Baryon spectrum and Magnetic Moments in Nonperturbative QCD

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- Baryon operators. \( Y \) or \( \Delta \)?
- Baryon Green’s function
- Field distribution in baryon
  - Confinement and string shape
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- How to calculate constituent quark mass and magnetic moments from string tension alone.
- Spectrum spin-averaged.
- Spin forces in baryon
- Hybrid baryons, 5q states etc.
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Introduction

The goal is — starting from known QCD vacuum properties derive baryon spectrum and structure.

Basics: Vacuum is described by correlators of gluonic field
\[ g^2 \langle F_{\mu\nu}(x) \otimes F_{\alpha\beta}(0) \otimes \rangle, \ldots g^n \langle F \otimes F \otimes F \otimes \ldots \otimes F \rangle \]
\[ \equiv \left( \delta_{\mu\nu} \delta_{\alpha\beta} - \delta_{\mu\alpha} \delta_{\nu\beta} \right) D(x) + \right. \]
\[ + \partial_{\mu} (x, D_2 \, \delta_{\nu\sigma} + \text{perm}) \left. \right| \quad 99\% \text{ of pert. and nonpert. contents is} \]
\[ \text{given by } D(x), D_1(x). \] (Lattice data: G. Bali, Delday)

String tension

\[ \sigma = \frac{1}{2} \int d^3 x \, D(x) \]

all spin-averaged spectrum defined by \( \sigma = 0.18 \text{ GeV}^2 \)

Spin splittings depend also on \( D_3, D_4 \).
\( D_3, D_4 \) fixed by lattice measurements
\( \chi_s(Q^2) \) — "current quark masses" fixed + theory.

No fitting parameters!
50 baryons

In the same way as for hybrids

\[ M(4q) = M_1 + 2.4 \text{ GeV} \quad (\delta = 0.16 \text{ GeV}^2) \]

\[ \Delta M_{5q} = 2.4 - 0.72 + 0.3 = 2 \text{ GeV} \]

Thus 5q states of this configuration should be in the region \( M_B \sim 3 \text{ GeV} \).

Color Coulomb correction and spin interaction can contribute around \(-0.5 \text{ GeV}\).

\[ M(5q) \geq 2.5 \text{ GeV} \]

Expected \( Z^+ (\bar{q} = 5) \) with decay \( Z^+ \to NK \)

From soliton models much lower mass.

(Diakonov et al., Weigel, Polyakov et al., ..)

\[ M_{Z^+} \sim 1.5 \text{ GeV} \]

But confinement is not implemented in chiral soliton models.
Hybrid baryons

\[ H_0 = M_0 + \text{ heavy-light hybrid } + \frac{P_1^2}{2\mu_1^2} + \frac{P_2^2}{2\mu_2^2} + \frac{P_3^2}{2\mu_3^2} + 6(y_{12} + y_{13}) \]

\[ \Delta H_{\text{self}} = -\frac{2\delta}{m} \quad \text{(for any } q, \bar{q}) \]

Heavy-light

\[ M = M_0 + \Delta H_{\text{self}} = 0.938 - 0.217 = 0.721 \text{ GeV} \]
\[ \delta = 0.16 \]

\[ M = M_0 + \Delta H_{\text{self}} = 1.355 \text{ GeV} \quad (L=0) \]
\[ \frac{0.373}{1.728 \text{ GeV}} \]

HL Hybrid radial excit.

\[ M_n = 1.728 \text{ GeV} + 0.74 \cdot n \quad n=0,1,2 \]

For Hybrid Baryon: add \( \Delta E \)

\[ \Delta E = E(\text{excitation in } 3q) - E(\text{exc. in HL}) \]

For radial, \( n=1 \)

\[ 0.87 \text{ GeV} \quad 0.563 \text{ GeV} \]

\[ \Delta E \sim 0.3 \text{ GeV} \]

Total excitation energy of hybrid

\[ \Delta M = 1.728 - 0.721 + 0.3 \approx 1.3 \text{ GeV} \]

Comp. to \( \Delta M \sim 0.93 \pm 0.1 \text{ GeV} \)

Capstick, Page (flux tube) for q\( \bar{q} \) lattice

\[ \Delta M \sim 1.3 \text{ GeV} \]
Spin forces

Known since '75 (De Rujula, Georgi, Gl.

To lowest order: \( O(\alpha_s) \)

\[ \frac{1}{r} \frac{dV}{dr} = 0 \]

\[ V_{SD}^{(e)} = \frac{2\alpha_s}{3} \sum_{i>j} \left[ \frac{(\vec{R}_{ij} \times \vec{P}_i) \vec{\sigma}_i}{4\mu_i^2 R_{ij}^3} + \frac{(\vec{R}_{ij} \times \vec{P}_j) \vec{\sigma}_j}{4\mu_j^2 R_{ij}^3} \right] \]

\[ V_{SD}^{(2)} = -\frac{2\alpha_s}{3(N_c-1)} \sum_{i>j} \left[ \frac{(\vec{R}_{ij} \times \vec{P}_i) \vec{\sigma}_i + (\vec{R}_{ij} \times \vec{P}_j) \vec{\sigma}_j}{\mu_i \mu_j R_{ij}^3} \right] \]

\[ V_4 (R_{ij}) = \frac{32\pi\alpha_s}{3} 8^{(3)} (\vec{R}_{ij}) \]

\[ V_3 (R_{ij}) = \frac{4\alpha_s}{R_{ij}^3} \]

\[ V_{SD}^{(pert)} = V_{SD}^{(e)} + V_{SD}^{(2)} + \sum_{i>j} \frac{\vec{b}_i \cdot \vec{b}_j}{12\mu_i \mu_j (N_c-1)} V_4 + S_{ij} V_3 \]

\[ S_{ij} = 3 \vec{b}_i \cdot \vec{b}_j \vec{n} - \vec{b}_i \cdot \vec{n} \]

\[ \vec{n} = \frac{\vec{R}_{ij}}{R_{ij}} \]
Spin forces (cont'd) Nonperturbative

\[ \frac{1}{2} \frac{dV}{dz} = - \int_0^\infty \frac{dr}{r} \int_{-\infty}^\infty dv \left[ D(0, v) + D_1(v, v) + x^2 \frac{\partial D_1}{\partial x^2} + \frac{2}{x^2} \right] \]

\[ = - \frac{2}{x^2} \int_0^{\infty} v^2 dv \int_0^\infty \frac{dr}{r} \frac{\partial D_1}{\partial x^2} \]

\[ \frac{1}{2} \frac{dV_2}{dz} = \int_0^1 \int_0^{\infty} d\beta d\rho \int_{-\infty}^\infty dv \left[ D(\hat{r}_{ij}, v) + D_1(\hat{r}_{ij}, v) + (\hat{r}_{ij}^2 + v^2) \frac{\partial D_1}{\partial v^2} \right] \]

\[ \hat{r}_{ij} = \vec{r}_i - \beta \vec{r}_j \]

\[ \frac{1}{2} \frac{d\Sigma}{dz} = \frac{1}{2} \Sigma \]

\[ V_3(u) = - \int_{-\infty}^{\infty} dv \ u^2 \ \frac{\partial D_1(u, v)}{\partial u^2} \]

\[ V_4(u) = \int_{-\infty}^{\infty} dv \left[ 3 D(u, v) + 3 D_1(u, v) + 2u^2 \ \frac{\partial D_1}{\partial u^2} \right] \]

\[ V_5 = - \int_0^1 \int_0^{\infty} d\beta d\rho \int_{-\infty}^\infty dv \ \frac{\partial D_1(\hat{r}_{ij}, v)}{\partial v^2} , \]

\[ D = D(\sqrt{r^2 + v^2}) , \ D_1(r, v) = D_1(\sqrt{r^2 + v^2}) \]

\[ D_1 = D_0 , D_1 \ \exp \left( - \frac{\sqrt{r^2 + v^2}}{T_0} \right) \]
Spin forces in a baryon

Nonperturbative were not known (except for asymptotics of one spin-orbit term - Thomas) precession.

New results (Yu.S. to be published)

\[ \hat{V}_{SD} = (v_l n)(v_l n) + (u_l n)(u_l n) + (u_l n)(u_l n) \]

The only approximation: lowest correlator \( \langle \delta F_6 F \rangle \) is retained. Expected accuracy: 1-2%

The dominant part is green (special case is possible for some states with red important)

\[ V_{SD} = \sum_{i=1}^{3} \frac{\delta (v_i n_i)}{2 \mu_i} \left( \frac{1}{z_i} \frac{dV_i}{dz_i} + \frac{1}{(z_i + \mu_i)} \frac{dE_i}{dz_i} \right) + \]

\[ + \frac{1}{N_c-1} \sum_{i<j} \frac{\delta (v_i n_j) + \delta (v_j n_i)}{2 \mu_i \mu_j} \frac{1}{z_i} \frac{dV_i}{dz_i} + \]

\[ + \frac{1}{N_c-1} \sum_{i<j} \frac{1}{12 \mu_i \mu_j} \left[ \delta (v_i v_j) + (3 \delta (v_i n_j + v_j n_i) - \delta (v_i n_j - v_j n_i)) \frac{1}{\sqrt{3}} \right] \]

\[ + \sum_{i<j} \left( v_i n_j \right) \frac{\delta (v_i n_j)}{(N_c-1) 2 \mu_i \mu_j} \]
Ground state\[ \frac{N+\Delta}{2} = 1.086 \text{ GeV (Exp)} \]

\[ M_0 = 6\mu_0 = 2.16 \text{ GeV} \quad (\sigma = 0.15 \text{ GeV}^2) \]
\[ \mu_0 = 0.36 \text{ GeV} \]

\[ \Delta H_{\text{self}} = -\frac{6\sigma}{\pi\mu_0} = -0.795 \text{ GeV} \]

\[ \Delta H_{\text{coul}} = -0.274 \text{ GeV} \quad (\alpha_s = 0.4) \]
\[ \Delta H_{\text{string}} = 0 \]

\[ M = M_0 + \Delta H_{\text{self}} + \Delta H_{\text{coul}} + \Delta H_{\text{string}} \]

\[ M (k=0) = 1.08 \text{ GeV} \]

Excitations:

<table>
<thead>
<tr>
<th>(k), (n)</th>
<th>(M)</th>
<th>((MN+M_\Delta)_{\text{exp}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k=n=0)</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td>(k=0, n=1) br.m</td>
<td>1.91</td>
<td>1.8</td>
</tr>
<tr>
<td>(k=0, n=2) br.m</td>
<td>2.62</td>
<td>?</td>
</tr>
<tr>
<td>(k=1, n=0)</td>
<td>1.63</td>
<td>1.6</td>
</tr>
<tr>
<td>(k=2, n=0)</td>
<td>2.04</td>
<td>?</td>
</tr>
</tbody>
</table>
Spectrum spin-averaged

restoring \( \mu_i \):

\[
H_0 = \sum_i \left( \frac{p_i^2}{2\mu_i} + \frac{\mu_i}{2} + \sigma_r \frac{\mu_i}{2} \right) \quad ; \quad H_0 \Psi = M_0 \Psi
\]

Hyperspherical method (Yu.S., A. Badalian + Yu.S. '66)

Hyperradius \( \rho^2 = \sum_{i<j} \beta_{ij} (\vec{r}_i - \vec{r}_j)^2 \)

Hyper momentum

\( \mathbf{K} \geq L \quad ; \quad \mathbf{K} = 0, 1, 2, \ldots \)

\[
\Psi = \frac{1}{\rho^{3/2}} \sum \psi_{K} \chi_{K} (\rho)
\]

\[
\frac{d^2}{dp^2} \chi_{K} + \left( 2pM_0 - \frac{(K + \frac{3}{2})(K + \frac{5}{2})}{\rho^2} \right) \chi_{K} = \sum \psi_{K} \chi_{K}
\]

For confining interaction convergence is excellent: \( M_0 (K=0) \) is accurate within 1%!

(By comparison to Green-function MC method)


\[
M_B^{(0)} = \frac{4}{24\sqrt{3}} (\lambda_L)^{3/4}
\]

\[
\lambda_L = 3 \left( \frac{0.566}{3} \right)^{2/3} \left( 1 + \frac{1}{\sqrt{3} L (L+1)} \right) \left[ L (L+1) \right]^{1/3}
\]

\[ L = K + \frac{3}{2} \]
Schematic derivation of $3q$ Hamiltonian.

\[ H = H_0 + \Delta H_{\text{string}} + \Delta H_{\text{Coul.}} + \Delta H_{\text{self}} + \Delta H_{\text{spin}}. \]

\[ H_0 = \sum_{i=1}^{3} \left\{ \sqrt{p_i^2 + m_i^2} + \sigma r_i \right\}; \quad \Delta H_{\text{Coul}} = -\sum_{i>j} \frac{3 \alpha(r_{ij})}{r_{ij}} \]

\[ \Delta H_{\text{self}} = -\sum_{i=1}^{3} \frac{4 \sigma \eta_i}{\pi \cdot 2 \mu_i} \quad \text{with} \quad \eta_i = 1, \quad m_i \to 0 \]

\[ \eta_i = 0, \quad m_i \to \infty \]

Baryon magnetic moments

Hamiltonian for the 3q baryon:

\[ H = \sum_{k=1}^{3} \left\{ \frac{m_k^2}{2\mu_k} + \frac{\vec{p}_k^2}{2\mu_k} + \sigma \sum |\vec{r}_k|/r \right\} \]

\( m_1, m_2, m_3 \) - current masses!

\( (m_k \rightarrow 0 \text{ for } N) \)

\[ M_n(\mu_k) = \frac{m_k^2}{2\mu_k} + \frac{\mu_k}{2} + E_n(\mu_k) \]

\[ \sum_k \left( \frac{\vec{p}_k^2}{2\mu_k} + \sigma |\vec{r}_k|/r \right) \Psi_n = E_n \Psi_n \]

\[ \Psi_n = \sum_{k, \nu_i} \Psi(k, \nu_i) \chi_{\nu_i}(\rho) \]

Lowest \( k = 0 \), \( \Psi_0 = \text{const.} \), \( \rho^2 = \sum (\vec{r}_k - \vec{r}_0)^2 \)

yields 98% of \( E_n \).

\[ \frac{\partial M_n(\mu_k)}{\partial \mu_k} = 0, \quad \mu_k = \mu_k^{(0)} = \sqrt{\sigma} C_k \]

\( \sigma = 0.15 \text{ GeV}^2; \quad m_5 = 0.175 \text{ GeV}, \quad m_n = m_d = 0 \)

The only input

\[ \mu_{\text{magn}} = \sum \frac{e_k \sigma_k^{(c)}}{2\mu_k}; \quad \langle \mu_{\text{magn}} \rangle = \langle \sum \mu \rangle \]

<table>
<thead>
<tr>
<th>Baryon</th>
<th>( \rho )</th>
<th>( n )</th>
<th>( \Lambda )</th>
<th>( \Sigma^- )</th>
<th>( \Sigma^0 )</th>
<th>( \Sigma^+ )</th>
<th>( \Xi^- )</th>
<th>( \Xi^0 )</th>
<th>( \Omega^- )</th>
<th>( \Delta^{++} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>theory</strong></td>
<td>2.54</td>
<td>-1.69</td>
<td>-0.69</td>
<td>-0.9</td>
<td>0.8</td>
<td>2.48</td>
<td>-0.63</td>
<td>-1.49</td>
<td>-2.04</td>
<td>4.36</td>
</tr>
<tr>
<td><strong>exp.</strong></td>
<td>2.79</td>
<td>-1.91</td>
<td>-0.61</td>
<td>-1.16</td>
<td>2.46</td>
<td>-0.65</td>
<td>-1.25</td>
<td>-2.02</td>
<td>4.52</td>
<td></td>
</tr>
</tbody>
</table>
Constituent mass and magnetic moments.

\[ \frac{8H}{8\mu_i} = 0 \rightarrow \mu_i = \sqrt{p_i^2 + m_i^2} \ - \text{operator.} \]

\[ H\psi = M_B\psi \]

Approximate method (accuracy checked ~ 5%)

\[ \frac{8M_B}{8\mu_i} = 0 ; \quad \mu_i = \sqrt{\beta} C \text{ (quantum number)} \]

For u, d quarks

\[ \mu_u = \mu_d = 2\sqrt{\frac{25}{7}} \left[ \frac{2}{3} \left( \frac{1 + \frac{2}{3\sqrt{3}}}{1 + \frac{2}{3\sqrt{3}}} \right) \right]^{\frac{3}{4}} = 0.376 \text{ GeV} \]

For s quark, \( m_s = 0.245 \text{ GeV} \)

\[ \mu_s = 0.46 \text{ GeV} \]

Magnetic moments

\[ \mu_B^z = \sum_{k=1}^{3} e_k \sigma_z^{(k)} \ ; \quad \mu_B = \langle \psi_B | \mu^z | \psi_B \rangle \]

\[ \mu^B_{\text{Proton}} = \frac{M_B}{\mu_u} \approx 2.54 \text{ (nuc.)} \ ; \quad \mu_{\text{Neutr.}} = -\frac{2}{3} \mu_u^d = -1.69 \]

\[ \mu^{\Sigma^-} = -\mu_{\Sigma^+} \left( 1 + \frac{3}{4} \frac{m_s^2}{c^2} - \frac{15}{32} \frac{m_s^4}{c^4} \right)^{-1} \]

\[ c = 0.957. \]
Properties of Hamiltonian.

- $H$ depends on $\sigma$ and current masses only.

- $L_i = 0$

\[
\frac{\delta H}{\delta \mu_i} = 0 \rightarrow \mu_i = \sqrt{p_i^2 + m_i^2}
\]

\[
\frac{\delta H}{\delta \nu_i} = 0 \rightarrow \nu_i = \sigma r_i
\]

\[
H_0 = \sum_{i=1}^{3} \left\{ \sqrt{p_i^2 + m_i^2} + \sigma r_i \right\}
\]

$\bar{r}_i = \left( \bar{z}_i - \bar{y} \right)$

Familiar rel. quark model Hamiltonian.

- $L_i > 0$, $L_i < 4$

Take string rotation as a correction:

\[
H = \sum_{i=1}^{3} \left\{ \sqrt{p_i^2 + m_i^2} + \sigma r_i + \Delta h_i^{(\text{string})} \right\}
\]

\[
\Delta h_i^{(\text{string})} = - \frac{2 \sigma^2 L_i (L_i + 1)}{M_0^4} \left( 1 + \frac{4}{3 (L_i + 1)} \right)
\]
Derivation of Hamiltonian

\[ \int (\mathrm{d}r)_x y e^{i \int \mathcal{L}(r, r) \, \mathrm{d}t} = \langle x | e^{-i H T} | y \rangle \]

\[ H = H(p, r), \quad p = \frac{\partial L}{\partial \dot{r}} \]

\[ H(p, r, \nu, \mu) = \sum_{i=1}^{3} \left\{ \frac{(\vec{p}_i \cdot \vec{r}_i)^2 + m_i^2}{2 \mu_i} + \frac{\mu_i}{2} + \frac{\hat{L}_i^2 / r_i^2}{2 [\mu_i + \int \beta_i^2 \nu_i(\beta_i) \, \mathrm{d}\beta_i]} + \frac{1}{2} \int_0^1 \frac{d\beta_i}{2 \nu_i(\beta_i)} + \frac{1}{2} \int_0^1 \frac{d\beta_i}{2 \nu_i(\beta_i)} \right\} \]

\[ \mu_i, \nu_i : \quad \frac{\delta H}{\delta \mu_i} = 0, \quad \frac{\delta H}{\delta \nu_i} = 0 \]

\[ p_i^2 = \frac{(\vec{p} \cdot \vec{r})^2}{r_i^2}, \quad p_{it}^2 = \frac{(\vec{p} \times \vec{r})^2}{r_i^2} = \]

\[ = (\mu_i + \int \beta_i^2 \nu_i \, \mathrm{d}\beta_i)^2 \frac{(\vec{r} \times \vec{r})^2}{r_i^2} \]

\[ \hat{L}_i^2 = (\vec{p} \times \vec{r})^2 = p_i^2 r_i^2 \]

string moment of inertia \[ \frac{1}{r_i^2} \]
Transformation of the action

$$Action = \sum_{i=1}^{3} \left( \mathcal{K}_i + \delta \left( S_i + \delta_{int} \right) \right)$$

$$\mathcal{K}_i = \frac{m_i^2}{2 \mu_i(t)} + \mu_i(t) \left( 1 + \tilde{\chi}_i^2 \right)$$

$$\delta S_i = \delta \int_0^T \int_0^1 \sqrt{w_i^2 w_i^2 - (\dot{w}_i \cdot \dot{w}_i)^2} \left\{ \mathfrak{L}(t) \right\}$$

**Approximation:**

Disregard string excitation since $\Delta M_{exc.} \approx 1 \text{ GeV}$

Take string straight.

$$\vec{w}_i(\beta, t) = \vec{z}_i(t) \beta_i + \vec{v}(t) (1 - \beta_i)$$

**Einstein variable:**

String-junction trajectory

$$e^{-\frac{1}{2} \int dt \int d\beta \sqrt{\dot{w}_i^2 \dot{w}_i^2 - \dot{w}_i^2}} = \int D\nu(t) e^{-\frac{\mathfrak{I}}{\nu_i} \int dt \int d\beta \mathcal{A}}$$

$$\mathcal{A}_i = \nu_i \left( \frac{w_i^2 - (\dot{w}_i \cdot \dot{w}_i)^2}{(\dot{w}_i)^2} \right) + \frac{\delta^2 w_i^2}{\nu_i}$$

$\nu_i$ (at static point) will be energy density of string.
Transformation of the Green's function

\[ G_{B} \sim \prod_{i=1}^{3} (m - \hat{D}_{i}) \int ds_{i} Dz^{(i)} e^{-\frac{K}{2} \langle WS e^{gSF} \rangle} \]

Introducing new mass variable!
(To be constituent mass at stationary point)

\[ \frac{d\Delta z_{4}}{d\tau} = 2\mu(\tau) = 2\mu(\Delta z_{4}) \]

\[ Dz_{4} = \prod_{n=1}^{N} \frac{d\Delta z_{4}(n)}{(4\pi\varepsilon)^{2}} \delta \left( \sum_{n=1}^{N} \Delta z_{4}(n) - \tau \right) \]

\[ \Delta z_{4}(n) = z_{4}(n) - \frac{z_{4}(n-1)}{\Delta t}, \quad N\varepsilon = \delta, \quad N\Delta t = 2 \]

\[ \delta \left( \sum \Delta z_{4}(n) - \tau \right) \rightarrow \delta \left( \int_{0}^{\infty} d\tau (2\mu(\tau) - \frac{\tau}{\Delta t}) \right) \rightarrow \delta (2\mu - \frac{\tau}{\Delta t}) \]

\[ ds Dz_{4}(t) = D\mu(t) ds \delta (2\mu - \frac{\tau}{\Delta t}) = \text{const} D\mu(t) \]

Action

\[ G_{B} \sim \prod (m - \hat{D}) \int D^{3}z(t) D\mu(t) e^{-\frac{K}{2} \langle WS e^{gSF} \rangle} \]

3d dynamics!
Figure 3: The lattice potential from [7] with $\beta = 5.8$ (points) and the potential $V_{\text{pert}}(r) + V_B(r)$ (solid curve) with $T_g = 0.12$ fm, $\sigma = 0.188$ GeV$^2$ and $\alpha_s = 0.4$.

**Points from Alexandru, De Forcrand et al.**
Figure 3: $V_Y^{(B)}(r)$ (solid curve) along with its constituents $3V^{(M)}(r)$ (dashed curve) and $V^{(hole)}(r)$ (dotted curve) in comparison with linear potentials $3\sigma R = \sqrt{3} \sigma r$ (solid gray curve) and $3/2 \sigma r$ (dashed-dotted gray curve). $\sigma = 0.21$ GeV$^2$ and $T_g = 0.2$ fm.
(a) QQQ on one line (string junction coincides with central Q)

(b) QQQ as quark–dyquark
QQQ-system

position of quark
Computing field structure of the string.

\[ S_{\mu\nu}(x) = \frac{\langle \text{tr} (W L P_{\mu\nu}(x) L^+) \rangle}{\langle \text{tr} W \rangle} - 1 \to a^2 \langle F_{\mu\nu}(x) \rangle_{Q}\bar{Q} \]

Using Stokes theorem, cluster theorem and keeping only bilocal corr.

\[ S_{\mu\nu}(x) = a^2 \int d\sigma_{x'}(x') \frac{D_{\rho_\lambda,\mu\nu}(x',x) + O(a^4)}{S} \]

\[ \frac{g^2}{N_c} \left( \frac{\delta\delta - \delta\delta}{2} \right) D(x-x') + \frac{i}{2} [\Theta\ldots] D_1 \]

\[ D = D(0) \exp\left(-\frac{ix_1}{\lambda}\right); \quad D_1 = \frac{1}{3} D(0) \exp\left(-\frac{ix_1}{\lambda}\right) \]

\[ \langle E_{1}(x, x') \rangle_{Q\bar{Q}} = \int dx' \int dx'' \left[ D(0) + D_1(0) - \frac{1}{2} \frac{h_1^2 + h_2^2}{2\lambda} D_1 \right] e^{-\frac{h}{2\lambda}} \]

\[ h = \sqrt{h_1^2 + h_2^2 + h_3^2}; \quad h_\mu = x_\mu - x'_\mu \]

probing plaquette \( P_{\mu\nu}(x) \).
\[
\langle W_3 \rangle = \text{tr}_Y \exp \sum_{n=0}^{\infty} \left( \frac{i g}{n!} \right)^n \int \langle \langle F(1) \ldots F(n) \rangle \rangle \, d\sigma(1) \ldots d\sigma(n) \sum S_i
\]

approximation (supported by lattice data): keep only lowest correlator
\[
\langle \langle F(1) F(2) \rangle \rangle \rightarrow D(x), D_4(x).
\]
Then also include spins: \[\exp \frac{\sqrt{2} F}{g} \]

\[
\langle W_3 \exp(g \sigma F) \rangle = \text{tr}_Y \exp \left[ \sum_{n=0}^{\infty} \left( \frac{i g}{n!} \right)^n \langle \langle F(1) \ldots F(n) \rangle \rangle \, d\sigma(1) \ldots d\sigma(n) \right]
\]

\[
\Rightarrow \text{tr}_Y \exp \left[ -\frac{g^2}{2} \int \langle \langle F(1) F(2) \rangle \rangle \, d\sigma(1) d\sigma(2) \right]
\]

\[
d\sigma^{(i)} = d\sigma^{(i)}(u) + \frac{1}{i} \delta^{(i)}_{\mu\nu} \, d\tau^{(i)}
\]

proper time of \( i \)-th quark.

New phenomenon: integrals over three lobes of minimal surface \( \sum S_i \).

\[
\frac{g^2}{2} \int d\sigma(u) \, d\sigma(v) \langle \langle F(u) F(v) \rangle \rangle = \int d\sigma(u+v) d\sigma(u-v) \frac{D(u-v)}{2}
\]

\[
= S_i \cdot \sigma \quad \text{size}(S_i) \gg T_F.
\]

\[ D(x) = D(0) \exp\left( -\frac{\alpha F}{g} \right) \]
Define $\text{in} \rangle$, $\text{out} \rangle$ states

$$
\Psi_{\text{in, out}} (x, y, z, \mathbf{Y}) = \Gamma_{\text{in, out}} B_{\mathbf{Y}} (x, y, z, \mathbf{Y})
$$

$$
G_B (x y z \mathbf{Y} | x' y' z' \mathbf{Y}') = \left\langle \text{tr}_Y \Gamma_{\text{out}} \frac{S(x, x') S(y, y') S(z, z')} {\mathbf{pp}} \Gamma_{\text{in}} \right\rangle
$$

$$
\text{tr}_Y = \frac{i}{2} \exp \int \mathcal{A}_m \, dz
$$

$$
S(x, x') = (m - D) \int_0^\infty d \mathbf{Z} e^{-K_{x, x'}} \Phi (x, x') e^{\oint_x^\mathbf{Z} \mathcal{S}_{\mathbf{M}} F_{\mathbf{M}} \, dz}
$$

$$
K = m^2 s + \frac{1}{4} \int_0^\infty \mathbf{Z}^2 \, dz , \quad \mathcal{S}_{\mathbf{M}} F_{\mathbf{M}} = \left( \begin{array}{cc} \partial \vec{B} & \vec{E} \\ \vec{E} & \partial \vec{B} \end{array} \right)
$$

\[ \text{closed Wilson loop} \]

$$
W_3 (x y z \mathbf{Y} | x' y' z' \mathbf{Y}')
$$

gauge invariant!
Other possible gauge-invariant baryons.

\[ g_\sigma(x) = \exp \nabla \phi(x) \]

5q baryons
For \( \bar{q} = \bar{s} \) one gets \( Z^+ \)

\( Z^+ \rightarrow K \bar{N} \)

Hybrid baryon:

valence gluon.
Baryon OPERATORS

More carefully: extended quark $q^\alpha(x,y) = q^{\alpha(x)} q^{\alpha(y)}$.

Baryon: $B_x(x,y,z,y) = \exp q^{\alpha(x)} q^{\alpha(y)} q^{\alpha(z)} q^{\alpha(y)}$.

New object:

$q_{\alpha\beta}(x) = \exp q^\alpha(x)$

Try to construct $B_{\Delta}$:

$B_{\Delta}(x,y,z) = q_{\alpha\beta}(x) \Phi_{\alpha\beta}^\alpha(x,y) q_{\gamma\delta}(y) \Phi_{\gamma\delta}^\beta(y,z) q_{\epsilon\rho}(z) \Phi_{\epsilon\rho}^\gamma(z)$

Not gauge-invariant!

$g^\alpha(x) \rightarrow V^\alpha_{\beta}(x) q^\beta(x)$

Physical reason: electric fluxes are not connected continuously.

For gluons (3g glueball) both $\gamma$-type and $\Delta$-type possible (Kuzmenko + Yu.S., 2002)

For baryons: only $\gamma$-type is possible.
Physical states

Elements:

Parallel transporter

\[ P_{xy} = P_{xy} \exp i g \int A_{ud} \, dz \]

\[ \begin{array}{c}
\text{string junction} \\
\text{antistring junction}
\end{array} \]

Hadrons: Principle of gauge invariance:

- connect elements in a white set (no color light seen!)

Meson

\[ \psi^M(x,y) = \psi(x) \Phi(x,y) \eta(y) \]

Baryon

\[ \psi^{B\alpha}(x,y,z) = \varepsilon^{abc} \psi^{\alpha}(x) \Phi(x,y) \Phi^{\beta}(y) \]

Hybrid

\[ \psi^H(x,y,z) = \psi(x) \Phi(x,y) \bar{\psi}(y) \Phi(x,z) \]

Mystery of confinement: how parallel transp become strings connecting qq with force of ??
This program: prediction of spin-averaged spectrum through $\sigma$ only, and spin-splittings via $\alpha_s$ and $D, D$, was done:

For heavy-light mesons:

For heavy quarkonia
A. Badalian et al. Phys.Rev.D ’99–’01

For light mesons
Yu.S. ’91 A. Badalian et al. hep-ph/0202246

For glueballs
A. Kaidalov + Yu.S. ’00 PLB

For glue lumps
Yu.S. ’00 Nucl. Phys. B.

For hybrids
Yu.S. ’91, Yu. Kalashnikova + D. Kuzmenko ’99–’02

For baryons without spin and string, selfenergy corrections

Agreement with exper. and lattice 5–10%
QCD vacuum and local Hamiltonian

Only pair correlations \( \langle F(x)F(0) \rangle \) are essential

Casimir scaling (Bali '00, Deldar)

\( T_g \approx 0.2 \text{ fm} \)

\( \langle F(x)F(0) \rangle = D(x) = e^{-\frac{|x|}{T_g}} \)

Lattice (Di Giacomo et al., Bali et al.)

Analytic (Dosch et al., Yu. S.)

Hybrid excitation energy is large –

\( \Delta M_h \approx 1 \div 1.3 \text{ GeV} \)

Lattice + analytic

This means that string excitation can be neglected (in first approximation).

Small \( T_g \) and large \( \Delta M_h \)

Local effective Hamiltonian

Spin-aver. spectrum dep. on \( \Theta \)

Spin splitting dep. on \( \alpha_s, T_g \)
Conclusions

- Nonperturbative QCD approach predicts spin-averaged baryon spectra without fitting parameters through $\beta$ only.

- Nonpert. spin interaction is now available, expressed through $D, \tilde{D}$.

- Spin interaction appears much more complicated than expected and used. Exact computations are needed.

- Origin of constituent quark and gluon masses is understood and they are computed through $\beta$ only together with magnetic moments.

- Negative energy states of quarks are strongly coupled by spin operators and may show up in some states (Roper?)

- Pions are emitted by quarks at the end of strings (Yu. S. hep-ph/0201170) and pion exchange forces should be added.