Recent Progress in Staggered Chiral Perturbation Theory

Weonjong Lee

Lattice Gauge Theory Research Center
Department of Physics and Astronomy
Seoul National University

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   - Pion Mass
   - Pion Decay Constants
   - $B_K$
   - $\pi - \pi$ Scattering Phase Shift

5 Summary and Conclusion
SWME Collaboration
1998 — Present
SWME Collaboration

- Seoul National University (SNU):
  Prof. Weonjong Lee
  Dr. Jon Bailey and Dr. Nigel Cundy (RA Prof.)
  11 graduate students.

- Brookhaven National Laboratory (BNL):
  Dr. Chulwoo Jung
  Dr. Hyung-Jin Kim (Postdoc)

- University of Washington, Seattle (UW):
  Prof. Stephen R. Sharpe.

- KISTI: Dr. Taegil Bae (Postdoc).

- University of Arizona, Tucson: Dr. Jongjeong Kim (Postdoc).
Lattice Gauge Theory Research Center (SNU)

- Center Leader: Prof. Weonjong Lee. (***)
- Research Assistant Prof.: Dr. Jon Bailey (***)
- Research Assistant Prof.: Dr. Nigel Cundy
- 11 graduate students (***)
- Secretary: Ms. Sora Park.
- More details on http://lgt.snu.ac.kr/.
Group Photo
Staggered Fermion Formulation
How to put quarks on the lattice?

- **Wilson Fermions:**
  1. Clover Action: (**)
  2. Twisted mass fermions: (**)
  3. Domain Wall Fermions: (***)
  4. Overlap Fermions:

- **Staggered Fermions:**
  1. Asqtad action: (*)
  2. HYP staggered fermions: (***)
  3. Fat7 staggered fermions:
  4. HISQ action: (***)
Cons and Pros for Staggered Fermions

Advantages (Pros):

1. Preserve part of exact chiral symmetries.
2. Numerically cheapest on the lattice.
3. No residual quark mass (no additive renormalization).
4. Easy to improve with almost no extra cost.
5. Staggered Chiral Perturbation Theory.

Possess 4 degenerate tastes (pure lattice artifacts).

Disadvantages (Cons):

1. Born with taste symmetry breaking by construction.
2. Theoretically more challenging to interpret the data.
Staggered Chiral Perturbation Theory
What is Staggered ChPT?

- ChPT designed to analyze the data produced using staggered fermions.
- Dual expansion in powers of $p^2 \approx m_q$ and $a^2$.
- It incorporates all the taste symmetry breaking effects into the LECs order by order in a perturbative series.
Birth of Staggered ChPT

- At the leading order of $p^2 \approx m_q \approx m_{\pi}^2 \approx a^2$, we can prove that the pion spectrum respects $SO(4)$ taste symmetry out of the full $SU(4)$ taste symmetry.

- Lee and Sharpe proved it for single flavor case (1999).

- Aubin and Bernard proved it for multiple flavor case (2003).

- Power counting rules are established through the numerical study on the lattice.
Splittings of Pion Multiplet Spectrum

(1) Coarse lattice ($a = 0.12$fm)

(2) Fine lattice ($a = 0.09$fm)
Scaling of the Splittings

\[ \Delta_B \left[ (\text{GeV})^2 \right] \]

\[ a^2 \alpha_s^2 \left[ 10^{-3} (\text{fm})^2 \right] \]

- \( \text{PxS} \)
- \( \text{PxV} \)
- \( \text{PxT} \)
- \( \text{PxA} \)
Sea quark mass dependence of splittings
We need to incorporate this effect of pion multiplet splittings into the data analysis.
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Staggered fermion formulation introduces mixing with extra operators in addition to the physical mixing. We can also incorporate this effect into the data analysis using SChPT.
Staggered Chiral Perturbation Theory (SChPT)

1. We need to incorporate this effect of pion multiplet splittings into the data analysis.

2. Staggered fermion formulation introduces mixing with extra operators in addition to the physical mixing. We can also incorporate this effect into the data analysis using SChPT.

3. The systematic tool is the SChPT.
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Staggered fermion formulation introduces mixing with extra operators in addition to the physical mixing. We can also incorporate this effect into the data analysis using SChPT.

The systematic tool is the SChPT.

Using the SChPT, we obtain the fitting functional form exactly order by order.
Staggered chiral perturbation theory

- **Power counting**
  \[ O(a^2 \Lambda_{\text{QCD}}^2) \approx O(p^2 / \Lambda_{\chi}^2) \approx O(m_\pi^2 / \Lambda_{\chi}^2) \approx O(m_q / \Lambda_{\text{QCD}}) \]

- **Lee & Sharpe Lagrangian for multiple flavors**
  
  \[ \mathcal{L}_{\text{LO}} = \frac{f^2}{8} \text{Tr}(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger) - \frac{1}{4} \mu f^2 \text{Tr}(M \Sigma + M \Sigma^\dagger) \]
  
  \[ + \frac{2m_0^2}{3} (U_I + D_I + S_I)^2 + a^2 \mathcal{V} \]

  - \( M = \text{diag}(m_u, m_d, m_s) \otimes \xi_I \)
  - \( \mathcal{V} : \) taste symmetry breaking potential

  \[ \text{SO}(4) \times \text{SU}(4)_T \xrightarrow{a \neq 0} \text{SW}_{4,\text{diag}} \]

  \[ \subset \]

  \[ p \ll \Lambda_{\chi} \]

  \[ \text{SO}(4) \times \text{SO}(4)_T \]
Application of SChPT
Pion Mass (Quark Mass)
Pion Flow Diagrams
Quark Flow Diagrams (1)
Quark Flow Diagrams (2)
Results at NLO

- Pion self energy:

\[ M_{\pi_F}^2 = m_{\pi_F}^2 + \Sigma(m_{\pi_F}^2) + \text{NNLO} \]

\[ \Sigma(p^2) = \frac{1}{(4\pi f)^2} [\sigma_{\text{conn}}(p^2) + \sigma_{\text{disc}}(p^2)] + \sigma_{\text{anal}}(p^2) \]

- Connected Part:

\[ \sigma_{\text{conn}} = a^2 \sum_B \left( \delta_{BF}^{\text{conn}} \ell(\pi_B^+) + \frac{\Delta_{BF}^{\text{conn}}}{48} [\ell(U_B) + 2\ell(\pi_B^+) + \ell(D_B)] \right) \]

- Disconnected Part:

\[ \sigma_{\text{disc}} = \frac{1}{12} \left[ 2(-12X_5 + a^4(\Delta_{VF}^{\text{conn}} + \ldots)\delta'_V \left( R_{X_\eta}^\pi (X_V)\tilde{\ell}(X_V) + \ldots \right) + \ldots \right] \]
Progress History (Pion Mass)


- Non-Goldstone pion sectors at NLO: Bailey & Kim & Lee (2012)

- Extension to mixed actions: underway by Yoon & Bailey & Lee (YBL) (2012)

  ※ Example of a mixed action:
  - valence quarks = HYP staggered fermions
  - sea quarks = asqtad staggered fermions

- Results have been used for the numerical study by MILC.

- We plan to apply the mixed action results to the data analysis.
Pion Decay Constants
Pion Flow Diagrams

Quark Flow Diagrams (Current Contribution)
Results at NLO

\textbf{Example:}

- Pion decay constant for fully dynamical case ($xy = ud$)
- SU(2) chiral perturbation theory ($m_u, m_d \ll m_s$)
- 2+1 flavors ($m_u = m_d = m_\ell \neq m_s$)

\[ f_{\pi_F} = f \left\{ 1 + \frac{1}{32\pi^2 f^2} \left[ -\frac{1}{4} \sum_B g_B \ell(\pi_B) + (4 - \Theta^V) \left\{ \ell(\pi_V) - \ell(\eta_V) \right\} + (V \rightarrow A) \right] + L_4 \frac{16\mu}{f^2} (2m_\ell + m_s) + L_5 \frac{16\mu}{f^2} m_\ell + a^2 F_F \right\} \]
Progress Report

- Extension to the mixed action: underway by YBL (2012)
- Results have been used for the numerical study by MILC.
- We plan to apply the mixed action results to our data analysis.
$B_K$ (Indirect CP Violation)
$B_K$ definition in standard model

\[
B_K = \frac{\langle \bar{K}_0 | \bar{s} \gamma_\mu (1 - \gamma_5) d | \bar{s} \gamma_\mu (1 - \gamma_5) d | K_0 \rangle}{\frac{8}{3} \langle \bar{K}_0 | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_\mu \gamma_5 d | K_0 \rangle}
\]

\[
\hat{B}_K = C(\mu) B_K(\mu),
\]

\[
C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]
\]
Pion Flow Diagrams for $B_K$

1. (20) OK
2. (21) OK
3. (22) OK
4. (23) X
5. (24) X
Quark Flow Diagrams for $B_K$
SU(2) Results at NLO

- $B_K$: ($m_u = m_d = m_\ell \ll m_s$)

$$B_K = d_1 Q_1 + d_2 \frac{X_P}{\Lambda^2_\chi} + d_3 \frac{L_P}{\Lambda^2_\chi} + \text{NNLO}$$

$$Q_1 = 1 + \frac{1}{32\pi^2 f^2} \left[ (L_I - X_I)\tilde{\ell}(X_I) + \ell(X_I) - 2 \sum_B \tau^B \ell(X_B) \right]$$

- $X_P = [m^{xx}_\pi(\xi_5)]^2$
- $L_P = [m^{\ell\ell}_\pi(\xi_5)]^2$
Progress History

- Extension to the mixed action : Sharpe (2008)
- BSM operators at NLO : Bailey & Kim & Lee & Sharpe (2012)
- Application to the numerical study : SWME (2010 ~ present)
- The SWME result of $B_K$ is posted to FLAG officially (2012).

$$\hat{B}_K = 0.727 \pm 0.004\text{(stat)} \pm 0.038\text{(sys)}$$

$$\varepsilon_K = (1.56 \pm 0.22) \times 10^{-3} \quad \text{(Exclusive } V_{cb})$$

$$= (1.88 \pm 0.22) \times 10^{-3} \quad \text{(Inclusive } V_{cb})$$

- We must reduce the errors of $B_K$ and $V_{cb}$ simultaneously.
$\pi - \pi$ Scattering
Application

$\pi - \pi$ Scattering and $S$–matrix

- Five channels of two pion states in staggered fermion formulation:

\[ \pi(P) - \pi(P), \quad \pi(A) - \pi(A), \quad \pi(T) - \pi(T), \]
\[ \pi(V) - \pi(V), \quad \pi(S) - \pi(S), \]

- The trouble is that their energy eigenvalues are non-degenerate.

- Recently, Hansen & Sharpe make it possible to study multi-channel scattering problem by modifying the Luscher formula.

- Now, it is possible to study the $N = 5$ multi-channel $\pi - \pi$ scattering problem on the lattice using staggered fermions.
Quark Flow Diagrams for $\pi - \pi$ Scattering
Unitarity Violation by Rooting Technique

- If the SU(4) taste symmetry is exactly conserved, then rooting cannot make a trouble of unitarity violation.
- However, if the SU(4) taste symmetry is broken, then rooting makes a unitarity violation.
- The staggered fermion formulation has taste symmetry breaking by construction.
- Hence, the rooting triggers the unitarity violation for staggered fermions.
- As a consequence, there are two kinds of unitarity violation on the lattice using staggered fermions: one from partially quenched QCD and the other from the rooting.
Fermion Determinant of Staggered Fermions:

\[ \int [d\psi][d\bar{\psi}] \exp[\int \bar{\psi}(D + m_1)\psi] = \det(D + m_1) \]

Here, the Dirac operator \((D + m_1)\) contains 4 copies of degenerate tastes.

In order to reduce the number of tastes to one, we use the rooting technique in the numerical study.

\[ \det(D + m_1) \rightarrow [\det(D + m_1)]^{1/4} \]

However, if the SU(4) taste symmetry is broken, then the rooting causes a unitarity violation since sea quarks and valence quarks have different Dirac operators.
How to get around the trouble: SChPT

- SChPT can, in principle, trace the rooting part and the unitarity violation terms.

- Hence, we fit the numerical data to the functional form suggested by SChPT.

- Then, we can remove the unitarity violating terms by hand.

- Then, the remaining part will be unitary, which corresponds to the S-matrix defined by Hansen & Sharpe.

- The SChPT calculation is underway by Yoon & Bailey & Lee.
Summary
# Summary of Current Status in SChPT

## Physics

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Weonjong Lee (SNU)

Lattice QCD

Chiral Dynamics 2012

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Sincere apologies for omitting some topics
Thank you very much !!!