Evolution of Efimov States in 2n Halo Nuclei:
A general study

Indranil Mazumdar

TUNL, Duke University, NC
&
Dept. of Nuclear & Atomic Physics,
Tata Institute of Fundamental Research,
Mumbai 400 005

Chiral Dynamics, 2012
Jefferson Lab
9th August, 2012
Plan of the talk

- Introduction to Efimov Effect
- The three-body formalism
- Searching for Efimov states in 2n halo nuclei:
  - $^{14}\text{Be}$, $^{19}\text{B}$, $^{20}\text{C}$, $^{22}\text{C}$, $^{38}\text{Mg}$, $^{32}\text{Ne}$ some results
- Summary and future scope
Collaborators

• V.S. Bhasin  
  *Delhi Univ.*

• V. Arora  
  *Delhi Univ.*

• A.R.P. Rau  
  *Louisiana State Univ.*

• *Phys. Rev. Lett.* 99, 269202
• *Nucl. Phys.* A790, 257
• *Phys. Rev. Lett.* 97, 062503
• *Phys. Rev. C69*, 061301(R)
• *Phys. Rev. C61*, 051303(R)
• *Phys. Rev. C56*, R5
• *Phys. Rev. C50*, R550
• *Few Body Systems*, 2009
• *Pramana*, 2010

\[
\begin{align*}
\text{Prog. Part. Nucl. Phys. 47,517 (2001) (Brown)} \\
\text{Rev. Mod. Phys. 76,(2004) 215(Jensen et al.)} \\
\text{Ann Rev. Nucl. Part. Sci. 45, 591(Hansen et al.)} \\
\text{Rev. Mod. Phys. 66 (1105)(K. Riisager)} \\
\text{Rev. Mod. Phys. 82 (2910) 1225 (C. Chin et al.)}
\end{align*}
\]
Efimov Effect

"A three-body system can support infinite bound states when none of the three pairs are bound, or one or two pairs are barely bound."


**Universality:**

Independent of the details of the 2-body interaction

Adjacent energy levels are related by

\[ \frac{E_{N+1}}{E_N} = e^{-2\pi} \]

Size of the Nth state is

\[ R_{\text{size}} \sim r_0 e^{N\pi} \]

The number of states decreases with increasing 2-body strength

**Necessary Conditions:**

- Low energy requirement
- Large scattering length

This scenario was predicted for three-body systems with

\[ a \gg r_0 \]

where

- \( a \): two-body scattering length
- \( r_0 \): two-body effective range

Note: modern helium pair potentials have \( a \approx 104 \text{Å} \)

Artificially weakening the pair interaction introduces up to infinitely many three-body bound states.

V. Efimov; Phys. Lett. 33B, 563 (1970)
V. Efimov; Comments Nucl. Part. Phys. 19, 271 (1990)
Efimov effect:

To

Efimov Physics

“From questionable to pathological to exotic to a hot topic ...”

Nature Physics 5, 533 (2009)

Vitaly Efimov

Univ. of Washington, Seattle

Efimov, 1990

Ferlaino & Grimm 2010
V. Efimov:
Nucl. Phys A 210 (1973)

Amado & Noble:
Phys. Lett. 33B (1971)
Phys. Rev. D5 (1972)

Fonseca et al.
Nucl. PhysA320, (1979)

Adhikari & Fonseca

Theoretical searches in Atomic Systems
T.K. Lim et al. PRL38 (1977)
T. Gonzalez-Lezana et al. PRL 82 (1999),

Diffraction experiments with transmission gratings
Carnal & Mlynek, PRL 66 (1991)
Hegerfeldt & Kohler, PRL 84, (2000)

Three-body recombination in ultra cold atoms

L.H. Thomas,
Phys.Rev.47,903(1935)
Evidence for Efimov quantum states in an ultracold gas of caesium atoms


Observation of an Efimov spectrum in an atomic system.


First two states of the Efimov spectrum in ultra cold 39K
Can we find Efimov Effect in the atomic nucleus?

Unlike cold atom experiments we have no control over the scattering lengths.

The discovery of 2-neutron halo nuclei, characterized by very low separation energy and large spatial extension are ideally suited for studying Efimov effect in atomic nuclei.
The Formalism

The 2-neutron halo nucleus $^{11}$Li is modeled as a three-body system consisting of a compact core of $^9$Li and two valence neutrons forming a halo around the core. We label the two neutrons and the core as 1,2,3 with momenta $P_1$, $P_2$, $P_3$ respectively. Assuming the core to be a structureless and spinless object, we write the Schrödinger equation in momentum space as

$$(T - E)\psi = -(V_{12} + V_{23} + V_{31})\psi$$

Where $E$ is the total energy (= binding energy, B.E.) and $T$ represents the kinetic energy such that

$$T - E = p_1^2/2m + p_2^2/2m + p_3^2/2m - E$$

$$= p_{ij}^2/2\mu_{ij} + p_{ik}^2/2\mu_{ik} - E$$

The three body bound state wave function in momentum space using the binary separable potentials

$$V_{12} = -\frac{\lambda_n}{2\mu_{12}}g(p_{12})g(p'_{12}),$$

$$V_{23} = -\frac{\lambda_c}{2\mu_{23}}g(p_{23})g(p'_{23}),$$

$$V_{31} = -\frac{\lambda_c}{2\mu_{31}}g(p_{31})g(p'_{31})$$

is

$$\psi(\vec{p}_{12}, \vec{p}_{3}; E) = D^{-1}(\vec{p}_{12}, \vec{p}_{3}; E)[g(p_{12})F(\vec{p}_{3}) + f(p_{23})G(\vec{p}_{1}) + f(p_{31})G(\vec{p}_{2})]$$

(1)

g(p) = 1/(p^2 + \beta_n^2), \quad f(p) = 1/(p^2 + \beta_c^2), \quad \lambda_n, \beta_n, \beta_c$$

reproduce spin singlet scattering length and effective range.

The spectator functions $F(p)$ and $G(p)$ satisfy the homogeneous coupled integral equations

$$[\Lambda_{n}^{-1} - h_n(p)]F(\vec{p}) = 2 \int dqK_1(\vec{p}, \vec{q})G(\vec{q})$$

(2)

$$[\Lambda_{c}^{-1} - h_c(p)]G(\vec{p}) = \int dqK_2(\vec{p}, \vec{q})F(\vec{q}) + \int dqK_3(\vec{p}, \vec{q})G(\vec{q})$$

(3)

After the angular integration, the two couple equation reduce to an integral equation in one variable. These equations are numerically computed as an eigenvalue problem.

Dasgupta, Mazumdar, Bhasin, Phys. Rev C50,550

Structural properties of $^{11}$Li
\[ V_{12} = -\frac{\hbar^2}{2\mu_{12}} g(p_{12})g(p'_{12}), \]
\[ V_{23} = -\frac{\hbar^2}{2\mu_{23}} f(p_{23})f(p'_{23}), \]
\[ V_{31} = -\frac{\hbar^2}{2\mu_{31}} f(p_{31})f(p'_{31}), \]
\[ g(p) = 1/(p^2 + \beta^2) \quad \text{and} \quad f(p) = 1/(p^2 + \beta_f^2). \]

Using the n-n and n-core potentials in the three-body Schrödinger equation,

\[ (T-E)\psi = -(V_{12} + V_{23} + V_{31})\psi, \]

the solution (three-body wave function) is expressed as

\[ \psi(p_{12}, p_{13}; E) = D^{-1}(p_{12}, p_{13}; E)[g(p_{12})F(p_{13}) + f(p_{23})G(p_{11}) + f(p_{31})G(p_{22})], \]

where

\[ D(p_{12}, p_{13}; E) \equiv \frac{p_{12}^2}{2\mu_{12}} + \frac{p_{13}^2}{2\mu_{12,3}} - E \]

The spectator functions \( F(p) \) and \( G(p) \) describe, respectively, the dynamics of the core and of the light halo particles; satisfy the homogeneous coupled integral equations

\[ [\Lambda_n^{-1} - h_n(p)]F(p) = 2 \int dq K_1(p, q)G(q), \]

\[ [\Lambda_c^{-1} - h_c(p)]G(p) = \int dq K_2(p, q)F(q) + \int dq K_3(p, q)G(q), \]

where \( \Lambda_n \equiv \lambda_n/2\mu_{12}, \quad \Lambda_c \equiv \lambda_c/2\mu_{13} \). The explicit expressions for the kernels \( K_1, K_2 \) and \( K_3 \) are given by

\[ K_1(p, q; E) = \frac{mg(q + p/2)f(q + aq)}{[q^2 + q \cdot p + p^2/2a - mE]}, \]

\[ K_2(p, q; E) = \frac{mg(p + q/2)f(q + ap)}{[p^2 + p \cdot q + q^2/2a - mE]}, \]

\[ K_3(p, q; E) = \frac{mg(q + p/2)f(p + bq)}{[q^2 + q \cdot p + p^2/2a - mE]}, \]

with

\[ a \equiv m_3/(m+m_3), \quad b \equiv m/(m+m_3) \quad \text{and} \quad d_3 \equiv (m+m_3)/(2m+m_3) \]

\[ h_n(p) = m \int dq g^2(q)/[q^2 + p^2/2a - mE], \]

\[ h_c(p) = m \int dq f^2(q)/[q^2 + p^2/2d - mE]. \]
\[ \tau_n^{-1}(p) F(p) \equiv \phi(p) \text{ and } \tau_c^{-1}(p) G(p) \equiv \chi(p) \]

Where

\[ \tau_n^{-1}(p) = \mu_n^{-1} - [ \beta_r (\beta_r + \sqrt{p^2/2a + \varepsilon^3})^2 ]^{-1} \]
\[ \tau_c^{-1}(p) = \mu_c^{-1} - 2a[ 1 + \sqrt{2a(p^2/4c + \varepsilon^3) } ]^{-2} \]

where \( \mu_n = \pi \lambda_n / \beta_1^2 \) and \( \mu_c = \pi \lambda_c / 2a \beta_1^3 \)

are the dimensionless strength parameters.

Variables \( p \) and \( q \) in the final integral equation are also now dimensionless,

\[ p/\beta_1 \to p \text{ & } q/\beta_1 \to q \]
and
\[ -mE/\beta_1^3 = \varepsilon_3, \text{ \( \beta_r = \beta/\beta_1 \) } \]

Factors \( \tau_n^{-1} \) and \( \tau_c^{-1} \) appear on the left hand side of the spectator functions \( F(p) \) and \( G(p) \) and are quite sensitive. They blow up as \( p \to 0 \) and \( \varepsilon_3 \) approaches extremely small value.
For our purpose of studying the sensitive computational details for the Efimov effect, the equations are suitably transformed in terms of $\phi(p)$ and $\chi(p)$, involving only the dimensionless quantities.

$$
\tau_n^{-1}(p) F(p) \equiv \phi(p) \quad \text{and} \quad \tau_c^{-1}(p) G(p) \equiv \chi(p)
$$

(1)

where

$$
\tau_n^{-1}(p) = \mu_n^{-1} - \left[ \beta_r (\beta_r + \sqrt{\frac{E}{\mu_n} + \varepsilon_3})^{-1},
$$

and

$$
\tau_c^{-1}(p) = \mu_c^{-1} - 2a[1 + \sqrt{2a(\frac{E}{\mu_c} + \varepsilon_3})]^{-2},
$$

with $\beta_r = \frac{\beta}{\beta_1}$, and $\mu_n = \pi^2 \lambda_n / \beta_1^3$ and $\mu_c = \pi^2 \lambda_c / 2a \beta_1^3$ are the dimensionless strength parameters.

The two coupled equations are now reduced to one integral equation for $\chi(p)$

$$
\Lambda_i \chi(p) = \int dq K_3(p, q, \varepsilon_3) \tau_c(q) \chi(q) + 2 \int dq dq' K_2(p, q, \varepsilon_3) \tau_n(q) \times K_1(q, q', \varepsilon_3) \tau_c(q') \chi(q').
$$

(2)

Here the kernels $K_1$, $K_2$, $K_3$ are essentially the same except that the variables $p, q$ etc. are now dimensionless quantities: $\frac{p}{\beta_1} \rightarrow p, \frac{q}{\beta_1} \rightarrow q$, and $\frac{m_E}{\beta_1} \equiv \varepsilon_3$.

The integral equation is basically an eigenvalue equation in $\Lambda_i$ is computed numerically to determine the three-body (core-n-n) ground-state energy as well as the Efimov states. It is to be noted that the factors $\tau_n$ and $\tau_c$ are quite sensitive particularly when the scattering lengths of the binary systems get large values.
TABLE I. $^{14}$Be ground and excited states three-body energy for different two-body input parameters.

<table>
<thead>
<tr>
<th>$n^{12}$Be Energy keV</th>
<th>$\lambda_1$</th>
<th>$a_2$ fm</th>
<th>$\epsilon_0$ keV</th>
<th>$\epsilon_1$ keV</th>
<th>$\epsilon_2$ keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>11.71</td>
<td>-21</td>
<td>1350</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.8</td>
<td>12.32</td>
<td>-61.6</td>
<td>1408</td>
<td>0.053</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12.46</td>
<td>-105</td>
<td>1450</td>
<td>2.56</td>
<td>0.061</td>
</tr>
<tr>
<td>1</td>
<td>12.52</td>
<td>-149</td>
<td>1456</td>
<td>3.8</td>
<td>0.22</td>
</tr>
<tr>
<td>0.1</td>
<td>12.62</td>
<td>-483</td>
<td>1488</td>
<td>6.1</td>
<td>0.62</td>
</tr>
<tr>
<td>0.05</td>
<td>12.63</td>
<td>-658</td>
<td>1490</td>
<td>6.4</td>
<td>0.68</td>
</tr>
<tr>
<td>0.01</td>
<td>12.65</td>
<td>-1491</td>
<td>1490</td>
<td>6.9</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Mazumdar and Bhasin, PRC 56, R5

First Evidence for low lying s-wave strength in $^{13}$Be
Fragmentation of $^{18}$O, virtual state with scattering length < 10 fm
Thoennessen, Yokoyama, Hansen
Phys. Rev. C 63, 014308
## Search for Efimov states in $^{19}$B, $^{22}$C, and $^{20}$C

<table>
<thead>
<tr>
<th>$n^{-17}$B energy (keV)</th>
<th>$\lambda_c/\alpha^3$</th>
<th>$a_s$ (fm)</th>
<th>$\epsilon_0$ (keV)</th>
<th>$\epsilon_1$ (keV)</th>
<th>$\epsilon_2$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>514.8</td>
<td>8.49</td>
<td>-6.515</td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>135.3</td>
<td>9.5</td>
<td>-12.71</td>
<td>728</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>10.0</td>
<td>-21.16</td>
<td>851</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.7</td>
<td>10.5</td>
<td>-53.2</td>
<td>978</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>0.67</td>
<td>10.75</td>
<td>-179.6</td>
<td>1042</td>
<td>5.4</td>
<td>0.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$n^{-20}$C energy (keV)</th>
<th>$\lambda_c/\alpha^3$</th>
<th>$a_s$ (fm)</th>
<th>$\epsilon_0$ (keV)</th>
<th>$\epsilon_1$ (keV)</th>
<th>$\epsilon_2$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>319</td>
<td>9.82</td>
<td>-8.23</td>
<td>1120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>127</td>
<td>10.5</td>
<td>-13.02</td>
<td>1287</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48.8</td>
<td>11.0</td>
<td>-21.0</td>
<td>1410</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.3</td>
<td>11.5</td>
<td>-48.2</td>
<td>15400</td>
<td>0.122</td>
<td></td>
</tr>
<tr>
<td>1.46</td>
<td>11.75</td>
<td>-121.5</td>
<td>1608</td>
<td>4.74</td>
<td>0.198</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$n^{-22}$C energy (keV)</th>
<th>$\lambda_c/\alpha^3$</th>
<th>$a_s$ (fm)</th>
<th>$\epsilon_0$ (keV)</th>
<th>$\epsilon_1$ (keV)</th>
<th>$\epsilon_2$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>15.51</td>
<td>20.38</td>
<td>3188.03</td>
<td>78.87</td>
<td>65.8</td>
</tr>
<tr>
<td>100</td>
<td>15.89</td>
<td>16.05</td>
<td>3291.54</td>
<td>115.72</td>
<td>100.09</td>
</tr>
<tr>
<td>113.2</td>
<td>16.0</td>
<td>15.15</td>
<td>3317.35</td>
<td>127.41</td>
<td>111.76</td>
</tr>
<tr>
<td>139.60</td>
<td>16.2</td>
<td>13.77</td>
<td>3371.24</td>
<td>150.32</td>
<td>135.29</td>
</tr>
<tr>
<td>168.59</td>
<td>16.4</td>
<td>12.64</td>
<td>3426.03</td>
<td>175.34</td>
<td>163.48</td>
</tr>
<tr>
<td>200</td>
<td>16.6</td>
<td>11.71</td>
<td>3482.95</td>
<td>202.15</td>
<td>194.15</td>
</tr>
</tbody>
</table>

Mazumdar, Arora Bhasin
Phys. Rev. C 61, 051303(R)
First evidence for a virtual $^{18}$B ground state

A. Spyrou$^{a,b,*}$, T. Baumann$^{a}$, D. Bazin$^{a}$, G. Blanchon$^{c}$, A. Bonaccurso$^{d}$, E. Breitbach$^{e}$, J. Brown$^{f}$, G. Christian$^{a,b}$, A. DeLine$^{g}$, P.A. DeYoung$^{h}$, J.E. Finck$^{g}$, N. Frank$^{i}$, S. Mosby$^{a,b}$, W.A. Peters$^{j}$, A. Russel$^{g}$, A. Schiller$^{k}$, M.J. Strongman$^{a,b}$, M. Thoennessen$^{a,b}$

dimensions, possibly of the order of 100 fm [1,10]. As shown by Mazumdar et al. [9], Efimov states would be expected in $^{19}$B only if the unbound subsystem $^{18}$B ($n-^{17}$B) would have a virtual ground state corresponding to a scattering length of a few hundred fm.

setup. An s-wave line shape was used to describe the experimental spectrum resulting in an upper limit for the scattering length of $-50$ fm which corresponds to a decay energy $<10$ keV. Observing an s-wave decay of $^{18}$B provides an experimental verification that the ground state of $^{19}$C includes a large s-wave component. The presence of this s-wave component shows that s–d mixing is still present in $^{18}$B and that the $s_{1/2}$ orbital has not moved significantly below the $d_{5/2}$ orbital.
The feature observed can be attributed to the singularity in the two body propagator \([\Lambda_c^{-1} - h_c(p)]^{-1}\).

There is a subtle interplay between the two and three body energies.

The effect of this singularity on the behaviour of the scattering amplitude has to be studied.
For $k \rightarrow 0$, the singularity in the two body cut does not cause any problem. The amplitude has only real part. The off-shell amplitude is computed by inverting the resultant matrix, which in the limit $a_o(p)_{p \rightarrow 0} \rightarrow -a$, the n-19C scattering length.

For non-zero incident energies the singularity in the two body propagator is tackled by the CSM.

$P \rightarrow p_1 e^{i\phi}$ and $q \rightarrow q e^{-i\phi}$

The unitary requirement is the $\text{Im}(f^{-1}_k) = -k$

Balslev & Combes (1971)
Matsui (1980)
Volkov et al.
\[ \epsilon_2 = 250 \text{ keV} \]
\[ \Gamma = 0.25 \text{ keV} \]
\[ E_{\text{res}} = 1.63 \text{ keV} \]

\[ \epsilon_2 = 300 \text{ keV} \]
\[ \Gamma = 0.27 \text{ keV} \]
\[ E_{\text{res}} = 1.7 \text{ keV} \]

\[ \epsilon_2 = 350 \text{ keV} \]
\[ \Gamma = 0.32 \text{ keV} \]
\[ E_{\text{res}} = 1.53 \text{ keV} \]

<table>
<thead>
<tr>
<th>n-^{18}\text{C Energy (keV)}</th>
<th>( \epsilon_3(0) ) (MeV)</th>
<th>( \epsilon_3(1) ) (keV)</th>
<th>( \epsilon_3(2) ) (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>3.00</td>
<td>79.5</td>
<td>66.95</td>
</tr>
<tr>
<td>100</td>
<td>3.10</td>
<td>116.6</td>
<td>101.4</td>
</tr>
<tr>
<td>140</td>
<td>3.18</td>
<td>152.0</td>
<td>137.5</td>
</tr>
<tr>
<td>180</td>
<td>3.25</td>
<td>186.6</td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>3.32</td>
<td>221.0</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>3.35</td>
<td>238.1</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>3.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>3.44</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Arora, Mazumdar, Bhasin, PRC 69, 061301
Ugo Fano  
(1912 – 2001)

Effects of Configuration Interaction on Intensities and Phase Shifts*

U. Fano  
National Bureau of Standards, Washington, D. C.  
(Received July 14, 1961)

Third highest in citation impact of all papers published in the entire Physical Review series.
Fitting the Fano profile to the $N^{-19}C$ elastic cross section for $n^{-18}C$ BE of 250 keV

\[\sigma = \sigma_0[(q + \varepsilon)^2/(1+\varepsilon^2)]\]

Mazumdar, Rau, Bhasin
The resonance due to the second excited Efimov state for n-^{18}C BE 150 keV. The profile is fitted by same value of \( q \) as for the 250 keV curve.
The calculation have been extended to

1) Two hypothetical cases: *very heavy core of mass* $A = 100$ (+ 2$n$) 
   *three equal masses* $m_1=m_2=m_3$

2) Two realistic cases of $^{38}\text{Mg}$ & $^{32}\text{Ne}$

$$
^{38}\text{Mg} \quad S_{2n} = 2570 \text{ keV} \quad n + \text{core} (^{37}\text{Mg}) \quad 250 \text{ keV (bound)} \\
^{32}\text{Ne} \quad S_{2n} = 1970 \text{ keV} \quad n + \text{core} (^{31}\text{Ne}) \quad 330 \text{ keV (bound)}
$$

$^{38}\text{Mg} (S_{2n})$ Audi & Wapstra (2003)  

$^{37}\text{Mg} \& ^{31}\text{Ne} \quad (S_n)$ Sakurai *et al.*, PRC 54 (1996),  
Jurado *et al.* PLB (2007), Nakamura *et al.* PRL (2009),  
Hamamoto PRC (2010),  
Urata, Hagino, Sagawa PRC (2011)

We have reproduced the ground state energies and have found at least two Efimov states that vanish into the continuum with increasing n-core interaction. They again show up as asymmetric resonances at around 1.6 keV neutron incident energy in the scattering sector.
TABLE: Ground and excited states for three cases studied, namely, mass 102 (columns 2, 3, 4), $^{38}\text{Mg}$ (column 5, 6, 7), and $^{32}\text{Ne}$ (columns 8, 9, 10) for different two body input parameters.

<table>
<thead>
<tr>
<th>n-Core Energy $\varepsilon_2$ keV</th>
<th>$\varepsilon_3(0)$ keV</th>
<th>$\varepsilon_3(1)$ keV</th>
<th>$\varepsilon_3(2)$ keV</th>
<th>$\varepsilon_3(0)$ keV</th>
<th>$\varepsilon_3(1)$ keV</th>
<th>$\varepsilon_3(2)$ keV</th>
<th>$\varepsilon_3(0)$ keV</th>
<th>$\varepsilon_3(1)$ keV</th>
<th>$\varepsilon_3(2)$ keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>4020</td>
<td>53.6</td>
<td>44.4</td>
<td>3550</td>
<td>61.3</td>
<td>49.9</td>
<td>3420</td>
<td>61.5</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>4080</td>
<td>70.4</td>
<td>61.7</td>
<td>3610</td>
<td>80.8</td>
<td>67.1</td>
<td>3480</td>
<td>81.0</td>
<td>67.2</td>
</tr>
<tr>
<td>80</td>
<td>4130</td>
<td>86.9</td>
<td>(78.4)</td>
<td>3670</td>
<td>99.2</td>
<td>84.16</td>
<td>3530</td>
<td>99.8</td>
<td>84.3</td>
</tr>
<tr>
<td>100</td>
<td>4170</td>
<td>103.1</td>
<td></td>
<td>3710</td>
<td>117</td>
<td>101.4</td>
<td>3570</td>
<td>117.5</td>
<td>101.5</td>
</tr>
<tr>
<td>120</td>
<td>4220</td>
<td>(119.3)</td>
<td></td>
<td>3750</td>
<td>134.5</td>
<td>(118.9)</td>
<td>3620</td>
<td>135</td>
<td>(118.9)</td>
</tr>
<tr>
<td>140</td>
<td>4259</td>
<td></td>
<td></td>
<td>3790</td>
<td>151.6</td>
<td></td>
<td>3650</td>
<td>152.5</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>4345</td>
<td></td>
<td></td>
<td>3860</td>
<td>185.6</td>
<td></td>
<td>3730</td>
<td>186.5</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>4460</td>
<td></td>
<td></td>
<td>3980</td>
<td></td>
<td></td>
<td>3852</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>4530</td>
<td></td>
<td></td>
<td>4040</td>
<td></td>
<td></td>
<td>3910</td>
<td></td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>4590</td>
<td></td>
<td></td>
<td>4120</td>
<td></td>
<td></td>
<td>3980</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Heavy Core

\[ \varepsilon_2 = 250 \text{ keV} \]
Core Mass = 100

\[ \varepsilon_2 = 150 \text{ keV} \]
Core Mass = 100

\[ \varepsilon_2 = 250 \text{ keV} \]
Core Mass = 36

\[ \varepsilon_2 = 150 \text{ keV} \]
Core Mass = 36

\[ \varepsilon_2 = 250 \text{ keV} \]
Core Mass = 30

\[ \varepsilon_2 = 150 \text{ keV} \]
Core Mass = 30

Mazumdar, Bhasin, Rau
Ground states for the two cases

- Equal mass case strikingly different from unequal (heavy core) case.

- Evolution of Efimov states in heavy core of 100 fully consistent with $^{20}\text{C}$ results.

Mazumdar
Few Body Systems, 2009
Summary

- A three body model with s-state interactions is exploited to search for Efimov states in 2n-halo nuclei.

- A virtual state of a few keV (2 to 4) energy corresponding to scattering length from -50 to -100 fm for the n-\(^{12}\)Be predicts the ground state and excited states of \(^{14}\)Be.

- \(^{19}\)B, \(^{22}\)C and \(^{20}\)C are investigated and it is shown that Borromean type nuclei are much less vulnerable to respond to Efimov effect.

- \(^{20}\)C is a promising candidate for Efimov states at energies below the n-(nc) breakup threshold.

- The bound Efimov states in \(^{20}\)C move into the continuum and reappear as Resonances with increasing strength of the binary interaction.

- Asymmetric resonances in elastic n+\(^{19}\)C scattering are attributed to Efimov states and are identified with the Fano profile. The conjunction of Efimov and Fano phenomena may lead to the experimental realization in nuclei.

- \(^{32}\)Ne & \(^{38}\)Mg exhibit very similar dynamical structure and are also candidates for probing Efimov states.
• We emphasize the cardinal role of channel coupling.

  There is also a definite role of mass ratios as observed numerically.

• However, channel coupling is an elegant and physically plausible scenario.

• Difference can arrive between zero range and realistic finite range potentials in non-Borromean cases.

  Note, that for n-^{18}C binding energy of 200 keV, the scattering length is about 10 fm while the interaction range is about 1 fm.

• The extension of zero range to finer details of Efimov states in non-Borromean cases may not be valid.

• The discrepancy observed in the resonance vs virtual states in ^{20}C clearly underlines the sensitive structure of the three-body scattering amplitude derived from the binary interactions.
Present Activities:

- Resonant states above the three body breakup threshold in $^{20}\text{C}$.
- Structure calculations for $^{12}\text{Be}$ & $^{14}\text{Be}$
- Role of Efimov states in Bose-Einstein condensation.
- Studying the proton halo ($^{17}\text{Ne}$) nucleus.
- Planning for possible experiments with $^{20}\text{C}$ beam

Epilogue

“the richness of understanding reveals even greater richness of ignorance”

D.H. Wilkinson
THANK YOU
Theoretical Models

• Shell Model
  Bertsch et al. (1990) PRC 41,42,
  Kuo et al. PRL 78,2708 (1997) 2 frequency shell model
  Brown (Prog. Part. Nucl. Physics 47 (2001)

  Talmi & Unna, PRL 4, 496 (1960) \(^{11}\)Be
  \(S_n = 820 \text{ keV}\)
  \(S_n = 504 \text{ keV}\)

Ab initio no-core full configuration calculation of light nuclei

Quantum Monte Carlo Calculations of Light Nuclei: \(^{4,6,8}\)He, \(^{6,7}\)Li, \(^{8,9,10}\)Be
Pieper, Wiringa

• Cluster model

• Three-body model ( for 2n halo nuclei )

• RMF model

• EFT
• System composed of ultra-cold potassium atoms \(^{(39}\text{K})\) with resonantly tunable two-body interaction.

• Atom-dimer resonance and loss mechanism

• Large values of \(a\) up to 25,000 \(a_0\) reached.

• First two states of an Efimov spectrum seen
Fedorov & Jensen
*PRL 71 (1993)*

Fedorov, Jensen, Riisager
*PRL 73 (1994)*

P. Descouvement

Conditions for occurrence of Efimov states in 2-n halo nuclei.
Striking Features:

- Extremely small separation energy $S_n$ or $S_{2n}$
- Very large matter radius
- Narrow momentum distribution of fragments
- Borromean property of many two neutron halos ($^6$He, $^{11}$Li, $^{14}$Be)

<table>
<thead>
<tr>
<th>Structure</th>
<th>Nucleus</th>
<th>$S_n$(keV)</th>
<th>$S_{2n}$(keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1n-halo</td>
<td>$^{11}$Be</td>
<td>504 ± 6</td>
<td>7317 ± 6</td>
</tr>
<tr>
<td></td>
<td>$^{19}$C</td>
<td>160 ± 120</td>
<td>4350 ± 110</td>
</tr>
<tr>
<td>2n-halo</td>
<td>$^6$He</td>
<td>1864 ± 1</td>
<td>974 ± 1</td>
</tr>
<tr>
<td></td>
<td>$^{11}$Li</td>
<td>330 ± 30</td>
<td>300 ± 30</td>
</tr>
<tr>
<td></td>
<td>$^{14}$Be</td>
<td>1850 ± 120</td>
<td>1340 ± 110</td>
</tr>
<tr>
<td></td>
<td>$^{19}$B</td>
<td>1030 ± 900</td>
<td>500 ± 430</td>
</tr>
</tbody>
</table>

$S_{2n} = 369.15 (0.65) \text{ keV}$

Accepted lifetime: $8.80 (0.14) \text{ ms}$  \hspace{1cm} $J^\pi = \frac{3}{2}^-$

$S_{2n} = 12.2 \text{ MeV for } ^{18}\text{O}$

Typical experimental momentum distribution of halo nuclei from fragmentation reaction

Kobayashi et al., PRL 60, 2599 (1988)
TABLE I. Parameters of the input two body \((n-n\) and \(n-^{9}\text{Li}\)) potentials. Given the \(^{11}\text{Li}\) binding energy, the strength parameter \(\lambda_c\) as obtained from the three-body equation is matched with the corresponding value obtained from the two-body analysis.

<table>
<thead>
<tr>
<th>B.E. of (^{11}\text{Li}) (MeV)</th>
<th>(\beta/\alpha)</th>
<th>(\lambda_n/\alpha^3)</th>
<th>(\beta_1/\alpha)</th>
<th>(\lambda_c/\alpha^3)</th>
<th>(\lambda_c/\alpha^3) three-body</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34</td>
<td>5.8</td>
<td>18.6</td>
<td>5.0</td>
<td>10.32</td>
<td>12.92</td>
</tr>
<tr>
<td>6.255</td>
<td>23.4</td>
<td>5.5</td>
<td>14.0</td>
<td>17.01</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>5.8</td>
<td>18.6</td>
<td>5.0</td>
<td>10.32</td>
<td>12.39</td>
</tr>
</tbody>
</table>

TABLE II. Values of the root mean square radii of neutron-neutron and neutron-\(^{9}\text{Li}\) separations calculated using Eqs. (18) and (19) for different binding energies of \(^{11}\text{Li}\).

<table>
<thead>
<tr>
<th>B.E. of (^{11}\text{Li}) (MeV)</th>
<th>(\bar{r}_{nn}) (fm)</th>
<th>(\bar{r}_{nn}) (fm) (from other model calculations [4])</th>
<th>(\bar{r}_{nc}) (fm) (from other model calculations [4])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>10.63</td>
<td>6.24 to 7.80</td>
<td>5.47 to 6.40</td>
</tr>
<tr>
<td>0.25</td>
<td>9.9</td>
<td>9.86</td>
<td></td>
</tr>
<tr>
<td>0.315</td>
<td>8.93</td>
<td>8.87</td>
<td></td>
</tr>
</tbody>
</table>

The rms radius \(r_{\text{matter}}\) calculated is \(\sim 3.6\) fm

\[
\langle r^2 \rangle_{\text{matter}} = A_c/A \langle r^2 \rangle_{\text{core}} + 1/A \langle \rho^2 \rangle \\
\rho^2 = r^2_{nn} + r^2_{nc}
\]

\textit{Dasgupta, Mazumdar, Bhasin} \\
PRC50, R550 \\
\textit{Fedorov et al (1993)} \\
\textit{Garrido et al (2002) (3.2 fm)}
Dasgupta, Mazumdar, Bhasin, PRC 50, R550

Data:
Ieki et al,
PRL 70, 1993

2-particle correlations:
Hagino et al., PRC, 2009

$^{11}\text{Li}$
$\text{Vs}$
$^6\text{He}$
Comparison between He and $^{20}\text{C}$ as three body Systems in atoms and nuclei
We emphasize the cardinal role of channel coupling.

There is also a definite role of mass ratios as observed numerically.

However, channel coupling is an elegant and physically plausible scenario.

Difference can arrive between zero range and realistic finite range potentials in non-Borromean cases.

Note, that for \( {n^{18}}C \) binding energy of 200 keV, the scattering length is about 10 fm while the interaction range is about 1 fm.

The extension of zero range to finer details of Efimov states in non-Borromean cases may not be valid.

The discrepancy observed in the resonance vs virtual states in \( {^{20}}C \) clearly underlines the sensitive structure of the three-body scattering amplitude derived from the binary interactions.
A possible experimental proposal to search for Efimov State in 2-neutron halo nuclei.

• Production of $^{20}$C secondary beam with reasonable flux
• Acceleration and Breakup of $^{20}$C on heavy target
• Detection of the neutrons and the core in coincidence
• Measurement of $\gamma$-rays as well

**The Arsenal:**
• Neutron detectors array
• Gamma array
• Charged particle array

**Another experimental scenario:**

$^{19}$C beam on deuteron target:
Neutron stripping reaction
Appearance of Resonance in $n^{-19}C$ Scattering

- The equation for the off-shell scattering amplitude in $n^{-19}C$ (bound state of $n^{-18}C$) can be written as

\[
4\pi \left( \frac{a}{d} \right) h(p^2, k^2; \alpha_2^2) a_k(\vec{p}) = (2\pi)^3 K_3(\vec{p}, \vec{k}) + 4\pi \int \frac{d\vec{q}K_3(\vec{p}, \vec{q})a_k(\vec{q})}{q^2 - k^2 - i\varepsilon} \cdot
\]

\[
+ 2(2\pi)^3 \int d\vec{q}K_2(\vec{p}, \vec{q})K_1(\vec{q}, \vec{k})\tau_n(q) \cdot
\]

\[
+ 2(4\pi) \int d\vec{q}K_2(\vec{p}, \vec{q})\tau_n(q) \int d\vec{q}' \frac{K_1(\vec{q}, \vec{q}')a_k(\vec{q}')}{q'^2 - k^2 - i\varepsilon} .
\]

- where $a_k(p)$ is the off-shell scattering amplitude, normalized such that

\[
a_k(\vec{p})|\vec{p}|=|\vec{k}| = f_k = \frac{e^{i\delta} \sin \delta}{k} .
\]