Heavy quark physics on the lattice

BARYONS2002

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**Lattice QCD**

Euclidean space-time lattice

+ QCD Lagrangian (discretised)

\[
\mathcal{L}_{QCD} = \mathcal{L}_g + \mathcal{L}_q \\
= \frac{1}{2g^2} Tr F_{\mu\nu}^2 + \bar{\psi} (\gamma \cdot D + m) \psi
\]

Parameters are those of QCD:
Bare gauge coupling $\beta = 6/g^2$
Quark masses, $m_i a$.
Lattice spacing ($a$) is implicit u.v. cutoff
Determine $a$ and fix $m_i$ from $1 + n_f$ hadron masses.
Difficulties

Systematic errors:

- Discretisation errors - physical results are dependent on $a$. Reduce the dependence and/or extrapolate to $a = 0$.

- Matching errors - must renormalise lattice matrix elements to obtain continuum results. Requires pert. or nonpert. matching calculation.

- Quenching errors - v. expensive to include light dynamical (sea) quarks. Error from using quenched approx. = 10-20% (??).
A short history of lattice QCD

- Invented 1974
  slow progress ....

- Renaissance in 1990s
  - Much improved understanding of systematic errors
  - 10-20% errors possible on spectrum, form factors etc

- Second lattice revolution - early 2000s
  - Teraflop computing power will enable simulations with ‘real’ dynamical quarks
  - carry improvement of systematic errors further
  - 2-3% errors will be possible
Why do you care?
Improved theoretical precision will add huge value to experiment, e.g. from B factories.

(CLEO-c - new expts will check lattice errors.)
Heavy Quark Physics

Heavy ($b$ and $c$) quarks ($m > \Lambda_{QCD}$) present special challenges to lattice QCD because $m_Q a \geq 1$. $m_b a \sim 2 - 3$, $m_c a \sim 0.5 - 1$.

$p^\parallel \approx m_Q$ very distorted and naive use of relativistic quark formulations (Wilson, clover) give large errors ($\propto m_Q a, (m_Q a)^2$).

BUT, $b$ and $c$ are non-relativistic in their bound states.

$m_Q$ and $p^\parallel \approx m_Q$ are irrelevant dynamical scales. Can treat $b$ and $c$ quarks accurately on the lattice with non-relativistic techniques.

Several ways to proceed:
1. **Static Quarks**: $m_Q = \infty$ limit. Spinless and flavourless. Quark prop $= \text{string of gluon fields in time dirn.}$ Useful limit for understanding HQET.

2. **NRQCD**: Non-relativistic effective theory.

$$L_Q = \bar{\psi}(D_t - \frac{\vec{D}^2}{2m_Qa} - c_B \frac{\vec{\sigma} \cdot \vec{B}}{2m_Qa} + \ldots) \psi$$

$\psi$ a 2-component spinor.

$m_Qa$ fixed by requiring one heavy hadron mass correct. $E_h(p) = E_h(0) + p^2 / 2m_h$.

$c_i$ fixed by pert. or nonpert. matching to QCD. $m_Q \to \infty$ is static.

Cannot take $a$ to 0 but improve until $a$-dependence small enough.
3. **Heavy Wilson quarks (FNAL method).**

\[ \mathcal{L}_Q = \bar{\psi}(\mathcal{D} + m_Q a - \frac{i a c_{sw}}{4} \sigma_{\mu\nu} F^{\mu\nu}) \psi \]

but interpret non-relativistically to fix \( m_Q a \). Match to QCD with \( m_Q a \)-dependent coefficients.
Small \( m_Q a \) limit is light quarks.
Large \( m_Q a \) limit is NRQCD.

4. **Wilson/clover quarks** : Same action as above but work only at small \( ma \) (OK for \( m_c \) ?).
Extrapolate to large \( m \) ⇒ large errors *and* expensive. Anisotropic lattices may be better.
(fine \( a_t \) ⇒ small \( ma_t \) even with large \( a_s \)). See X. Liao parallel talk.
Results on the spectrum

1. $b\bar{b}$ ($\Upsilon$) spectrum (UKQCD collaboration - Marcantonio et al)

Radial and Orbital Splittings in the $b\bar{b}$ system

--- : Experiment
○ : NRQCD for the $b$, glue: $n_f = 0, \beta = 6.0$.
● : NRQCD for the $b$, glue: $n_f = 2$, $clove, \kappa = 0.135, \beta = 5.2$, UKQCD ensemble.
Fine Structure in the $b\bar{b}$ system

--- : Experiment

○ : NRQCD for the $b$, glue: $n_f = 0, \beta = 6.0$.

● : NRQCD for the $b$, glue: $n_f = 2, \text{clover}, \kappa = 0.135, \beta = 5.2$, UKQCD ensemble.

■ : extrapolate to light dynamical mass and to $n_f = 3$.

Hyperfine splitting sees dynamical quarks. Predict $m(\Upsilon) - m(\eta_b) = 60 \pm 15$ MeV. Aim for 1 - 2 % error for CLEO-c.
2. $c\bar{c}$ ($\psi$) spectrum

Columbia results (Chen et al) on anisotropic lattices in the QA, including $c\bar{c}g$ hybrids.
3. $b$-light ($B$) spectrum

NRQCD results in the QA (Hein et al).
4. \textit{b}-light-light baryon spectrum

NRQCD results in the QA (Ali Khan et al) for \textit{udb, usb, ssb} states.
Quark masses

$m_b$ fixed s.t. $B$ or $\Upsilon$ mass correct. Best determination is from $B$ in static limit in QA.

$$\overline{m}_b(\overline{m}_b) = Z_{\text{cont}}(m_B - E_B(\vec{p}^2 = 0) + E_0)$$

$Z_{\text{cont}}$ and $E_0$ known in pert. th. through $\alpha_s^3$. 'World average' is 4.30(10) GeV. (Ryan, LAT01)

$m_b \approx 50$ MeV smaller unquenched (?)

New non-pert methods in progress (Sommer et al).

$m_c$ from $\psi$ and $\alpha_s^2$ lattice mass renorm for non-rel. case (LAT01: Juge et al)

$m_c(M_c) = 1.28(4)$ GeV, QA

$m_c$ from $D_s$ and non-pert renorm. for rel. case (Becirevic et al)

$m_c(m_c) = 1.26(13)$ GeV, QA.
Determining $B$ matrix elements

1. $f_B$
   Simplest 2pt m.e. for $B$ leptonic decay.

\[ \langle 0 | A_\mu | B \rangle = p_\mu f_B \]

Has improved a lot with time (QA).
Match $A_\mu$ on lattice to continuum. Done to $O(\alpha_s, 1/m_Q, a)$ for NRQCD and heavy Wilson. Typical error ‘budget’:

<table>
<thead>
<tr>
<th>Source</th>
<th>percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistical + interp.</td>
<td>3</td>
</tr>
<tr>
<td>disc. $O((a\Lambda)^2)$</td>
<td>4</td>
</tr>
<tr>
<td>pert. $O(\alpha_s^2, \alpha_s^2/(aM))$</td>
<td>7</td>
</tr>
<tr>
<td>NRQCD $O((\Lambda/M)^2, \alpha_s\Lambda/M)$</td>
<td>2</td>
</tr>
<tr>
<td>light quark mass</td>
<td>±4</td>
</tr>
<tr>
<td>$a^{-1}(m_\rho)$</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
</tr>
</tbody>
</table>

World averages (Ryan, LAT01)

$f_B^{(QA)} = 173 \pm 23$ MeV, 20% larger unq. ?

$f_{D_s}^{(QA)} = 203 \pm 14$ MeV

$f_{B_s}/f_B = 1.15(5)$; $f_{D_s}/f_D = 1.16(4)$

Need to reduce pert. errors by $O(\alpha_s^2)$ calcs/non-pert. techniques.
2. $B_B$
Matrix element for 'box' diagram for $B^0 - \overline{B}^0$ mixing – becomes m.e. of 4-q operator from $H_W$. Convenient to take ratio to $f_B^2$, call result $B_B$.
$\Delta m_B \propto |V_{tb}^* V_{tq}|^2 f_B^2 B_B$. 

\[
\begin{align*}
B^0 & \quad \overline{B}^0 = \\
H_W &
\end{align*}
\]
World averages (Ryan, LAT01, Bernard, LAT00):

\[ \hat{B}_{B_d} = 1.30(12)(13); \ f_{B_d}\sqrt{\hat{B}_{B_d}} = 230(40) \text{ MeV}. \]

\[ \hat{B}_{B_s}/\hat{B}_{B_d} = 1.01(3); \]

\[ \xi \equiv \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} = 1.16(5) \]
3. \( B \) SL decay.

Matrix elements needed for determination of \( V_{ub}, V_{cb} \).

\( B \rightarrow D(*) \) decay.

Heavy quark symmetry very useful here \( \rightarrow \) study matrix element as function of \( \omega = v_B \cdot v_D \).
Form factor then has universal shape to a good approx. = Isgur-Wise function.
exptl rate($v_B \cdot v_D = 1$) = $|V_{cb}|^2 \mathcal{F}(1)$.

FNAL LAT01 (Hashimoto et al):

$\mathcal{F}_{B \to D^*}(1) = 0.913(30)$. (QA)

gives $V_{cb} \times 10^3 = 38.7 \pm 1.8 \pm 1.5$ (LEP).
$B \to \pi, \rho$ decay.

Lattice calculations (all QA) work at small $\vec{p}_\pi$, far from physical region. Require extrapolation, interpolation, etc. Smooths out very rough raw data. More work is needed.

Soft pion theorem $f_0(q^2_{max}) = f_B/f_\pi$ doesn’t work well. Chiral extrapolations to light quark masses very important.
Future

- MILC collab. using improved staggered quarks $\rightarrow$ can simulate light dyn. quarks with Tflop computers in next few years.

- CLEO-c will determine $\Upsilon, \psi, D$ physics to high precision.

Lattice calcs must improve systematic errors to 2-3%.

- Improve heavy quark actions, higher rel. corrns and better match to continuum.

- Improve matching of matrix elements using automated pert. th. (Trottier, Horgan)

- Simulate with light dynamical quarks and match to chiral pert. theory.