Baryon spectroscopy in the Quark Model

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• What is a quark model?
  - Effective degrees of freedom, properties

• How should we treat confinement?
  - Recent guidance from lattice QCD

• What are the q-q “residual” interactions
  - One-gluon, one-boson, instanton-induced?

• What should we do about qqq(qq) states?
  - Self energies, mixings

• Conclusions
The Cork Model

Up  Charm  Top

Down  Strange  Bottom
Effective degrees of freedom

• **Constituent quarks**?
  - Have effective masses
  - Not point-like (have EM and strong form factors)

• **Di-quark cluster + quark**?
  - Two degrees of freedom at low energy \(\Rightarrow\) fewer excited states

• **Collective excitations of string-like model**?
  - Algebra-based models: radial excitations from rotations and vibrations of strings \(\Rightarrow\) more excited states

• **Light quarks in a bag, pions coupled to surface**?
  - Difficult to describe highly excited states

\(\Rightarrow\) **Constituent quarks + flux tubes**
What are their properties?

- Dynamically generated constituent masses, which can run with $Q^2$
  - Evidence from lattice that at low $Q^2$, $m \approx \Lambda_{QCD}$
  - Similar results from Dyson-Schwinger Bethe-Salpeter studies of hadrons, quark propagators

- Models with $K=(p^2+m^2)^{1/2}$ could use $0<m_{ud}<250$ MeV
  - Isospin-violating mass splittings $\Rightarrow K_d-K_u \approx 5-10$ MeV
  - From current algebra $\Delta m = m_d-m_u \approx 5-10$ MeV
  - $K_d-K_u = (p^2+[m_u+\Delta m]^2)^{1/2} - (p^2+m_u^2)^{1/2} \approx (m_u/K_u)\Delta m$ if $\Delta m<<K_u$

- So $m_u \approx K_u \approx \Lambda_{QCD} \approx 200$ MeV!
  - Strange quark has $m_s \approx m_u + 150 - 200$ MeV
...What are their properties?

• Effective sizes, form factors
  - Strong form factors: Gaussian (convenience), monopole
    • Make finite contact interactions $\propto \delta^3(r_i-r_j)$
    • Heavier quarks more point-like
  - EM form factors required to fit nucleon $G_E$, $G_M$
    • Even in relativistic (light-cone) calculations
    • E.g. $F_1^i(Q^2) = e_i/(1+Q^2/\Lambda_1^2)$, $F_2^i(Q^2) = \kappa_i/(1+Q^2/\Lambda_2^2)^2$
    • The $\kappa_i$ should be environment sensitive
      - Lattice (Leinweber and Voloshyn), BM loops
        ⇒ See talk by P. Gonzalez, Tues.@3:00!
  - Strong and EM sizes should be similar
How should we treat confinement?

• Quenched lattice measurement of $QQQ$ potential
• Takahashi, Matsufuru, Nemoto and Suganuma, PRL 86 (2001) 18.
• Measure potential with $3Q$-Wilson loop (static quarks) for $0 < t < T$
• Also fit $QQ$ potential to compare $\sigma$ and Coulomb terms
... How should we treat confinement?

- Fit 16 QQQ configurations to
  \[ V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{|r_i - r_j|} + \sigma_{3Q} L_{\text{min}} + C_{3Q} \]

- \(L_{\text{min}} = \text{min. length Y-shaped string}\)

- 3Q, QQ string tensions similar

- Coulomb terms in OGE ratio \(\frac{1}{2}\)

### TABLE I.

<table>
<thead>
<tr>
<th>(\sigma)</th>
<th>(A)</th>
<th>(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3Q</td>
<td>0.1524(28)</td>
<td>0.1331(66)</td>
</tr>
<tr>
<td>(Q-\bar{Q})</td>
<td>0.1629(47)</td>
<td>0.2793(116)</td>
</tr>
</tbody>
</table>

- \(\sigma\) is in lattice units \(a^{-2}\)
- Meson string tension 0.89 GeV/fm (\(a=0.19\) fm)
... How should we treat confinement?

- Also tried fit to function
  \[ V_{3Q} = -A_\Delta \sum_{i<j} \frac{1}{|r_i - r_j|} + \sigma_\Delta \sum_{i<j} |r_i - r_j| + C_\Delta \]

- Fit worse: \( \chi^2 \) per d.f. 3.8 \( \Rightarrow 10.1 \)
- Result is a reduced string tension \( \sigma_\Delta = 0.53 \sigma \)
  - Simply a geometrical factor
  - Perimeter \( P \) satisfies \( 1/2 < L_{\text{min}}/P < 1/(3)^{1/2} = 0.58 \)
  - Accidentally close to ratio \( \langle \Lambda_i \cdot \Lambda_j \rangle_{\text{baryons}} / \langle \Lambda_i^* \cdot \Lambda_j \rangle_{\text{mesons}} \)
    but confinement is not (colored) vector exchange!

\( \Rightarrow \) string-like potential good for QQQ baryons!
\( \Rightarrow \) Assume it is good for qqq baryons
Flux-tube model

- Based on strong-coupling lattice QCD
  - Color fields confined to narrow tubes, energy $\propto$ length
  - Junction, to maintain global color gauge invariance
  - Plaquette operator from lattice action:
    - Moves tubes transverse to their original direction
    - Moves junction, but leaves string excited
Model confining interaction

- Flux tubes, combined with adiabatic approx.
  - confining interaction: minimum length string
  - $V_B(r_1, r_2, r_3) = \sigma(l_1 + l_2 + l_3) = \sigma L_{\text{min}}$
    - note $\sigma$ is meson string tension
    - linear at large q-junction separations
  - Strings should be allowed to be dynamical
    - See talk by Philip Page, Thus.@11:15!
...Model confining interaction

- Fix quark positions $r_i$, allow flux tubes to move
  - Junction moves relative to its equilibrium position
  - Strings move transverse to their equilibrium directions
- Ground state of string defines adiabatic potential
  - $V_B(r_1, r_2, r_3) = \sigma (l_1 + l_2 + l_3) = \sigma \text{L}_{\text{min}}$, plus zero point motion
...Adiabatic potentials

- plot of $V_B - b \sum_i l_i$ as a function of $\rho, \lambda$, with $\cos(\theta_{\rho\lambda}) = 0$: 
What are their residual interactions?

- Ground-state spectrum suggests flavor-dependent short-range (contact) interactions
- One-gluon exchange: good fit to ground states with (color-magnetic dipole-dipole), e.g. \( \Sigma-\Lambda \Rightarrow \) DeRujula, Georgi, Glashow

\[
M = \sum_{i=1}^{3} m_i + \frac{2\alpha_s}{3} \frac{8\pi}{3} \langle \delta^3(r) \rangle \sum_{i<j=1}^{3} \frac{S_i \cdot S_j}{m_im_j}
\]

- Gives, e.g. \( (m_\Sigma-m_\Lambda)/(m_\Delta-m_N)=2/3 \) \((1-m_{u,d}/m_s) \approx 2/3 \) \((0.6)=0.27, \) expt. \( (1193-1116)/(1232-939)=0.26 \)
- Explains regularities in meson spectrum (e.g. evolution of vector-pseudoscalar splitting with quark mass)
  - Unclear why this should work for light quarks...
- Taken at face value predicts tensor interaction

\[
H^{ij}_{\text{hyp}} = \frac{2\alpha_s}{3m_im_j} \left\{ \frac{8\pi}{3} S_i \cdot S_j \delta^3(r_{ij}) + \frac{1}{r_{ij}^3} \left[ 3(S_i \cdot r_{ij})(S_j \cdot r_{ij}) - S_i \cdot S_j \right] \right\}
\]

- And spin-orbit interactions, at a level not present in analyses
Residual interactions...

- **Contact splitting active in** L=1 **excited states**
- **Characteristic splitting is** \((m_\Delta - m_N)/2\)
- Add consistent tensor interaction
- No strong evidence for tensor from spectrum
- Best evidence from decays, \(S_{11}(1535) \to N\eta\)

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**Isgur and Karl**

PRD20, (1979) 768
Residual interactions...

- Also applied to $L=1$ strange baryons
- Degenerate lightest $\Lambda^{1/2-}$ and $\Lambda^{3/2-}$
- Data sparse, analyses even less certain
- Consistent (OGE plus confinement) spin-orbit cannot explain $\Lambda^{3/2-}(1520)$-$\Lambda^{1/2-}(1405)$
Residual interactions…

- Model has been applied to all baryons (Isgur, Karl,...)
- Variational calculation in large HO basis (SC, N. Isgur)
  - String confinement, plus associated spin-orbit
  - Include OGE Coulomb, contact, tensor, spin-orbit
  - Relativistic KE, relativistic corrections in potentials, e.g.
    \[
    \left( \frac{m_i m_j}{E_i E_j} \right)^{\frac{1}{2}+\epsilon_{\text{cont}}} \left[ \frac{8\pi}{3} \alpha_s(r_{ij}) \frac{2}{3} \mathbf{S}_i \cdot \mathbf{S}_j \left[ \frac{3}{8\pi} \frac{\sigma_{ij}^3}{m_i m_j} e^{-\frac{\sigma_{ij}^2}{r_{ij}^2}} \right] \right] \left( \frac{m_i m_j}{E_i E_j} \right)^{\frac{1}{2}+\epsilon_{\text{cont}}}
    \]
  - Photocouplings calculated with $H_{\text{int}}$ expanded to $O(p^2/m^2)$
  - Strong decays calculated in pair creation ($^3P_0$) model (with W. Roberts)
- Reasonable agreement; allows prediction of favorable channels to find 'missing' baryons
- Puzzles: Roper mass; $\Lambda_{3/2}^{-}(1520)$-$\Lambda_{1/2}^{-}(1405)$; $L=1$ too light by 50 MeV, positive parity too massive by 50 MeV,...
Residual interactions...

- Another possibility: should light quarks exchange pions? Robson; Buchmann, Faessler, ...
- Gluons not active in light-quark hadrons: flavor dependence through exchange of octet of pseudoscalars (GBE)
- Contact interaction:
  \[ H_X \sim - \sum_{i<j} \frac{V(r_{ij})}{m_im_j} \lambda^F_i \cdot \lambda^F_j \sigma_i \cdot \sigma_j \]
- Order of states inverted? Natural with GBE ⇔ Glozman & Riska (GR)
Residual interactions...

- GR fit radial matrix elements of $V(r_{ij})$ to spectrum
- **Calculated** in variational H.O. basis with consistent tensor Glozman, Plessas, Theussl, Wagenbrunn, & Varga
- add nonets of exchange vector mesons and scalars
  - Relativistic K.E., string confinement; calculate decays
Residual interactions...

- Another flavor-dependent possibility: instanton-induced interactions \( \Rightarrow \) see talks by: Ulrich Löring, Tues.@5:30; D. Diakonov, Wed.@8:30

\[
\langle q^2; S, L, T|W|q^2; S, L, T \rangle = -4g \delta_{S,0} \delta_{L,0} \delta_{T,0} W
\]

- Present if \( qq \) in S-wave, \( I=0, S=0 \) state
- \( W \) is a contact interaction
- Applied to excited states Blask, Bohn, Huber, Metsch & Petry
  - Efficient model—few parameters
  - Small splittings in P-wave \( \Sigma \), positive-parity states generally too heavy, by 250 MeV
  - Works reasonably in ground states, P-wave N and \( \Lambda \)
Baryon spectroscopy requires a decay model

• It is not enough to give the masses of your states!
  - Analyses of data generally have fewer states in them than in models (some model states are “missing”)
    • But see talk by Javier Vijande, Thus.@2:15!
  - You must also predict which of your states are likely to be seen in analyses of scattering data
  - $^3P_0$ is popular phenomenological decay model
  - See talks by P. Gonzalez & B. Desplanques, Tues@3:00, 3:15
    • Has advantage that emitted mesons have structure
    ⇒ Can correlate many decays with very few parameters
Nucleon model states and $N\pi$ couplings

SC and N. Isgur, PRD34 (1986) 2809;
Δ model states and Nπ couplings

Δ experimental and model states below 2200 MeV

PDG mass range

N=3 band

N=0,1,2 bands

Nπ amplitudes

0 5 \geq 10 \text{ MeV}^{1/2}
Unquenching the quark model

- In QCD $qqq(qq)$ configurations possible in baryons: effect on CQM?
  - Model with baryon-meson intermediate states, loops $\Rightarrow$ self energies
  - High-momentum part of loops contains OBE
Unquenching the quark model...

To calculate self energies and mixing:

- Need model of $B \rightarrow B'$ $M$ vertices and their momentum dependence
- Need model of spectrum (including states not seen in experiment) $\leftrightarrow$ wave functions $\leftrightarrow$ vertices

- Solve $E + \Sigma_B(E) = M_B$ self-consistently to find $E$ (bare mass) at which sum is physical mass $M_B$
- Find effects on spectrum by examining splittings in bare masses
Unquenching the quark model...

• Baryon self energy due to individual B'M loops comparable to widths – convergence?
• Best calculations applied to $N$, $\Delta$, $\Lambda$, $\Sigma$, ground and $L=1$ states
• Intermediate states $B'M$
  - Ground state mesons $M \in \{\pi, K, \eta, \eta', \rho, \omega, K^*\}$
  - Ground states baryons $B \in \{N, \Delta, \Lambda, \Sigma, \Sigma^*\}$
  - $\pi$, $K$, $\eta$: Blask, Huber & Metsch ZPA326 (1987) 413
Unquenching the quark model...

- **Couple mesons to baryons**
  - Point-like elementary particles
  - Using pair creation ($^3P_0$) or similar model ✔

- **Either time-ordered perturbation theory**

\[
\Sigma_R(E) = \sum_B \int d^3p \frac{V_{\pi R N}^\dagger(p)V_{\pi R N}(p)}{E - E_R(p) - E_\pi(p)}
\]

- **Or dispersion relation**

\[
m_A^2 - (m_A^0)^2 = \sum_I w_I^A \int_{S_{thr}^i} \frac{\rho(s, m_B, m_M)}{m_A^2 - s} ds
\]

⇒ Zenczykoswki calculates $\rho$ using $^3P_0$
Unquenching the quark model…

• Effects on spectrum are substantial (≈ 50-100 MeV):
  - Zenczykowski finds many mass splittings close to analyses without qqq residual interactions
  - Other calculations show splittings in bare masses which resemble spin-orbit effects
    • Solution to spin-orbit problem in baryons?

• Lack self-consistent treatment of external and intermediate states—converged?
  • Such convergence slower in $^3P_0$ NRQM: mesons, Geiger and Isgur
  • Faster in covariant model based on Schwinger-Dyson Bethe-Salpeter approach: SC, Pichowsky, Walawalkar
Unquenching the quark model…

- Essential problem: there are lots of $B'M$ thresholds nearby in energy
  - $N_\rho$, $\Delta_\rho$ similar thresholds to $N(1535)\pi$, $\Lambda(1116)K$, etc.
  - Cannot study spectrum by restricting $M$ to $\pi$ (or even all pseudoscalars) or $B'$ to $N$, $\Delta$ (or even all octet and decuplet ground states)
Unquenching the quark model…

- Zencyzkowski:
  - assume $SU(3)_f \otimes SU(2)_{\text{spin}}$, only ground state $B$ and $M$ exist
  - all octet and decuplet ground state baryons have mass $M_B$ and same wavefns.
  - all pseudoscalar and vector ground state mesons have mass $M_M$ and same wavefns.

- All loop integrals now the same, apart from $SU(6)$ factor at vertices

$\Rightarrow$ Sum of loops for $N$ and $\Delta$ same only if include all $B'M$ combinations consistent with quantum numbers!
Unquenching the quark model…

- Convergence examined using unmixed oscillator wave functions for intermediate baryons
  - Brack and Bhaduri, PRD35 8451 ('87)

\[
\Sigma_{\pi}(i) = -\frac{1}{4\pi^2} \sum_B \sum_P \int_0^\infty \frac{k^2 |\mathcal{M}_{Bi}|^2 F_{\pi}^2(k)dk}{\omega_k [\omega_k - (E_B - E_i)]}
\]

- \(B_{\pi}\), with \(B \in \{N=0,1,2 \text{ & 3 band } N, \Delta \text{ states}\}\)
- Find splitting \(\Delta-N\) converges by \(N=3\), but those of negative-parity states do not
Self energies in relativized $^3P_0$ model

• PhD thesis of Danielle Morel (FSU)
  ⇨ See her talk, Tues@6:00!

• Calculate vertices as a function of loop momentum using $^3P_0$ model (analytic, Maple)
  - Use mixed relativized-model wavefunctions (expanded up to N=7 band)
  - Include intermediate states BM with
    • Mesons $M \in \{\pi, K, \eta, \eta', \rho, \omega, K^*\}$
    • Baryons $B \in \{N, \Delta, \Lambda, \Sigma, \Sigma^*\}$, including all excitations up to N=3 band
Self energies in relativized $^{3}\text{P}_0$ model…

- Integrate over loop momentum, over pole if above threshold

$$\Sigma_{N^*}(E) = \sum_{BM} \int d^3k \frac{|M_{N^* \rightarrow BM}(k)|^2}{E - \sqrt{m_M^2 + k^2} - \sqrt{M_B^2 + k^2}}$$

- Usual $^{3}\text{P}_0$ model gives vertices that are too hard, loops get large contributions from high momenta
  - Geiger and Isgur give pair-creation operator a form factor $\sim \exp(-f^2[p_q - p_{\bar{q}}]^2)$, with $f^2=3.0\;\text{GeV}^{-2}$
  - Gives vertex a spatial size of $\sim0.35\;\text{fm}$

- Silvestre-Brac and Gignoux use similar form factor $\Rightarrow$ reasonable splittings of $-ve$ parity states
Self energies and mixing

- **Self energies are not diagonal!**
  - States with same quantum numbers mix through BM loops
  - Need to **diagonalize** self energy matrix
  - E.g. N and Roper, 2x2 mixing through all $B_\pi$ intermediate states:
    - Need $N \leftrightarrow B_\pi \leftrightarrow N$, $R \leftrightarrow B_\pi \leftrightarrow R$, $N \leftrightarrow B_\pi \leftrightarrow R$, $R \leftrightarrow B_\pi \leftrightarrow N$
    - Energy dependent 2 x 2 matrix $\Sigma(E)$
  - How can we find the self energies of the un-mixed states? (Thanks to Mike Pichowsky...)
Self energies and mixing…

• Poles in Green’s function are at zeros of $G_0^{-1}-\Sigma(E)$

• quadratic equation for counter terms

$$\begin{vmatrix}
E - M_N + i\epsilon & 0 \\
0 & E - M_R + i\epsilon
\end{vmatrix} - \begin{pmatrix}
\Sigma_{NN}(E) & \Sigma_{NR}(E) \\
\Sigma_{RN}(E) & \Sigma_{RR}(E)
\end{pmatrix} - \begin{pmatrix}
\delta M_N & 0 \\
0 & \delta M_R
\end{pmatrix} = 0$$

• Solve at $E = M_N$ and $E = M_R$: \(\Rightarrow\) two equations in two unknowns, find \(\delta M\)'s
Conclusions

• Quark models of baryons undergoing renaissance!
• Lattice QCD, Schwinger-Dyson approaches:
  - Can help identify degrees of freedom and nature of confinement
  - Able to calculate ground and some first excited states
  - Can work together with quark models (extrapolation to light quark masses)
• The next Fock-space component is likely more important than differences among qqq models
  - calculating its effects requires:
    • Use of full SU(6)-related set of intermediate states, spatially-excited intermediate baryons
    • Realistic model of off-shell vertex amplitudes
    • Careful treatment of mixing effects
Tallahassee may be a small city…

…but people there have heard about baryons