FDC Capacitance Calculations – v2.0

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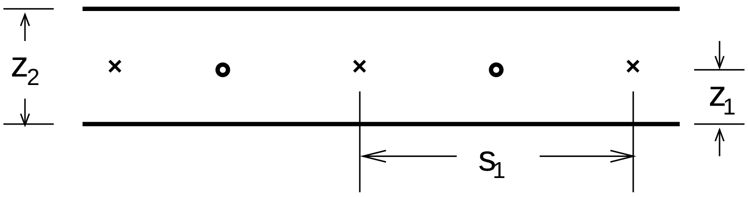
1 Induced Electrode Charges

Consider a chamber (or electrode structure) as shown in the figure below with the geometry parameters as defined. Here the wire plane consists of alternating sense and field wires sandwiched between two uniform cathode planes. The goal is to compute the relative charges on each of the different electrodes (sense wires, field wires, and cathode surfaces).



PARAMETERS

z_1=5 mm
s_1=10 mm
R_s=0.01 mm
R_f=0.05 mm
z_2=10 mm

1.1 Potential Equations

The computation of the relative charges follows from the definitions of the capacitances. The form of the potential equations for the sense, field, and cathodes is given by [1]:

$$1). \quad V_s = \frac{\lambda_s z_1}{\epsilon_0 s_1}\left(1 - \frac{s_1}{2\pi z_1}\ln\frac{2\pi R_s}{s_1}\right) + \frac{\lambda_f z_1}{\epsilon_0 s_1}\left(1 - \frac{s_1}{2\pi z_1}\ln 2\right) + \frac{\sigma_c z_1}{\epsilon_0} \tag{1}$$

$$2). \quad V_f = \frac{\lambda_s z_1}{\epsilon_0 s_1}\left(1 - \frac{s_1}{2\pi z_1}\ln 2\right) + \frac{\lambda_f z_1}{\epsilon_0 s_1}\left(1 - \frac{s_1}{2\pi z_1}\ln\frac{2\pi R_f}{s_1}\right) + \frac{\sigma_c z_1}{\epsilon_0} \tag{2}$$

$$3). \quad V_c = \frac{\lambda_s z_1}{\epsilon_0 s_1} + \frac{\lambda_f z_1}{\epsilon_0 s_1} + \frac{\sigma_c z_2}{\epsilon_0}. \tag{3}$$

Here λ_s and λ_f represent the charge per unit length on the sense and field wires, respectively, and σ_c is the surface charge density on the cathode planes. The terms λ_s/s_1 and λ_f/s_1 may be written as σ_s and σ_f and represent the surface charge density of the sense and field wires, respectively.

Then the potentials V_s, V_f, and V_c can be represented by the matrix equation:

$$
\begin{pmatrix} V_s \\ V_f \\ V_c \end{pmatrix} = A \begin{pmatrix} \sigma_s \\ \sigma_f \\ \sigma_c \end{pmatrix} \qquad \sigma = \text{surface charge density} \tag{4}
$$

$$
A = \frac{1}{\epsilon_0} \begin{pmatrix} z_1 - \frac{s_1}{2\pi} \ln \frac{2\pi R_s}{s_1} & z_1 - \frac{s_1}{2\pi} \ln 2 & z_1 \\ z_1 - \frac{s_1}{2\pi} \ln 2 & z_1 - \frac{s_1}{2\pi} \ln \frac{2\pi R_f}{s_1} & z_1 \\ z_1 & z_1 & z_2 \end{pmatrix} \tag{5}
$$

where A is the potential coefficient matrix (units of $\mathrm{mm}^2/\mathrm{pF}$) and $\epsilon_0 = 8.85 \times 10^{-3}$ pF/mm.

1.2 Matrix Elements

For the geometry considered above, the individual matrix elements are given by:

$$
\begin{aligned}
A_{11} &= \frac{1}{8.85 \times 10^{-3}\ \mathrm{pF/mm}} \left(5\ \mathrm{mm} - \frac{10\ \mathrm{mm}}{2\pi} \ln \frac{2\pi 0.01\ \mathrm{mm}}{10\ \mathrm{mm}}\right) = 1476.72\ \mathrm{pF} && (6) \\
A_{12} &= \frac{1}{8.85 \times 10^{-3}\ \mathrm{pF/mm}} \left(5\ \mathrm{mm} - \frac{10\ \mathrm{mm}}{2\pi} \ln 2\right) = 440.32\ \mathrm{pF} && (7) \\
A_{13} &= \frac{1}{8.85 \times 10^{-3}\ \mathrm{pF/mm}} 5\ \mathrm{mm} = 564.97\ \mathrm{pF} && (8) \\
A_{21} &= A_{12} = 440.32\ \mathrm{pF} && (9) \\
A_{22} &= \frac{1}{8.85 \times 10^{-3}\ \mathrm{pF/mm}} \left(5\ \mathrm{mm} - \frac{10\ \mathrm{mm}}{2\pi} \ln \frac{2\pi 0.05\ \mathrm{mm}}{10\ \mathrm{mm}}\right) = 1187.28\ \mathrm{pF} && (10) \\
A_{23} &= A_{13} = 564.97\ \mathrm{pF} && (11) \\
A_{31} &= A_{13} = 564.97\ \mathrm{pF} && (12) \\
A_{32} &= A_{13} = 564.97\ \mathrm{pF} && (13) \\
A_{33} &= \frac{1}{8.85 \times 10^{-3}\ \mathrm{pF/mm}} 10\ \mathrm{mm} = 1129.94\ \mathrm{pF} && (14)
\end{aligned}
$$

$$
A = \begin{pmatrix} 1476.72 & 440.32 & 564.97 \\ 440.32 & 1187.28 & 564.97 \\ 564.97 & 564.97 & 1129.94 \end{pmatrix} \mathrm{pF}. \tag{15}
$$

Inverting this matrix gives::

$$
A^{-1} = \epsilon_0 \begin{pmatrix} 96.849 & -16.895 & -39.977 \\ -16.895 & 127.831 & -55.468 \\ -39.977 & -55.468 & 147.723 \end{pmatrix} \mathrm{pF/m^2} \quad \text{with } \epsilon_0 = 8.85\ \mathrm{pF/m^2}. \tag{16}
$$

- The solution for the charges on the different electrodes is given by:

$$\begin{pmatrix} \sigma_s \\ \sigma_f \\ \sigma_c \end{pmatrix} = A^{-1} \begin{pmatrix} V_s \\ V_f \\ V_c \end{pmatrix}. \tag{17}$$

In comparison with the definition of capacitance ($C = Q/\Delta V$), the matrix A^{-1} is defined as the *capacitance matrix.*

1.3 Induced Charges

• Now calculate the induced charges on the cathode and field wire using the specific condition:

$$V_s = V,\ V_f = V_c = 0.$$

Then: $\sigma_s = A_{11}^{-1} \cdot V_s, \quad \sigma_f = A_{21}^{-1} \cdot V_s, \quad \sigma_c = A_{31}^{-1} \cdot V_s$

- Charge induced on each cathode is in ratio: $\sigma_c/\sigma_s = A_{31}^{-1}/A_{11}^{-1} = 41.3\%$
- Charge induced on field wire is in ratio: $\sigma_f/\sigma_s = A_{21}^{-1}/A_{11}^{-1} = 17.4\%$

For the alternative geometry with only sense wires and cathode planes (i.e. no field wires), the charge induced on each cathode is in the ratio:

$$\sigma_c/\sigma_s = 50\%.$$

2 Strip-to-Strip Capacitance

The primary design issue for the front-end electronics for the `FDC` system is to maintain a signal-to-noise ratio of better than 100:1 in the presence of a large detector capacitance. Capacitance measurements in detectors are very important because they serve to determine the performance of the detector in terms of the charges induced on the cathodes, gain variations, detector efficiency, etc. They also play an important role in determining the characteristics of the readout electronics. Accounting for the total capacitance of the system then forms a basic requirement for the input capacitance for the front-end electronics performance. An additional concern for the `FDC` design is the coupling of the signals between adjacent strips. Studies of the PHENIX CSCs have shown that appropriate setting of the integration times of the electronics is important to decrease sensitivity to distortions in the induced charge distribution due to inter-strip coupling.

For the `FDC` chambers the dominant contribution to the capacitance seen by the preamplifiers will be the strip-to-strip capacitance. The capacitance between adjacent strips has been estimated in several previous `FDC` documents using a form devised by the PHENIX collaboration [1]. This form is given by:

$$C(\text{pF/cm}) = 0.12t/w + 0.09(1+k)log_{10}(1 + 2w/s + w^2/s^2), \tag{18}$$

where t is the strip thickness, w is the strip width, s is the strip separation, and k is the dielectric constant of the backing material (here it is envisioned to be kapton). For the `FDC` design that is presently considered, the capacitance using this expression is 60 pF/m (assuming t=18.56 μm for 0.5 oz of Cu/ft^2, w=4 mm, s=1 mm, and k=3.5 for kapton).

Gerard Visser at IUCF has considered this formula in more detail [2]. He cannot make full sense of it as the PHENIX note itself does not provide a reference for this expression. Gerard believes that the above form may be intended to compute the capacitance between two strips in isolation on an infinitely thick dielectric.

However if this is true, the assumption is a poor one given that the dielectric is not thick compared with the strip dimensions. Gerard claims that it is probably a better approximation to consider an infinitely thin dielectric than an infinitely thick one. Under this assumption in the above equation (i.e. using k=1), C=25.2 pF/m. This gives a capacitance to ground (i.e. between the cathode strip and all other strips, wires, and ground planes) of ~50 pF/m. However this is probably an overestimate. With a grounded strip on either side of a given strip, each partially shields the other. So the total capacitance is not twice the capacitance of two isolated strips.

Gerard believes that a better approximation is to use the formula derived for a "coplanar waveguide" which consists of a strip of width w with gaps of width s on either side. Surrounding this are semi-infinite ground planes. This should be a good estimate of the total capacitance from one of the cathode strips to all of the others on a given plane. Mostly, however, this capacitance goes to the two neighboring strips. The associated form for the total capacitance per unit length is given by:

$$C(\text{pF/cm}) = \frac{4\pi\epsilon_0}{-\ln\left[q\left(\frac{w}{w+2s}\right)\right]}, \tag{19}$$

where $q(k)$ is a theta function (in Mathematica this is the built-in function `EllipticNomeQ`). This form is also contained in a similar manner in Appendix B of Ref. [3]. With w=4 mm and s=1 mm, this form gives C=41.4 pF/m. Again, this represents the capacitance from one strip to all others on the cathode plane. From this we can estimate the interstrip capacitance as ~20.7 pF/m. Gerard has compared the above formula to measurements carried out on a test chamber at IUCF and found agreement at the 10% level.

3 Wire Capacitance

The sense wire capacitance can be calculated using the argument from Sauli [4]. For a chamber consisting of an anode wire plane at potential V_0 sandwiched between two grounded cathode planes, the capacitance per unit length is given by:

$$C = \frac{2\pi\epsilon_0}{\left(\frac{\pi\ell}{s}\right) - \ln\left(\frac{2\pi a}{s}\right)}. \tag{20}$$

Here a is the anode wire radius, ℓ is the distance between the wire plane and the cathode plane, and s is the separation between the anode wires. Using a=0.01 mm, ℓ=5 mm, and s=10 mm, we can find the capacitance is given by: C=8.37 pF/m.

4 Capacitance From Strip to Ground

A simple-level estimate of the capacitance from a strip to the wire plane can be determined using the form of capacitance of a parallel-plate capacitor. This is given by:

$$C = \epsilon_0 \frac{A}{d} = \epsilon_0 \frac{L \cdot w}{d}. \tag{21}$$

Here A represents the strip area, L the strip length, w the strip width, and d is the separation of the strip from the wire plane. The capacitance per unit length is then:

$$\frac{C}{L} = \epsilon_0 \frac{w}{d} = 8.85 \text{ pF/m} \frac{4 \text{ mm}}{5 \text{ mm}} = 7.1 \text{ pF/m} \tag{22}$$

In computing the capacitance between the strip and ground, the definition from Gerard Visser is really the capacitance between the strip and everything else, i.e. all of the other strips, wires, and ground planes. This is (to first order) the parameter which is important for the preamplifier noise optimization. For full cross-talk simulations, we will eventually need to split this up into capacitances to other channels (which are virtual grounds) and capacitances to ground.

In the computation of the capacitance between a cathode strip and ground, the different contributions to the capacitance add in series. In this the capacitance to the wires dominates. We can thus estimate the capacitance between a cathode strip and ground to be ~7.1 pF/m.

5 Total Strip Capacitance

A final category to consider is the capacitance between the cathode strips and the various connections in the chamber. Gerard Visser has estimated this to be about 8 pF [5]. With this we can collect all of the various contributions to the cathode strip capacitance included in the various sections of this document.

- 41.4 pF/m from a strip to all other strips.
- 7.1 pF/m from a strip to the ground plane.
- 7.1 pF/m from a strip to the anode wire plane.
- 8 pF for connections.

This amounts to a total of 75 pF for the longest strips in the full-scale `FDC chamber` which are estimated to be 1.2 m long, or 22 pF for the longest strips in the prototype which are estimated to be 0.25 m long.

References

[1] B.V. Dinesh, PHENIX report phenix-muon-96-5, http://www.phenix.bnl.gov/phenix/WWW/muon/phnotes/PN125/PN-125.html

[2] Gerard Visser, e-mail communication dated May 26, 2005.

[3] E. Gatti, A. Longoni, and P. Semenza, "Optimum Geometry for Strip Cathodes or Grids in MWPC for Avalanche Localization Along the Anode Wires", Nucl. Inst. and Meth. **163**, 83 (1979).

[4] F. Sauli, "Principles of Operation of Multiwire Proportional and Drift Chambers", CERN internal note 77-09, (1977).

[5] Gerard Visser, "Noise Comparison of Some Preamps Relevant to FDC/Prototype Work", GlueX-doc-520-v1, (2005).