Hadron Structure in AdS/QCD

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Outline

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  - EOM for the bulk profiles: mass spectra and wave functions
  - Inclusion of high Fock states
  - Consistency with large $N_c$
  - Inclusion photons

- Applications
  - Electromagnetic form factors
  - Generalized parton distributions
  - Roper resonance $N(1440)$
Introduction

- **AdS/QCD ≡ Holographic QCD (HQCD)** – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – **anti-de Sitter (AdS) space**

- **HQCD models** reproduce main features of QCD at low and high energies

- **Motivation:** **AdS/CFT correspondence** 1998 (Maldacena, Polyakov, Witten et al)
  
  Dynamics of the superstring theory in AdS\(_{d+1}\) background is encoded in \(d\) conformal field theory living on the AdS boundary.

  **More general:** extra-dimensional (ED) theories including gravity are holographically equivalent to the gauge theories living on the boundary of ED space

- **Symmetry arguments:** Conformal group acting in boundary theory isomorphic to **SO(4, 2)** – the isometry group of **AdS\(_5\)** space
**Introduction**

- **AdS metric**  \( ds^2 = \frac{R^2}{z^2} \left( dx_\mu dx^\mu - dz^2 \right) \)  \( \text{Poincaré form} \)

  \( z \) is extra dimensional (holographic) coordinate; \( z = 0 \) is UV boundary

**UV asymptotics**  Klebanov, Witten

\[
\Phi(x, z) \bigg|_{z \to 0} \to z^{d-\Delta} \left[ \Phi_0(x) + O(z^2) \right] + z^\Delta \left[ \Phi_{ph}(x) + O(z^2) \right]
\]

\( \Phi_0(x) \) is source of the CFT operator \( \hat{O} \),
\( \Phi_{ph}(x) \sim \langle \hat{O} \rangle \) is physical fluctuation

**AdS/CFT dictionary**

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operator ( \hat{O} )</td>
<td>Bulk field ( \Phi(x, z) )</td>
</tr>
<tr>
<td>( \Delta ) — scaling dimension of ( \hat{O} )</td>
<td>( m ) — mass of ( \Phi(x, z) )</td>
</tr>
<tr>
<td>Source of ( \hat{O} )</td>
<td>Non-normalizable bulk profile near ( z = 0 )</td>
</tr>
<tr>
<td>( \langle \hat{O} \rangle )</td>
<td>Normalizable bulk profile near ( z = 0 )</td>
</tr>
</tbody>
</table>
Introduction

• **Top-down approaches**  Low-energy approximation of string theory trying to find a gravitational background with features similar to QCD (e.g. Sakai-Sugimoto model)

• **Bottom-up approaches**  More phenomenological use the features of QCD to construct 5D dual theory including gravity on AdS space

• **Towards to QCD:**
  – Break conformal invariance and generate mass gap
  – Tower of normalized bulk fields (Kaluza-Klein modes) ↔ Hadron wave functions
  – Spectrum of Kaluza-Klein modes ↔ Hadrons spectrum

• **Hard-wall:**
  AdS geometry is cutted by two branes UV \((z = \epsilon \to 0)\) and IR \((z = z_{IR})\)
  Analogue of quark bag model, linear dependence on \(J(L)\) of hadron masses

• **Soft-wall:**
  Soft cuttoff of AdS space by dilaton field \(e^{-\varphi(z)}\)
  Analytical solution of EOM, Regge behavior \(M^2 \sim J(L)\)
Introduction

Building HW/SW models

Erlich, Karch, Katz, Son, Stephanov, Da Rold, Pomarol, ... 

- Geometry: AdS space near $z = 0$ corresponds to QCD conformal in the UV

- Truncate AdS space by two branes:
  UV ($z = 0$) and IR ($z = z_{\text{IR}}$) in HW or (dilaton) in SW

- 5D gauge groups
  
  Isospin: $\text{SU}(2)$ isospin $\rightarrow$ 5D $\text{SU}(2)$ gauge group
  
  Vector mesons $\rightarrow$ Vector KK gauge bosons

  Chiral symmetry: $\text{SU}(2) \times \text{SU}(2)$ $\rightarrow$ $\text{SU}(2) \times \text{SU}(2)$ 5D gauge group
  
  Axial mesons $\rightarrow$ Axial KK gauge bosons

  $\text{EB} \chi S$ $\rightarrow$ non-normalizable $z$-profile of scalar 5D field $S_0^{\text{non}}(z) = \frac{\hat{m}}{2} z$

  $\text{SB} \chi S$ $\rightarrow$ normalizable $z$-profile of scalar 5D field $S_0^{\text{nor}}(z) = \frac{\langle \bar{q}q \rangle}{2} z^3$
Introduction

- Scalar and Pseudoscalar fields
  \[ X(x, z) = \left( S(x, z) + S_0(z) \right) e^{\pi(x, z)\vec{\pi}/F_\pi} \]
  where \( S_0(z) = S_0^{\text{non}}(z) + S_0^{\text{nor}}(z) \)

- Pseudoscalar mesons are dual to the mixing of \( \pi_i(x, z) \) and \( \phi_i(x, z) \) – longitudinal component of axial field
  \[ A_\mu^i(x, z) = A_\mu^i\perp(x, z) + \partial_\mu \phi^i(x, z) \]

- Action

\[
S = \int d^5x e^{-\varphi(z)} \sqrt{g} \left\{ -\frac{1}{4g_5^2} (F_L^2 + F_R^2) - |DX|^2 + 3|X|^2 \right\}
\]

where

\[ \varphi(z) = \kappa^2 z^2, \quad g_5^2 = 12\pi^2 / N_c, \]
\[ V = \frac{1}{2}(A_L + A_R), \quad A = \frac{1}{2}(A_L - A_R), \]
\[ F_{LM,N}^{M,N} = \partial^M A_{L,R}^N - \partial^N A_{L,R}^M - i[A_{L,R}^M, A_{L,R}^N], \]
\[ D^M X = \partial^M X - iA_L^M X + iX A_R^M \]
Introduction

• **How it works:** Mass spectrum and WFs of vector mesons

• Apply axial gauge $V_z = 0$ and derive EOM for the 4D-transverse component $V_{\mu}^\perp$

\[
e^{\varphi(z)} \partial_z \left[ \frac{e^{-\varphi(z)}}{z} \partial_z V_{\mu}^\perp(x, z) \right] - \partial^2 V_{\mu}^\perp(x, z) = 0
\]

• KK expansion

\[
V_{\mu}^\perp(x, z) = \sum_n V_{\mu,n}(x) v_n(z)
\]

\[
e^{\varphi(z)} \partial_z \left[ \frac{e^{-\varphi(z)}}{z} \partial_z v_n(z) \right] + M_{V_n}^2 v_n(z) = 0
\]

• After the substitution $v_n(z) = e^{\varphi(z)/2} \sqrt{z} \psi_n(z)$ derive **Schrödinger EOM**

\[
-\partial^2_z \psi_n(z) + U(z) \psi_n(z) = M_{V_n}^2 \psi_n(z)
\]

where $U(z) = \kappa^2 z^2 + \frac{3}{4z^2}$ dilaton potential

• **Analytical solutions**

\[
M_{V_n}^2 = 4\kappa^2 (n + 1), \quad \psi_n(z) = \sqrt{\frac{2n!}{(n+1)!}} \kappa^2 z^{3/2} e^{-\kappa^2 z^2/2} L_n^1(\kappa^2 z^2)
\]
Main results in meson sector

- Reproduces consequences of chiral symmetry:
  \[ M_\pi^2 F_\pi^2 = 2\hat{m} \langle \bar{q}q \rangle \]

- Properties of correlators of vector and axial current:
  \[ \Pi_V(Q^2) \to -\frac{N_c}{24\pi^2} \log Q^2 \quad \text{at} \quad Q^2 \to \infty \]
  \[ \Pi_A(Q^2) \to \frac{F_\pi^2}{Q^2} \quad \text{at} \quad Q^2 \to 0 \]

  VV (AA) Correlators are expanded as a sum over vector (axial) meson resonances

- Mass spectrum (Regge-like behavior)

- Form factors with correct power scaling, distribution functions, etc.
Baryons in soft-wall model


- **SW holographic approach** for baryons with inclusion of high Fock states dual to bulk fermion fields of higher dimension.

- **Objective:** Application to nucleon form factors, GPDs, nucleon resonances (Roper)
Baryons in soft-wall model

- **Bulk fermion fields**
  \( \Psi_+ (x, z) \) and \( \Psi_- (x, z) \) dual to \( O_R = (p_R, n_R) \) and \( O_L = (p_L, n_L) \)

- **Bulk fermion mass**  \( \pm m = \pm (\Delta - 3/2) \), where \( \Delta \) - scaling dimension

- **Scaling dimension**  \( \equiv \) Twist-dimension  \( \tau = N + L \),
  \( N \) - number of partons, \( L = \max |L_z| \)

- **Action** for the fermion field of twist \( \tau \)
  \[
  S_\tau = \int d^4x dz \sqrt{g} e^{-\varphi(z)} \sum_{i=\pm} \bar{\Psi}_{i,\tau}(x, z) \hat{D}_i(z) \Psi_{i,\tau}(x, z),
  \]
  \( \hat{D}_\pm(z) = \frac{i}{2} \Gamma^M \frac{\partial}{\partial M} \mp \frac{m + \varphi(z)}{R} \)

- **dilaton**  \( \varphi(z) = \kappa^2 z^2 \) (Regge behavior of hadron masses)

- **metric**  \( g_{MN}(z) = \epsilon^a_M(z) \epsilon^b_N(z) \eta_{ab} \),  \( g = |\det g_{MN}| \)

- **vielbein**  \( \epsilon^a_M(z) = e^{A(z)} \delta^a_M, \quad A(z) = \log(R/z) \) (conformal)

- **interval**  \( ds^2 = g_{MN} dx^M dx^N = e^{2A(z)} (g_{\mu\nu} dx^\mu dx^\nu - dz^2) \)
Baryons in soft-wall model

- **P-transformations**

\[
U_P^{-1} \Psi_{\tau, \pm}(t, \vec{x}, z) U_P = \pm \gamma^0 \gamma^5 \Psi_{\tau, \mp}(t, -\vec{x}, z)
\]

\[
U_P^{-1} \bar{\Psi}_{\tau, \pm}(t, \vec{x}, z) U_P = \pm \bar{\Psi}_{\tau, \mp}(t, -\vec{x}, z) \gamma^0 \gamma^5
\]

\[
\pm U_P^{-1} \bar{\Psi}_{\pm, \tau}(t, \vec{x}, z) \Psi_{\pm, \tau}(t, \vec{x}, z) U_P = \mp \bar{\Psi}_{\mp, \tau}(t, -\vec{x}, z) \Psi_{\mp, \tau}(t, \vec{x}, z)
\]

\[
U_P^{-1} S_{\tau}^\pm U_P = S_{\mp}^\tau
\]

- **C-transformations**

\[
U_C^{-1} \Psi_{\pm}(x, z) U_C = \mp C \gamma^5 \bar{\Psi}_{\mp}^T(x, z)
\]

\[
U_C^{-1} \bar{\Psi}_{\pm}(x, z) U_C = \pm \Psi_{\mp}^T(x, z) \gamma^5 C
\]

\[
\pm U_C^{-1} \bar{\Psi}_{\pm}(x, z) \Psi_{\pm}(x, z) U_C = \mp \bar{\Psi}_{\mp}(x, z) \Psi_{\mp}(x, z)
\]

\[
U_C^{-1} S_{\tau}^\pm U_C = S_{\mp}^\tau
\]
Baryons in soft-wall model

- **Redefinition**
  \[ \Psi_{i,\tau}(x, z) = e^{\varphi(z)/2 - 2A(z)} \psi_{i,\tau}(x, z) \]

- **Expansion on left- and right-chirality components (eigenstates of \( \gamma^5 \))**
  \[ \psi_{i,\tau}(x, z) = \psi_{i,\tau}^L(x, z) + \psi_{i,\tau}^R(x, z) \]

- **Kaluza-Klein expansion**
  \[ \psi_{i,\tau}^{L/R}(x, z) = \frac{1}{\sqrt{2}} \sum_n \psi_{n}^{L/R}(x) f_{i,\tau,n}^{L/R}(z), \]

- **Relations between bulk profiles**
  \[
  f_{\tau,n}^R(z) \equiv f_{+,\tau,n}^R(z) = - f_{-,\tau,n}^L(z), \\
  f_{\tau,n}^L(z) \equiv f_{+,\tau,n}^L(z) = f_{-,\tau,n}^R(z).
  \]

- **EOM**
  \[
  \left[ -\partial_z^2 + \kappa^4 z^2 + 2\kappa^2 \left( m \mp \frac{1}{2} \right) + \frac{m(m \pm 1)}{z^2} \right] f_{\tau,n}^{L/R}(z) = M_{n\tau}^2 f_{\tau,n}^{L/R}(z),
  \]
Baryons in soft-wall model

- Solutions

\[ f_{\tau,n}^L(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau)}} \kappa^\tau z^{\tau-1/2} e^{-\kappa^2 z^2/2} L_n^{\tau-1}(\kappa^2 z^2), \]

\[ f_{\tau,n}^R(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}} \kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2} L_n^{\tau-2}(\kappa^2 z^2) \]

and

\[ M_{n\tau}^2 = 4\kappa^2 \left( n + \tau - 1 \right) \]

with

\[ \int_{0}^{\infty} dz \; f_{\tau,n_1}^L(z) f_{\tau,n_2}^R(z) = \delta_{n_1 n_2} \]
Baryons in soft-wall model

- Inclusion of high Fock states

\[ S = \sum_{\tau} c_{\tau} S_{\tau} \]

\( c_{\tau} \) - set of free parameters

- Integration over \( z \) using normalization condition for \( f^{L/R} \)

\[ S = \int d^4x \bar{\psi}_n(x) \left[ \sum_{\tau} c_{\tau} i \not{\partial} - \sum_{\tau} c_{\tau} M_{n\tau} \right] \psi_n(x). \]

Correct normalization of kinetic term of 4D spinor field

\[ \sum_{\tau} c_{\tau} = 1, \quad \sum_{\tau} c_{\tau} M_{n\tau} = M_n \text{ (baryon mass)} \]
Baryons in soft-wall model

- Large $N_c$ expansion
  \[ M_n = \sum_{\tau} c_{\tau} M_{n\tau} \sim \kappa \cdot \sqrt{\frac{n + \tau - 1}{N_c}} \sim N_c \]

- $\kappa \sim \sqrt{N_c}$ consistent with LFH (Brodsky-Teramond)
  \[ F_\pi = \frac{\sqrt{3}}{8} \kappa = 83 \text{ MeV} \sim \sqrt{N_c} \]

- Dilaton can be identified with VEV of the scalar bulk field with dimension-2, which is holographically dual to the dimension-2 gluon operator $A^2_{\mu}$.

- $\kappa^2 \sim N_c$ is related to the vacuum expectation value (VEV) $\langle \alpha_s A^2_{\mu} \rangle \sim N_c$

- $A^2_{\mu}$ has been discussed in the literature in detail:
  Celenza-Shakin; Chetyrkin-Narison-Zakharov; Gubarev-Stodolsky-Zakharov; Dorokhov-Broniowski; Arriola-Bowman-Broniowski

- Quadratic form of the dilaton profile is not unique

  See: Liu-Tseytlin model (top-down AdS/QCD approach)
  \[ e^{\varphi(z)} = 1 + qz^4, \text{ where } q \text{ related to dim-4 condensates } \langle \alpha_s G^2_{\mu\nu} \rangle \sim N_c \text{ and } \langle \alpha_s G_{\mu\nu} \tilde{G}_{\mu\nu} \rangle \sim N_c \]
Baryons in soft-wall model

- Holographic variable $z$ (as conjugate to $\kappa$) scales as $z \sim \frac{1}{\sqrt{N_c}}$

- Physical interpretation:
  - Limit $N_c \to \infty$ means $z = 0$ or approaching to UV boundary
    baryons are bound states of infinitely number of $N_c$ quarks
  - Limit $N_c \to 0$ means $z \to \infty$
    no bound states of quarks due to confinement imposed by the dilaton

see also  Brodsky, Huet PLB 417 (1998) 145
Electromagnetic structure of nucleons

• Abidin-Carlson: First application of SW model (3q configurations)

• Coupling of bulk vector and fermion fields

\[
\mathcal{L}_{\text{int}}(x, z) = \sum_{i=+, -} \sum_{\tau} c_{\tau} \bar{\Psi}_{i, \tau}(x, z) \hat{V}_i(x, z) \Psi_{i, \tau}(x, z)
\]

\[
\hat{V}_\pm(x, z) = Q \Gamma^M V_M(x, z) \pm \frac{i}{4} \eta_V \left[ \Gamma^M, \Gamma^N \right] V_{MN}(x, z) \pm g_V \tau_3 \Gamma^M i \Gamma^z V_M(x, z)
\]

\[
\langle p' | J^\mu(0) | p \rangle = \bar{u}(p') \left[ \gamma^\mu F_1^N(t) + \frac{i}{2m_N} \sigma^{\mu\nu} q_\nu F_2^N(t) \right] u(p)
\]

\[
\begin{align*}
F_1^p(Q^2) &= C_1(Q^2) + g_V C_2(Q^2) + \eta_V^p C_3(Q^2) \\
F_2^p(Q^2) &= \eta_V^p C_4(Q^2) \\
F_1^n(Q^2) &= -g_V C_2(Q^2) + \eta_V^n C_3(Q^2) \\
F_2^n(Q^2) &= \eta_V^n C_4(Q^2)
\end{align*}
\]
Electromagnetic structure of nucleons

- \( C_1(Q^2) = \frac{1}{2} \int_0^\infty dz \, V(Q, z) \sum_\tau c_\tau \left( [f^L_\tau(z)]^2 + [f^R_\tau(z)]^2 \right) \)
  \[ = \sum_\tau c_\tau \, B(a + 1, \tau) \left( \tau + \frac{a}{2} \right) \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}} \]

- \( C_2(Q^2) = \frac{1}{2} \int_0^\infty dz \, V(Q, z) \sum_\tau c_\tau \left( [f^R_\tau(z)]^2 - [f^L_\tau(z)]^2 \right) \)
  \[ = \frac{a}{2} \sum_\tau c_\tau \, B(a + 1, \tau) \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}} \]

- \( C_3(Q^2) = \frac{1}{2} \int_0^\infty dz \, \partial_z V(Q, z) \sum_\tau c_\tau \left( [f^L_\tau(z)]^2 - [f^R_\tau(z)]^2 \right) \)
  \[ = a \sum_\tau c_\tau \, B(a + 1, \tau + 1) \frac{a(\tau-1)-1}{\tau} \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}} \]

- \( C_4(Q^2) = 2m_N \int_0^\infty dz \, V(Q, z) \sum_\tau c_\tau \, f^L_\tau(z) f^R_\tau(z) \)
  \[ = \frac{2m_N}{\kappa} \sum_\tau c_\tau \, (a + 1 + \tau) \, B(a + 1, \tau + 1) \sqrt{\tau - 1} \sim \sum_\tau \frac{c_\tau}{a^{\tau}} \]

- \( a = \frac{Q^2}{4\kappa^2} \), \( B(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)} \) is the Beta function.
Electromagnetic stucture of nucleons

- $V(Q, z)$ – propagator of trans. massless vector field (analogue of EM field)

\[
V(Q, z) = \Gamma \left( 1 + \frac{Q^2}{4\kappa^2} \right) U \left( \frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2 \right) \\
= \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x \frac{Q^2}{4\kappa^2} e^{-\frac{\kappa^2 z^2 x}{1-x}}
\]

Radyushkin-Grigoryan (integral repr.)
Brodsky-Teramond (identify $x$ with LC momentum fraction)

\[
V(0, z) = 1, \quad V(Q, 0) = 1, \quad V(Q, \infty) = 0.
\]

At $Q^2 = 0$ functions $C_i(Q^2)$ are normalized as

\[
C_1(0) = 1, \quad C_2(0) = C_3(0) = 0, \quad C_4(0) = \frac{2m_N}{\kappa} \sum_\tau c_\tau \sqrt{\tau - 1}
\]
Electromagnetic structure of nucleons

Choice of free parameters

- $\kappa = 383$ MeV, $c_3 = 1.25$, $c_4 = 0.16$, $g_V = 0.3$
- $c_5$ is expressed through $c_3$ and $c_4$
  
  $$c_5 = 1 - c_3 - c_4 = -0.41$$

- $c_3$, $c_4$ are constrained by the nucleon mass
- $\kappa$ is fixed by the nucleon mass and nucleon electromagnetic radii
- $g_V$ is fixed by fine tuning of the neutron electromagnetic radii
- Nonminimal couplings $\eta^{p,n}_V$ from nucleon magnetic moments
  
  $$\eta^p_V = \frac{\kappa (\mu_p - 1)}{2m_N C_0} = 0.30, \quad \eta^n_V = \frac{\kappa \mu_n}{2m_N C_0} = -0.32, \quad C_0 = \sqrt{2} c_3 + \sqrt{3} c_4 + 2c_5$$
# Electromagnetic structure of nucleons

## Mass and electromagnetic properties of nucleons

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Our results</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_p$ (GeV)</td>
<td>0.93827</td>
<td>0.93827</td>
</tr>
<tr>
<td>$\mu_p$ (in n.m.)</td>
<td>2.793</td>
<td>2.793</td>
</tr>
<tr>
<td>$\mu_n$ (in n.m.)</td>
<td>-1.913</td>
<td>-1.913</td>
</tr>
<tr>
<td>$r^P_E$ (fm)</td>
<td>0.840</td>
<td>0.8768 ± 0.0069</td>
</tr>
<tr>
<td>$\langle r^2_E \rangle^n$ (fm$^2$)</td>
<td>-0.117</td>
<td>-0.1161 ± 0.0022</td>
</tr>
<tr>
<td>$r^P_M$ (fm)</td>
<td>0.785</td>
<td>0.777 ± 0.013 ± 0.010</td>
</tr>
<tr>
<td>$r^n_M$ (fm)</td>
<td>0.792</td>
<td>0.862$^{+0.009}_{-0.008}$</td>
</tr>
<tr>
<td>$r_A$ (fm)</td>
<td>0.667</td>
<td>0.67±0.01</td>
</tr>
</tbody>
</table>
Electromagnetic structure of nucleons

\[ \frac{G_E(Q^2)}{G_D(Q^2)} \]

\[ Q^2 \text{ (GeV}^2) \]

Berger et al. (1971)
Price et al. (1971)
Hanson et al. (1973)
Simon et al. (1980)
Milbrath et al. (1999)
Jones et al. (2000)
Dieterich et al. (2001)
Gayou et al. (2002)
Electromagnetic structure of nucleons

\[ \frac{G_E^p(Q^2)}{G_M^p(Q^2)} \]

- Walker et al. (1994)
- Punjabi et al. (2005)
- Gayou et al. (2002)
- Ron et al. (2011)
- Puckett et al. (2010)
- Puckett et al. (2012)
Electromagnetic structure of nucleons

\[ G_M(Q^2)/\mu_p G_D(Q^2) \]

- Janssens et al. (1966)
- Litt et al. (1970)
- Berger et al. (1971)
- Bartel et al. (1973)
- Hanson et al. (1973)
- Hoehler et al. (1976)
- Sill et al. (1993)
- Andivahis et al. (1994)
- Walker et al. (1994)
Electromagnetic structure of nucleons

\[ G_M(Q^2) / (\mu_n, G_D(Q^2)) \]

- Rock et al. (1982)
- Lung et al. (1993)
- Markowitz et al. (1993)
- Anklin et al. (1994)
- Gao et al. (1994)
- Bruins et al. (1995)
- Anklin et al. (1998)
- Xu et al. (2000)
- Kubon et al. (2002)
- Xu et al. (2003)

\[ Q^2 (\text{GeV}^2) \]
Electromagnetic structure of nucleons

\[ G_{E}^{n}(Q^{2}) \]

- p.27
Electromagnetic structure of nucleons

\[
\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)}
\]

\[
Q^2 \text{ (GeV}^2)\]

- Walker et al. (1994)
- Punjabi et al. (2005)
- Gayou et al. (2002)
- Ron et al. (2011)
- Puckett et al. (2010)
- Puckett et al. (2012)
Electromagnetic structure of nucleons

\[ G_E^{n}(Q^2) / G_M^{n}(Q^2) \]

- \( n(\bar{e}, e'\bar{n}) \)
- \( ^2H(\bar{e}, e'\bar{n}) \): PWBA
- \( ^2H(\bar{e}, e'\bar{n}) \): FSI+MEC+IC+RC

\[ Q^2 \text{ (GeV}^2) \]

\[ G_E^{n}(Q^2) / G_M^{n}(Q^2) \]
Electromagnetic structure of nucleons

\[ Q^4 F^p(Q^2) \]

\[ Q^2 \text{ (GeV}^2) \]

\[ 0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \quad 30 \quad 35 \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]
Electromagnetic structure of nucleons

\[ Q^4 F_{11}(Q^2) \]

\[ Q^2 (\text{GeV}^2) \]

\[ -0.6 \]

\[ 0 \]

\[ 0.1 \]

\[ 0.2 \]

\[ 0.3 \]

\[ 0.4 \]

\[ 0.5 \]

\[ 0.6 \]
Electromagnetic structure of nucleons

\[ Q^2 F_2^p(Q^2)/F_1^p(Q^2) \]

\( Q^2 \) (GeV\(^2\))

- JLab Hall A GEp(I)
- JLab Hall A GEp(II)
- JLab Hall C GEp(III)
Electromagnetic structure of nucleons

\[ \frac{Q^2 F_2(Q^2)}{F_1(Q^2)}/\log^2(\frac{Q^2}{\Lambda^2}) \]

\[ \Lambda = 0.3 \text{ GeV} \]

- JLab Hall A GEp(I)
- JLab Hall A GEp(II)
- JLab Hall C GEp(III)
Nucleon GPDs

- Sum rules relating EM FF and GPDs  \( \text{Ji, Radyushkin} \)

\[
F^p_1(t) = \int_0^1 dx \left( \frac{2}{3} H_v^u(x, t) - \frac{1}{3} H_v^d(x, t) \right)
\]

\[
F^n_1(t) = \int_0^1 dx \left( \frac{2}{3} H_v^d(x, t) - \frac{1}{3} H_v^u(x, t) \right)
\]

\[
F^p_2(t) = \int_0^1 dx \left( \frac{2}{3} E_v^u(x, t) - \frac{1}{3} E_v^d(x, t) \right)
\]

\[
F^n_2(t) = \int_0^1 dx \left( \frac{2}{3} E_v^d(x, t) - \frac{1}{3} E_v^u(x, t) \right)
\]

- Grigoryan-Radyushkin integral representation for bulk-to-boundary propagator

\[
V(Q, z) = \kappa^2 z^2 \int_0^1 dx \frac{Q^2}{4\kappa^2} e^{-\frac{x}{1-x}\kappa^2 z^2}
\]

- LF mapping (Brodsky-Teramond): \( x \) is equivalent to LC momentum fraction
Nucleon GPDs

• GPDs \( H_v^q(x, Q^2) = q(x) x \frac{Q^2}{4 \kappa^2} \), \( E_v^q(x, Q^2) = e^q(x) x \frac{Q^2}{4 \kappa^2} \)

• Distribution functions \( q(x) \) and \( e^q(x) \)

\[
q(x) = \alpha^q \gamma_1(x) + \beta^q \gamma_2(x), \quad e^q(x) = \beta^q \gamma_3(x)
\]

Flavor couplings \( \alpha^q, \beta^q \) and functions \( \gamma_i(x) \) are written as

\[
\alpha^u = 2, \quad \alpha^d = 1, \quad \beta^u = 2 \eta_p + \eta_n, \quad \beta^d = \eta_p + 2 \eta_n
\]

and

\[
\begin{align*}
\gamma_1(x) & = \frac{1}{2} (5 - 8x + 3x^2) \\
\gamma_2(x) & = 1 - 10x + 21x^2 - 12x^3 \\
\gamma_3(x) & = 24(1 - x)^2
\end{align*}
\]
Nucleon GPDs

\[ H^u_{v}(x, Q^2) \]

\[ H^d_{v}(x, Q^2) \]

\[ E^u_{v}(x, Q^2) \]

\[ -E^d_{v}(x, Q^2) \]
Nucleon GPDs

- Nucleon GPDs in impact space  Burkardt, Miller, Diehl, Kroll et al

\[ q(x, b_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2} H_q(x, k_\perp^2) e^{-ib_\perp k_\perp} = q(x) \frac{\kappa^2}{\pi \log(1/x)} e^{-\frac{b_\perp^2 \kappa^2}{\log(1/x)}} \]

\[ e^q(x, b_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2} E_q(x, k_\perp^2) e^{-ib_\perp k_\perp} = e^q(x) \frac{\kappa^2}{\pi \log(1/x)} e^{-\frac{b_\perp^2 \kappa^2}{\log(1/x)}} \]

- Parton charge and magnetization densities in transverse impact space

\[ \rho^N_E(b_\perp) = \sum_q e^N_q \int_0^1 dx q(x, b_\perp) = \frac{\kappa^2}{\pi} \sum_q e^N_q \int_0^1 dx \frac{q(x)}{\log(1/x)} e^{-\frac{b_\perp^2 \kappa^2}{\log(1/x)}} \]

\[ \rho^N_M(b_\perp) = \sum_q e^N_q \int_0^1 dx e^q(x, b_\perp) = \frac{\kappa^2}{\pi} \sum_q e^N_q \int_0^1 dx \frac{e^q(x)}{\log(1/x)} e^{-\frac{b_\perp^2 \kappa^2}{\log(1/x)}} \]

where  \[ e^p_u = e^n_d = 2/3 \]  and  \[ e^u_n = e^p_d = -1/3 \]
Nucleon GPDs

• Transverse width of impact parameter dependent GPD

\[
\langle R_{\perp}^2(x) \rangle_q = \frac{\int d^2 b_{\perp} b_{\perp}^2 q(x, b_{\perp})}{\int d^2 b_{\perp} q(x, b_{\perp})} = -4 \left( \frac{\partial \log H_v^q(x, Q^2)}{\partial Q^2} \right)_{Q^2=0} = \frac{\log(1/x)}{\kappa^2}
\]

• Transverse rms radius

\[
\langle R_{\perp}^2 \rangle_q = \frac{\int d^2 b_{\perp} b_{\perp}^2 \int_0^1 dx q(x, b_{\perp})}{\int d^2 b_{\perp} \int_0^1 dx q(x, b_{\perp})} = \frac{1}{\kappa^2} \left( \frac{5}{3} + \frac{\beta q}{12 \alpha_q} \right) \simeq 0.527 \text{ fm}^2
\]
Nucleon GPDs

Plots for $q(x, b_\perp)$ for $x = 0.1$: $u(x, b_\perp)$ - upper panels, $d(x, b_\perp)$ - lower panels
Roper resonance $N(1440)$

- Put $n = 1$ and get solutions dual to Roper:

$$M_R \simeq 1440 \text{ MeV}$$

- $N \to R + \gamma$ transition

$$M^\mu = \bar{u}_R \left[ \gamma_\perp^\mu F_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{M_R(q^2)} F_2(q^2) \right] u_N, \quad \gamma_\perp^\mu = \gamma^\mu - \frac{q^\mu \not{q}}{q^2}$$

- Helicity amplitudes

$$H_{\pm \frac{1}{2} 0} = \sqrt{\frac{Q_-}{Q^2}} \left( F_1 M_+ - F_2 \frac{Q^2}{M_R} \right)$$

$$H_{\pm \frac{1}{2} \pm 1} = -\sqrt{2Q_-} \left( F_1 + F_2 \frac{M_+}{M_R} \right)$$

- Alternative set [Weber, Capstick, Copley et al]

$$A_{1/2} = -b H_{\frac{1}{2} 1}, \quad S_{1/2} = b \frac{|P|}{\sqrt{Q^2}} H_{\frac{1}{2} 0},$$

$$Q_\pm = M_\pm^2 + Q^2, \quad M_\pm = M_R \pm M_N, \quad b = \sqrt{\frac{\pi \alpha}{2EM_R M_N}}$$
Roper resonance $N(1440)$

Helicity amplitudes $A_{1/2}^{N}(0), S_{1/2}^{N}(0)$

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Our results</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{1/2}^{p}(0)$ (GeV$^{-1/2}$)</td>
<td>-0.065</td>
<td>-0.065 ± 0.004</td>
</tr>
<tr>
<td>$A_{1/2}^{n}(0)$ (GeV$^{-1/2}$)</td>
<td>0.040</td>
<td>0.040 ± 0.010</td>
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<tr>
<td>$S_{1/2}^{p}(0)$ (GeV$^{-1/2}$)</td>
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<tr>
<td>$S_{1/2}^{p}(0)$ (GeV$^{-1/2}$)</td>
<td>-0.040</td>
<td></td>
</tr>
</tbody>
</table>
Roper resonance $N(1440)$

Helicity amplitude $A_{1/2}^p(Q^2)$

Data: CLAS Coll at JLab, Mokeev et al, 1205.3948 [nucl-ex]
Roper resonance $N(1440)$

Helicity amplitude $S_{1/2}^P(Q^2)$

Data: CLAS Coll at JLab, Mokeev et al, 1205.3948 [nucl-ex]
Roper resonance $N(1440)$

Tiator-Vanderhaeghen:

Quark transition charge density in the transverse plane

$$\rho_0(\vec{b}_\perp) = \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \ e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \ \frac{1}{2P^+} \left\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda \right| J^+(0) \left| P^+, -\frac{\vec{q}_\perp}{2}, \lambda \right\rangle$$  \hspace{1cm} [unpol.]

$$\rho_T(\vec{b}_\perp) = \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \ e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \ \frac{1}{2P^+} \left\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp \right| J^+(0) \left| P^+, -\frac{\vec{q}_\perp}{2}, s_\perp \right\rangle$$  \hspace{1cm} [T – pol.]
Roper resonance $N(1440)$
Roper resonance $N(1440)$
Roper resonance $N(1440)$
Summary

- **AdS/QCD** \(\equiv\) **Holographic QCD (HQCD)** – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – **anti-de Sitter (AdS) space**

- **HQCD models** reproduce main features of QCD at low and high energies

- We develop a soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances

- Mesons, baryons and exotic states from unified point view and including high Fock states

- Future work: nucleon TMDs, DVCS