SIDIS Overview

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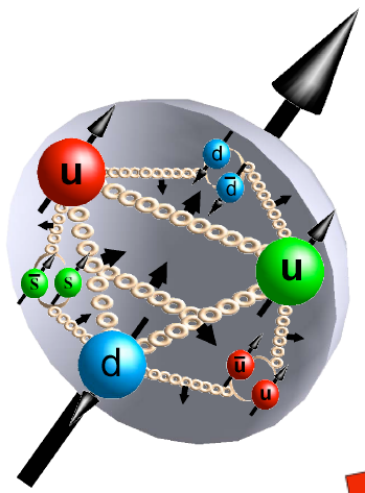
Summer Workshop on Physics Program at Hall C
Jefferson Lab, Newport News, VA, August 27, 2010

Exploring the Nucleon

Know what we are made of!
Building blocks of all known elements

Exploring QCD and strong interaction
Confinement,
Lattice QCD,
Asymptotic freedom
Perturbative QCD,…

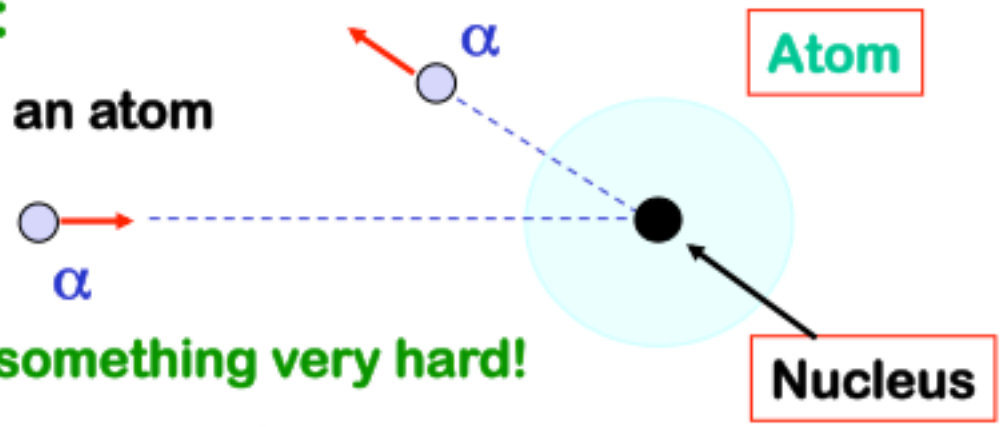


Tool for discovery:
Colliding high energy nucleons
LHC, Tevatron,
RHIC, HERA, …

Fundamental properties:
Spin decomposition,
3D quark-gluon structure,
Quark-gluon correlation,
Role of glue,
Color distribution inside a nucleus.
…

"see" substructure of a nucleon?

- **Rutherford experiment:**
  - to see the substructure of an atom

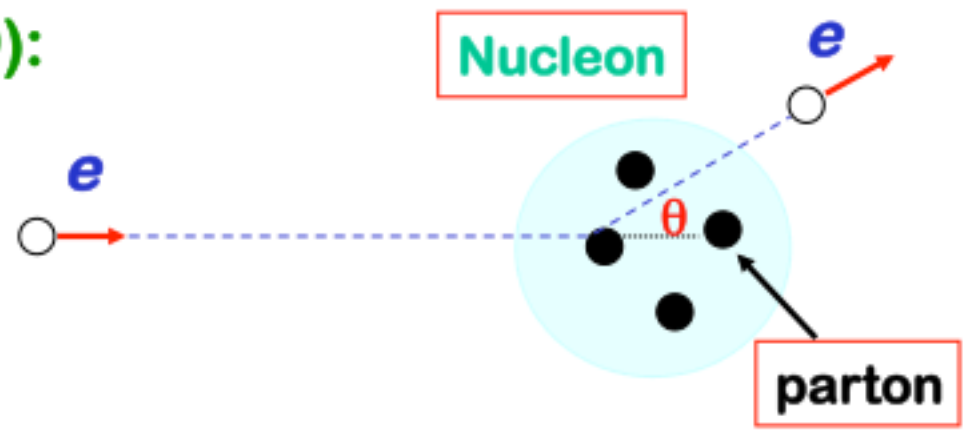


High energy α bounce off something very hard!

→ Discovery of nucleus inside an atom

- **SLAC experiment (1969):**

Lepton-nucleon deeply inelastic scattering (DIS)



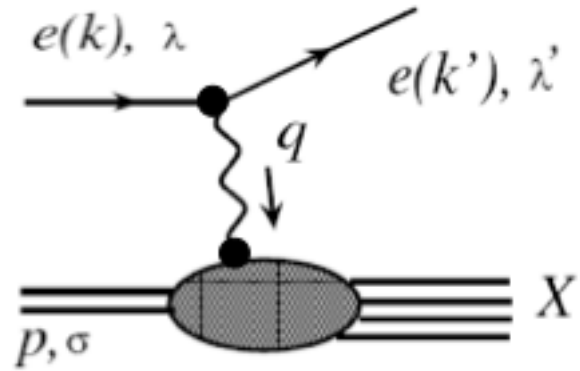
Scattering information on the θ-distribution

→ Discovery of the point-like spin-1/2 "partons"

Callan-Gross relation

Inclusive lepton-hadron DIS

- **Cross section:**



$$E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$

- **Hadronic tensor:**

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4z\ \mathrm{e}^{iq\cdot z}\ \langle p,S|J_\mu^\dagger(z)J_\nu(0)|p,S\rangle$$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1(x_B,Q^2) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2(x_B,Q^2)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1(x_B,Q^2) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2}g_2(x_B,Q^2)\right]$$

- **Structure functions – infrared sensitive:**

$$F_1(x_B,Q^2), F_2(x_B,Q^2), g_1(x_B,Q^2), g_2(x_B,Q^2)$$

NO detailed QCD dynamics needed to derive this decomposition!

QCD at work

- **QCD is a quantum field theory of quarks and gluons:**

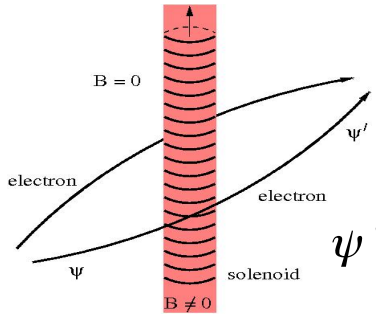
  But, no free quarks and gluons have ever been detected!

- **QCD Asymptotic freedom:**

  Perturbative QCD works for short-distance infrared safe quantities!

- **QCD is a gauge theory:**

  A phase or a gauge link is always associated with the motion of a charge particle!



$$\psi' = e^{ie\int ds.A}\psi$$

- **PQCD cannot calculate cross section with identified hadron:**

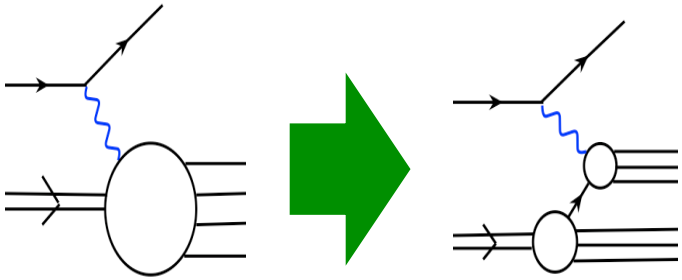
  Dynamics at hadronic scale cannot be calculated perturbatively!

- **The question:**

  How to test QCD – a theory of quarks and gluons and to map out hadron structure without seeing the quarks and gluons?

Connecting hadrons to the partons

❑ QCD factorization – separation of momentum scales:



+…

(Diagrams with more active partons from each hadron!)

❑ Leading Power – an approximation:

Single active parton from each hadron!

Short-distance – probability

Power corrections – parton correlations

$$\sigma_{\text{phy}}(Q,1/R) \sim \widehat{\sigma}(Q) \otimes \varphi(1/R) + O(1/QR)$$

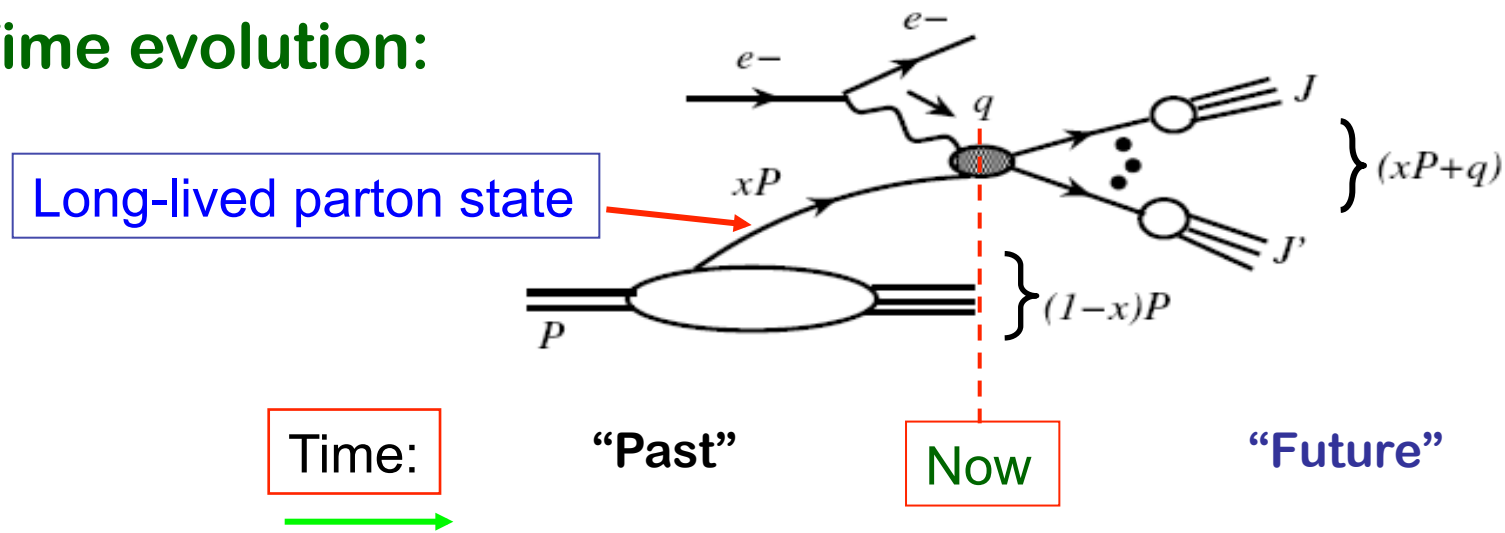
Measured

Long-distance – probability

Factorization ⟷ needs a "long-lived" parton state

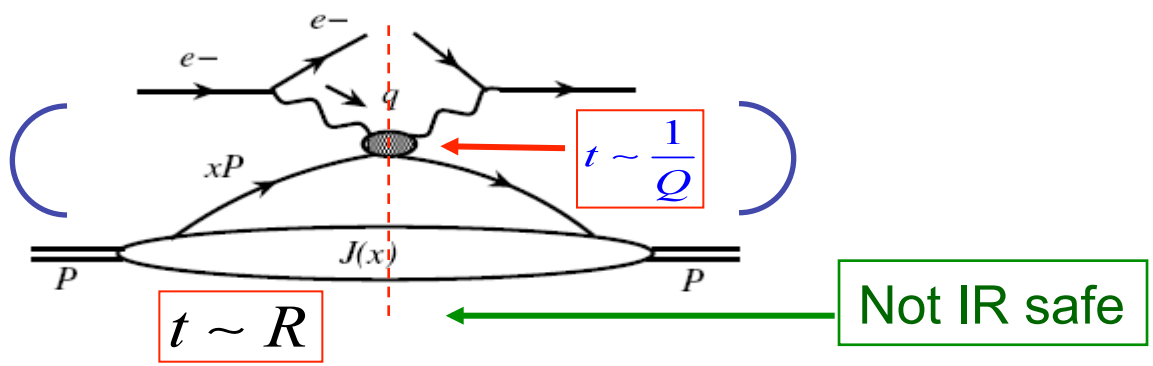
Picture of factorization for DIS – one hadron

❑ Time evolution:



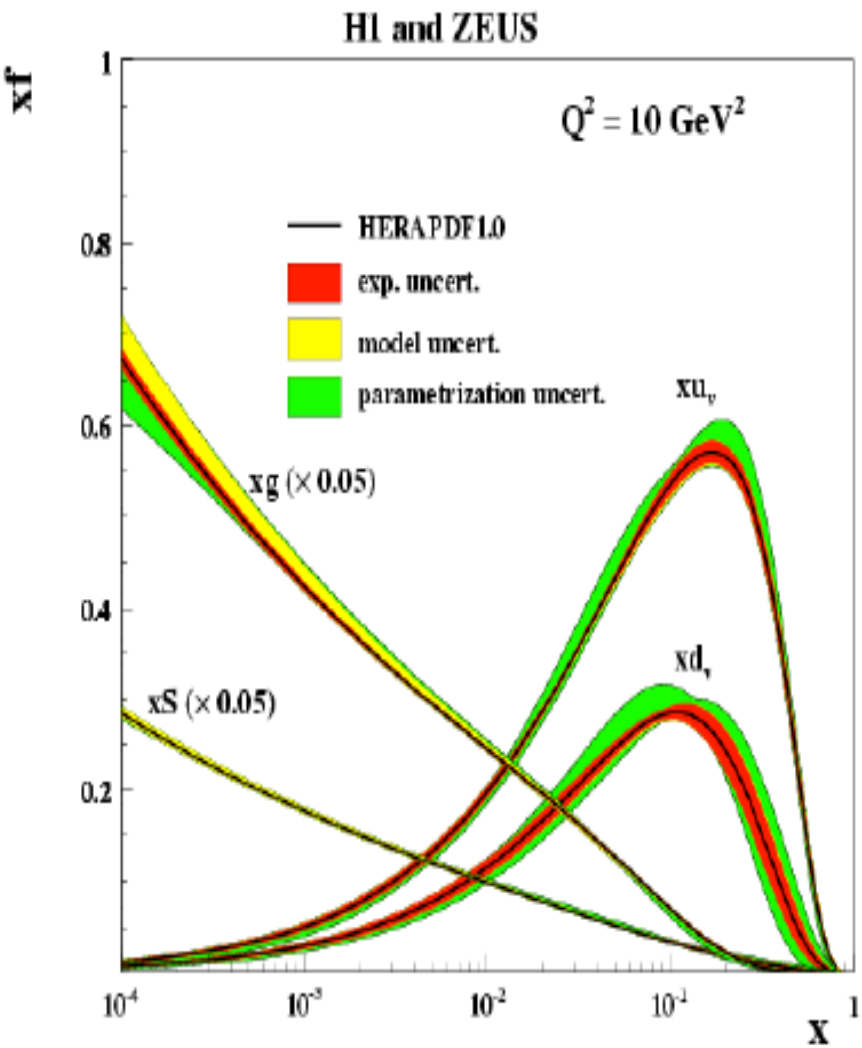
❑ Inclusive cross section – summing over all hard jets:

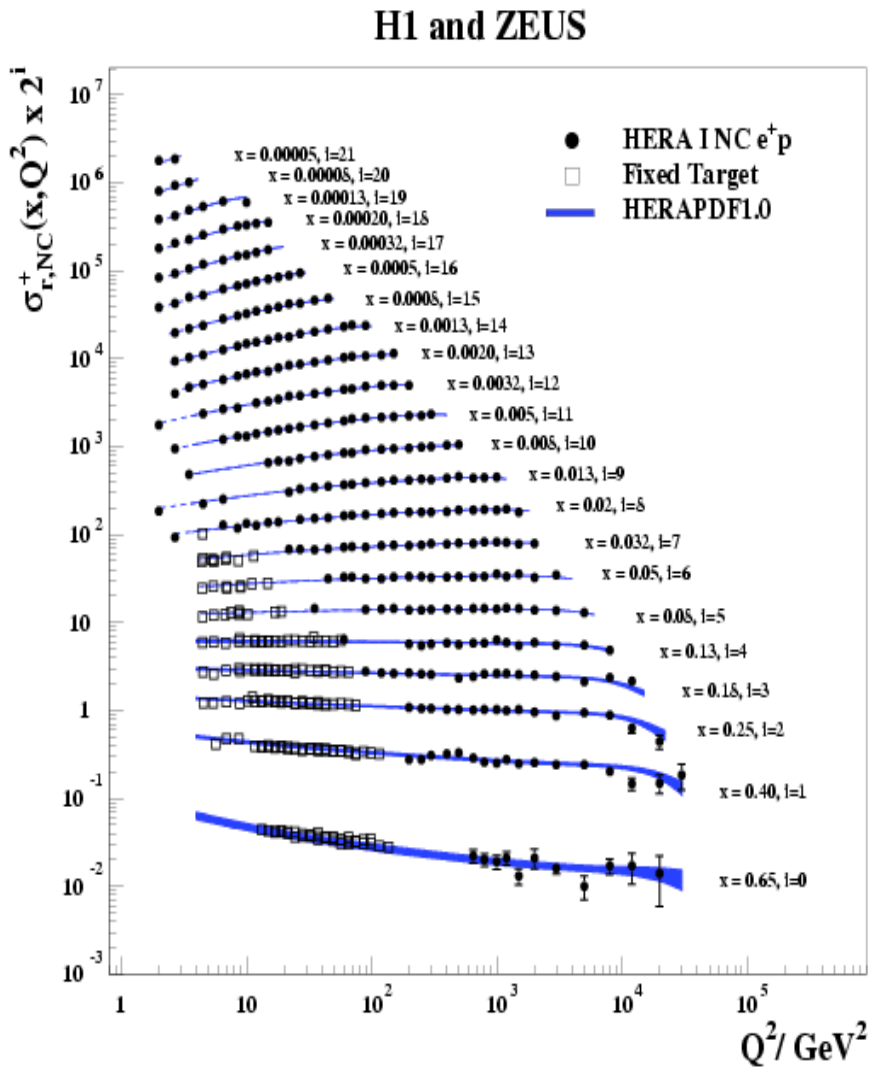
$$\sigma_{\rm tot}^{\rm DIS} \propto \mathrm{Im}\left(\right.$$



$$= \sum_f \int dx\, \phi_f(x, \mu_F^2)\, \hat{\sigma}_f(x_B/x, Q^2/\mu_F^2, \alpha_s(\mu)) + \mathcal{O}(1/Q)$$

PDFs from HERA: H1 + ZEUS



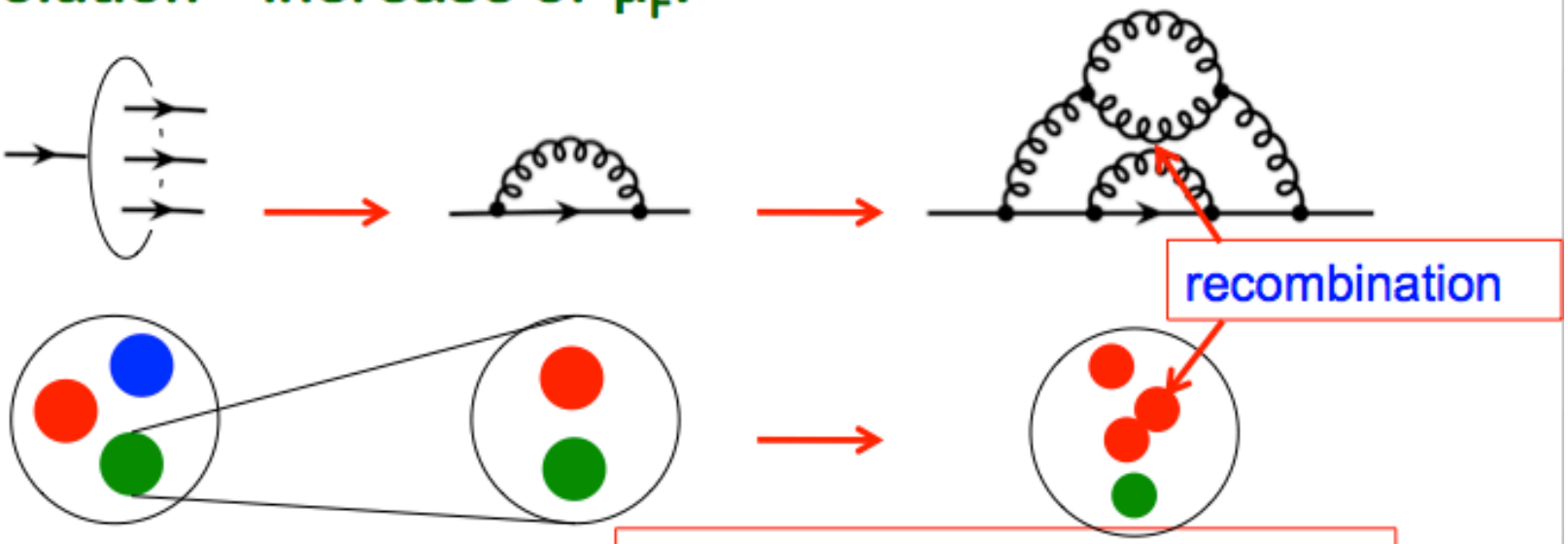


O(1%) precision for $10 < Q^2 < 100\ GeV^2$

JHEP 1001:109,2010

Factorization scale dependence

- **Resolution – increase of μ_F:**

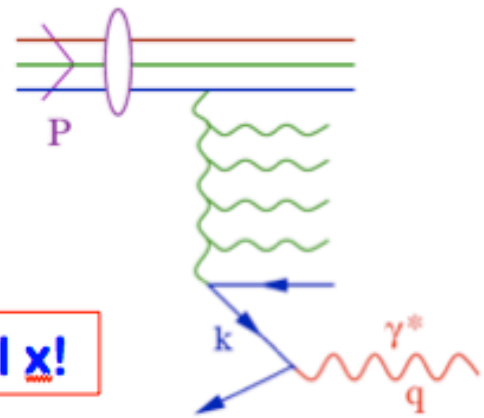


More partons at a smaller $1/\mu_F$

- **DGLAP evolution:**

$$\frac{\partial \phi_g(x,\mu^2)}{\partial \ln(\mu_F^2)} = P_{gg}(x) \otimes \phi_g(x,\mu^2) + \ldots$$

$$P_{gg}(x) \propto \frac{1}{x}$$



Large number gluons (sea quarks) at small x!

Negative glue at a low Q^2

- Negative gluon density at low x and low Q

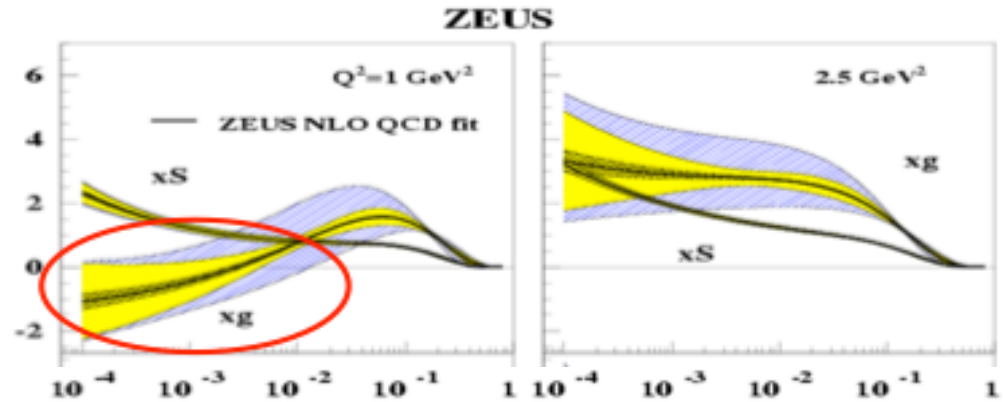
Does it mean that we have no gluon for $x < 10^{-3}$ at 1 GeV?

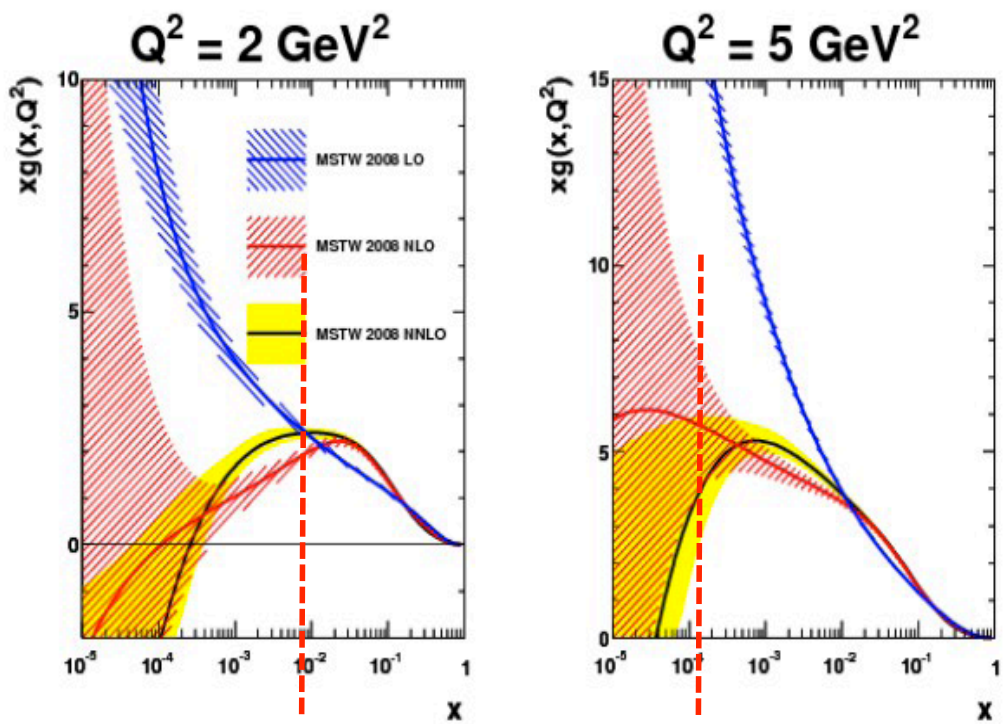
No!

- DGLAP evolution:

$$P^{(1)}_{g/g}(x) \propto \frac{1}{x}$$

Gluon recombination slows down small-x evolution



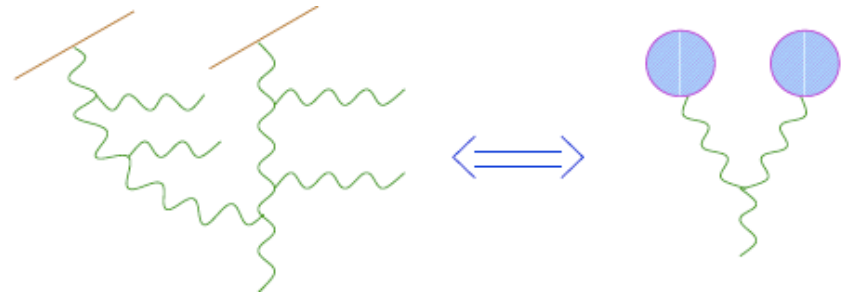


RHIC LHC MSTW 2008

Parton recombination

❑ Parton recombination:

Gribov, Levin and Ryskin, 83



❑ Modified evolution:

Mueller, Qiu, 86

$$\frac{\partial \phi_g(x,\mu^2)}{\partial \ln(\mu^2)} = P_{gg}(x) \otimes \phi_g(x,\mu^2) - \frac{C}{Q^2 R^2} \mathcal{P}_{ggg}(x) \otimes [\phi_g(x,\mu^2)]^2 + \ldots$$

Only valid when the 2nd term is relatively small

Slow down the evolution
Prevent the gluon density to become negative

❑ Power corrections:

$$F(x_B, Q^2) = \sum_f c_f^{(2)}(x_B/x, Q^2/\mu^2) \otimes \phi_f(x,\mu^2) + \frac{1}{Q^2} c_f^{(4)}(x_B/x, Q^2/\mu_F^2) \otimes \phi_f^{(4)}(x,\mu^2) + \ldots$$

Saturation - Color Glass Condensate

- **Saturation:** **Radiation = Recombination**

Mueller, Qiu, 86

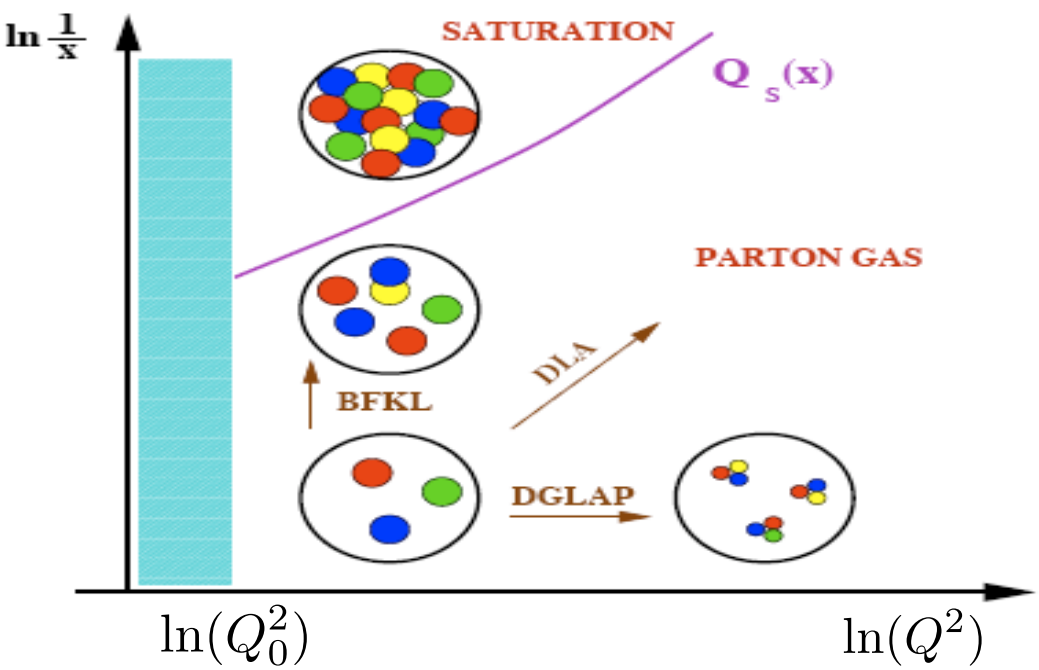
Estimate: $$1 \approx \frac{8\pi\alpha_s}{3Q^2R^2}\, xG(x,Q^2)$$

McLerran, Venugopalan, 94
And …

- **Saturation scale:** $$Q_s^2(x) \simeq \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2} \sim \frac{1}{x^\lambda}$$

$Q^2 \gg Q_s^2(x)$: Dilute regime (rapid growth: BFKL, DGLAP)

$Q^2 \lesssim Q_s^2(x)$: Saturation: $n \sim 1/\alpha_s$ (large but constant)



✧ **CO gluon distribution is scheme dependent!**

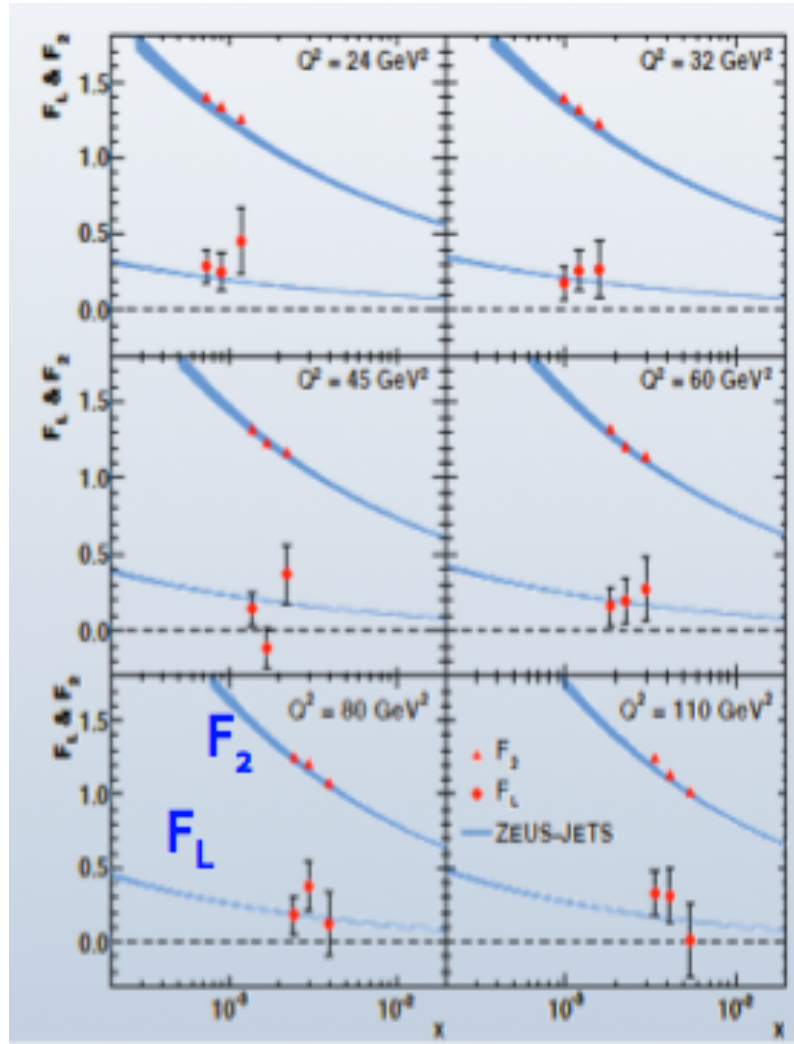
NO gluonic contribution to F_2 in DIS scheme!

✧ **DIS is too inclusive to separate the various flavor contribution**

Measurement of F_L

- F_L starts at $O(\alpha_s)$:

$$F_L = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{dz}{z^3} \left[\frac{16}{3} F_2(z) + 8 \sum_q e_q^2 \left(1 - \frac{x}{z}\right) z g(z) \right]$$



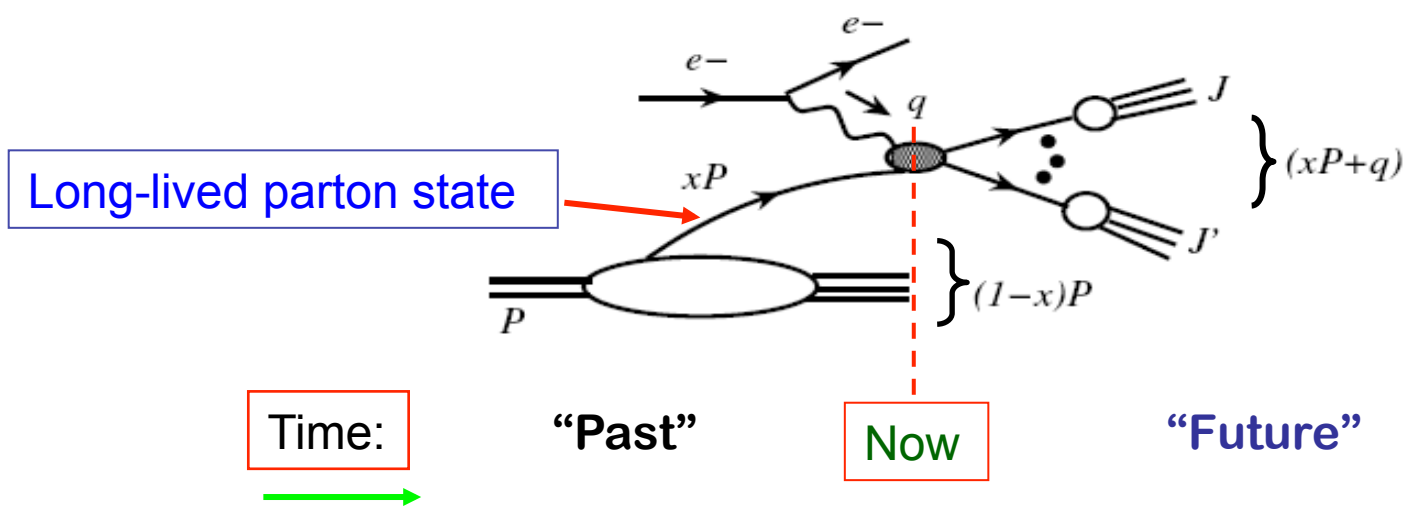
HERA data does not give new additional constraints on gluon

SIDIS could help!

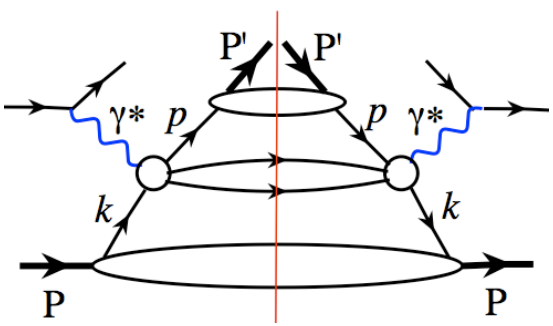
F_2(charm/bottom), two jets, …

Semi-inclusive DIS

- **SIDIS = DIS with an identified final-state**



- **Single hadron SIDIS cross section:**

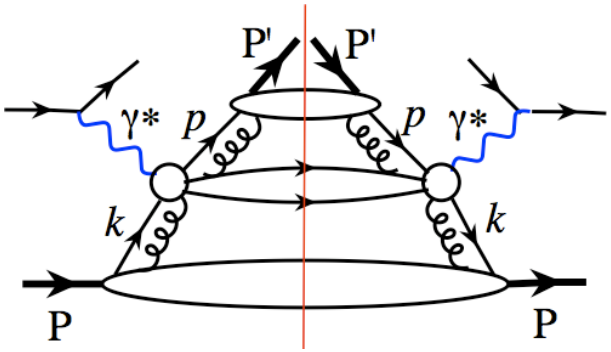


$$d\sigma_{\gamma^* h \to h'} \propto \phi_{f/h} \otimes d\hat{\sigma}_{\gamma^* f \to f'} \otimes D_{f' \to h'}$$

- **"Total c.m. energy":** $s_{\gamma^* p} = (p+q)^2 \approx Q^2 \left[\frac{1-x_B}{x_B} \right] \approx \frac{Q^2}{x_B}$

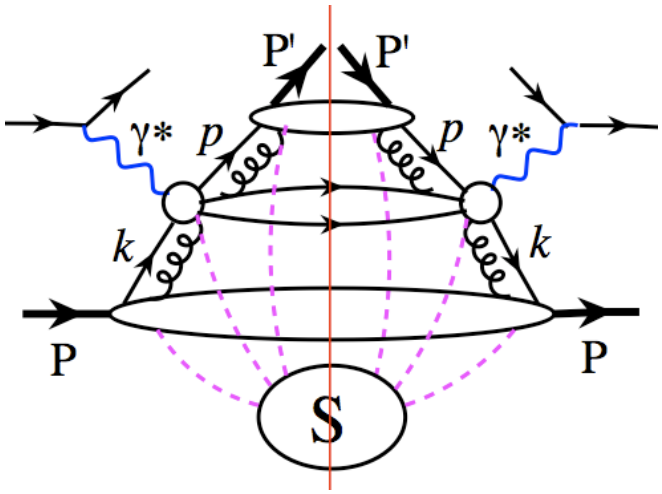
QCD Factorization

❑ Collinear gluons:



Collinear longitudinally polarized gluons do not change the collinear collision kinematics

❑ Soft interaction:

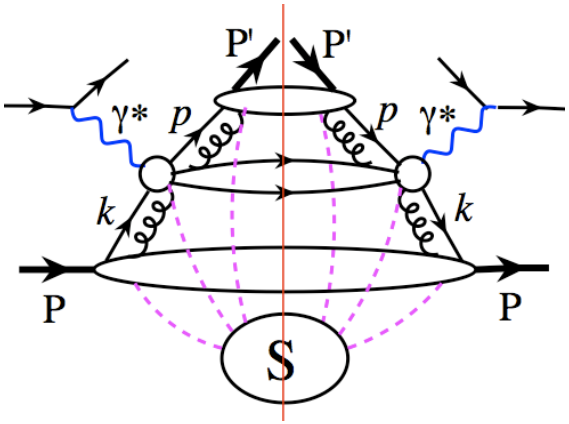


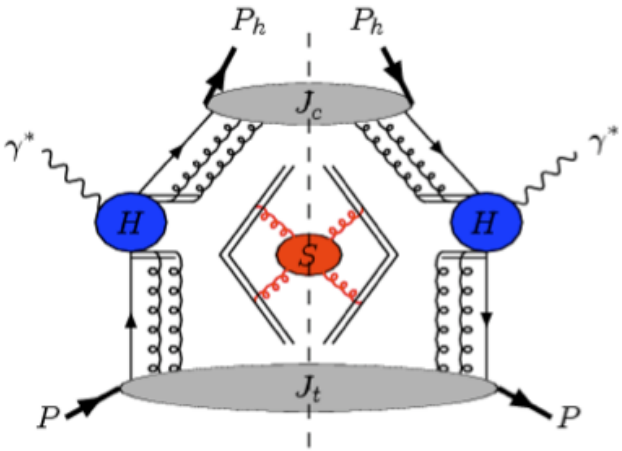
If the interaction between two jet functions can resolve the "details" of the jet functions, the jet functions could be altered before hard collision – break of the factorization

QCD factorization for SIDIS

❑ Factorization:

Ji, Ma, Yuan





❑ Low P_{hT} – TMD factorization:

$$\sigma_{\rm SIDIS}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f \otimes \mathcal{D}_{f\to h} \otimes \mathcal{S} + \mathcal{O}\left(\frac{P_{h\perp}}{Q}\right)$$

❑ High P_{hT} – Collinear factorization:

$$\sigma_{\rm SIDIS}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

❑ P_{hT} Integrated - Collinear factorization:

$$\sigma_{\rm SIDIS}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

Advantages of SIDIS

- **SIDIS complements to DIS:**

  Flavor separation of partonic contribution to inclusive DIS

  Example: F2(charm) – direct information on the glue

- **Needs unknown fragmentation functions:**

  Opportunity to study hadronization

  Better controlled probe for nuclear medium properties

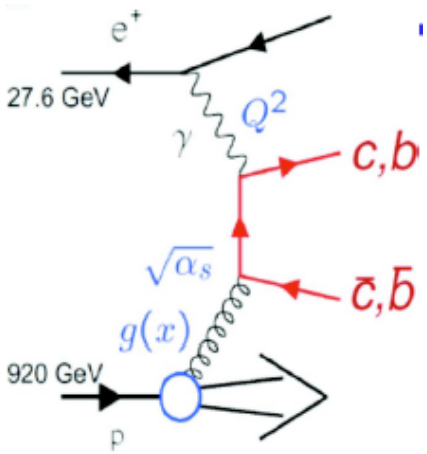
- **Leading power TMD factorization works:**

  Opportunity to study parton transverse motion directly

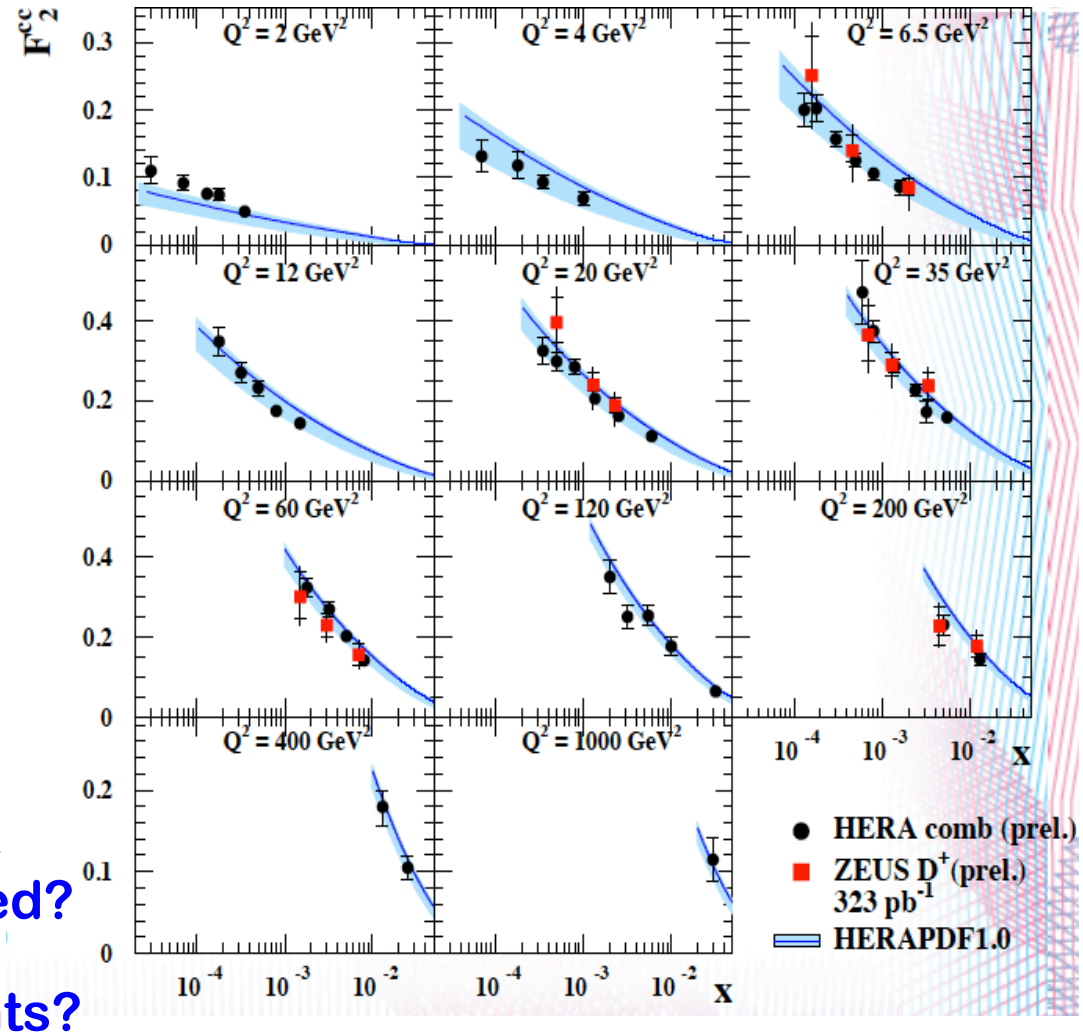
  Complements to Drell-Yan and e+e- to two jets (hadrons) for TMD distributions. Actually, is better (knowing the lepton plane exactly)

Direct information gluons

❑ Heavy flavor production in SIDIS:



H1+ZEUS



Gluon continues to grow at $x=10^{-5}$ and $Q^2=2$ GeV^2

No sign of saturation!

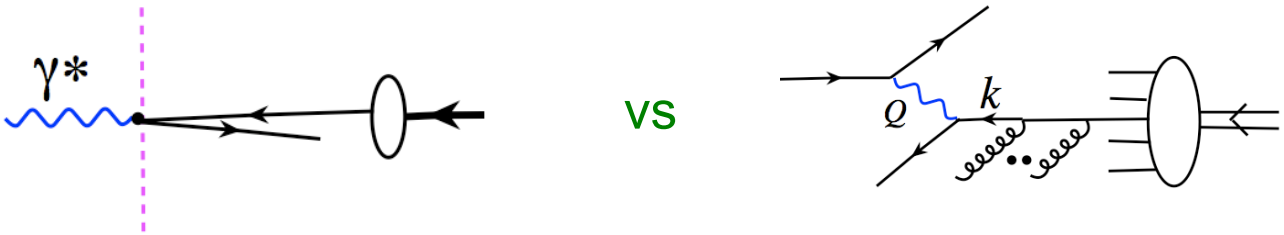
What would happen if a nuclear target/beam is used?

What about two-jets events?

Single hadron production at low p_T

❑ Unique kinematics - unique event structure:

Briet frame: Large Q^2 virtual photon acts like a "wall"



High energy low p_T jet (or hadron) - ideal probe for parton's transverse motion!

See Schweitzer's talk

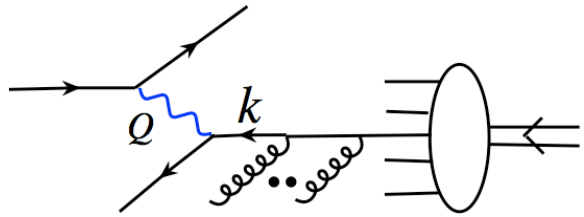
- ✧ Hard scale: Q assures a hard collision and pQCD calculation
- ✧ Soft Scale: p_T probes parton's transverse momentum at the collision point

❑ Parton's transverse momentum at the hard collision:

- ✧ is not equal to 1/fm – typical scale in hadron wave function
- ✧ Gluon shower from both initial state and final-state partons, and soft interaction between them can all change the p_T

Gluon shower in QCD and resummation

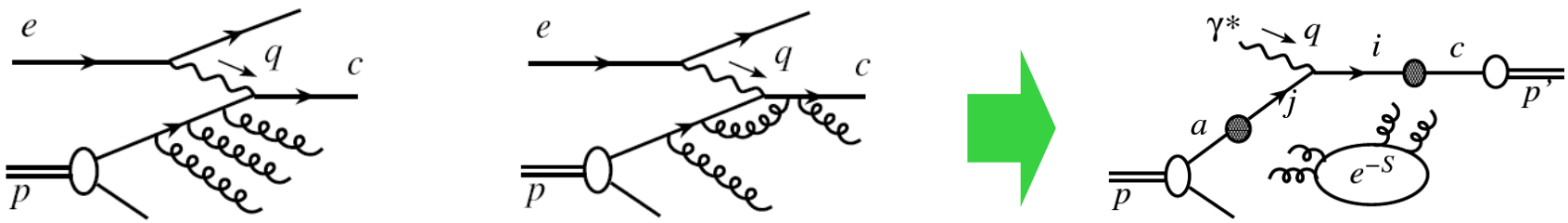
- **When q_T is small, fixed order calculation diverges:**



LO: $\frac{\alpha_s}{q_T^2}\left[a + b\log(Q^2/q_T^2)\right] \to \infty$ as $q_T^2 \to 0$

initial-state and final-state soft gluon radiations generate large logarithms: $\frac{1}{q_T^2}\alpha_s^n \log^{2n-1}(Q^2/q_T^2)$

- **QCD resummation:**

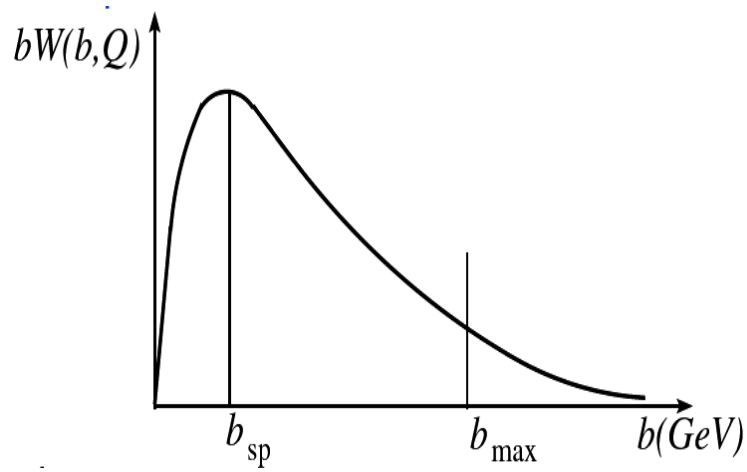


Calculation in the b-space

❑ Resummed x-section:

$$\frac{d\sigma^{(\mathrm{resum})}_{A\to h}}{dx_B\, dQ^2\, dz\, dq_T^2} \propto \int \frac{d^2 b}{(2\pi)^2}\, \mathrm{e}^{i\vec{q}_T\cdot\vec{b}}\, W(b,x,z,Q)$$

$$W = \begin{cases} W^{\mathrm{pert}}(b,x,z,Q) & b \le b_{max} \\ W^{\mathrm{pert}}(b_{max},x,z,Q)\, F^{NP}(b,Q;b_{max}) & b > b_{max} \end{cases}$$



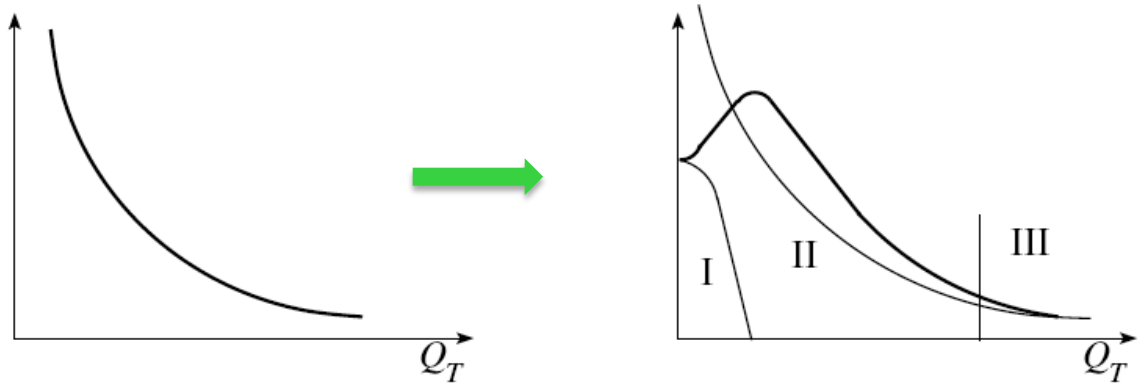
$$W^{\mathrm{pert}}(b,x,z,Q) = \sum_j e_j^2 \left[f_{a/A} \otimes C^{in}_{a\to j}\right] \left[C^{out}_{j\to c} \otimes D_{b\to h}\right] \times \mathrm{e}^{-S(b,Q)}$$

❑ Features:

- Sudakov form factor $\rightarrow\ b_{sp} \propto (\frac{\Lambda_{\mathrm{QCD}}}{Q})^{\lambda}, \lambda \sim 0.5$
- evolution of $f_{a/A}$ and $D_{c\to h}$ also moves b_{sp}
  smaller $\xi \Rightarrow \mu\frac{\partial}{\partial\mu} f_{a/A}(\xi) > 0 \Rightarrow$ lower b_{sp}

Resummed p_T distribution

❑ Remove the divergence:



❑ Features:

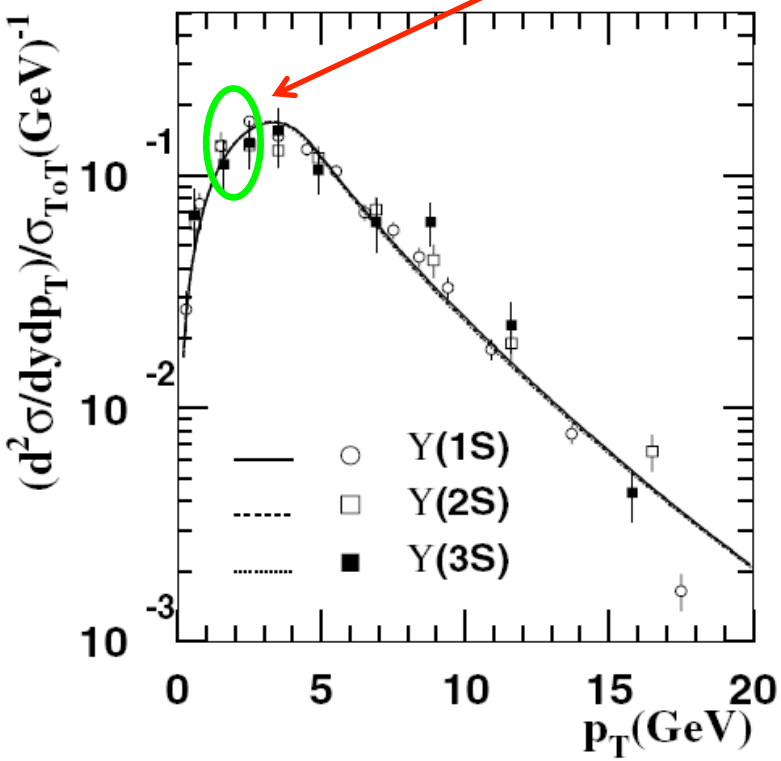
- (I): dominated by intrinsic k_T (Gaussian type)
- (II): pQCD soft-gluon resummation ($q_T \leq Q$)
- (III): pQCD fixed order calculation ($q_T \sim Q$)
- relative size of three regions depend on Q^2 and S
- large Q^2 and large $S \Rightarrow$ smaller region (I)
- smaller $Q^2 \rightarrow$ smaller logs $\rightarrow$ smaller region (II)

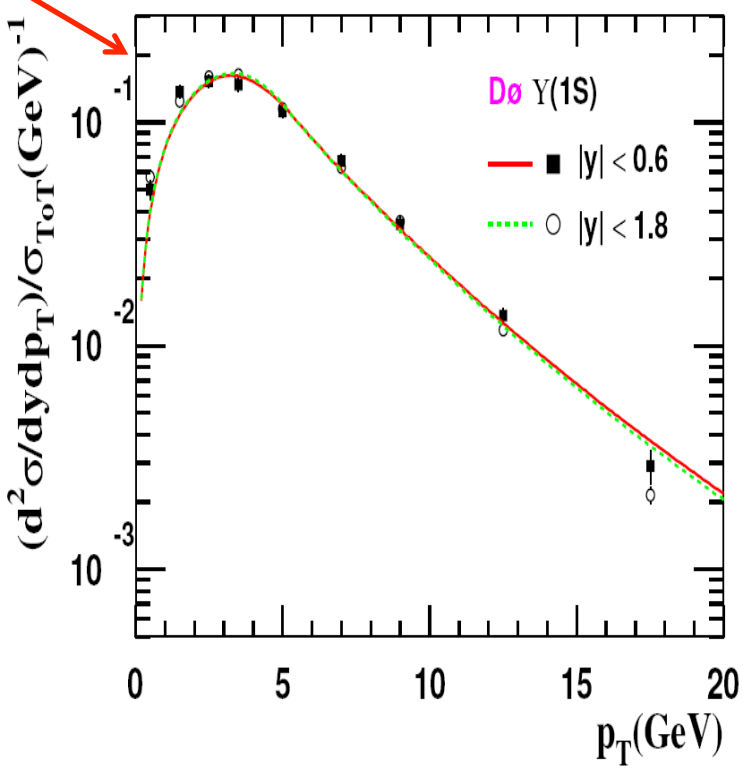
Works for heavy boson production

- **Upsilon at Tevatron:**

Berger, Qiu, Wang, 2005

Dominated by perturbative small-b contribution in its Fourier conjugate space





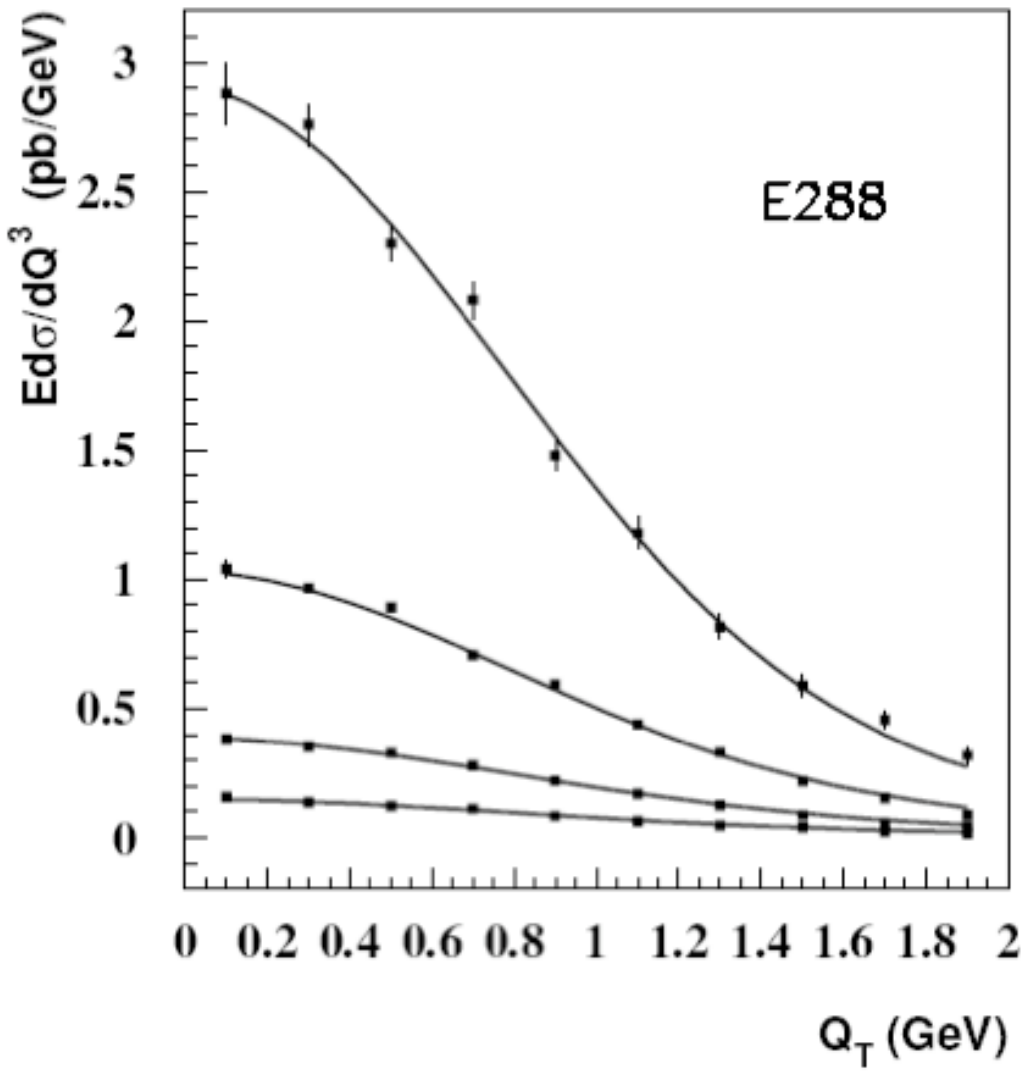
- **Works better for W/Z, also works for Drell-Yan, …**

Qiu, Zhang, 2001

Comparison with Fermilab Drell-Yan data

- **Low energy data:**

Qiu, Zhang, 2001



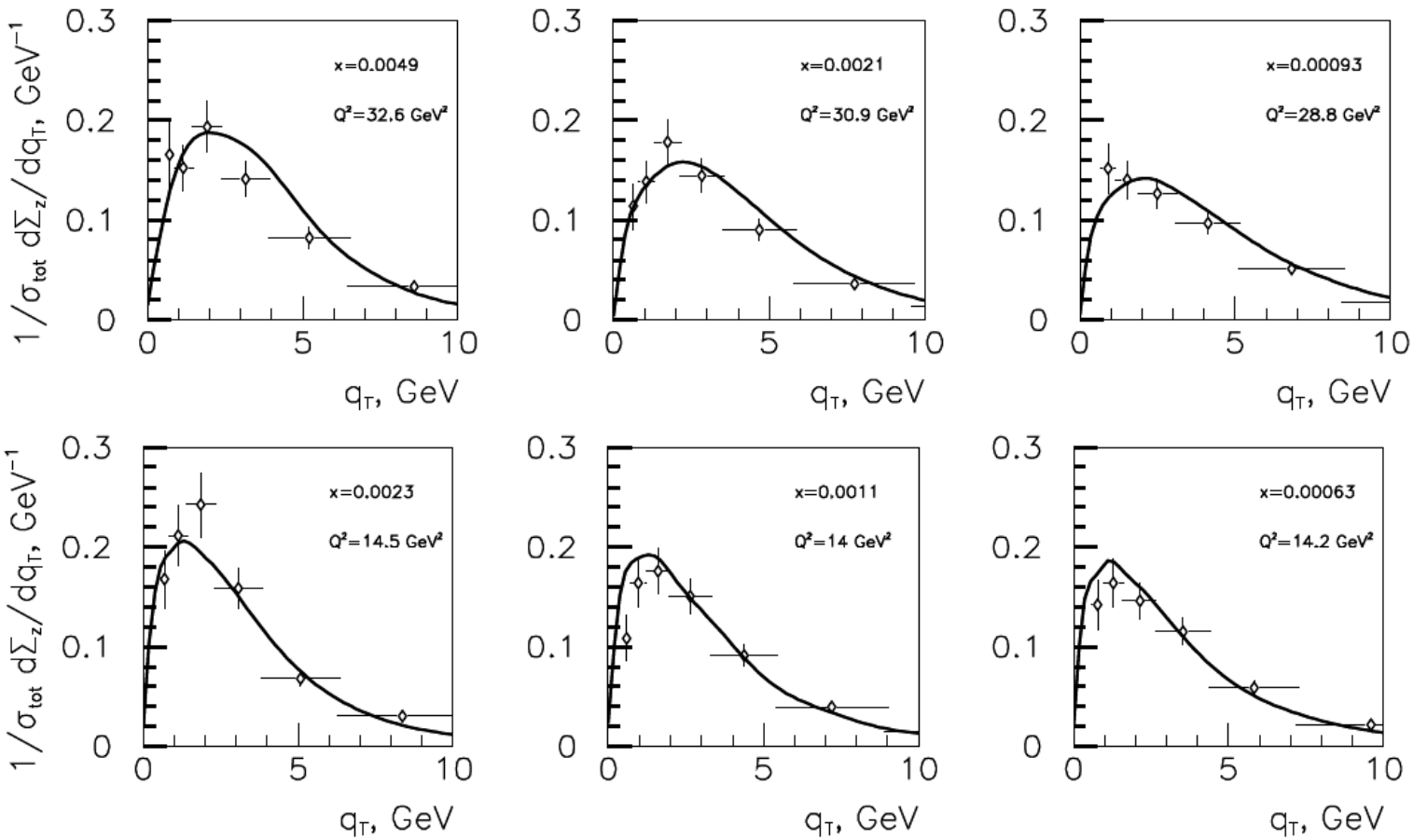
E288 data
P_{beam} = 400 GeV

Much larger power corrections

Also works for HERA data

$$\sum \int dz\, z\, \frac{d\sigma_{A\to h}}{dx_B\, dQ^2\, dz\, dq_T^2}$$

Nadolsky, et al, 1999, 2001



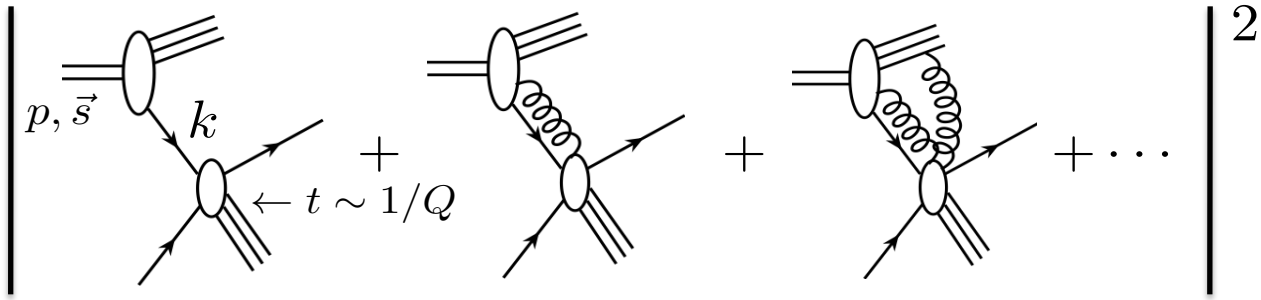
It will be interested to see if it works for Jlab energies and future EIC

Parton's transverse momentum distributions

Going beyond LP collinear factorization

❑ We measure cross sections – single hard scale:

$\sigma(Q,\vec{s}) \propto$



$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

Too large to compete?

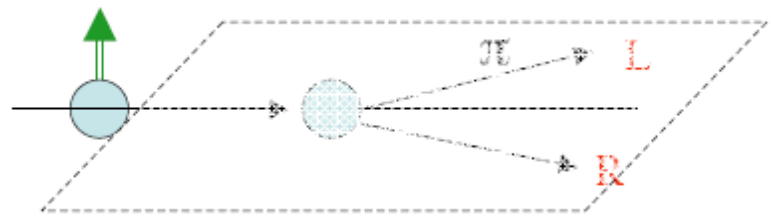
❑ Explore new observables:

- **Spin asymmetry:** $\sigma_{AB}(Q,\vec{s}) - \sigma_{AB}(Q,-\vec{s})$ **if the 1st term cancels**
- **Small-x probes – hard probe is NOT local – size (or A)-dependence!**
- **Multiple observed scales – TMD, GPD, …** $2R \gg \frac{1}{xp} \gtrsim 2R\frac{m}{p}$

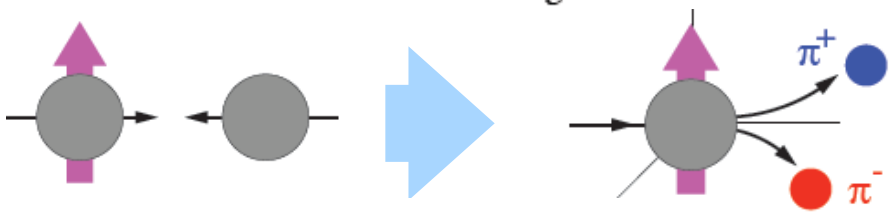
$$Q \gg Q' \gtrsim 1/\mathrm{fm} \sim \Lambda_{\mathrm{QCD}}$$

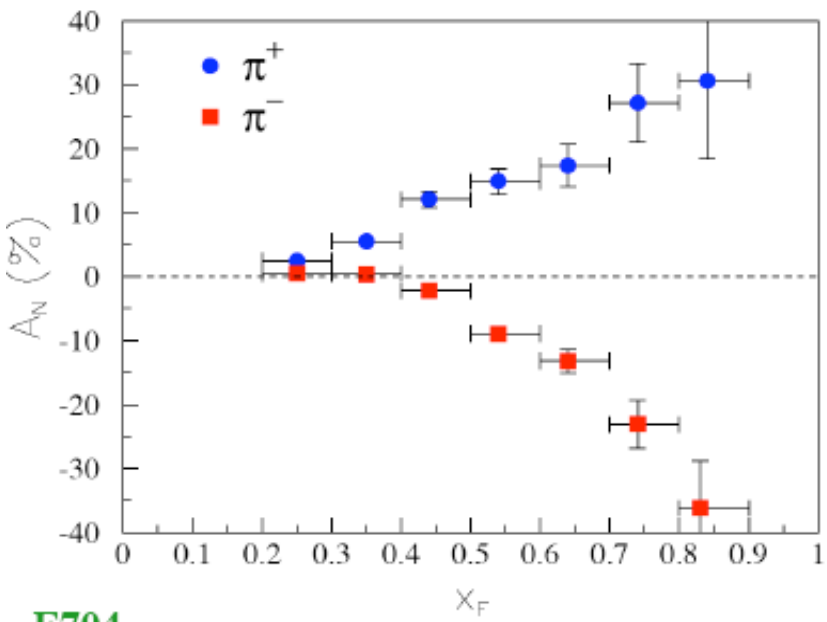
Large SSA in hadronic collisions

- **Hadronic** $p\uparrow + p \rightarrow \pi(l)X$:

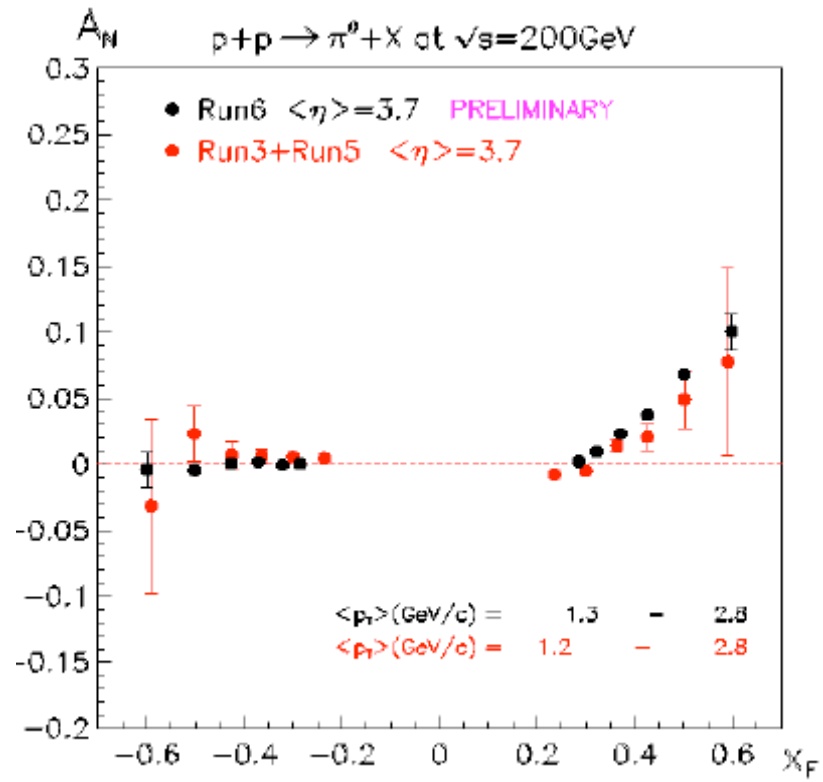


$$A_N = \frac{1}{P_{\text{beam}}} \frac{N^{\pi}_{\text{left}} - N^{\pi}_{\text{right}}}{N^{\pi}_{\text{left}} + N^{\pi}_{\text{right}}}$$





E704

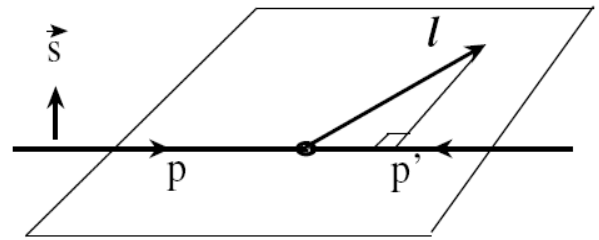


STAR (BRAHMS, too)

Minimum conditions for A_N =\= 0

- **SSA corresponds to a naively T-odd triple product:**

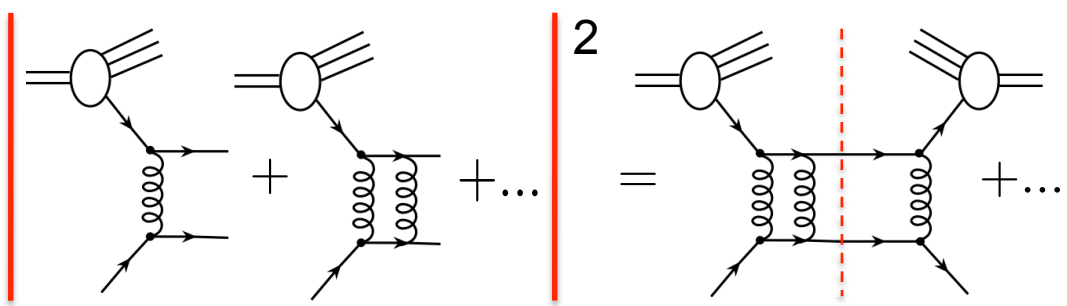
$$A_N \propto i\,\vec{s}_p \cdot (\vec{p} \times \vec{\ell}) \;\Rightarrow\; i\,\epsilon^{\mu\nu\alpha\beta}\, p_\mu s_\nu \ell_\alpha p'_\beta$$



Novanish A_N requires a phase, enough vectors to fix a scattering plan, and a spin flip at the partonic scattering

- **Leading power in QCD:**

$$\sigma_{AB}(p_T, \vec{s}) \propto$$



$$\propto\; \alpha_s \frac{m_q}{p_T}$$

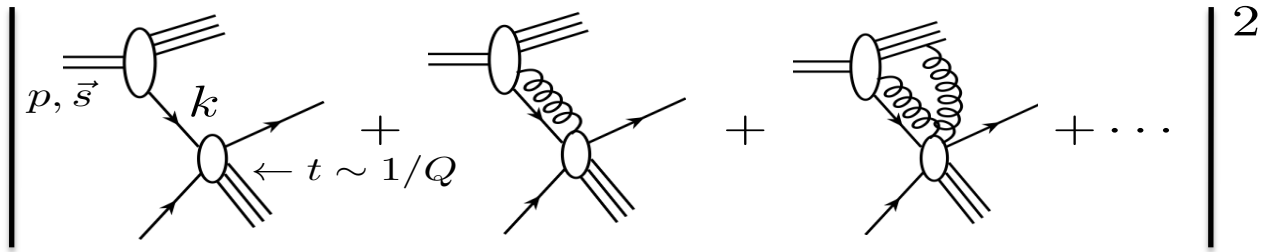
Kane, Pumplin, Repko, PRL, 1978

A_N=\=0 in collinear factorization

Efremov, Teryaev, 82; Qiu, Sterman, 91

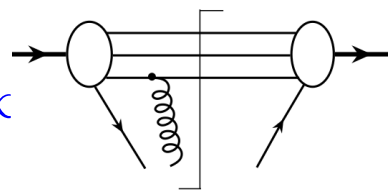
❑ A_N – twist-3 effect:

$\sigma(Q, \vec{s}) \propto$



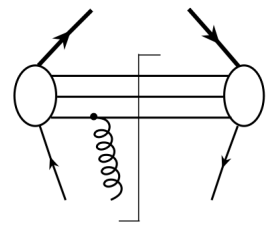
$$\Delta(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D_f(z) + \delta q_f(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z)$$

$T^{(3)}(x,x) \propto$



Qiu, Sterman, 1991

$D^{(3)}(z,z) \propto$



Kang, Yuan, Zhou, 2010

❑ Spin flip:

– Interference of single parton and a two-parton composite state

❑ The phase:

– Interference of Real and Imaginary part of scattering amplitude

– gluonic pole: $\propto T^{(3)}(x,x)$

– fermionic pole contribution: $\propto T^{(3)}(x,0)$ or $T^{(3)}(0,x)$

Features of A_N in collinear factorization

Qiu, Sterman, 91

❑ Factorization is valid (as good as leading power):

$$\Delta\sigma_{AB\to h}(p_T,\vec{s}_T) = \sum_{abc} T^{(3)}_{a/A}(x,\vec{s}_T)\otimes f_{b/B}(x')\otimes\hat{\sigma}_{ab\to c}(p_T,\vec{s}_T)\otimes D_{c\to h}(z)$$

Qiu, Sterman, 1991,98

$$+\sum_{abc}\delta q^{(2)}_{a/A}(x,\vec{s}_T)\otimes f_{b/B}(x')\otimes\hat{\sigma}'_{ab\to c}(p_T,\vec{s}_T)\otimes D^{(3)}_{c\to h}(z)$$

Kang, Yuan, Zhou, 2010

Kanazawa, Koike, 2000

$$+\sum_{abc}\delta q^{(2)}_{a/A}(x,\vec{s}_T)\otimes f^{(3)}_{b/B}(x')\otimes\hat{\sigma}''_{ab\to c}(p_T,\vec{s}_T)\otimes D_{c\to h}(z)$$

❑ Generic features:

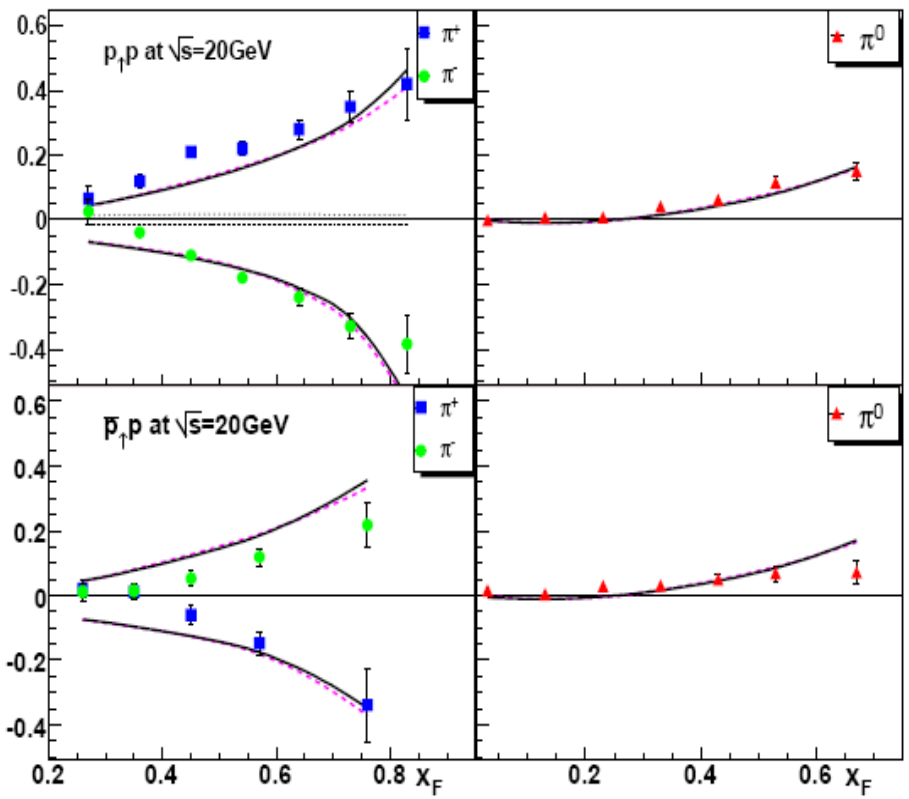
$$A_N \propto \frac{\epsilon_\perp^{\alpha\beta}s_\alpha p_{T\beta}}{-\hat{t}}\left[-x\frac{d}{dx}T^{(3)}(x,x)\right] \propto \frac{\epsilon_\perp^{\alpha\beta}s_\alpha p_{T\beta}}{p_T^2}\left[\frac{n}{1-x}\right]$$

$$\text{if } T^{(3)}(x,x)\propto q(x)\propto(1-x)^n$$

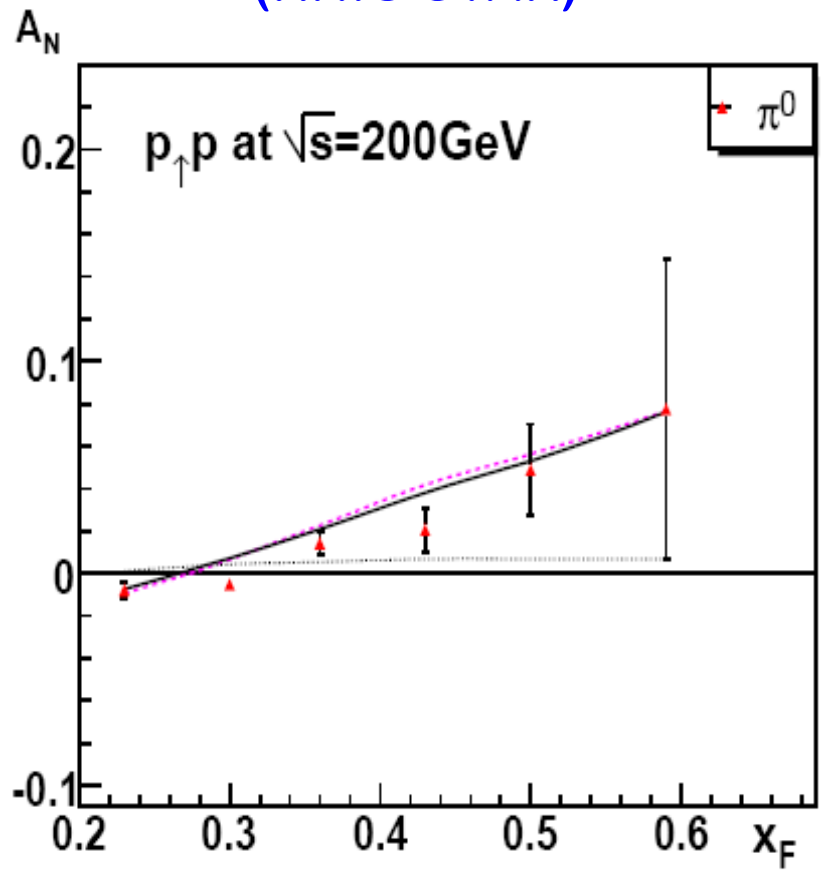
- A_N falls as $1/p_T$ if p_T is large
- A_N increases as x_F if x_F is large

Asymmetries from the $T_F(x,x)$

(FermiLab E704)



(RHIC STAR)



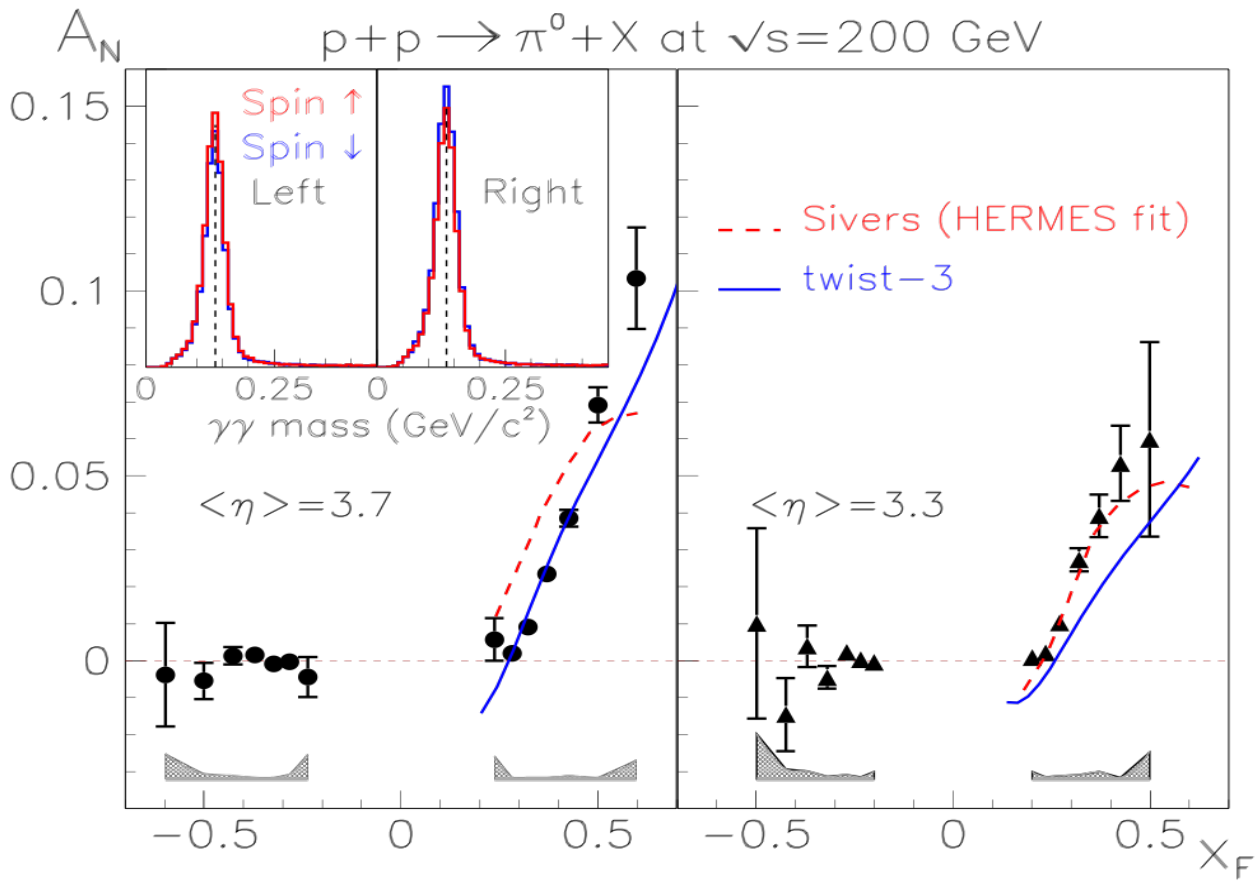
Kouvaris,Qiu,Vogelsang,Yuan, 2006

Nonvanish twist-3 function → Nonvanish transverse motion

A_N at RHIC energy

❑ STAR Run 6 inclusive π^0:

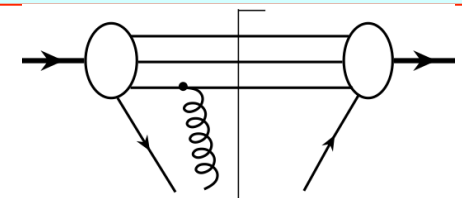
PRL 101, 222001 (2008)
arXiv:0801.2990v1 [hep-ex]



U. D'Alesio, F. Murgia
Phys. Rev. D 70, 074009 (2004)
arXiv:hep-ph/0712.4240

C. Kouvaris, J. Qiu, W. Vogelsang, F. Yuan,
Phys. Rev. D 74, 114013 (2006).

Twist-3 distributions relevant to SSA



❑ Two-sets Twist-3 correlation functions:

$$\widetilde{\mathcal{T}}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{\mathcal{T}}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{\mathcal{T}}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{\mathcal{T}}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

No probability interpretation!

Kang, Qiu, 2009
Braun, et al 2009

❑ Twist-2 distributions:

- **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

- **Polarized PDFs:**

$$\Delta q(x) \propto \langle P, S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_\parallel\rangle$$
$$\Delta G(x) \propto \langle P, S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P, S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

Interpretation of twist-3 distributions?

- Quark-gluon correlation as an example:

$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-} \times \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^+(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

- Normal twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-} \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

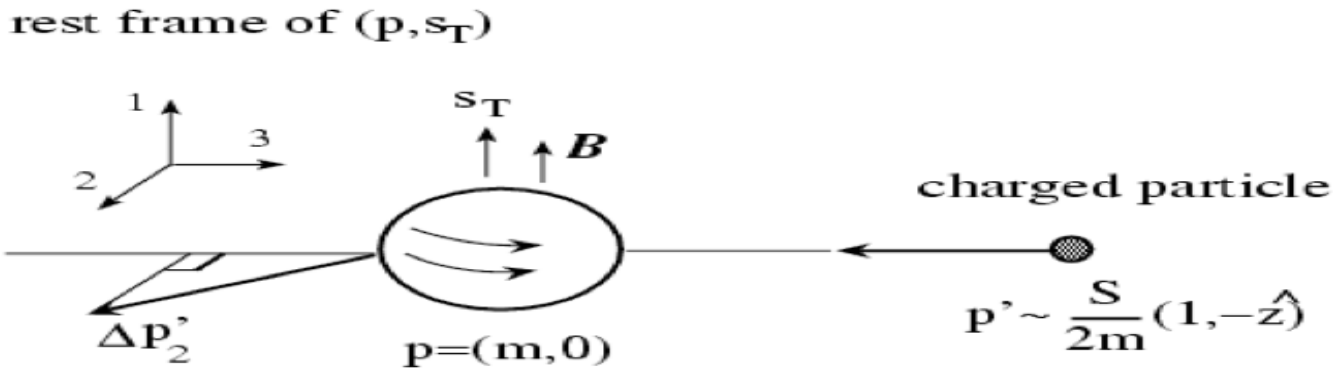
- Difference – the operator in Red:

$$\left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^+(y_2^-) \right]$$

How can we interpret the "expectation value" of this operator?

What the twist-3 distribution can tell us?

- The operator in Red – a classical Abelian case:



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

- In the c.m. frame:

$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+$$

- The total change:

$$\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y^-)$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Multi-gluon correlation functions

❑ Diagonal tri-gluon correlations:

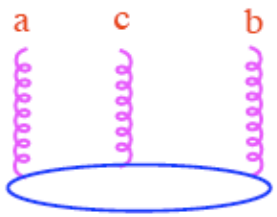
Ji, PLB289 (1992)

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+y_1^-} \times \frac{1}{xP^+} \langle P, s_\perp | F^+{}_\alpha(0) \left[\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma{}^+(y_2^-)\right] F^{\alpha+}(y_1^-) | P, s_\perp \rangle$$

❑ Two tri-gluon correlation functions – color contraction:

$$T_G^{(f)}(x,x) \propto i f^{ABC} F^A F^C F^B = F^A F^C (\mathcal{T}^C)^{AB} F^B$$

$$T_G^{(d)}(x,x) \propto d^{ABC} F^A F^C F^B = F^A F^C (\mathcal{D}^C)^{AB} F^B$$



Quark-gluon correlation: $T_F(x,x) \propto \overline{\psi}_i F^C (T^C)_{ij} \psi_j$

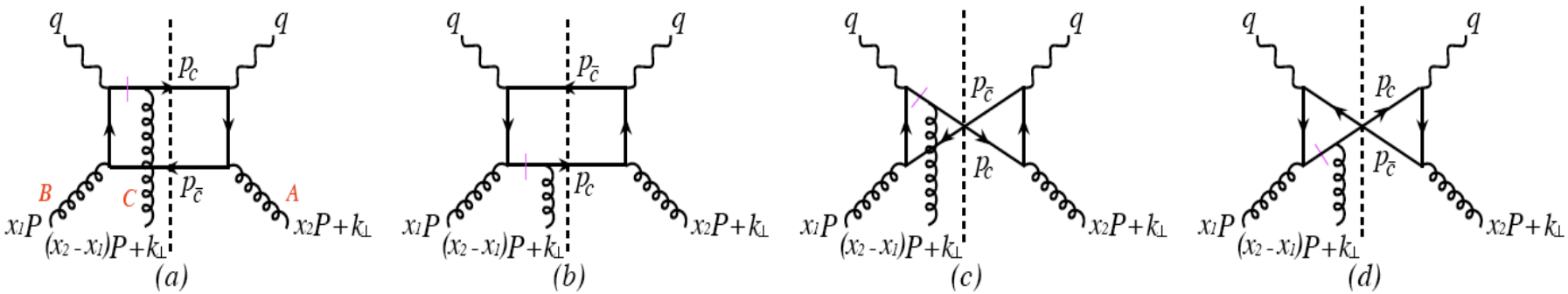
❑ D-meson production at EIC:

- ❖ Clean probe for gluonic twist-3 correlation functions
- ❖ $T_G^{(f)}(x,x)$ could be connected to the gluonic Sivers function

D-meson production at future EIC

Kang, Qiu, PRD, 2008

❑ Dominated by the tri-gluon subprocess:



- Active parton momentum fraction cannot be too large
- Intrinsic charm contribution is not important
- Sufficient production rate

❑ Single transverse-spin asymmetry:

$$A_N = \frac{\sigma(s_\perp) - \sigma(-s_\perp)}{\sigma(s_\perp) + \sigma(-s_\perp)} = \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} \Bigg/ \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}$$

- SSA is directly proportional to tri-gluon correlation functions
- Any small A_N discovers the tri-gluon correlation!

Features of the SSA in D-production at EIC

- **Dependence on tri-gluon correlation functions:**

$$D-\text{meson} \propto T_G^{(f)} + T_G^{(d)} \qquad \overline{D}-\text{meson} \propto T_G^{(f)} - T_G^{(d)}$$

Separate $T_G^{(f)}$ and $T_G^{(d)}$ by the difference between D and $\bar{D}$

- **Model for tri-gluon correlation functions:**

$$T_G^{(f,d)}(x,x) = \lambda_{f,d} G(x) \qquad \lambda_{f,d} = \pm\lambda_F = \pm 0.07\text{GeV}$$

- **Kinematic constraints:**

$$x_{min} = \begin{cases} x_B\left[1 + \frac{P_{h\perp}^2 + m_c^2}{z_h(1-z_h)Q^2}\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \geq 1 \\ x_B\left[1 + \frac{2m_c^2}{Q^2}\left(1 + \sqrt{1 + \frac{P_{h\perp}^2}{z_h^2 m_c^2}}\right)\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \leq 1 \end{cases}$$

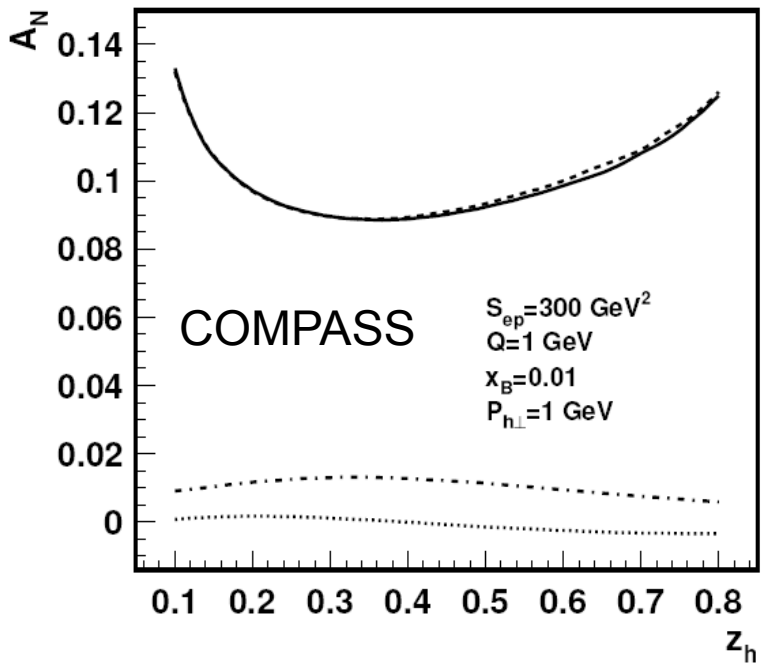
Note: The $z_h(1-z_h)$ has a maximum

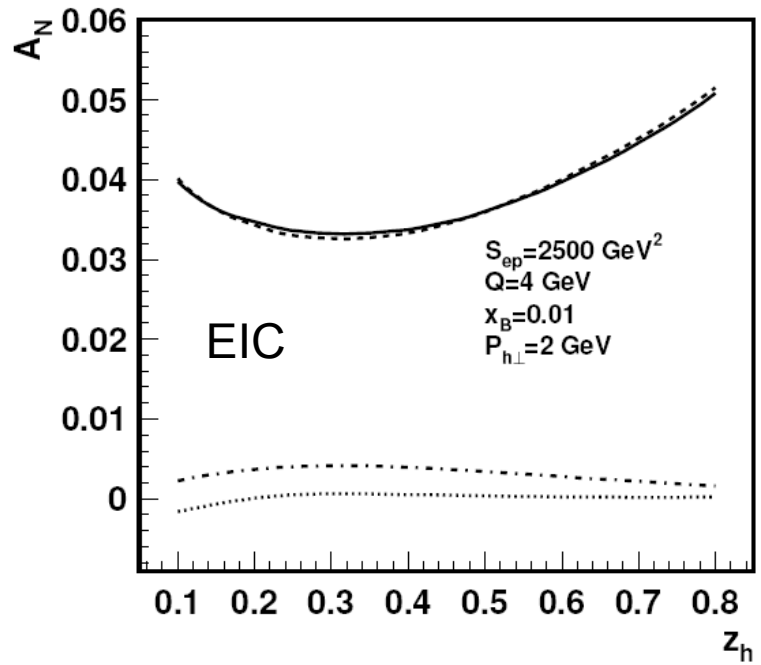
SSA should have a minimum if the derivative term dominates

Minimum in the SSA of D-production at EIC

Kang, Qiu, PRD, 2008

- SSA for D^0 production (λ_f only):





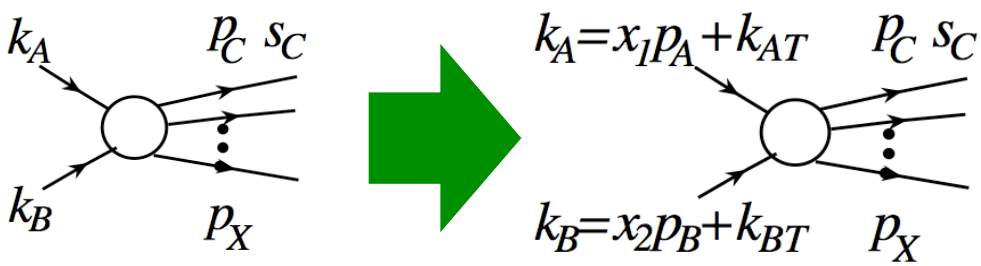
- Derivative term dominates, and small φ dependence
- Asymmetry is twice if $T_G^{(f)} = +T_G^{(d)}$, or zero if $T_G^{(f)} = -T_G^{(d)}$
- Opposite for the $\bar{D}$ meson
- Asymmetry has a minimum ~ z_h ~ 0.5

TMD factorization approach to SIDIS

- **More relevant to observables with two very different momentum scales:** $Q_1 \gg Q_2 \sim \Lambda_{\rm QCD}$

  $Q_1 \gg \Lambda_{\rm QCD}$ **Makes it possible to have pQCD factorization**

  $Q_2 \sim \Lambda_{\rm QCD}$ **Sensitive to parton's transverse motion**



On-shell active partons

$$d\sigma_{AB}(Q) = f_{a/A}(x_1, k_{AT}, s_A) \otimes f_{b/B}(x_2, k_{BT}, s_B) \otimes d\hat{\sigma}_{ab}(Q, \alpha_s) + \mathcal{O}(1/Q)$$

Complication: soft gluon interactions between hadrons

- **Valid for processes involving only two hadrons:**

e^+e^-: $e^+ + e^- \to h_1(p_1) + h_2(p_2)$ where p_1, p_2 almost back-to-back

SIDIS: $e(l) + h(p) \to e(l') + h'(p') + X$ with $Q \gg q_T$

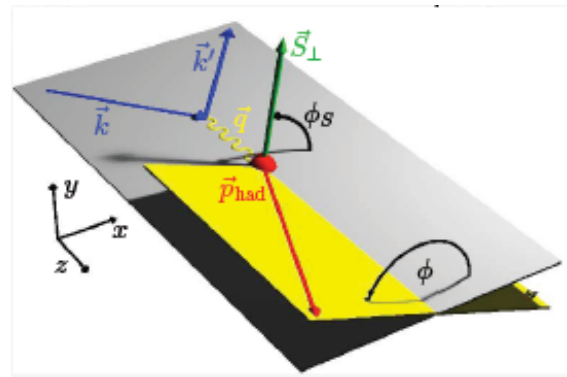
Drell-Yan: $h_1(p_1) + h_2(p_2) \to ll'(q) + X$ with $Q = \sqrt{q^2} \gg q_T$

Key: **color flow + locality** Collins, Qiu; Yuan, Vogelsang; Rogers, Mulder, …

Asymmetry of single particle production

- **SIDIS complements to DY and e+e-:**

SIDIS has uniquely determined leptonic plane + hadronic plane



- **General expression for cross section:**

$$d\sigma = d\sigma^0_{UU} + \cos 2\phi\, d\sigma^1_{UU} + \frac{1}{Q}\cos\phi\, d\sigma^2_{UU} + \lambda_e \frac{1}{Q}\sin\phi\, d\sigma^3_{LU}$$

$$+S_L\left\{\sin 2\phi\, d\sigma^4_{UL} + \frac{1}{Q}\sin\phi\, d\sigma^5_{UL} + \lambda_e\left[d\sigma^6_{LL} + \frac{1}{Q}\cos\phi\, d\sigma^7_{LL}\right]\right\}$$

$$+S_T\left\{\sin(\phi-\phi_S)\, d\sigma^8_{UT} + \sin(\phi+\phi_S)\, d\sigma^9_{UT} + \sin(3\phi-\phi_S)\, d\sigma^{10}_{UT}\right.$$

$$+\frac{1}{Q}\left(\sin(2\phi-\phi_S)\, d\sigma^{11}_{UT} + \sin\phi_S\, d\sigma^{12}_{UT}\right)$$

$$\left.+\lambda_e\left[\cos(\phi-\phi_S)\, d\sigma^{13}_{LT} + \frac{1}{Q}\left(\cos\phi_S\, d\sigma^{14}_{LT} + \cos(2\phi-\phi_S)\, d\sigma^{15}_{LT}\right)\right]\right\}$$

σ_{XY}

Beam Target

Polarization

Most notable TMD distributions - I

- Sivers function – transverse polarized hadron:

Sivers function

$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$
$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

- Boer-Mulder function – transverse polarized quark:

$$f_{q,\boldsymbol{s}_q/p}(x,\boldsymbol{k}_\perp) = \frac{1}{2} f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q^\uparrow/p}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$
$$= \frac{1}{2} f_{q/p}(x,k_\perp) - \frac{1}{2}\frac{k_\perp}{M} h_1^{\perp q}(x,k_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

Boer-Mulder function

Affect angular distribution of Drell-Yan lepton

Most notable TMD distributions - II

- Collins function – FF of a transversely polarized parton:

$$D_{h/q,s_q}(z,\boldsymbol{p}_\perp) = D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

$$= D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\, M_h} H_1^{\perp q}(z,p_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

Collins function

- Fragmentation function to a polarized hadron:

$$D_{\Lambda,\boldsymbol{s}_\Lambda/q}(z,\boldsymbol{p}_\perp) = \frac{1}{2} D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{\Lambda^\uparrow/q}(z,p_\perp)\, \boldsymbol{S}_\Lambda \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

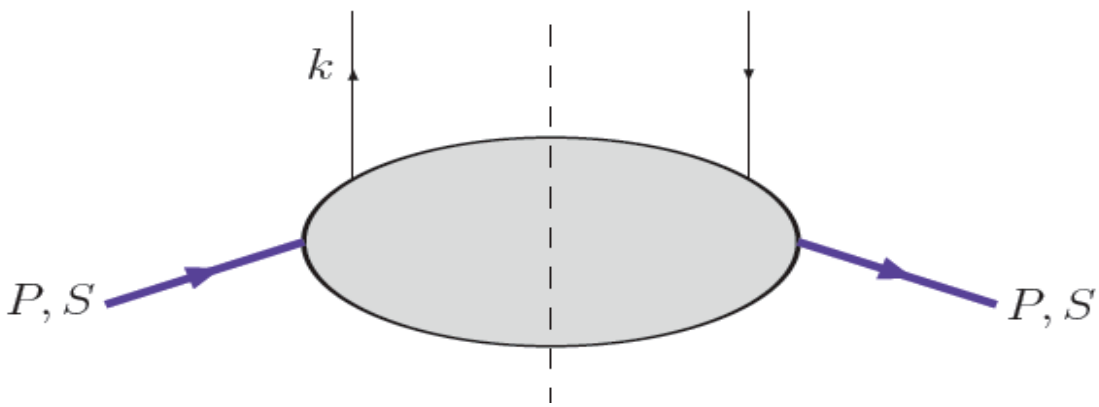
$$= \frac{1}{2} D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\, M_\Lambda} D_{1T}^{\perp q}(z,p_\perp)\, \boldsymbol{S}_\Lambda \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

Unpolarized parton fragments into a polarized hadron - Λ

Other TMD distributions

❑ Quark TMD distributions:

$$\Phi(x, \boldsymbol{k}_\perp) = \frac{1}{2}\Big[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \Big(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \Big) \gamma^5 \not{n}_+ + h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \Big(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \Big) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} + h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Big]$$



Total 8 TMD quark distributions

❑ Gluon TMD distributions, …

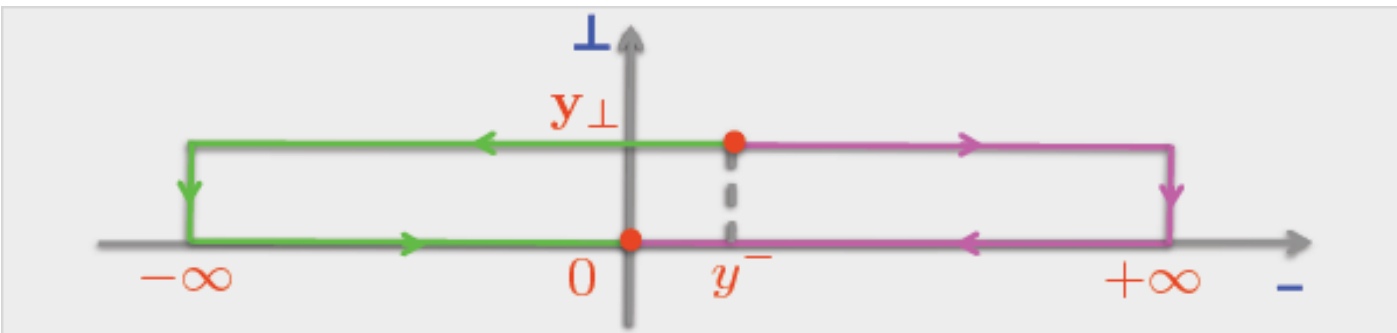
Production of quarkonium, two-photon, …

Process dependence of TMDs

- The form of gauge link is a result of factorization:

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\,\boxed{\text{Gauge link}}\,\frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

- SIDIS: $\Phi_n^\dagger(\{+\infty,0\},\mathbf{0}_\perp)\Phi_{\mathbf{n}_\perp}^\dagger(+\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\Phi_n(\{+\infty,y^-\},\mathbf{y}_\perp)$
- DY: $\Phi_n^\dagger(\{-\infty,0\},\mathbf{0}_\perp)\Phi_{\mathbf{n}_\perp}^\dagger(-\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\Phi_n(\{-\infty,y^-\},\mathbf{y}_\perp)$



$$f_{q/h^\uparrow}^{\text{SIDIS}}(x,\mathbf{k}_\perp,\vec{S}) \neq f_{q/h^\uparrow}^{\text{DY}}(x,\mathbf{k}_\perp,\vec{S})$$

Collinear factorized PDFs are process independent

Critical test of TMD factorization

- **Parity – Time reversal invariance:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x, \mathbf{k}_\perp, \vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x, \mathbf{k}_\perp, -\vec{S})$$

- **Definition of Sivers function:**

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x, k_\perp)\, \vec{S} \cdot \hat{p} \times \hat{\mathbf{k}}_\perp$$

- **Modified universality:**

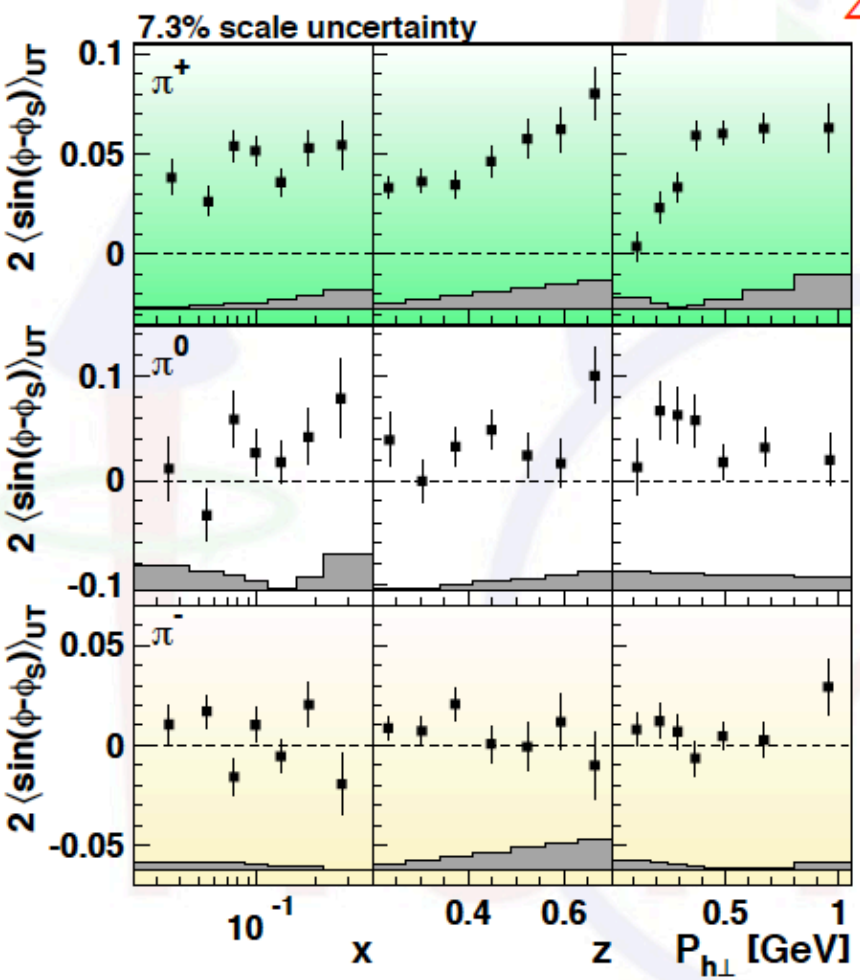
$$\Delta^N f_{q/h^\uparrow}^{\rm SIDIS}(x, k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\rm DY}(x, k_\perp)$$

Same applies to TMD gluon distribution

Spin-averaged TMD is process independent

Sivers functions from HERMES

$$2\langle \sin(\phi - \phi_S)\rangle_{UT} = -\frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x,p_T^2) \otimes_{\mathcal{W}} D_1^q(z,k_T^2)}{\sum_q e_q^2 f_1^q(x,p_T^2) \otimes D_1^q(z,k_T^2)}$$



π^+ dominated by u-quark scattering:

$$\simeq -\frac{f_{1T}^{\perp,u}(x,p_T^2) \otimes_{\mathcal{W}} D_1^{u\to\pi^+}(z,k_T^2)}{f_1^u(x,p_T^2) \otimes D_1^{u\to\pi^+}(z,k_T^2)}$$

☛ u-quark Sivers DF < 0

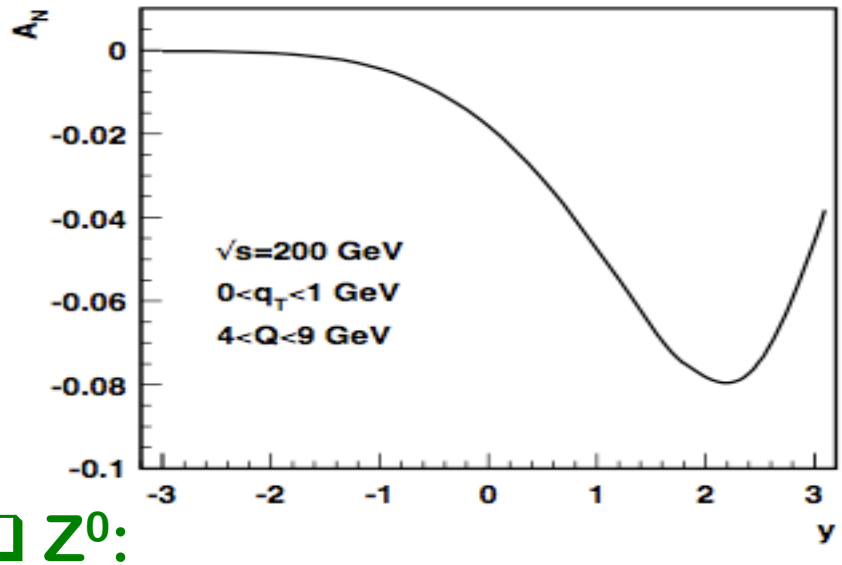
☛ d-quark Sivers DF > 0 (cancelation for π^-)

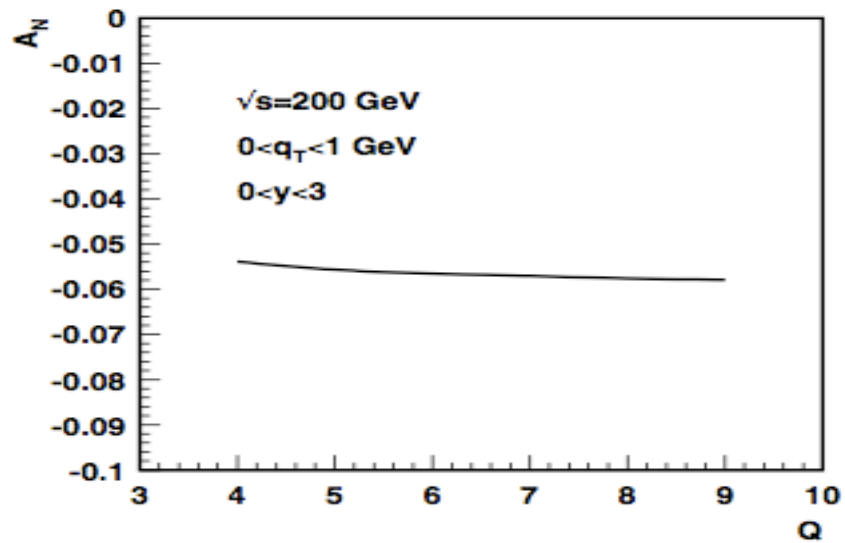
The modified universality – Drell-Yan/Z^0

- **Drell-Yan:**

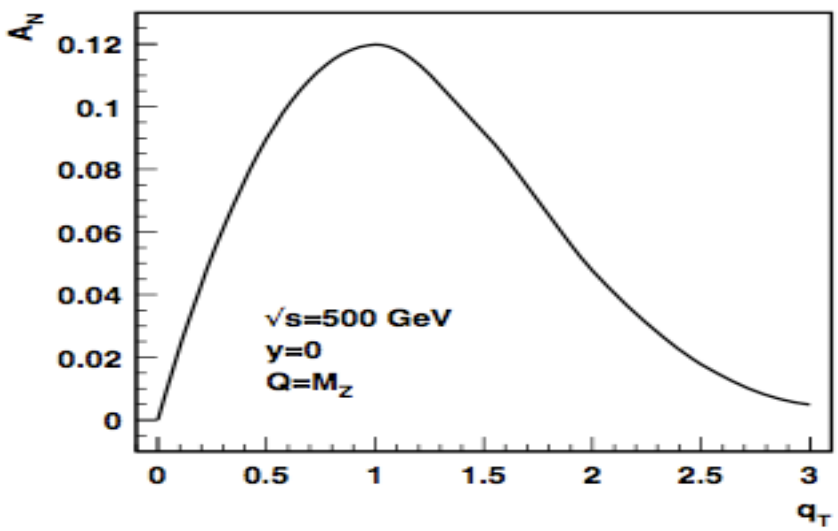
$$A_N^{\sin(\phi-\phi_s)} = -A_N$$

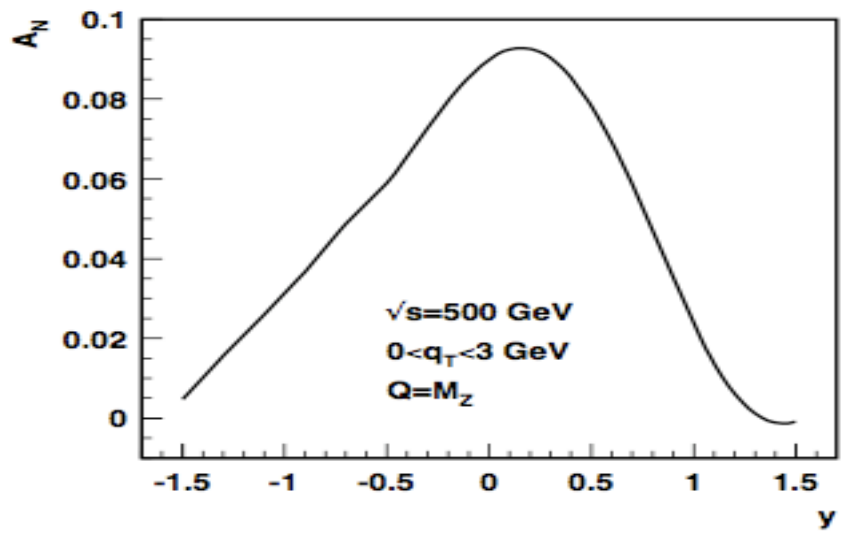
Collins et al. 2006
Kang, Qiu, 2009





- **Z^0:**

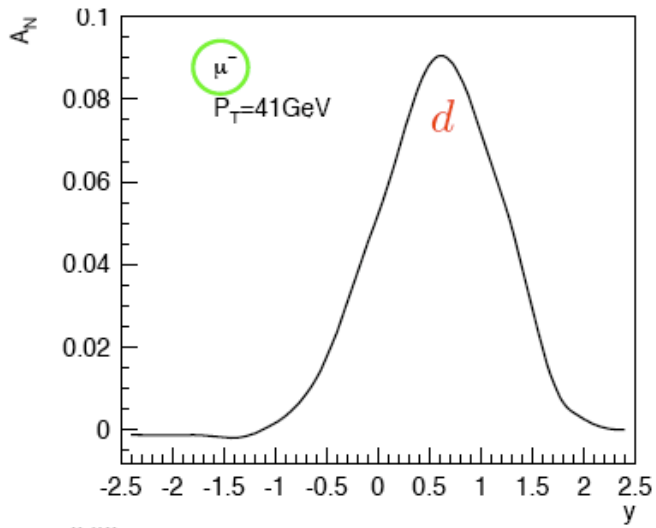


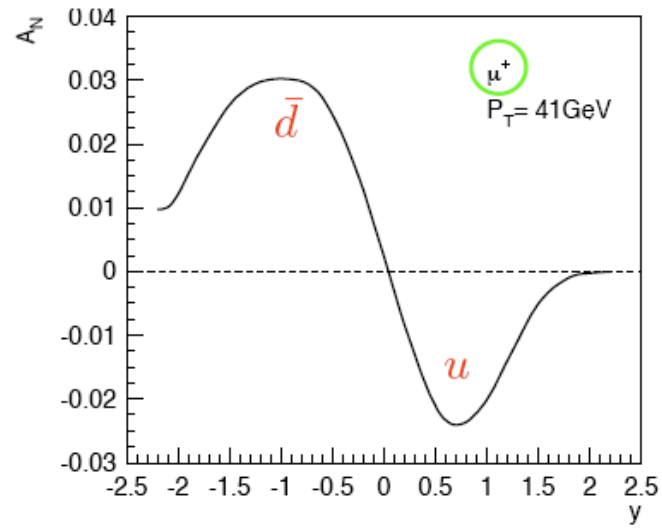


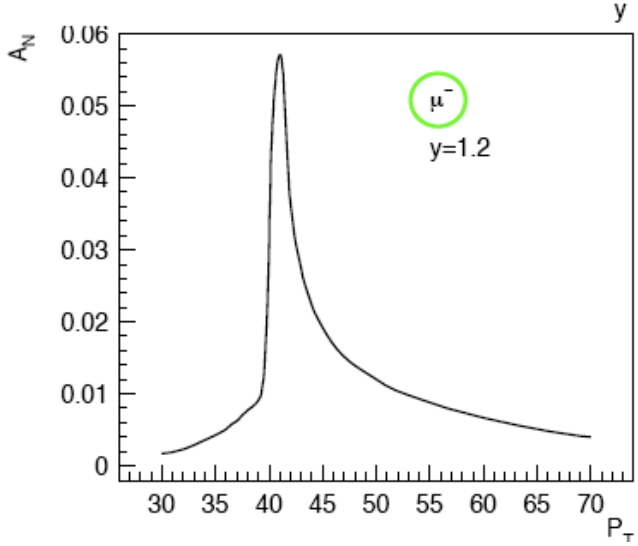
SSA of lepton from W-decay

Kang, Qiu, PRL 2009

❑ Lepton SSA is diluted from the decay:







- flavor separation
- asymmetry gets smaller due to dilution should still be measurable by current RHIC sensitivity

Complimentary to Drell-Yan/Z^0 production

TMD vs collinear factorization

- **TMD factorization and collinear factorization cover different regions of kinematics:**

  Collinear: $Q_1 \ldots Q_n \gg \Lambda_{QCD}$

  TMD: $Q_1 \gg Q_2 \sim \Lambda_{QCD}$

  - ✧ **One complements the other, but, cannot replace the other!**
  - ✧ **Predictive power of both formalisms relies on the validity of their own factorization**

  Consistency check – overlap region – perturbative region

- **"Formal" operator relation between TMD distributions and collinear factorized distributions:**

  spin-averaged:
  $$\int d^2k_\perp \Phi_f^{\rm SIDIS}(x,k_\perp) + {\rm UVCT}(\mu_F^2) = \phi_f(x,\mu_F^2)$$

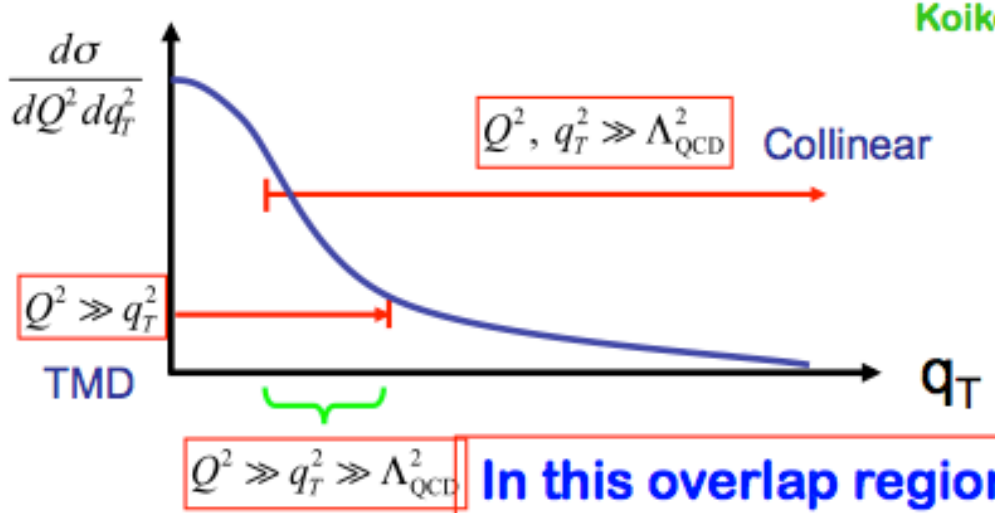
  Transverse-spin:
  $$\frac{1}{M_P}\int d^2k_\perp\, \vec{k}_\perp^2\, q_T(x,k_\perp) + {\rm UVCT}(\mu_F^2) = T_F(x,x,\mu_F^2)$$

  But, TMD factorization is only valid for low k_T– TMD PDFs at low k_T

The consistency check

- IF both factorizations are proved to be valid,
  - both formalisms should yield the same result in overlap region
  - Case studies – Drell-Yan/SIDIS

Ji, Qiu, Vogelsang, and Yuan
Koike, Vogelsang, and Yuan



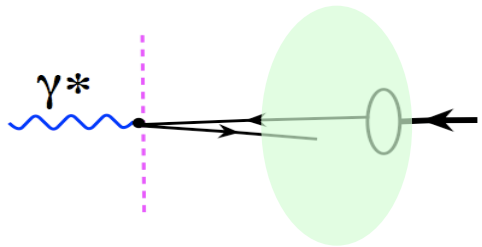
In this overlap region, both formalisms indeed give the same result

- TMD factorization fails for processes involving three or more identified hadrons!

Collins, Qiu, 2007
Vogelsang, Yuan, 2007, Collins, 2007
Rogers, Mulders, 2010

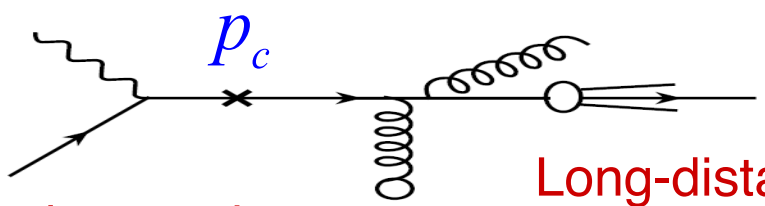
SIDIS: probe for nuclear medium effect

- **Scattered parton interacts with the medium coherently:**



- **Multiple scattering with/out radiation:**

Guo & Wang PRL 2000, …
Wang & Wang, PRL 2002, …



Long-distance fragmentation

Short-distance hard scattering

$p_c^2 \ll O(Q^2)$ → Induced radiation modifies the fragmentation

→ First scattering determines the parton production rate

$\ln(Q^2/p_c^2)$ → Modifies *DGLAP evolution of the Fragmentation function – "energy loss"*

Energy loss in cold nuclear matter

❑ Radiation energy loss:

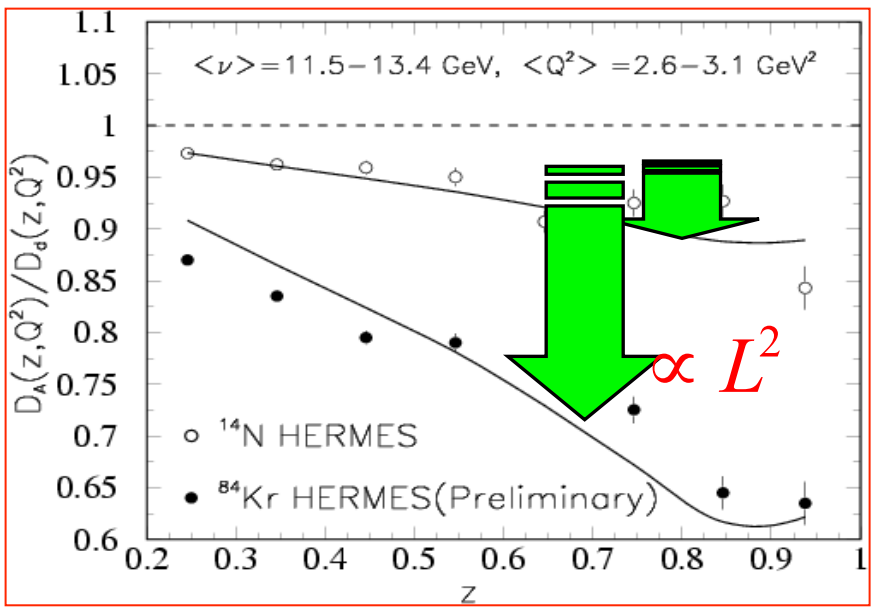
$$\Delta E = \nu \langle \Delta z \rangle = (E - E')\langle \Delta z \rangle$$

$\langle \Delta z \rangle$ - radiative energy loss fraction

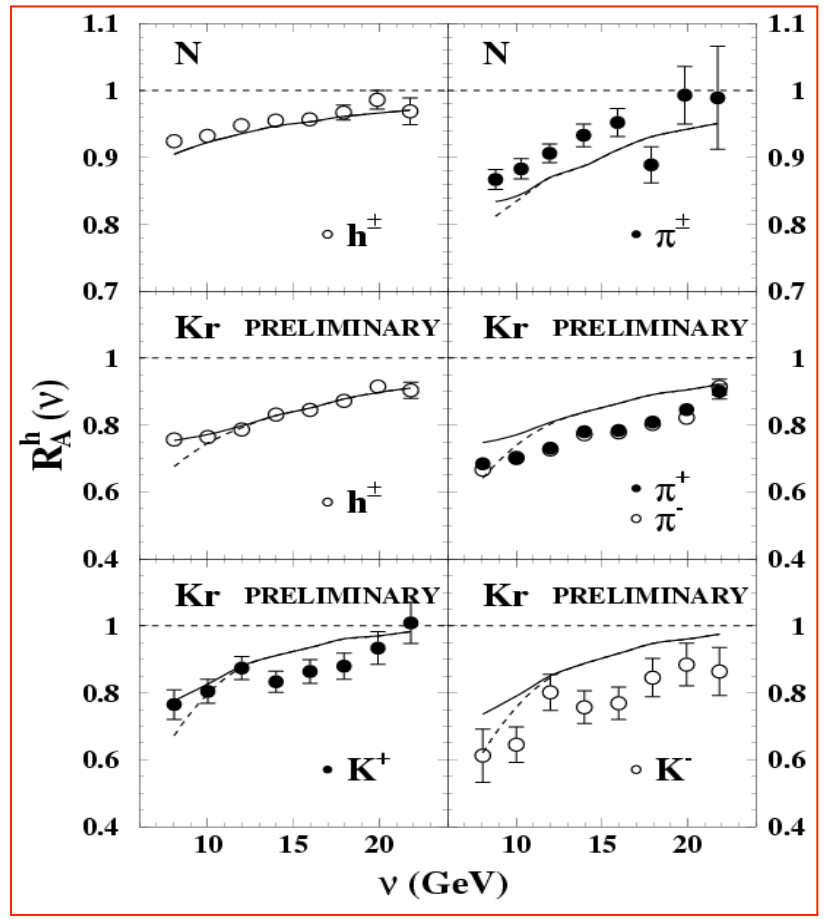
$$x_B = Q^2/(2p\cdot q), \quad x_A = 1/(m_N R_A)$$

$$\langle \Delta z \rangle = \tilde{C}(Q^2)\frac{C_A \alpha_s^2(Q^2)}{N_c}\frac{x_B}{Q^2 x_A^2} 6 \ln \frac{1}{2x_B}$$

$$\langle -dE/dL \rangle_{\text{cold}} \approx 0.5 - 0.6 \text{ GeV/fm}$$



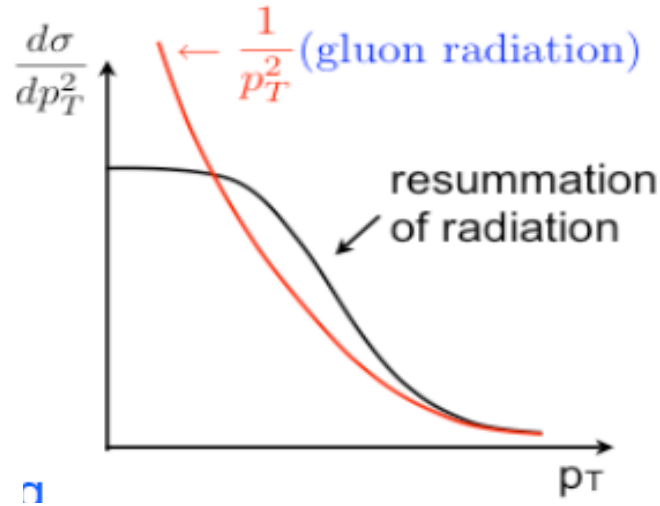
E.Wang, X.-N.Wang, PRL 89 (2002)



F.Arleo, Eur.Phys.J. C30 (2003)

Transverse momentum broadening

- Low p_T distribution is ill-defined in the fixed order calculation
  - All order resummation (CSS formalism)



- Multiple scattering in medium:
  - Each scattering is too soft to calculate perturbatively
  - Resummation + multiple scattering (not yet achieved)

- Moment of p_T-distribution is perturbative calculable:

$$\langle p_T^2(Q^2)\rangle = \int dp_T^2\, p_T^2 \frac{d\sigma}{dQ^2 dp_T^2} \Big/ \int dp_T^2 \frac{d\sigma}{dQ^2 dp_T^2}$$

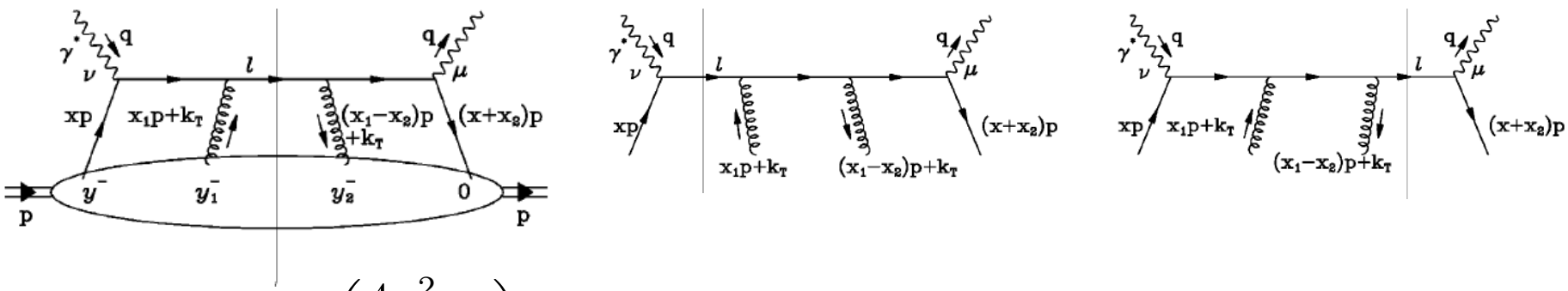
- observed particles only

- Momentum broadening: $\Delta\langle p_T^2(Q^2)\rangle = \langle p_T^2(Q^2)\rangle_{eA} - \langle p_T^2(Q^2)\rangle_{ep}$
  - Easy to measure, sensitive to the medium properties
  - Perturbatively calculable $\propto A^{1/3}$

Luo, Qiu, Sterman, 1994

Leading hadron broadening in SIDIS

- **Broadening by double scattering:**



$$\Delta\langle l_T^2\rangle_{\rm SIDIS} = \left(\frac{4\pi^2\alpha_s}{3}\right)\lambda^2_{\rm SIDIS}A^{1/3}$$

Guo, 1998; Fries 2003

- **Same as that for Drell-Yan at this order:**

$$\Delta\langle q_T^2\rangle_{\rm DY} = \left(\frac{4\pi^2\alpha_s}{3}\right)\lambda^2_{\rm DY}A^{1/3}$$

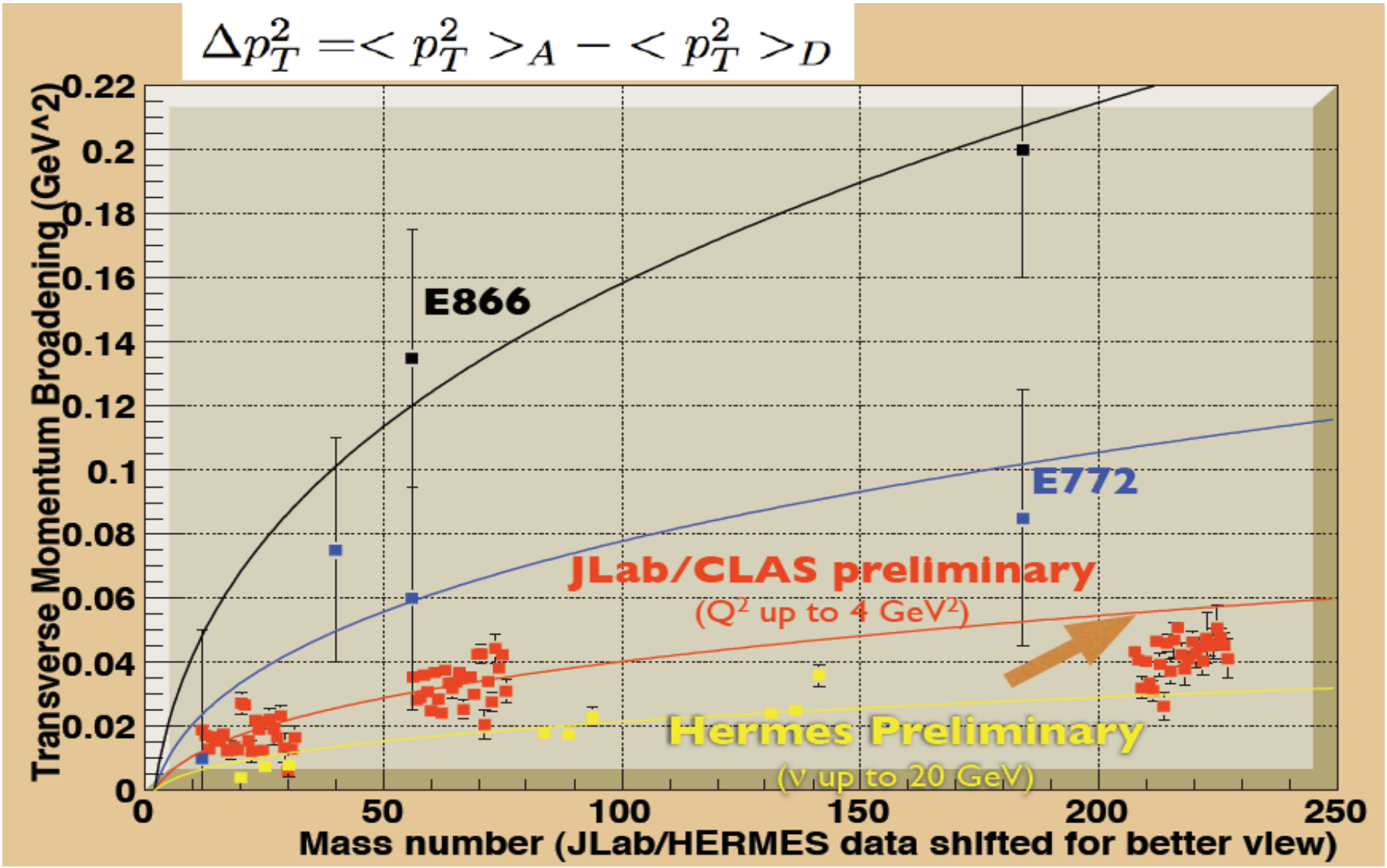
- **Medium density dependence:**

$\lambda^2_{\rm DY}$ **and** $\lambda^2_{\rm SIDIS}$ **could be different**

$A^{1/3}$ **is a consequence of the assumption – "no" color flow between nucleons inside a nucleus**

Broadening in color nuclear matter

W. Brooks, QM2009



Universality in nuclear dependence

❑ Nuclear modification factor:

$$\frac{d\sigma^{hA}}{dQ^2 dq_T^2} = A^{\alpha(q_T)} \times \frac{d\sigma^{hN}}{dQ^2 dq_T^2}$$

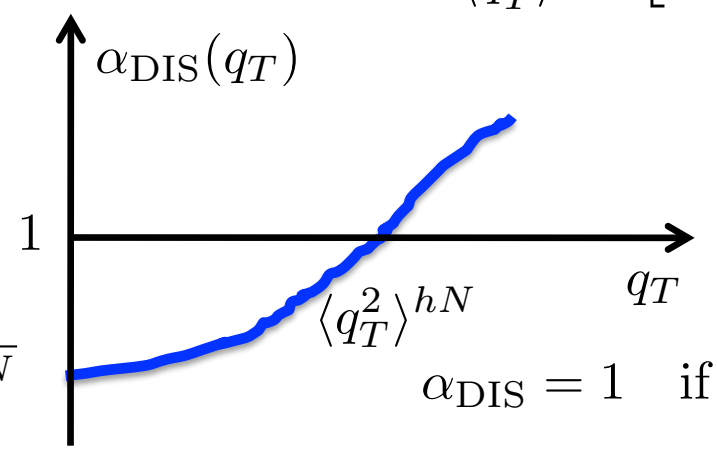
For pion production in SIDIS, q_T in photon-hadron frame

❑ Universal form of $\alpha(q_T)$:

Measured broadening: $\Delta\langle q_T^2\rangle_{\rm DIS} \equiv b_{\rm DIS} A^{1/3}$

Guo, et al. 2000

Modification factor: $\alpha_{\rm DIS}(q_T) \approx 1 + \frac{b_{\rm DIS}}{\langle q_T^2\rangle^{hN}}\left[-1 + \frac{q_T^2}{\langle q_T^2\rangle^{hN}}\right]$



$\alpha_{\rm DIS}(q_T = 0) \approx 1 - \frac{b_{\rm DIS}}{\langle q_T^2\rangle^{hN}}$

$\alpha_{\rm DIS} = 1 \quad \text{if} \quad q_T^2 = \langle q_T^2\rangle^{hN}$

Generalized parton distribution (GPD)

❑ Generalized quark distribution:

$$F_q(x,\xi,t,\mu^2) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P'|\bar{\psi}_q(\lambda/2)\frac{\gamma\cdot n}{2P\cdot n}\psi_q(-\lambda/2)|P\rangle$$

$$\equiv H_q(x,\xi,t,\mu^2)\left[\bar{\mathcal{U}}(P')\gamma^\mu\mathcal{U}(P)\right]\frac{n_\mu}{2P\cdot n}$$

$$+ E_q(x,\xi,t,\mu^2)\left[\bar{\mathcal{U}}(P')\frac{i\sigma^{\mu\nu}(P'-P)_\nu}{2M}\mathcal{U}(P)\right]\frac{n_\mu}{2P\cdot n}$$

with $\xi = (P'-P)\cdot n/2$ and $t = (P'-P)^2 \Rightarrow -\Delta_\perp^2$ if $\xi \to 0$

❑ Connection to normal quark distribution:

$$H_q(x,0,0,\mu^2) = q(x,\mu^2)$$

❑ Parton's orbital motion:

$$J_q = \frac{1}{2}\lim_{t\to 0}\int dx\, x\,[H_q(x,\xi,t) + E_q(x,\xi,t)]$$

$$= \frac{1}{2}\Delta q + L_q$$

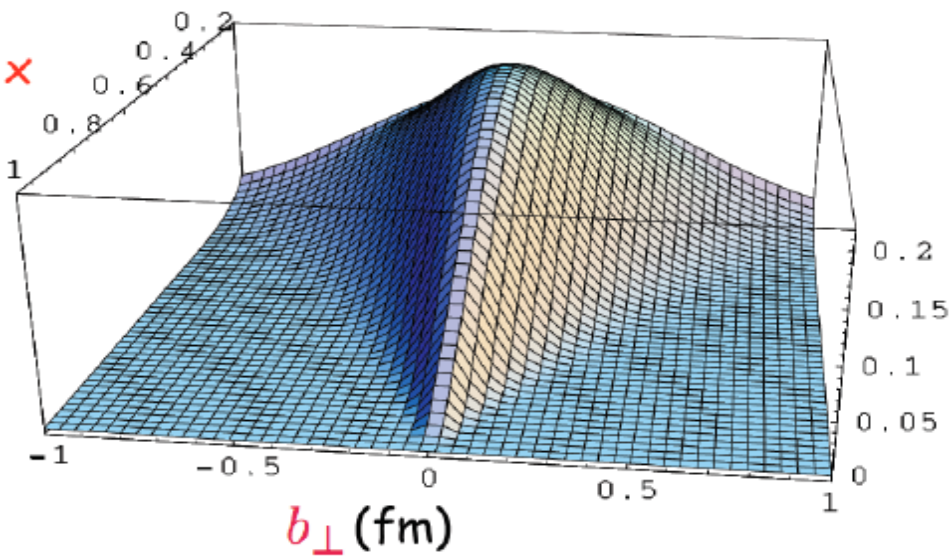
Ji, PRL78, 1997

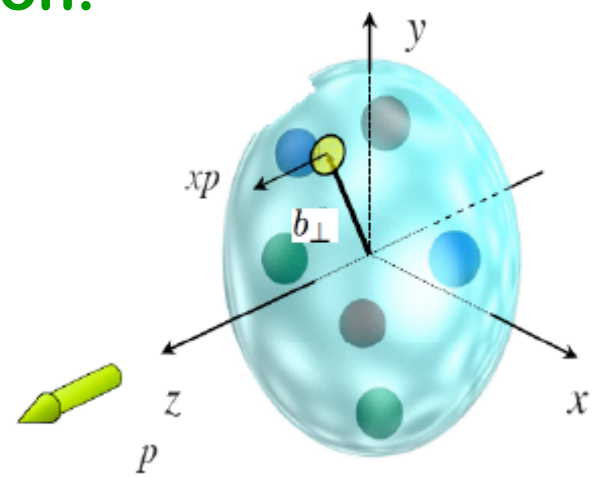
Tomographic images of the nucleon

❑ Spatial dependent quark distribution:

Burkardt, NPA711, 2002

$$\int d^2\mathbf{\Delta}_\perp \, e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \, H_q(x,\xi=0,-\mathbf{\Delta}_\perp^2)$$
$$= \; q(x,b_\perp)$$



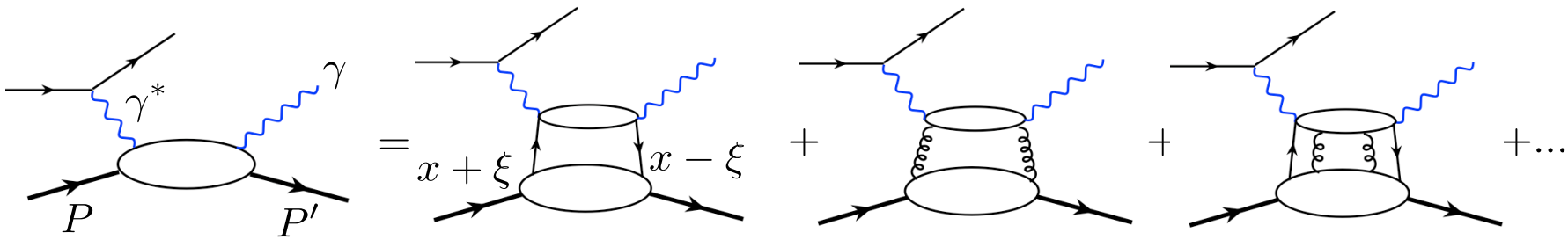


Note: $$q(x,b_\perp) \neq \int \frac{d^2k_\perp}{(2\pi)^2} e^{-i\vec{k}_\perp\cdot\vec{b}_\perp} \mathrm{TMD}(x,\vec{k}_\perp)$$

Key: connect the GPDs to physical observables

Exclusive processes with DIS kinematics

- **DVCS – exchange vacuum quantum number:**



- **DVCS – Factorization is ok if** $Q^2 \gg -t^2$

ξ – Does not evolve

- **Evolution from color singlet ladder diagrams:**

x – convolution

- **DVCS – meson production:**

Factorization requires a sufficient separation between the meson and scattered proton: $Q^2 \gg -t^2 \gg \Lambda^2_{\rm QCD}$

Summary

- QCD is very rich in dynamics, much more than QED, while QED is the underline theory of all excitements of CMP, …

- After 35 years, we have learned only a very small part of QCD dynamics: short-distance - less than 0.1 fm, although we have been successful

- SIDIS offers better observables to probe the partonic structure of a hadron

- Future EIC and its SIDIS program could offer many new opportunites to test QCD dynamics

Thank you!