Physics Overview of Inclusive Electron-Nucleus Scattering at \( x > 1 \)

Donal Day
University of Virginia
**Introduction**

Inclusive electron scattering can be labeled as old-fashioned but it is clear that inclusive electron scattering from nuclei provides a rich, yet complicated mixture of physics that has yet to be fully exploited.

**Compelling Physics to be Studied**

- Momentum distributions and the spectral function $P(k,E)$.
- Short Range Correlations and Multi-Nucleon Correlations
- Scaling ($x, y, \varphi', \xi$)
- Medium Modifications -- tests of EMC; 6-quark admixtures
- Parton Recombination
- Duality
- Structure Function $Q^2$ dependence and Higher Twists
Inclusive Quasielastic and Deep Inelastic Scattering at High Momentum Transfers

Two distinct processes

- **Quasielastic from the nucleons in the nucleus**

\[
\vec{e} \rightarrow \vec{e}', M_A - \vec{k}, \quad \vec{k} + \vec{q}, \quad W^2 = M^2
\]

- **Inelastic, Deep Inelastic from the quark constituents of the nucleon.**

\[
W^2 \geq (M_n + m_\pi)^2
\]

Inclusive final state does means no separation of two dominant processes

\[ \chi > 1 \quad \chi < 1 \]
The two processes share the same initial state

QES in PWIA
\[
\frac{d^2\sigma}{dQd\nu} \propto \int dk \int dE \sigma_{ei} \left[ S_i(k, E) \delta() \right] 
\]

Spectral function

\[ n(k) = \int dE S(k, E) \]

DIS
\[
\frac{d^2\sigma}{dQd\nu} \propto \int dk \int dE W_{1,2}^{(p,n)} S_i(k, E) 
\]

Spectral function

However they have very different \(Q^2\) dependencies

\(\sigma_{ei}\) goes as the elastic (form factor)\(^2\)

\(W_{1,2}\) scale with \(\ln Q^2\) dependence

Nonetheless there is a rich, if complicated, blend of nuclear and fundamental QCD interactions available for study from these types of experiments.
The shape of the low $\nu$ cross section is determined by the momentum distribution of the nucleons.

As the momentum transfer increases inelastic scattering from the nucleons begins to dominate.

We can use $x$ and $Q^2$ as knobs to dial the relative contribution of QES and DIS.

The quasielastic peak (QE) is broadened by the Fermi-motion of the struck nucleon.

The quasielastic contribution dominates the cross section at low energy loss ($\nu$) even at moderate to high $Q^2$. 

• The shape of the low $\nu$ cross section is determined by the momentum distribution of the nucleons.
• As the momentum transfer increases inelastic scattering from the nucleons begins to dominate
• We can use $x$ and $Q^2$ as knobs to dial the relative contribution of QES and DIS.
**Short Range Correlations (SRCs)**

Mean field contributions: \(k < k_F\)
Well understood
High momentum tails: \(k > k_F\)
Calculable for few-body nuclei, nuclear matter.
Dominated by two-nucleon short range correlations

Isolate short range interactions (and SRC's) by probing at high \(p_m\)

Poorly understood part of nuclear structure

Significant fraction have \(k > k_F\)

Uncertainty in SR interaction leads to uncertainty at \(k \gg\), even for simplest systems
Short Range Correlations

For a nucleon at rest, \( x < 1 \)
as \( x = 1 \) is the elastic limit

For e-A scattering \( x \) is not so restricted; \( x > j-1 \)
where \( j \) is the number of nucleons coming together.
Recall for \( k = k_F \), \( x \leq 1.2 \).

\[ x > 1 \Rightarrow 2 \text{ nucleons close together} \]
\[ x > 2 \Rightarrow 3 \text{ nucleons close together} \]

Further, when \( j \) nucleons are close together the \( A-j \)
nucleons have little influence.

The Spectral Function with a high-\( k \) nucleon can be
represented as a sum over 2,3 ... nucleon correlations;
one must account for the CM motion of the correlation.
Short Range Correlations

In the region where correlations should dominate, large \( x \),

\[
\sigma(x, Q^2) = \sum_{j=1}^{A} \frac{1}{A} a_j(A) \sigma_j(x, Q^2)
\]

\[
= \frac{A}{2} a_2(A) \sigma_2(x, Q^2) + \frac{A}{3} a_3(A) \sigma_3(x, Q^2) + \cdots
\]

\( a_j(A) \) are proportional to finding a nucleon in a \( j \)-nucleon correlation. It should fall rapidly with \( j \) as nuclei are dilute.

\[
\sigma_2(x, Q^2) = \sigma_{eD}(x, Q^2) \quad \text{and} \quad \sigma_j(x, Q^2) = 0 \quad \text{for} \quad x > j.
\]

\[
\Rightarrow \quad \frac{2 \sigma_A(x, Q^2)}{A \sigma_D(x, Q^2)} = a_2(A) \quad \bigg|_{1 < x \leq 2}
\]

\[
\frac{3 \sigma_A(x, Q^2)}{A \sigma_{A=3}(x, Q^2)} = a_3(A) \quad \bigg|_{2 < x \leq 3}
\]

In the ratios, off-shell effects and FSI largely cancel.

\( a_j(A) \) is proportional to probability of finding a \( j \)-nucleon correlation
Short Range Correlations

\[ \frac{2 \sigma_A}{A \sigma_D} = a_2(A); \quad (1.4 < x < 2.0) \]

\( a_j(A) \) is proportional to probability of finding a \( j \)-nucleon correlation
Arguments about role of FSI

Benhar et al.: FSI includes a piece that has a weak $Q^2$ dependence

There is the cancellation of two large factors ($\approx 3$) that bring the theory to describe the data. These factors are $Q^2$ and $A$ dependent

The solution

- Direct ratios to $^2H$, $^3He$, $^4He$ out to large $x$ and over wide range of $Q^2$
- Study $Q^2$, $A$ dependence (FSI)
- Verify ratios
- Absolute Cross section to test exact calculations and FSI
- Extrapolation to NM
Sensitivity to SRC

We want to be able to isolate and probe two-nucleon and multi-nucleon SRCs.

Dotted = mean field approx.
Solid = +2N SRCs.
Dashed = +multi-nucleon.

11 GeV can reach $Q^2 = 20(13)$ GeV$^2$ at $x = 1.3(1.5)$ — very sensitive, especially at higher $x$ values.
Scaling

- Scaling refers to the dependence of a cross section, in certain kinematic regions, on a single variable. If the data scales in the single variable then it validates the assumptions about the underlying physics and scale-breaking provides information about conditions that go beyond the assumptions.

- Scaling of DIS at SLAC in 1960's in terms of the Bjorken $x$-variable provided evidence for the existence of quarks.

In a proton (or neutron), $x = \frac{Q^2}{2mv}$ is restricted $< 1$ as single quark can carry, at most, the total momentum of the proton.

- At moderate $Q^2$ inclusive data from nuclei has been well described in terms $y$-scaling, one that arises from the assumption that the electron scatters from a quasi-free nucleons.

- We expect that as $Q^2$ increases we should see for evidence (x-scaling) that we are scattering from a quark that has obtained its momenta from interactions with partons in other nucleons. These are super-fast quarks.
Scaling in Nature

Swimming speed of a freshwater fish, the dace.

Scaling: Why is animal size so important?
Kurt Schmidt Nielsen
Assumption: scattering takes place from a quasi-free proton or neutron in the nucleus.

\[ y \approx -q/2 + mv/q \]
x and ξ scaling

An alternative view is suggested when the data (deuteron) is presented in terms of scattering from individual quarks

\[ x = \frac{Q^2}{2M\nu} \]

\[ \nu W_A^2 \text{ versus } x \]

\[ \xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}} \rightarrow x \]

\[ \nu W_A^2 \text{ versus } \xi \]

\[ \nu W_A^2 = \nu \cdot \frac{\sigma^{\text{exp}}}{\sigma_M} \left[ 1 + 2\tan^2(\theta/2) \right] \cdot \left( \frac{1 + \nu^2/Q^2}{1 + R} \right)^{-1} \]
The Nachtmann variable (fraction $\xi$ of nucleon light cone momentum $p^+$) has been shown to be the variable in which logarithmic violations of scaling in DIS should be studied.

Local duality (averaging over finite range in $x$) should also be valid for elastic peak at $x = 1$ if analyzed in $\xi$.

$$F_2^A(\xi) = \int_{\xi}^{A} dz F(z) F_2^n(\xi/z)$$
\[ F(y) = \frac{\sigma^{\exp}}{Z\sigma_p + N\sigma_n} \cdot K \]

Preliminary Results - Deuteron

\[ \nu W_A^2 \text{ versus } x \]

\[ \nu W_A^2 \text{ versus } \xi \]
$F(y) = \frac{\sigma^{\text{exp}}}{(Z\sigma_p + N\sigma_n)} \cdot K$

$\nu W^A_2$ versus $x$

$\nu W^A_2$ versus $\xi$
Preliminary Results - $^{12}\text{C}$

$$F(y) = \frac{\sigma^{\text{exp}}}{(Z\sigma_p + N\sigma_n)} \cdot K$$

$\nu W^A_2$ versus $x$

$\nu W^A_2$ versus $\xi$
Preliminary Results - $^{197}$Au

\[ F(y) = \frac{\sigma^{\text{exp}}}{(Z \tilde{\sigma}_p + N \tilde{\sigma}_n)} \cdot K \]
Medium Modifications generated by high density configurations

Nucleons are already closely packed in nuclei
Ave. separation ~1.7 fm in heavy nuclei
nucleon charge radius ~ 0.86 fm

Nucleon separation is limited by the short range repulsive core

Even for a 1 fm separation, the central density is ~4x nuclear matter

Comparable to neutron star densities!

High enough to modify nucleon structure?
Sensitivity to non-hadronic components

Valence quark distribution

\[ x < 1 \]

5% 6-quark bag

\[ x > 1 \]

5% 6-quark bag

Ratio: With/Without

\[ \frac{F_{2}}{F_{2 \text{ only p+n}}} \]

Ratio: With/Without

\[ \frac{F_{2 \text{ with 6q}}}{F_{2 \text{ only p+n}}} \]
Quark distributions at $x > 1$
Two measurements (very high $Q^2$) exist so far:

**CCFR (ν-C):** $F_2(x) \propto e^{-sx}$  
$s = 8$

**BCDMS (μ-Fe):** $F_2(x) \propto e^{-sx}$  
$s = 16$

Limited $x$ range, poor resolution
Limited $x$ range, low statistics

With 11 GeV beam, we should be in the scaling region up to $x \approx 1.4$
Quark Distribution Functions

Deuterium

\[ \xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}} \rightarrow x \]

\[ \nu W_2^A \text{ versus } \xi \]
Approach to Scaling - Deuteron

Dashed lines are arbitrary normalization (adjusted to go through the high $Q^2$ data) with a constant value of $\frac{d\ln(F_2)}{d\ln(Q^2)}$

Filled dots - this experiment

Next slide
Approach to Scaling (Deuteron)

Scaling appears to work well even in regions where the DIS is not the dominate process. We can expect that any scaling violations will melt away as we go to higher $Q^2$.

Convolution model
QES
RR ($W^2 < 4$)
DIS ($W^2 > 4$)
Approach to Scaling (Carbon)

**Convolution model**
- QES
- RR \((W^2 < 4)\)
- DIS \((W^2 > 4)\)

**Scaling appears to work well even in regions where the DIS is not the dominate process**

We can expect that any scaling violations will melt away as we go to higher \(Q^2\)

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**Figure**
- Preliminary E02-019
- \(\theta = 32, E = 5.766\)
- Carbon

**Graph**
- Cross section vs. \(\xi\)
- Data points and fitted curves for different Q^2 values (5.2 GeV/c^2 and 7.4 GeV/c^2)
- Comparison of QES, RR, and DIS regions.
Predictions for 11 GeV

Quark distributions at \( x > 1 \)

13.2 (GeV/c)^2

Deuteron is worst case as narrow QE peak makes for larger scaling violations

17.3 (GeV/c)^2
Convolution model
QES
DIS + RR

Quark distributions at $x > 1$

Predictions for 11 GeV

$13.2 \text{(GeV/c)}^2$

$17.3 \text{(GeV/c)}^2$
Inclusive DIS at $x > 1$ at 12 GeV

- New proposal for next JLAB PAC
- Extend measurements to large enough $Q^2$ to fully suppress the quasielastic contribution
- Extract structure functions at $x > 1$
- $Q^2 \approx 20$ at $x=1$, $Q^2 \approx 12$ at $x = 1.5$
Kinematic range to be explored

- Black - 6 GeV
- Red - CLAS
- Blue - 11 GeV

- SRC, n(k), FSI, $\sigma$
- Super-fast quarks, quark distribution functions, medium modifications
- HMS, SHMS, $Q^2 (\text{GeV}/c)^2$, $x$

Black - 6 GeV, red - CLAS, blue - 11 GeV
Summary

• High $Q^2$ scattering at $x>1$ holds great promise and is not nearly fully exploited.
• Window on wide variety of interesting physics.
• Provides access to SRC and high momentum components through $y$-scaling, ratios of heavy to light nuclei, $\varphi'$-scaling
• Testing ground for EMC models of medium modification, quark clusters, and other non-hadronic components
• Moment analysis of structure functions
• DIS is does not dominant over QES at 6 GeV but should be at 11 GeV and at $Q^2 > 10 - 15 \ (GeV/c)^2$.
• Experiments are relatively straightforward. JLAB at 12 GeV will significantly expand the coverage in $x-Q^2$
Quasielastic Electron Nucleus Scattering Archive

Welcome to Quasielastic Electron Nucleus Scattering Archive

In connection with a review article (Quasielastic Electron-Nucleus Scattering, by O. Benhar, D. Day and I. Sick) to be submitted to Reviews of Modern Physics, we have collected here an extensive set of quasielastic electron scattering data in order to preserve and make available these data to the nuclear physics community.

We have chosen to provide the cross section only and not the separated response functions. Unless explicitly indicated the data do not include Coulomb corrections.

Our criteria for inclusion into the data base is the following:

2. Radiative corrections applied to data.
3. No known or acknowledged pathologies

At present there are about 600 different combinations of targets, energies and angles consisting of some 19,000 data points.

In the infrequent event that corrections were made to the data after the original publications, we included the latest data set, adding an additional reference, usually a private communication.

As additional data become known to us, we will add to the data sets.

If you wish to be alerted to changes in the archive or to the inclusion of new data, send an email to me. Send any comments or corrections you might have as well.