{\psi}(0^-) \psi(0^-, 0^-) | p, \vec{S} \rangle \]

- Parity + Time-reversal invariance:

\[ f_{q/h^\uparrow}^{\text{SIDIS}}(x, k_\perp, \vec{S}) \neq f_{q/h^\uparrow}^{\text{DY}}(x, k_\perp, \vec{S}) \]

- For a fixed spin state:

\[ f_{q/h^\uparrow}^{\text{Sivers}}(x, k_\perp)^{\text{SIDIS}} = - f_{q/h^\uparrow}^{\text{Sivers}}(x, k_\perp)^{\text{DY}} \]

It is a critical test of TMD factorization approach.
Transition from low $p_T$ to high $p_T$

- Two-scale becomes one-scale:

  \[ A_N(Q^2, p_T) \]

  \[ p_T \ll Q \quad \sim Q_s \quad p_T \sim Q \]

  TMD \quad \text{Collinear Factorization}

- TMD factorization to collinear factorization:

  Two approaches are consistent in the overlap region:

  \[ \Lambda_{QCD} \ll p_T \ll Q \]

  $A_N$ finite – requires correlation of multiple collinear partons

  New opportunities!
Collinear factorization for SSA

- Collinear factorization beyond leading power:
  \[\sigma(Q, \bar{s}) \propto t \sim 1/Q \]
  \[2 \left( \frac{\langle k_\perp \rangle}{Q} \right)^n \text{ – Expansion} \]

\[\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + \left( \frac{1}{Q} \right) H_1 \otimes f_2 \otimes f_3 + O\left( \frac{1}{Q^2} \right)\]

Too large to compete!  Three-parton correlation

- Single transverse spin asymmetry:
  \[\Delta \sigma(s_T) \propto T^{(3)}(x, x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, z) + \ldots\]

\[T^{(3)}(x, x) \propto \]

Qiu, Sterman, 1991, …

\[D^{(3)}(z, z) \propto \]

Kang, Yuan, Zhou, 2010

Integrated information on parton’s transverse motion!
$A_N$ from LO quark-gluon correlation

(FermiLab E704)  (RHIC STAR)

Kouvaris, Qiu, Vogelsang, Yuan, 2006

Nonvanish twist-3 function  $\rightarrow$  Nonvanish transverse motion
Role of color magnetic force

- **Two-sets Twist-3 correlation functions:**

  No probability interpretation!

\[
\tilde{T}_{q,F} = \int \frac{dy_1^- dy_2^\ast}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T | \bar{\psi}_q(0) \frac{\gamma^+}{2} [e^{x_1sT\sigma\vec{n}} F_{+}^{\sigma}(y_2^-)] \psi_q(y_1^-) | P, s_T \rangle
\]

\[
\tilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^\ast}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) [e^{x_1sT\sigma\vec{n}} F_{+}^{\sigma}(y_2^-)] F^{+\lambda}(y_1^-) | P, s_T \rangle (-g_{\rho\lambda})
\]

\[
\tilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^\ast}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T | \bar{\psi}_q(0) \frac{\gamma^+\gamma_5^\rho}{2} [i s_T^\sigma F_{+}^{\sigma}(y_2^-)] \psi_q(y_1^-) | P, s_T \rangle
\]

\[
\tilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^\ast}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) [i s_T^\sigma F_{+}^{\sigma}(y_2^-)] F^{+\lambda}(y_1^-) | P, s_T \rangle (i\epsilon_{\perp\rho\lambda})
\]

- **Twist-2 distributions:**

  - **Unpolarized PDFs:**

    \[ q(x) \propto \langle P | \bar{\psi}_q(0) \frac{\gamma^+}{2} \psi_q(y) | P \rangle \]

    \[ G(x) \propto \langle P | F^{+\mu}(0) F^{+\nu}(y) | P \rangle (-g_{\mu\nu}) \]

  - **Polarized PDFs:**

    \[ \Delta q(x) \propto \langle P, S_{\parallel} | \bar{\psi}_q(0) \frac{\gamma^+\gamma_5^\rho}{2} \psi_q(y) | P, S_{\parallel} \rangle \]

    \[ \Delta G(x) \propto \langle P, S_{\parallel} | F^{+\mu}(0) F^{+\nu}(y) | P, S_{\parallel} \rangle (i\epsilon_{\parallel\perp\mu\nu}) \]

- **Twist-3 fragmentation functions:**

  See Kang, Yuan, Zhou, 2010, Kang 2010
Predictive power of TMD and CO approach

- Universality of the nonperturbative functions
  
  The sign change of Sivers function is a critical test for TMD approach

- Ability to calculate and control the high order contribution
  
  Factorization naturally introduces the factorization scale dependence

  \[
  \sigma_{\text{phy}}(Q, \Lambda_{\text{QCD}}) \approx \sum_f \hat{\sigma}_f(Q, \mu) \otimes \phi_f(\mu, \Lambda_{\text{QCD}})
  \]

  \[
  \rightarrow \frac{d}{d\mu} \sigma_{\text{phy}}(Q, \Lambda_{\text{QCD}}) = 0
  \]

  Scaling violation of nonperturbative functions

  NLO contribution is critical!

  Major theory effort in studying the scale-dependence of TMDs and twist-3 correlation functions

  Boer, ...
Early surprise: a sign “mismatch”

- Sivers function and twist-3 correlation:

\[ gT_{q,F}(x, x) = - \int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^q(x, k_\perp)|_{SIDIS} + \text{UVCT} \]

- “direct” and “indirect” twist-3 correlation functions:

Calculate \( T_{q,F}(x, x) \) by using the measured Sivers functions.
Possible interpretations

- A node in $k_T$-distribution:
  - Like the DSSV’s $\Delta G(x)$
  - HERMES vs COMPASS
  - Physics behind the sign change?

EIC can measure TMDs for a wide range of $k_T$

- Large twist-3 fragmentation contribution in RHIC data:
  - If Sivers-type initial-state effect is much smaller than fragmentation effect and two effects have an opposite sign
  - Can be tested by $A_N$ of single jet or direct photon at RHIC

- A node in $x$-dependence of Sivers or twist-3 distributions

Physics behind the node if there is any

Kang, Qiu, Vogelsang, Yuan, 2011
Boer, …
Scaling violation of twist-3 correlations

- Evolution equation is a consequence of factorization:

  \[ \Delta \sigma(Q, s_T) = \frac{1}{Q} H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F) \]

  **Factorization:**

  \[ \frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F) \]

  **DGLAP for \( f_2 \):**

  \[ \frac{\partial}{\partial \ln(\mu_F)} f_3 = \left( \frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3 \]

  **Evolution for \( f_3 \):**

- Evolution kernel is process independent:

  - Calculate directly from the variation of process independent twist-3 distributions
    
    Kang, Qiu, 2009
    Yuan, Zhou, 2009

  - Extract from the scale dependence of the NLO hard part of any physical process
    
    Vogelsang, Yuan, 2009

  - Renormalization of the twist-3 operators
    
    Braun et al, 2009
Scaling violation for twist-3 correlations

- **Evolution kernels – “DGLAP”:**

  Leading order evolution kernels for all channels have been derived!

  \[
  \frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left\{ P_{qq}(z) T_{q,F}(\xi, x, \mu_F) \right. \\
  \left. + \frac{C_A}{2} \left[ \frac{1 + z^2}{1 - z} [T_{q,F}(\xi, x, \mu_F) - T_{q,F}(\xi, x, \mu_F)] + z T_{q,F}(\xi, x, \mu_F) \right] + \frac{C_A}{2} T_{\Delta q,F}(x, \xi, \mu_F) \right. \\
  \left. + P_{qq}(z) \left( \frac{1}{2} \right) \left[ T_{G,F}^{(d)}(\xi, x, \mu_F) + T_{G,F}^{(f)}(\xi, x, \mu_F) \right] \right\}
  \]

- All kernels are infrared safe
- Diagonal contribution is the same as that of DGLAP
- Quark and antiquark evolve differently – caused by tri-gluon

- **What are urgently needed:**

  NLO partonic contributions to SSA of all measureable observables!

- **A completely new domain to test QCD!**

  From paton’s transverse motion to direct QCD quantum interference
Follow DGLAP at large $x$

Large deviation at low $x$ (stronger correlation)

We have all the tools to do NLO calculations, but, need man power!
Propose new observables for ep collisions

Process:  \( e(\ell) + h(p) \rightarrow \text{jet}(p_j)(\text{or } \pi, ...) + X \)

Lepton-hadron scattering without measuring the scattered lepton

Single hard scale:  \( p_{jT} \)  in lepton-hadron frame

Complement to SIDIS:  \( e(\ell) + h(p) \rightarrow e'(\ell') + \text{jet}(p_j)(\text{or } \pi, ...) + X \)

Two scales:  \( Q, p_{jT} \)  in virtual-photon-hadron frame

Key difference in theory treatment:

Collinear factorization for  \( e(\ell) + h(p) \rightarrow \text{jet}(p_j)(\text{or } \pi, ...) + X \)

TMD factorization for  \( e(\ell) + h(p) \rightarrow e'(\ell') + \text{jet}(p_j)(\text{or } \pi, ...) + X \)

Test the consistency between TMD and Twist-3 to SSA in the same experimental setting

Jlab, Compass, Future EIC, ...
Factorization is valid:

Same as hadron-hadron collision to jet + X

\[
\frac{d\sigma^{lh\rightarrow jet(P_J)X}}{dP_{JT}dy} \approx \sum_{ab} \int dx f^a_{1/l}(x, \mu) \int dx' f^b_{1/h}(x', \mu) \frac{d\sigma^{ab\rightarrow Jet(P_J)X}}{dP_{JT}dy}(x, x', P_{JT}, y, \mu)
\]

\[a = l, \gamma, q, \bar{q}, g\]

\[b = q, \bar{q}, g\]

Leading order results:

\[
P^0 \frac{d^3\sigma}{d^3P_J} = \frac{\alpha^2_{em}}{s} \sum_a \frac{e_a^2}{(s + t)x} \left\{ f^a_1(x) H_{UU} + \lambda_l \lambda_p g^a_1(x) H_{LL} \right\}
\]

\[+ 2\pi M \varepsilon^i_j S^i_T P^j_{JT} \left[ T^a_F(x, x) - x \frac{d}{dx} T^a_F(x, x) \right] \hat{s} \cdot \hat{t} u \] \[H_{UU} \]

\[+ \lambda_l 2M \vec{S}_T \cdot \vec{P}_{JT} \left[ (\vec{g}^a(x) - x \frac{d}{dx} \vec{g}^a(x)) \hat{s} \cdot \hat{t} u \right] \]

\[
\lambda_l, \lambda_p : \text{Lepton, hadron helicity, respectively}
\]

\[
\vec{S}_T : \text{Hadron’s transverse spin vector}
\]
Numerical results

- Asymmetries:
  \[ A_{LL} = \frac{\sigma_{LL}}{\sigma_{UU}}, \quad A_{UT} = \frac{\sigma_{UT}}{\sigma_{UU}}, \quad A_{LT} = \frac{\sigma_{LT}}{\sigma_{UU}} \]

- Double spin asymmetries – very small:

Wandzura-Wilczek approximation:

\[ g_T(x) \approx \int_x^1 \frac{dy}{y} g_1(y) \quad \tilde{g}(x) \approx x \int_x^1 \frac{dy}{y} g_1(y) \]

\[ \rightarrow A_{LT} \sim 0.001 \]
Independent check of the “sign mismatch”:

Red line: $T_F(x, \mu)$ extracted from fitting SSA in hadronic collisions

Blue line: $\pi T_F(x, x) = - \int d^2k_T \frac{k_T^2}{2M^2} f_{1T}(x, k_T^2)\Big|_{DIS}$ Sivers function

Excellent test for the mechanism of SSA possibly at JLab, surely at future EIC
More on future directions

- RHIC spin, JLab at 12 GeV, possibly at Compass, ...

- Future EIC:
  - a dedicated QCD machine for the visible matter

  Yellow book on EIC physics from INT workshop is available:
  
  arXiv: submit/0295324 [nucl-th]

- Physics opportunities at EIC:
  
  - Inclusive DIS – Spin, \( F_L \), ...
  - SIDIS – TMDs, spin-orbital correlations,
  - One jet or particle inclusive – multiparton quantum correlation, ...
  - GPDs – parton spatial distributions
  - ...

- More spin physics opportunities at low energy hadron machines around the world
## Golden PDF measurements at EIC

<table>
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<th>Science Deliverable</th>
<th>Basic Measurement</th>
<th>Uniqueness Feasibility Relevance</th>
<th>Requirements</th>
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<tr>
<td>spin structure at small $x$</td>
<td>inclusive DIS</td>
<td>🟢</td>
<td>need to reach $x=10^{-4}$ large $x,Q^2$ coverage about $10^{fb^{-1}}$</td>
</tr>
<tr>
<td>contribution of $\Delta g$, $\Delta \Sigma$ to spin sum rule</td>
<td></td>
<td></td>
<td>very similar to DIS</td>
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<td>full flavor separation in large $x,Q^2$ range</td>
<td>semi-inclusive DIS</td>
<td>🟢</td>
<td>excellent particle ID</td>
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<td>strangeness, $s(x)-\bar{s}(x)$ polarized sea</td>
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<td></td>
<td>improved FFs (Belle, LHC, ...)</td>
</tr>
<tr>
<td>electroweak probes of proton structure flavor separation electroweak parameters</td>
<td>inclusive DIS at high $Q^2$</td>
<td>🟢</td>
<td>20x250 to 30x325 positron beam? polarized $^3$He beam?</td>
</tr>
</tbody>
</table>

Stratmann’s talk to EICAC review, also in INT Yellow Book
The “money” plot – inclusive DIS

- Precision of $\Delta g(x,Q^2)$:

  $$\frac{d g_1}{d \ln Q^2} \propto \Delta g(x, Q^2)$$

  Expectation:

  $$\int_0^1 dx \Delta g(x, Q^2)$$ to 10% level?

- Questions for theorists:

  ✧ Physics behind the node?
  ✧ Factorization at small-x?
  ✧ Dominance of leading power when it is so small?
  ✧ …

INT Yellow Book
Integrate over final-state hadron’s transverse momentum:

One hard scale – collinear factorization

\[ h = \pi, K, \ldots \]

Separation of parton flavors

Strangeness distributions:

NuTeV anomaly on Sin\(^2\) \(\theta_W\)

Tension with the 1st moment
QCD factorization/calculation have been very successful in interpreting HEP scattering data.

What about the hadron structure?

Not much!

Scattering with a polarized hadron beam opens up many new ways to test QCD and to study hadron structure: TMDs, GPDs, quark-gluon correlations, …

EIC is the future for spin physics.

The challenge for theorists is to indentify new, measurable, and factorizable observables that carry rich information on hadron’s partonic structure.

Thank you!
Backup slices
EIC Kinematics

- **DIS kinematics:**
  \[ Q^2 = -q^2 = x_B y S \]
  \[ x_B = \frac{Q^2}{2p \cdot q} \]
  \[ y = \frac{p \cdot q}{p \cdot k} \]
  \[ S = (p + k)^2 \]

- **EIC (eRHIC – ELIC) basic parameters:**
  - "localized" probe: \[ Q^2 \gtrsim 1 \text{ GeV} \]
  - \[ x_{\text{min}} \sim 10^{-4} \]
  - Luminosity \( \sim 100 \times \text{HERA} \)
  - Polarization, heavy ion beam, …
EIC advantages

- Inclusive DIS – Spin:

```
(\infty \to \infty)
```

Forward scattering matrix elements:

```
(\infty \to \infty)
```

- SIDIS – Best place to measure TMDs:
  - TMD Factorization
  - Naturally two very different scales: $Q, p_T$
  - Well-defined lepton-plane and hadron-plane – separation of TMDs
First hint of tri-gluon correlation

- PHENIX data on J/psi:
  - Collinear factorization:
    - tri-gluon correlation
    - direct quantum interference

- Challenges:
  - J/psi production mechanism
  - Initial- vs final-state effect
  - Connection to Gluon Sivers function

PHENIX data on open charm:

Collins, Qiu, Vogelsang, Yuan, Rogers, Mulder, ...
Gauge link – QCD phase:

Summation of leading power gluon field contribution produces the gauge link:

$$\Phi_n(\infty, y^-) = \mathcal{P} \exp \left( -ig \int_{y^-}^{\infty} d\lambda \, n \cdot A(\lambda n) \right)$$

Gauge invariant PDFs:

$$\phi(x, p, s) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p, s | \bar{\psi}(0) \hat{\Gamma}_{ji} \Phi_n^\dagger(\infty, 0) \Phi_n(\infty, y^-) \psi_i(y^-) | p, s \rangle$$

Collinear PDFs: “Localized” operator with size $\sim 1/xp \sim 1/Q$ “localized” color flow

Gauge link should be process dependent – color flow!