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Beam energy combinations which provide simultaneous longitudinal electron polarization in two experimental halls

A large fraction of the approved experimental program for the CEBAF accelerator requires longitudinal beam polarization at the experimental target. In addition, although only a fraction of the hall B program requires polarized beam, hall B has stated that they would prefer to operate with longitudinal beam polarization for as large a fraction of their beam time as possible. In TN-96-032, I identified the beam energy combinations which would provide simultaneous longitudinal polarization in halls A and C, but I incorrectly stated that the conditions providing longitudinal polarization in hall B and either of the other halls were few. In this TN, I identify all those combinations of beam energies which provide longitudinal polarization in two experimental halls simultaneously, for 5-pass accelerator energies between 2 and 6 GeV. There are over 400 discrete energy combinations which provide longitudinal beam polarization in any two halls simultaneously.

As in the earlier TN, I use the following physical constants: m = 0.51099906 MeV/c^2; and (g-2)/2 = 0.001159652. Following the equations in TN-96-032, we can write the total precession from the injector to each of the three halls as:

$$q_A = P\left[2n_A^2 - n_A\left(1-2a-\frac{1}{2.4}\right) - a\left(1-\frac{1}{4.8}\right)\right]p \equiv m_A p$$

$$q_B = P\left[2n_B^2 - n_B(1-2a) - a\right]p \equiv m_B p$$

$$q_C = P\left[2n_C^2 - n_C\left(1-2a+\frac{1}{2.4}\right) - a\left(1+\frac{1}{4.8}\right)\right]p \equiv m_C p$$

where P = $\left(\frac{E_l}{m}\right)\left(\frac{g-2}{2}\right)$, E_l is the energy of a single linac, α = 0.1125 is the ratio of the injector energy to the linac energy, and n_A, n_B, and n_C are the number of recirculation passes delivered to the indicated hall. Both linacs are assumed to operate at the same energy, and thus the energy of the beam in any particular hall is given by:

$$E_{A,B,C} = \left(2n_{A,B,C} + a\right)E_l$$

To find beam energy combinations which will provide simultaneous longitudinal polarization in any two halls, we simply require that the difference in the precessions to the two halls in question be an integral multiple of π. In these cases, we can find a single orientation of the polarization at the injector which will arrive longitudinal in each of the two halls. The general form of the precession difference equation is:

$$\frac{q_s - q_t}{p} = Pf(n_s, n_t) = m_s - m_t$$

For each particular choice of two halls, σ and τ, there are 21 possible values for the function f. I refer to this function as the precession difference function, and note that it depends only upon the number of the recirculation pass delivered to each of the two halls. In TN-96-032 these values were given for the case of halls A and C. Below, we give the 21 values of f for the cases of halls A and B, and for halls C and B, as well as repeating the previously presented values for halls A and C for completeness. Each three line entry in the tables below gives, in addition to the value of the function $f(n_s, n_t)$, the range of values of $m_s - m_t$ which can be obtained by operation of the accelerator with five pass energies between 2 and 6 GeV, and the total number of such values over this energy range. It is the fact that the values of f are in general large - i.e. that in general there is a large precession difference between any two halls - that permits so many energy combinations to provide simultaneous longitudinal polarizations to these two halls.

To illustrate the use of these tables to find beam energy combinations which provide simultaneous longitudinal polarization in two particular halls, consider the case of providing 3 pass beam to hall A and 5 pass beam to hall C, with a five pass energy close to 4 GeV. From the table for halls A and C, we obtain the value of $f(n_A = 3, n_C = 5)$ = -27.069792. Allowable values for $m_A - m_C$ can range between -13 and -36. Since we want an energy close to 4 GeV, which is midrange between 2 and 6 GeV, we guess that a value of $m_A - m_C$ close to the midrange of the allowed values, e.g. -24 or -25, will provide the best choice. Working through the numbers for these two cases, we find that $m_A - m_C$ = -24 corresponds to a linac energy of 390.6778 MeV, and $m_A - m_C$ = -25 corresponds to 406.9560 MeV. These values for the linac energy give five pass energies of 3.9507 GeV and 4.1153 GeV, respectively. The remaining step is to calculate the total precession to either one of the halls (remember that the difference in precession between the two halls is an integer), and thus determine what polarization orientation must be set at the injector to obtain longitudinal polarization in the hall. For the case with the linac energy of 390.6778 MeV, we find a total precession of 14.926694 π to hall A. Thus, to obtain longitudinal polarization in hall A, we need to add 0.073306 π = 13.195° to the polarization direction at the injector, in the horizontal plane. The skeptic is encouraged to work through these numbers, and check that the result also gives longitudinal polarization in hall C.

As a final note, it is worth remembering that for many cases where longitudinal polarization is required in the experimental halls, there is often not a genuine requirement for *exact* longitudinal polarization. Polarization orientations other than pure longitudinal will also have a transverse component, which will reverse as the longitudinal component is reversed. While one might be reluctant to have this situation present during a parity violation measurement, there are cases, such as when longitudinally polarized targets are used, when it is difficult to imagine that a small transverse component of polarization would be in any way problematic. To the extent that the condition for exact longitudinal polarization in a particular hall may be relaxed somewhat, the energy combinations calculated from the exact formulas may also be relaxed.

	$n_A = 1$	$n_A = 2$	$n_A = 3$	$n_A = 4$	$n_A = 5$
$n_B = 1$	Condition Not Allowed	6.081771 3 to 8 6 values	15.723438 8 to 21 14 values	29.365104 14 to 39 26 values	47.006771 22 to 64 43 values
$n_B = 2$	-4.784896 -3 to -6 4 values	Condition Not Allowed	10.498438 5 to 14 10 values	24.140104 11 to 32 22 values	41.781771 19 to 56 38 values
$n_B = 3$	-14.009896 -7 to -19 13 values	-8.368229 -4 to -11 8 values	Condition Not Allowed	14.915104 7 to 20 14 values	32.556771 15 to 44 30 values
$n_B = 4$	-27.234896 -13 to -37 25 values	-21.593229 -10 to -29 20 values	-11.951563 -6 to -16 11 values	Condition Not Allowed	19.331771 9 to 26 18 values
$n_B = 5$	-44.459896 -21 to -60 40 values	-38.818229 -18 to -52 35 values	-29.176563 -14 to -39 26 values	-15.534896 -8 to -21 14 values	2.106771 1 to 2 2 values

Table 1. Precession Difference Functions for Halls A and B

	$n_C = 1$	$n_C = 2$	$n_C = 3$	$n_C = 4$	$n_C = 5$
$n_B = 1$	Condition Not Allowed	4.368229 2 to 5 4 values	13.176563 6 to 17 12 values	25.984896 12 to 35 24 values	42.793229 20 to 58 39 values
$n_B = 2$	-5.665104 -3 to -7 5 values	Condition Not Allowed	7.951563 4 to 10 7 values	20.759896 10 to 28 19 values	37.568299 18 to 51 34 values
$n_B = 3$	-14.890104 -7 to -20 14 values	-10.081771 -5 to -13 9 values	Condition Not allowed	11.534896 6 to 15 10 values	28.343229 13 to 38 26 values
$n_B = 4$	-28.115104 -13 to -38 26 values	-23.306771 -11 to -31 21 values	-14.498438 -7 to -19 13 values	Condition Not Allowed	15.118229 7 to 20 14 values
$n_B = 5$	-45.340104 -21 to -38 41 values	-40.531771 -19 to -55 37 values	-31.723438 -15 to -43 29 values	-18.915104 -9 to -25 17 values	-2.106771 -1 to -2 2 values

Table 2. Precession Difference Functions for Halls B and C

	$n_A = 1$	$n_A = 2$	$n_A = 3$	$n_A = 4$	$n_A = 5$
$n_C = 1$	Condition Not Allowed	6.521875 3 to 8 6 values	16.163542 8 to 22 15 values	29.805208 14 to 40 27 values	47.446875 22 to 64 43 values
$n_C = 2$	-3.928125 -2 to -5 4 values	Condition Not Allowed	11.355208 6 to 15 10 values	24.996875 12 to 34 23 values	42.638542 20 to 58 39 values
$n_C = 3$	-12.736458 -6 to -17 12 values	-7.094792 -4 to -9 6 values	Condition Not Allowed	16.188542 8 to 22 15 values	33.830208 16 to 46 31 values
$n_C = 4$	-25.544792 -12 to -34 23 values	-19.903125 -10 to -27 18 values	-10.261458 -5 to -13 9 values	Condition Not Allowed	21.021875 10 to 28 19 values
$n_C = 5$	-42.353125 -20 to -57 38 values	-36.711458 -17 to -49 24 values	-27.069792 -13 to -36 24 values	-13.428125 -7 to -18 12 values	4.213542 2 to 5 4 values

Table 3. Precession Difference Functions for Halls A and C