Recent Successes and Open Issues in Baryon Structure

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- QCD
  - Confinement
  - Vacuum
  - Test and Refine Models

- Baryon Structure
  - Lattice QCD
  - Baryon Properties
  - Spectroscopy

- Dense Matter
  - Nuclear Structure
  - Bound Nucleon
  - Baryons In-Medium
QCD

\[ \mathcal{L} = \bar{\psi}i\hat{D}\psi - \frac{G^2}{4} \]

- Bizarre Properties:
- Asymptotic Freedom
  i.e. Quarks feel almost no strong force when close together
- Confinement
  i.e. Quarks can never be separated
  — Restoring Force of 10 TONNES no matter how far apart they are!
QCD Vacuum

- Non-zero gluon energy density

\[ \epsilon_{\text{vac}} = -\frac{9}{32} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle \approx -0.5 \text{GeV/fm}^3 \]

Shifman, Vainstein & Zakharov

- This is large!

c.f. MIT bag: \( \frac{B}{|\epsilon_{\text{vac}}|} \sim 0.1 \)

\implies \text{little change from inside to outside hadron?}

- ALSO important topological structure:
  - e.g. \text{instantons} ....
  - linked to chiral symmetry breaking (?)

Slides: Leinweber, Bonnet, Thompson, Detmold ......
Covariant Models

- Schwinger-Dyson Equations
  (Roberts, Williams, Maris, Tandy, Alkofer, Cahill, ...)
- Very impressive phenomenology
- Truncate infinite set integral equations
- Pion form factor & structure function
- Vector mesons
- Nucleon form factors
- Can now test and refine against lattice data
- Gluon propagator
- Quark propagator
- Eventually quark-gluon vertex
Theoretical Uncertainties

- Sensitivity to MEC, spin-orbit effects, distorted waves etc. — very slight in ratio:

![Graph showing experimental data points and theoretical analysis]

Theoretical analysis of Udias et al., Laget etc.
First Evidence for Change of Bound Nucleon Structure

- Ratio of electric to magnetic form factor of the proton in $^4\text{He}$


- Full distorted wave analysis by Udias et al.
Major Experimental Advance: \((\bar{e}, e' \bar{p})\)

- **Mainz/Jefferson Laboratory**: High intensity, polarised, 100% duty factor electrons
- **Quasi-free scattering**: Use polarised beam and measure polarisation of knocked-out proton

- **Measure \(G_E/G_M\)**: Using \((\bar{e}, e' \bar{p})\), measure ratio of electric to magnetic proton form factor, compared to free-space ratio, in a specific shell-model state!
Finite Nuclei

(P. Guichon, K. Saito, E. Rodionov and A.W. Thomas,

- Born-Oppenheimer Approximation:
  Composite particles in relativistic, external fields is very complicated
  Expect good to about 3% to use Born-Oppenheimer

- Obtain Shell Model:
  "Nucleon" internal structure self-consistently adjusts to local mean-field, for each single-particle state

- MAJOR CONCEPTUAL CHANGE:
  What occupies shell-model states are NOT NUCLEONS but "nucleon-like" quasi-particles with DIFFERENT mass, magnetic moment, charge radii......

- How to Test?
ATOMIC NUCLEI

- Traditional picture:
  "Point-like" protons and neutrons moving in strong Lorentz scalar and vector mean-fields (e.g. QHD)

- Role of Internal Structure?
  Energy to excite nucleon, 300-500 MeV: same as scalar mean-field!

- QMC: Quark-Meson Coupling model
  Self-consistently solve for the change of the internal structure of the nucleon – which is in turn the source of the scalar mean-field

- Nuclear Matter:
  Natural Saturation Mechanism
  At present requires hadron model
  In principle could calculate scalar response of nucleon and hence nuclear structure directly from QCD!
"Roper"

Most models predict the first radial excitation of the nucleon around 1600 MeV - *What is the Roper (1440)?*

"Bad" operator... Recall that the "bad" operator vanishes in NR limit.

Overlap on radial excitation

![LHPC/UKQCD Preliminary](image)

\[ \beta = 6.2 \]

Poor signal at lightest masses
Hadron Spectroscopy: SUMMARY

- Low lying, +ve parity states NOT found by Sasaki et al. or Melnitchouk et al. BUT c.f. Richards et al.....different source?

- Similarity to constituent quark model in large mass regime is clear
  - e.g. equal spacing of major shells......

- Splitting of -ve parity states clear
  - State with S=0 di-quark lower in mass

- Major worry: lattices are all small
  - Sasaki et al. and Richards et al. 1.6 fm
  - Melnitchouk et al. 2.0 fm

c.f. QQCD proton radius about 0.65 fm

- Of course, issue of chiral extrapolation remains an open problem......
Strangeness -2

- Same shell structure as $N, \Sigma$ and $\Lambda$

From Melnitchouk et al., hep-lat/0202022
Little overlap with $\Lambda(1405)$

From Melnitchouk et al., hep-lat/0202022
**Strangeness -1**

- Yellow dot: Excited $\Sigma$ similar to $N$:
- Red dot: As for $N$, baryon with $S = 0$ di-quark lower in mass

From Melnitchouk et al., hep-lat/0202022
NUCLEON EXCITED STATES

- Note similarity to oscillator model
- Splitting of $\frac{1}{2}^-$ states clear

From Melnitchouk et al., hep-lat/0202022;
very similar to Sasaki et al., hep-lat/0102010
Hadron Spectroscopy on the Lattice

- There are new techniques to produce information on excited baryons

  $N'(\frac{1}{2}^+)$, Wilson fermions

- Lee & Leinweber, Nucl. Phys. B(PS) 73 (1999) 258:
  $N^*(\frac{1}{2}^-), N^*(\frac{3}{2}^-)$, $D\chi$34 fermions

- Sasaki, Blum & Ohta, hep-lat/0102010:
  $N'(\frac{1}{2}^+), N^*(\frac{1}{2}^-)$, Domain Wall Fermions

- Richards et al., JLAB-THY-01-38:
  $N^*(\frac{1}{2}^-), \Delta^*(\frac{3}{2}^-)$, non-perturbative clover

- Melnitchouk et al., hep-lat/0202022:
  Octet $\frac{1}{2}^+$ and $\frac{1}{2}^-$, FLIC action

- So far, all calculations based on QCD
Nucleon and $N'(\frac{1}{2}^+)$

Summary of various actions:

From Melnitchouk et al., hep-lat/0202022.
Nucleon and $N^*(\frac{1}{2}^-)$

Summary of various actions:

From Melnitchouk et al., hep-lat/0202022.
Hadron Spectroscopy - II

- Are some baryons NOT primarily 3-quark states?
  - e.g. Roper (1440):
  - $2\hbar \omega$ BUT below $1\hbar \omega$!
  - Is it a breathing mode?

Guichon....

- Is it channel coupling effect?

Speth, Hanhart...Afnan....

- Similarly $\Lambda(1405)$
  - Is it $\bar{K}N$ bound state?

Dalitz, Yuan.....

- OR $\Sigma \pi - \bar{K}N$ coupled channel effect?

Veit, Jennings.... Weise.....

- There has been tremendous progress on such questions on the Lattice......
Exciting new prospects at JLab

Are there states of pure glue?

Are there hybrid states
- e.g. BNL group

- $\pi_1(1370) \rightarrow \eta\pi, \eta'\pi$
- $\pi_1(1640) \rightarrow \eta'\pi, \rho\pi, f_1\pi$
Connection Between QQCD and QCD?

- Values of $\alpha, \beta \sim$ same as $\tilde{\alpha}, \tilde{\beta}$:
  
  - $N$: $[1.24(1), 0.91(3)]$ c.f. $[1.23(2), 0.85(6)]$
  - $\Delta$: $[1.44(3), 0.75(7)]$ c.f. $[1.45(4), 0.71(11)]$

![Graph showing baryon mass vs. $m_n^2$](image)

Common $\Lambda = 0.8$ GeV


(Analysis: Young et al., hep-lat/0111041)
Connection of Quenched to Full QCD

- Suggests LNA and NLNA loops are the major difference between FULL and Quenched QCD

- Thus QQCD is NOT an "uncontrolled approximation"
  BUT may be a very cost efficient source of information on hadron structure and spectroscopy

- Need to test on other octet baryons

- Need better QQCD data at low mass

eg. FLIC actions (Zanotti et al., hep-lat/0110216)
Now fit QQCD data using:

\[ M_B = \tilde{\alpha}_B + \tilde{\beta}_B m_\pi^2 + \tilde{\Sigma}_B(m_\pi, \Lambda) \]

Use same \( \Lambda \) as in full QCD

(Consistent with QQCD data on rms radii)

NOTE: major difference in behaviour for \( N \) and \( \Delta \)

(From Young et al., hep-lat/0111041)
For the $\Delta$ major change

Several contributions are repulsive rather than attractive – as in full QCD

Self-energy contributions to the $\Delta$

(From Young et al., hep-lat/0111041)
Baryon Masses in QQCD – 3

- Adopt same approach with QQCD data
  \[ M_B = \tilde{\alpha}_B + \tilde{\beta}_B m^2_\pi + \tilde{\Sigma}_B (m_\pi, \Lambda) \]

- \( \tilde{\Sigma}_B \) sum of QQCD self-energy terms yielding LNA and NLNA behaviour


Self-energy contributions to the nucleon
(From Young et al., hep-lat/0111041)
Success of describing $N$ and $\Delta$ masses in full QCD using:

$$M_B = \alpha_B + \beta_B m_\pi^2 + \Sigma_B(m_\pi, \Lambda)$$

$\Sigma_B$ is sum of the self-energy terms yielding **LNA** and **NLNA** behaviour.

From Young et al., hep-lat/0111041. $\Lambda = 0.92$ GeV
Baryon Masses in Quenched QCD

- Chiral behaviour of baryon masses is quite different in QQCD

- $\eta'$ is also a Goldstone boson $\Rightarrow$

$$M_B = M_B^{(0)} + c_1^B m_\pi + c_2^B m_\pi^2 + c_3^B m_\pi^3 + \ldots$$

- Coefficients of $m_\pi$, $m_\pi^3$ and $m_\pi^4 \ln m_\pi$ are **model independent**

- $m_\pi$ term **unique to QQCD**: arises from (b):

  ![Diagram](a)

  ![Diagram](b)
Parton Distributions vs Mass

Can use fit to plot $x(u_V - d_V)$ as a function of quark mass:

Note *constituent-quark* behaviour as mass passes strange quark mass

Comparison with Data

- Can use lowest 4 moments to reconstruct $x$-dependence of $u_v(x) - d_v(x)$

![Graph showing different extrapolation methods: average parameter, improved extrapolation, linear extrapolation × 1/2]

Detmold, Melnitchouk...

- Dark shading is world data
- Light shading chiral extrapolation
Moments of $u - d$

Chiral extrapolation is essential:

Detmold, Melnitchouk, Negele, Renner & Thomas,

hep-lat/0103006: $a + bm^2 + ac_{\text{LNA}}m^2\pi \ln(m^2_\pi/(m^2_\pi + \mu^2))$
Chiral Behaviour

- For Gottfried sum rule violation:
  \[ \bar{d} - \bar{u}|_{\text{LNA}} \sim c_{\text{LNA}} m_{\pi}^2 \ln m_{\pi} \]

  Thomas, Melnitchouk & Steffens, PRL 85 (2000) 2892

- For \( u - d \):

  \[ \begin{align*}
  \begin{array}{c}
  \text{(a)} \\
  \text{(b)} \\
  \text{(c)} \\
  \text{(d)} \\
  \text{(e)}
  \end{array}
  \end{align*} \]

- \( < x^n > = a_n + b_n m_{\pi}^2 + a_n c_{\text{LNA}} m_{\pi}^2 \ln \frac{m^2}{m_{\pi}^2 + \mu^2} \)

- \( c_{\text{LNA}} \) from chiral perturbation theory

  Ji & Chen and Arndt & Savage

- reproduces CBM calculation over full range of \( m_{\pi}^2 \)

- \( \mu \) is scale at which chiral behaviour turns off: expect \( \sim 500 \text{ MeV} \)
Moments of Parton Distributions

- Can only compute low moments on lattice:

\[
< x^n > = \int_0^1 dx x^n [q(x) + (-)^{n+1} \bar{q}(x)]
\]

\[
\sim < N | \bar{\psi} \gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_{n+1}} \psi | N > (1)
\]

- Compute \(< x^n > \) for \( u - d \)

⇒ NO "disconnected diagrams"

- Until now, discrepancies with data of 50 % or more......
\[
\langle r^2 \rangle = \frac{c_1 + \chi_i}{1 + c_2 \cdot m^2_{\pi}}
\]

Table 1
Baryon electric charge radii and the quark sector contributions. The latter are defined on the left-hand sides of Eqs. (6)–(11). One-loop corrected estimates of \(\alpha_i^{(p)}\) (in Eq. (1)) and \(\chi_i\) (in units of fm\(^2\)) for each octet baryon are indicated. For each extrapolation, the fit parameters, \(c_1\) and \(c_2\), and the predicted value of \(\langle r^2 \rangle\) at the physical pion mass are reported. Asterisks denote the squared charge radii reconstructed from the sum of separate quark sector extrapolations. (The units are such that the pion mass is in GeV and the squared charge radius in fm\(^2\)).

<table>
<thead>
<tr>
<th>Baryon or quark sector</th>
<th>(\alpha_i^{(p)})</th>
<th>(\chi_i)</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(\langle r^2 \rangle)</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>(\frac{1}{6}) (\frac{5}{3}(D + F)^2)</td>
<td>0.174</td>
<td>0.34</td>
<td>0.30</td>
<td>0.68(8)</td>
<td>0.740(15) [14]</td>
</tr>
<tr>
<td>(u_p)</td>
<td>(\frac{2}{3}\left[ \frac{1}{6} \frac{5}{3}(D + F)^2 \right] )</td>
<td>0.116</td>
<td>0.52</td>
<td>0.73</td>
<td>0.74(11)</td>
<td></td>
</tr>
<tr>
<td>(d_p)</td>
<td>(\frac{1}{3}\left[ \frac{1}{6} \frac{5}{3}(D + F)^2 \right] )</td>
<td>0.058</td>
<td>0.18</td>
<td>1.38</td>
<td>0.06(5)</td>
<td></td>
</tr>
<tr>
<td>(*p)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.68(10)</td>
<td>0.740(15) [14]</td>
</tr>
<tr>
<td>(n)</td>
<td>(\frac{1}{6} + \frac{5}{3}(D + F)^2)</td>
<td>0.174</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
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<td>(\frac{1}{3}\left[ \frac{1}{6} \frac{5}{3}(D + F)^2 \right] )</td>
<td>0.058</td>
<td>0.26</td>
<td>0.73</td>
<td>0.37(6)</td>
<td></td>
</tr>
<tr>
<td>(*n)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.25(8)</td>
<td>0.113(4) [15]</td>
</tr>
<tr>
<td>(\Lambda)</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda^\prime)</td>
<td>0</td>
<td>0</td>
<td>0.15</td>
<td>0.97</td>
<td>0.14(3)</td>
<td></td>
</tr>
<tr>
<td>(\Sigma^+)</td>
<td>(\frac{1}{3} \frac{5}{3} \left( \frac{D^2}{3} + F^2 \right) )</td>
<td>0.138</td>
<td>0.68</td>
<td>2.03</td>
<td>0.92(11)</td>
<td></td>
</tr>
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<td>(\frac{1}{3} \frac{5}{3} \left( \frac{D^2}{3} + F^2 \right) )</td>
<td>0.138</td>
<td>0.58</td>
<td>0.93</td>
<td>0.83(8)</td>
<td></td>
</tr>
<tr>
<td>(\Sigma^\prime)</td>
<td>0</td>
<td>0</td>
<td>0.06</td>
<td>0.17</td>
<td>0.06(1)</td>
<td></td>
</tr>
<tr>
<td>(\Sigma_0)</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td>0.77(8)</td>
<td></td>
</tr>
<tr>
<td>(\Sigma^0)</td>
<td>0</td>
<td>0</td>
<td>0.19</td>
<td>1.48</td>
<td>0.18(2)</td>
<td></td>
</tr>
<tr>
<td>(\Sigma^0)</td>
<td>0</td>
<td>0</td>
<td>0.06</td>
<td>0.17</td>
<td>0.06(1)</td>
<td></td>
</tr>
<tr>
<td>(\Sigma^-)</td>
<td>(\frac{1}{2} + \frac{5}{3} \left( \frac{D^2}{3} + F^2 \right) )</td>
<td>0.138</td>
<td>0.25</td>
<td>0.08</td>
<td>0.52(3)</td>
<td>0.69(16) [16]</td>
</tr>
<tr>
<td>(\Sigma^-)</td>
<td>(\frac{1}{3} + \frac{5}{3} \left( \frac{D^2}{3} + F^2 \right) )</td>
<td>0.138</td>
<td>0.21</td>
<td>0.37</td>
<td>0.47(3)</td>
<td>0.91(72) [17]</td>
</tr>
<tr>
<td>(\Sigma^-)</td>
<td>0</td>
<td>0</td>
<td>0.06</td>
<td>0.17</td>
<td>0.06(1)</td>
<td>0.54(3) [16]</td>
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<td>0</td>
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<td>0.91(72) [17]</td>
<td></td>
</tr>
</tbody>
</table>

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Analogy to Charge Radius in Lattice QCD

- Same asymptotic behaviour

\[(\langle r^2 \rangle_E = \frac{c_1 \pm \chi N}{2} \log \frac{m^2}{\mu^2 + m^2} \frac{m^2}{1 + c_2 m^2}}{1 + c_2 m^2}\]

Fit to lattice QCD data for charge radius of the proton
To build a very effective approximation, ensure exact large and small mass limits.

\[ S_{\text{int}} = -\frac{e^2 B^2}{8\pi^2} \left( \frac{d_1 + \frac{1}{3} \log\left(\frac{m^2}{m^2 + eB}\right) - d_2 \frac{m^2}{eB}}{1 + 45d_2 \left(\frac{m^2}{eB}\right)^3} \right) \]

Two parameter representation of the exact result:
accurate to better than 10% for all \( m \)
Either asymptotic limit is a poor approximation to exact solution

\[ -8\pi^2S/(e^2B^2) \]

- Exact Euler-Heisenberg Result
- \( -1/45 \cdot (eB/m^2)^2 \)
- \( 0.763968 + 1/3 \cdot \log(m^2/eB) \)
Euler-Heisenberg Problem

- **Exactly soluble EFT**
  (Dunne, Thomas, Wright: hep-th/0110155)

- \[ S = -i \ln \det(i \mathcal{D} - m) \]

- **Exact solution:**
  \[ S = -\frac{e^2 B^2}{8\pi^2} \int_0^\infty \frac{ds}{s^2} \left( \coth s - \frac{1}{s} - \frac{s}{3} \right) e^{-m^2 s/(eB)} \]

- **Asymptotic expansion for large \( m \):**
  \[ S = -\frac{e^2 B^2}{8\pi^2} \left[ -\frac{1}{45} \left( \frac{eB}{m^2} \right)^2 + \frac{4}{315} \left( \frac{eB}{m^2} \right)^4 \ldots \right] \]

- **Non-analytic small \( m \) expansion:**
  \[ S = -\frac{e^2 B^2}{8\pi^2} \left\{ \frac{1}{3} \log \frac{m^2}{eB} + 0.763969 + O \left( \frac{m^2}{eB} \right) \right\} \]
Charge Radii

- **Diverge in chiral limit:**
  As pion mass $m_\pi \sim m_q^{1/2} \to 0$, charge radii must diverge.

- **Pion loop:**
  gives contribution that behaves like:
  $\ln m_\pi \to \infty$ as $m_\pi \to 0$

- **Such behaviour can never arise in constituent quark model.**
Future Developments

- Need \textbf{100's T\text{flops}} to compute hadron properties at $m_q = 5$ MeV

- Will NOT happen for at least \textbf{10 years}

\textbf{BUT}

- Several groups will have \textbf{10's T\text{flops}} within \textbf{2–3 years}

- Advances in controlled chiral extrapolation should be sufficient for accurate \textbf{physical} hadron properties!
Overview of Hadron Masses

- Behave like constituent quark model for $m_\pi$ above 400–500 MeV:
OVERVIEW

- Lattice data $\bar{m} > 60$ MeV:
  - (i.e. $m_\pi \geq 400$–500 MeV)
  - Hadron properties smooth, slowly varying functions of the constituent quark mass ($M \sim M_0 + cm$):
    - $M_{N,\Delta} \sim 3M \sim (a + bm^2_\pi)$
    - $M_{\rho,\omega} \sim 2M \sim (a' + \frac{2}{3} bm^2_\pi)$
    - $\mu_H \sim 1/M$

- BUT at small $\bar{m} (< 60$ MeV):
  - (i.e. $m_\pi \leq 400$–500 MeV)
  - Chiral Symmetry $\Rightarrow$
  - rapid, non-analytic variation with $\bar{m}$:
    - $\delta M_H \sim \bar{m}^{3/2}$
    - $\delta \mu_H \sim \bar{m}^{1/2}$
    - $\delta < r^2 >_{ch} \sim \ln \bar{m}$

- Models (like CBM) yield a natural explanation of transition from LNA to smooth behaviour
Non-Analytic Behaviour (cont.)

\[ \sigma_{NN} = -\frac{3q_A^2}{32\pi f^2} m_\pi^3 \]
\[ = -5.6m_\pi^3 \text{ GeV} \]
\[ = -17 \text{ MeV at } m_{\pi}^{\text{phys}} = 140 \text{ MeV} \]

BUT

\[ = -460 \text{ MeV at } m_{\pi} = 420 \text{ MeV} \]

- where lowest lattice data exists!
- Thus non-analytic behaviour is a huge problem for chiral extrapolation

OR

- Major opportunity analogous to study versus \( N_c \), to learn from study of QCD as a function of \( m_q \)
**Common Lattice Method:**

- Formula: \( M_B = \alpha + \beta m^2 + \gamma m^3 \)

- Tiny coefficient of \( m^3 \) - incompatible with \( \chiPT \)

\[ \gamma = -0.761 \quad \text{c.f.} \quad \gamma = -5.60 \text{ for } \chiPT \]
Non-Analytic Behaviour

- Dynamical symmetry breaking
  \[ \Rightarrow \text{dependence of all hadron properties on } m_q \text{ is not analytic} \]
- e.g. Nucleon Mass

\[ N \to N\pi \to N:\]
\[ \delta M_N = -\frac{3g_A^2}{16\pi^2 f^2_\pi} \int_0^\infty dk k^2 w^2(k) \frac{k^2 + m_\pi^2}{k^2 + m_\pi^2} \]

\[ = m_0 + c_2 m_\pi^2 + c_3^{\text{LNA}} m_\pi^3 + c_4 m_\pi^4 + \ldots \]
(chiral perturbation theory)

- \[ c_3^{\text{LNA}} = -\frac{3g_A^2}{32\pi f^2_\pi} \] is model independent
  - coming from pion pole

- \[ c_3^{\text{LNA}} m_\pi^3 \propto m_q^3 \] !!
- \[ \Delta \to N\pi \to \Delta \] gives \[ m_\pi^4 \ln m_\pi \]
Non-Analytic Behaviour

- Dynamical symmetry breaking
  ⇒ dependence of all hadron properties on $m_q$ is not analytic
- e.g. Nucleon Mass

$$\delta M_N = -\frac{3g_A^2}{16\pi^2 f_\pi^2} \int_0^\infty \frac{dk}{k^2 + m_\pi^2}$$

$$= m_0 + c_2 m_\pi^2 + c_3^\text{LNA} m_\pi^3 + c_4 m_\pi^4 + .......
$$

(chiral perturbation theory)

- $c_3^\text{LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}$ is model independent
  - coming from pion pole

- $c_3^\text{LNA} m_\pi^3 \propto m_q^{\frac{3}{2}}$

- $\Delta \rightarrow N\pi \rightarrow \Delta$ gives $m_\pi^4 \ln m_\pi$
Relevance for Lattice QCD

- **Time:**
  Small quark masses increase time for calculations:

\[ t \propto \frac{1}{m_q^{3.0-3.5}} \]

- **Hence extrapolations:**
  Therefore in practice compute at large quark mass and extrapolate to physical value

- **State-of-the-art:**
  CP-PACS (Tsukuba):
  \( M_N \): above 40MeV
  \( \mu_H \): above 90 MeV
  c.f. 5–8 MeV physical values.....

- That is state-of-the-art involves quark masses a factor of 8–20 too large!
Quark Propagator

- Fix Landau gauge numerically
  (Leinweber, Skullerud, Williams... also Aoki et al.)

\[ S_E(p) = \frac{Z(p^2)}{i \gamma_\mu p_\mu + M(p^2)} \]

- Asymptotic form: \(- \frac{4 \pi \alpha_s}{3 p^2} \langle \bar{q}q \rangle \)
- "Constituent" mass \( M(0) \sim 300 \text{ MeV} \)
- very similar to DSE phenomenology
Gluon Propagator

- Fix Landau gauge numerically
  (Leinweber, Williams, Bonnet ...)
- pQCD $\Rightarrow D(q^2) \sim \frac{1}{q^2}$
- DATA $\Rightarrow$ Non-Perturbative for $q < 2$

- No infra-red enhancement
- **disagrees** SDE phenomenology
- **BUT** agrees Alkofer et al.:
  ghosts enhance quark-gluon vertex
- to be tested soon .......
The ratio $\mu_p G_E / G_M$:

Oettel et al. (2001)
Nucleon electric form factors:

- compressed
  \[ q - q \rightarrow \text{hard form factor} \]

- realistic
  "diquark" size:
  \[ \ell \approx 0.3 \text{ fm} \]

- \( \rho - \omega \):
  20–25\% to \((r_p)^2_{el}\)

- photon resolves
  \[ q - q \rightarrow \text{quenched form factor} \]

- (e.m.'ally)
  compact dq \(\rightarrow\) mimics
  \(\pi\) cloud?

* Oettel, Alkofer, v. Smekal, EPJA8 (2000), 553
Quark–photon vertex II:

Fit off-shell behaviour to the $\pi$ form factor:

$p - \omega$ contributions:

- mainly small $Q^2$
- 25% to $r_{\pi}^2$ (model dependent!)
Further Tests at Higher-$Q^2$

More recent measurements JLAB E93-049:

![Graph showing data points and lines representing different calculations.]

Preliminary results: R. Ransome & C. Glashauser
Quark Condensate

- Need Hartree + Fock + RPA to understand

Cotanch et al., Krein et al., ........

\[ \langle \bar{u} u \rangle \approx \langle \bar{d} d \rangle \approx (-240 \text{MeV})^3 \]

- Pion is a Goldstone excitation:

\[ m_{\pi}^2 \propto \frac{m_u + m_d}{2} = m_q \]

- Dynamical chiral symmetry breaking also implies quark has large effective (constituent) mass at low momentum:

\[ m(p^2) \to m_q \quad \text{as} \quad p^2 \to \infty \]

\[ m(p^2) \to M(0) \quad \text{as} \quad p^2 \to 0. \]