Baryons on the Lattice

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• Review lattice methodology
• Study of confinement mechanisms
• Spectroscopy results:
  – Quenching effects
  – Quenched and dynamical light quark spectroscopy
  – Importance of chiral extrapolations
  – Adding electromagnetic interactions
• Computing resources
Regularization of QCD on a lattice

- Approximate continuous space-time with a 4-dim lattice, and derivatives by finite differences.
- Quarks are put on sites, gluons on links. Gluons represented by 3x3 complex unitary matrices $U_{\mu}(x) = \exp(iga A_{\mu}(x))$ elements of the group SU(3).

$$\left\langle \mathcal{O} \left( U, \psi, \bar{\psi} \right) \right\rangle = \frac{1}{Z} \int dU_{\mu} d\bar{\psi} d\psi \, \mathcal{O} \left( U, \psi, \bar{\psi} \right) e^{-S_{G}(U)+\bar{\psi}M(U)\psi}$$

$$\quad \rightarrow \frac{1}{Z} \int dU_{\mu} \, \mathcal{O} \left( U, M^{-1}(U) \right) \det \left( M \left( U \right) \right) e^{-S_{G}(U)}$$

- Gaussian integration over anti-commuting fermion fields $\psi$ resulted in $\det(M(U))$ and $M^{-1}(U)$ factors.
- Gauge action composed of $U$ fields. Approximates continuum:

$$S_{G} = \frac{1}{4} \int d^{4}x \, F_{\mu\nu}^{a} F_{\mu\nu}^{a} + O(a^{2})$$
Monte Carlo Methods

• On a finite lattice need to compute integral over large, but finite, number $U$-fields. Can be done numerically, though not by direct integration.

• Stochastic Monte Carlo method: generate series of configurations $U^{(i)}(x)$ distributed with probability $\exp(-S_G(U))\det(M(U))/Z$ and compute expectation values as averages over those configurations:

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(U^{(i)}, M^{-1}(U^{(i)}))$$

• Statistical errors go like $1/\sqrt{N}$, for $N$ configurations

• The $\det(M(U))$ factor is a big computational cost since the matrix $M$ is order $V \times V$, though it is sparse.

• Quenched approximation: set $\det(M) = 1$, e.g., neglect internal quark loops.
Statistical and Systematic Uncertainties

• Procedure is in principle exact after systematic errors are controlled

• **Statistical uncertainties:**
  – Statistical errors go like $1/\sqrt{N}$, for $N$ configurations.
  – Including determinant, cost of producing each configuration $O(100)$ times more expensive.

• **Systematic uncertainties:**
  – *Finite volume:* lattice box must hold a hadron state, typically $L \sim 2\text{fm}$ or more. Need $M_\pi L \sim 4$ (several pion Compton wavelengths)
  – *Chiral extrapolations:* calculations with small quark masses expensive – extrapolate observables to physical quark mass region (*delicate!*).
  – *Discretization effects:* inherent $O(a)$ or $O(a^2)$ lattice uncertainty. Must extrapolate to *continuum limit* ($a \to 0$) to recover physical quantities.
Confinement and Model Predictions - Static Quark Potentials

- Many models propose different mechanisms for confinement
- Static quark potential (potential between infinitely massive quarks forming mesons) in different representations can discriminate among the models
- Perturbative Casimir scaling hypothesis well describes non-perturbative lattice data:

\[ V(R) = d_D V_F(R), \quad d_D = C_D / C_F \]

for Casimir $C_D$ in representation $D=3,6,8,…$

- Claimed to rule out models like Bag and Instanton – scaling different
- Flux tube counting also inconsistent

Bali, 99
Static Baryon Potential

For SU(N) baryons, form N quarks in a gauge invariant quark state.

What is the area law behavior? Test two ansatze: Y-law and Δ-law

- **Y-law**: energy comes from flux tubes of shortest length join quarks. Area looks 3-bladed (N=3) joining at center. Looks like a Y. Length of flux tubes $L_Y$.

- **Δ-law**: energy composed of surfaces among all quark line pairs. Looks a delta. Length $L_\Delta$.

$$V_{Nq} = \lim_{r \to \infty} \frac{1}{T} \log(W_{Nq})$$

Data consistent with Δ-law – sum of 2 body quark potentials. Similar result for N=4.

Simonov argues *impossible*: field strength depleted near Y junction – lowers potential.

Should check using adjoint sources!

Alexandrou, DeForcrand Tsapalis, 2001; Kuzmenko, Simonov, 2002
Hadron Spectrum – Benchmark of Lattice QCD

- Spectrum of lowest lying states is the benchmark of LQCD
- Most extensively pursued lattice calculation
- Quenched spectrum agrees with experiment to 10%
- Inconsistency in meson sector apparently resolved in full QCD
- Systematic uncertainties:
  - *Finite volume*: $V \to \infty$
  - *Continuum extrapolation*: $a \to 0$
  - *Chiral extrapolations*: $M_{PS} \to M_{\pi}$
- Quenching effects – to some degree controllable/understandable???
Problem of Chiral Symmetry

• Naive lattice discretization of free Dirac operator

\[ \sum_\mu \gamma_\mu \partial_\mu \rightarrow \frac{1}{2a} \sum_\mu \gamma_\mu \left[ \delta_{x+a\hat{\mu},y} - \delta_{x-a\hat{\mu},y} \right] \]

• In momentum space

\[ G^{-1}(p) = \frac{i}{a} \sum_\mu \gamma_\mu \sin(ap_\mu) = i \sum_\mu \gamma_\mu p_\mu \left[ 1 + O((ap_\mu)^2) \right] \]

• Has additional zeros at all corners of the Brillouin zone, e.g. \( a*p_\mu = 0, \ldots, \pi \) – infamous doubling problem. Can *lift doublers* – add a Laplacian term that breaks chiral symmetry.

• Nielson-Ninomia no-go theorem – cannot avoid both doubling and chiral symmetry breaking with a local, hermitian action analytic in gauge fields. *Major theoretical problem.*

• Problem has been solved with recent advent of *chiral fermion actions* (e.g., Domain-Wall fermions). Crucial for matrix elements. Needed for spectroscopy??
Scaling – Continuum Limit

- Renormalization theory tells us that breaking a symmetry leads to induced quantum terms in an action.
- Wilson fermion action has $O(a)$ scaling from breaking chiral symmetry.
- Can rigorously add a dimension 5 operator (hyper-fine term) to improve scaling from $O(a)$ to $O(a^2)$
- Scaling violations dramatically reduced – mostly from improving chiral symmetry.
Quenched Pathologies in Hadron Spectrum

- How well is QCD described by an effective chiral theory of interacting particles (e.g., pions in chiral dynamics)?
- Suppressing fermion determinant leads to well known pathologies as studied in chiral pertubation theory (Bernard, Golterman, Sharpe)
- Missing vacuum contributions to disconnected piece of singlet correlator
  \[
  \langle \bar{\psi}(x)\gamma_5\psi(x)\bar{\psi}(y)\gamma_5\psi(y) \rangle = \langle \text{Tr}[\gamma_5 G(x,y)\gamma_5 G(y,x)]_{c,s} \rangle \\
  - N_f \langle \text{Tr}[\gamma_5 G(x,x)]_{c,s} \text{Tr}[\gamma_5 G(y,y)]_{c,s} \rangle
  \]
- Manifested in \(\eta'\) propagator missing vacuum contributions. E.g.,
  \[
  \hspace{1cm} \mathcal{O} + \mathcal{O} + \mathcal{O} \ldots
  \]
  \[
  \langle \text{Tr}\gamma_5 G(x,x)\text{Tr}\gamma_5 G(0,0) \rangle \rightarrow f_p \frac{1}{p^2 + m_\pi^2} m_0^2 \frac{1}{p^2 + m_\pi^2} f_p + \ldots
  \]
- New divergences arise. One idea is to incorporate knowledge of divergences in calculations and then extract useful information
Anomalous Chiral Behavior

- Compute $\eta'$ mass insertion from behavior in $Q\chi$PT
- Hairpin correlator fit holding $m_\pi$ fixed - well described by simple mass insertion
  \[ \langle \text{Tr} \gamma_5 G(x,x) \text{Tr} \gamma_5 G(0,0) \rangle \]
  \[ \to f_P \frac{1}{p^2 + m_\pi^2} m_0^2 \frac{1}{p^2 + m_\pi^2} f_P \]
  \[ f_P^{\text{quenched}} = \left( \frac{1}{m_\pi^2} \right)^\delta \tilde{f}_P \]
- $f_P$ shows diverging term. Overall $\delta = 0.059(15)$
- $m_0 = 680(30)\text{ MeV}\,$, $\chi$PT gives $850\text{MeV}.\,$ Still $O(a)$ errors
“Decay” in Quenched Approximation

- Dramatic behavior in Isotriplet scalar particle $a_0 \rightarrow \eta - \pi$ intermediate state
- Construct $a_0$ propagator from chiral lagrangian including couplings between $\eta - \pi$ states and rescattering states which can be resummed
- Lightest $a_0$ propagator fairly well described by 1-loop resummed bubble term with $\eta$ mass insertion fixed
- Find mass $a_0 = 1.34(9)$ GeV. Still $O(a)$ errors

Bardeen, Duncan, Eichten, Thacker, 2000
Decay in Full QCD

- MILC: evidence of (S-wave) decay in $a_0 \rightarrow \eta \pi$ in a $N_f=2+1$ calculation
- 3-flavor mass follows quenched then drops below. Decay not computed
- What is a decay? Subtle
  - Most straightforward way is for mass exactly at threshold
  - Compute all 2-point correlators $C_{H,H'}$ for $H=a_0$, $\eta\pi$
  - For $m(a_0) = m(\eta\pi)$, can compute transition amplitude $<a_0|\eta\pi>$ for large time separations from ratios of $C_{H,H'}$
Quenched Spectroscopy

Quenched XPT predictions for pseudoscalar, vector mesons, and decuplet baryons

\[ m_{PS,i2}^2 = A(m_1 + m_2) \left\{ 1 - \delta \left[ \ln \left( \frac{2A m_i}{\Lambda_X^2} \right) \right] + m_2 / (m_2 - m_1) \ln \left( \frac{m_2}{m_1} \right) \right\} + B(m_1 + m_2)^2 + O(m^3) \]

\[ m_H(m_{PS}) = m_0 + C_{1/2} m_{PS} + C_1 m_{PS}^2 + C_{3/2} m_{PS}^3, \quad C_{1/2} \propto \delta \]

- Large Wilson fermion calculation by CPPACS (Tsukuba) (99)
- Some evidence for quenching effects: more clearly seen in pseudoscalar channel
- Masses computed at 4 lattice spacings. Lattice sizes ranging up to 64^3 x 112 for a 3.2fm box.
Mass Predictions from Quenched Spectroscopy

- After chiral extrapolation, another extrapolation to continuum limit
- Fix scale at each coupling from experimental $\pi, \rho, \text{and } K$ (or $\phi$) masses
- Quenched spectrum systematically deviates from experiment. Typically 5% too small.
- Calculation $\sim 50$ Gigaflop-year

![Graphs showing mass predictions versus lattice spacing](image-url)
Meson Mass Predictions from Dynamical Fermions

- CPPACS: $N_f=2$ dynamical calculation. 4 quark masses at 3 couplings. Box sizes about 2.5 fm.
- Consistent results between original quenched calculations. Systematic deviation from experiment.
- $N_f=2$ calculation agrees within 1% of experiment – sea quark effects important.
- Increased hyperfine splitting consistent with suppressed spin-spin coupling in quenched from faster running of coupling.
- Calculation $\sim 1$ Teraflop-year
Baryon Mass Predictions from Dynamical Fermions

- CPPACS: Nf=2 dynamical sea quark effects not as apparent
- N and Δ mass high, but other masses consistent.
- Box sizes about 2.5fm. Worry finite-volume effects large.
- Octet and decuplet chiral extrapolations have many parameters.
Improved Chiral Extrapolations

• Adelaide group – extensively studying higher order chiral PT effects on hadronic quantities

• Basic upshot – naïve chiral extrapolations just too naïve!!

• Incorporate leading non-analytic behavior from heavy baryon χPT: $B \rightarrow B' \, \pi \rightarrow B$ for $B=N, \Delta$

\[
M_B = \alpha_B + \beta_B + \Sigma_B \left( m_\pi, \Lambda \right)
\]

• Leading order:

\[
M_B = M_B^0 + c^B_1 m_\pi + c^B_2 m_\pi^2 + c^B_3 m_\pi^3 + c^B_4 m_\pi^4 + c^B_4 m_\pi^4 \log(m_\pi) + \ldots
\]

• Bad approx at moderate $m_\pi$. Use form-factor

Leinweber, Thomas, Tsushima, Wright, 99
Comparing Quenched and Full QCD Chiral Extrapolations

- Compare MILC quenched and $N_f=2+1$ staggered spectrum:
  \[ M_B = \alpha_B + \beta_B + \Sigma_B \left( m_\pi, \Lambda \right) \]

- Note: $\Sigma_B$ not 0 in chiral limit. Fit parameters for quenched and full. Here $\alpha$ is not chiral limit mass. In chiral limit, full N mass still near 1 GeV

- Supports claim dominant effects of quenching attributed to first order meson loop corrections

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_N$ (GeV)</th>
<th>$\beta_N$</th>
<th>$\alpha_\Delta$ (GeV)</th>
<th>$\beta_\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>1.24(2)</td>
<td>0.92(5)</td>
<td>1.43(3)</td>
<td>0.75(8)</td>
</tr>
<tr>
<td>Quenched</td>
<td>1.23(2)</td>
<td>0.85(6)</td>
<td>1.45(4)</td>
<td>0.71(11)</td>
</tr>
</tbody>
</table>

Young, Leinweber, Thomas, Wright, 2001
(Quenched) Electromagnetic Splittings

- Determination of N-P splitting long standing problem.
- Virtual photon effects mass splittings within isomultiplets comparable to up-down quark mass difference.
- Accurate computations of isospin splitting must include EM effects.
- A first generation calculation (quenched) including U(1) gauge fields.
  - Assign electric charges to quarks
  - Use $\chi$PT in both SU(3) and U(1). Scale electric charge large(r)
  - Estimate final volume and meson cloud effects from $\chi$PT
- Results surprisingly reasonable. Finite volume corrections large.

<table>
<thead>
<tr>
<th>Level splitting</th>
<th>Raw lattice</th>
<th>Finite volume</th>
<th>Meson cloud</th>
<th>Total lattice</th>
<th>Physical splitting</th>
</tr>
</thead>
<tbody>
<tr>
<td>N - P</td>
<td>2.83(56)</td>
<td>-0.75</td>
<td>-0.53</td>
<td>1.55(56)</td>
<td>1.293</td>
</tr>
<tr>
<td>$\Sigma^0 - \Sigma^+$</td>
<td>3.43(39)</td>
<td>-0.80</td>
<td>-0.16</td>
<td>2.47(39)</td>
<td>3.18(10)</td>
</tr>
<tr>
<td>$\Sigma^- - \Sigma^0$</td>
<td>4.04(36)</td>
<td>+0.86</td>
<td>-0.27</td>
<td>4.63(36)</td>
<td>4.88(10)</td>
</tr>
<tr>
<td>$\Sigma^+ + \Sigma^- - 2\Sigma^0$</td>
<td>0.61(19)</td>
<td>+1.66</td>
<td>-0.11</td>
<td>2.16(19)</td>
<td>1.70(15)</td>
</tr>
<tr>
<td>$\Xi^- - \Xi^0$</td>
<td>4.72(24)</td>
<td>+0.86</td>
<td>+0.10</td>
<td>5.68(24)</td>
<td>6.4(6)</td>
</tr>
</tbody>
</table>

Duncan, Eichten, Thacker, 96
Excited Baryons

- Describing N* spectrum gives vital clues about dynamics of QCD and hadronic physics
  - Role of excited glue
  - Quark-diquark picture
  - Quark interactions

- Open mysteries:
  - Nature of \textit{Roper}?  
  - \(\Lambda(1405)\) mass?
  - Missing resonances?

- History of lattice studies of excited baryons quite brief. Recent work using improved gauge and fermion actions

- As spin increases, baryon spin rep. occurs in multiple lattice representations.

Lattice Representations
Continuum spin reducible under three irreducible ray representations of the cubic group

<table>
<thead>
<tr>
<th>Rep.</th>
<th>Continuum spin reps</th>
</tr>
</thead>
<tbody>
<tr>
<td>G(_1)</td>
<td>1/2, 7/2, …</td>
</tr>
<tr>
<td>H</td>
<td>3/2, 5/2, 7/2, …</td>
</tr>
<tr>
<td>G(_2)</td>
<td>5/2, 7/2, …</td>
</tr>
</tbody>
</table>
Parity

Negative parity interpolation operators:

- Measure three local interpolating operators for the proton:
  \[
  \begin{align*}
  N_{1/2}^{1/2+} &= \epsilon_{ijk}^{} (u_i^T C \gamma_5 d_j) u_k \\
  N_{2}^{1/2+} &= \epsilon_{ijk}^{} (u_i^T C d_j) \gamma_5 u_k \\
  N_{3}^{1/2+} &= \epsilon_{ijk}^{} (u_i^T C \gamma_4 \gamma_5 d_j) u_k
  \end{align*}
  \]

- \( N_1(N_2) \) connects upper (lower) spinor components in diquark piece - \( N_2 \) vanishes in NR limit

- \( \Delta^{3/2,1/2} = \epsilon_{ijk}^{} (u_i^T C \gamma_\mu u_j) u_k \) - Spin projection

- Quark model suggests a better interpolation operator for \( N^{1/2} \) - have a covariant derivative in the valence quark

- On lattice (anti)-periodic in time, have both positive and negative parity states. Fit proton correlation function at each end of lattice to obtain the respective masses.
How Crucial is Chiral Symmetry?

- If we had unbroken chiral symmetry, $N^{1/2+}$ and $N^{1/2-}$ would be degenerate.
- Chiral symmetry crucial: *Do we require “lattice” chiral symmetry?*
  - *Apparently no* – compare Wilson with $O(a)$ $\chi$ breaking terms; Non-Perturbatively imp. Clover with $O(a^2)$ $\chi$ breaking terms; Domain Wall (almost) no $\chi$ breaking terms, but $O(a^2)$ discretization errors.
  - Some mixing of results with discretization errors here – similar mass splitting of 400 MeV for each action.
  - In large $N_c$, mass splitting comes from $l=1$ to $l=0$ (S-P splitting) – reproduced by $O(a)$ Wilson action.

Various Adelaide, Jlab, UKQCD, 2001
Roper

- Compare $N^{1/2+}$ with first (radially) excited state (Roper)
- Large mass splitting. Excited state higher than $N^{1/2-}$. Other channels similar.
- Possible (but unlikely) strong $m_\pi$ dependence. More likely bad overlap of $N_2$ with excited state. Excited state masses notoriously difficult. Need anisotropic lattices – greatly improves signal
- Concern: small physical volume of 1.6fm for DWF and 2.0fm for others – can squeeze up excited state since larger in size

Various Adelaide, Jlab, UKQCD, 2001
Other Excited States

- In recent works, generically see too large splitting among pos. parity – excited state and too small in neg. parity. Quite possibly too small volume (2fm) – by quark model $l=1$ twice as large as $l=0$.
- Splitting not as large in $\Lambda$ channel
Delta

- I=3/2, J=3/2 Delta (2.1fm). Splitting probably also high – new chiral extrapolation??!!

Jlab/UKQCD, 2001
Exotics and Hybrids

- **Exotics**: big focus of JLab (and lattice group!)
  - Spin exotic mesons are $J^{PC}$ states not accessible in quark model
  - Characterized by excited glue or perhaps four-quark states
- Several lattice calculations of heavy hybrid and exotic meson states
- Lattice calculations of light exotic meson states still first generation *(noisy)*!
  - Lightest $1^{-+}$ exotic roughly 2GeV
  - Considerably higher than experimental candidates 1.4, 1.6 GeV

- No baryon exotics!
- Baryon hybrids? Model questions.
  - Gluonic versions of baryons one of many states induced
  - Study of baryon potentials might provide good insight
Lattice Hadron Physics Collaboration (LHPC)

- JLab/MIT and 8 other universities. Over 20 senior physicists
- Four identified physics goals:
  - Nucleon structure
  - Spectroscopy – N*, Hybrids, glueballs
  - Hadron-Hadron interaction
  - Fundamental aspects of QCD (e.g., mechanisms of confinement)
- Computing resources:
  - Small clusters of workstations and a QCDSP
  - Currently awaiting purchase of 200 node box dual Pentium 4 cluster – expect to sustain > 200 Gigaflop/sec
SciDAC

Scientific Discovery through Advanced Computing

- DOE program supporting national effort by US lattice community to develop software and hardware infrastructure for next generation computers
- Physics efforts centered around JLab (hadron physics), FNAL (weak matrix elements), BNL (high temperature)
- Current funding only supports software developers – around $2M to Jlab over 3 years
- Currently awaiting purchase of 200 node box dual Pentium 4 cluster – expect to sustain > 200 Gigaflop/sec
- Goal is a coordinated three 10 Terflop/sec computing facilities for national community by 2005.

- US lattice resources greatly lagging other countries!
Conclusions

• First generation lattice calculations of excited baryon spectroscopy
• Precise calculations commmensurate with experimental program require:
  – Measure of large number of correlators
  – Sufficiently light pions to resolve pion cloud
  – Large physical volumes
  – Continuum extrapolation
  – Full QCD
• State-of-the-art calculations require \( \sim 100 \text{ Gigaflop-year} \) in Quenched QCD and \( \sim 1 \text{ Teraflop-year} \) in full QCD.
  – Required resources not available to US lattice community
  – Focus of interest on weak-matrix element calculations