Model independent form factor relations at large $N_c$

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Quantum chromodynamics — theory of strong interaction
  - gauge theory of quarks and gluons based on $SU(N_c = 3)$ symmetry
introduction

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Practical problem — QCD is strongly coupled at low energies
- conventional perturbative expansion is not applicable
- expansion around non-interacting theory
- corrections in the powers of coupling constant
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- Practical problem — QCD is strongly coupled at low energies
  - conventional perturbative expansion is not applicable
  - expansion around non-interacting theory
  - corrections in the powers of coupling constant

- Some useful approaches
  - expansion around large-$N_c$ limit
  - expansion around massless-quark (chiral) limit
introduction — two limits of QCD

- Large $N_c$ world
  - number of colors $N_c$ is a hidden free parameter of QCD
  - simplifies substantially in the limit $N_c \to \infty$
    - due to combinatorics properties of diagrams

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  - QCD possesses a new symmetry if quark masses are zero — chiral symmetry
  - again, leads to simplification of the problem
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- **Promising idea — develop models of QCD in these two limits**
  - even though these limits do not completely describe the real world, they are believed to capture many of its (at least qualitative) details.
  - systematic procedure how to include corrections in the powers of $m_\pi$ or $1/N_c$
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- **Double limit is not uniform and ordering of limits does matter (for certain observables).**
introduction — baryon models

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  - QCD becomes weakly interacting theory of mesons
  - Baryons emerge as solitons-like configurations of meson fields
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- large $N_c$ — encoded in the very core of the models, in the semiclassical treatment
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  - attracted wide interest recently
  - large $N_c$ — encoded in the very core of the models ($N_c \rightarrow \infty$ taken first)
  - looks totally different (if nothing else they are formulated in five dimensions)

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introduction — baryon models

- Important to check, if large $N_c$ and chiral physics are encoded correctly
  - of course, there is more to modeling QCD than getting large $N_c$ and chiral behavior right
  - however, there is, in principle, infinite number of models
  - simple method to check the inclusion of large $N_c$ and chiral physics is important
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  - the need for new model-independent relation
model-independent relation

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  - use position-space electric and magnetic form factors (Fourier transforms of standard momentum-space ones\(^{(4)}\))

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\lim_{r \to \infty} \frac{r^2 \tilde{G}^{l=0}_E}{\tilde{G}^{l=0}_M} \frac{\tilde{G}^{l=1}_E}{\tilde{G}^{l=1}_M} = 18
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- isoscalar electric \( \tilde{G}_E^{l=0} \)
- isoscalar magnetic \( \tilde{G}_M^{l=0} \)
- isovector electric \( \tilde{G}_E^{l=1} \)
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The relation was previously derived in the context of chiral soliton models. It is plausible to believe it is model-independent, i.e., it does NOT depend on any details of the model. In the past, all such relations derived in the chiral soliton models turned out (after deeper investigation) to be model independent. The purpose of this work is to prove the relation in a model-independent way. Not all models on the market satisfy it; something is wrong with the Sakai-Sugimoto ("top-down") model. The underlying reason for this appears to be due to a failure of the flat-space instanton approximation.
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- The relation is proved in the large \( N_c \) chiral perturbation theory
Features of large $N_c \chi$PT

- baryon mass is parametrically large (of order $N_c$) — heavy baryon approximation
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Features of large $N_c \chi$PT

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- Large $N_c$ also eliminates diagrams suppressed by factor $1/N_c$.
- Large $N_c$ consistency relations implies that the $\Delta$ is degenerate with nucleon (generally whole tower of $I = J$ isobars) — $\Delta$ must be included in the calculation.
  - Mass difference $\Delta = M_\Delta - M_N$ is of order $1/N_c$ and serves as a new low energy constant.
inputs of the calculation

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    - mass difference \( \Delta = M_\Delta - M_N \) is of order \( 1/N_c \) and serves as a new low energy constant
  - the form of pion-baryon-baryon’ vertex is determined by the large \( N_c \) consistency relations
inputs of the calculation

- Feynman rules for vertices
  - photon-two pions: $\epsilon_{a3b} A_\mu (p_a^\mu + p_b^\mu)$
    - key for isovector current, see $\epsilon_{a3b}$
Feynman rules for vertices

- Photon-two pions: \( \epsilon_{a3b} A_\mu (p^\mu_a + p^\mu_b) \)
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- Photon-three pions: \( \frac{1}{12 \pi^2 f_3^2} \epsilon_{abc} \epsilon^{\mu \nu \kappa \lambda} A_\mu p^\nu_a p^\kappa_b p^\lambda_c \)
  - Key for isoscalar current, see \( \epsilon_{abc} \)
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  - **pion-baryon-baryon**: \( \frac{g_A}{2f_\pi} \sqrt{\frac{2J(B') + 1}{2J(B) + 1}} \tau_a^{BB'} \sigma_i^{BB'} p_i \)
    - determined by the consistency relations of large \( N_c \) QCD
    - matrices \( \tau^{BB'} (\sigma^{BB'}) \) act in isospin (spin) space, they are a generalization of Pauli matrices, which appear in the pion-nucleon-nucleon vertex
Coupling matrices

\[
\tau_1^{(NN)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2^{(NN)} = i \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \tau_3^{(NN)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

\[
\tau_1^{(N\Delta)} = \begin{pmatrix} -\sqrt{\frac{3}{2}} & 0 \\ 0 & -\sqrt{\frac{1}{2}} \end{pmatrix}, \quad \tau_2^{(N\Delta)} = i \begin{pmatrix} \sqrt{\frac{3}{2}} & 0 \\ 0 & \sqrt{\frac{1}{2}} \end{pmatrix}, \quad \tau_3^{(N\Delta)} = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}
\]

\[
\tau_1^{(\Delta N)} = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \end{pmatrix}, \quad \tau_2^{(\Delta N)} = i \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix}, \quad \tau_3^{(\Delta N)} = \ldots
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\[
\tau_1^{(\Delta\Delta)} = \begin{pmatrix} 0 & \sqrt{\frac{3}{5}} & 0 & 0 \\ \sqrt{\frac{3}{5}} & 0 & \frac{2}{\sqrt{5}} & 0 \\ 0 & \frac{2}{\sqrt{5}} & 0 & \sqrt{\frac{3}{5}} \\ 0 & 0 & \sqrt{\frac{3}{5}} & 0 \end{pmatrix}, \quad \tau_2^{(\Delta\Delta)} = i \begin{pmatrix} 0 & -\sqrt{\frac{3}{5}} & 0 & 0 \\ \sqrt{\frac{3}{5}} & 0 & -\frac{2}{\sqrt{5}} & 0 \\ 0 & \frac{2}{\sqrt{5}} & 0 & -\sqrt{\frac{3}{5}} \\ 0 & 0 & \sqrt{\frac{3}{5}} & 0 \end{pmatrix}, \quad \tau_3^{(\Delta\Delta)} = \ldots
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Feynman rules for vertices

- photon-two pions: $\epsilon_{a3b} A_\mu (p_a^\mu + p_b^\mu)$
  - key for isovector current, see $\epsilon_{a3b}$
- photon-three pions: $\frac{1}{12\pi^2 f_\pi^3} \epsilon_{abc} \epsilon^{\mu\nu\kappa\lambda} A_\mu p_{a\nu} p_{b\kappa} p_{c\lambda}$
  - key for isoscalar current, see $\epsilon_{abc}$
- pion-baryon-baryon: $\frac{g_A}{2f_\pi} \sqrt{\frac{2J(B') + 1}{2J(B) + 1}} \tau_a^{(BB')} \sigma_i^{(BB')} p_i$
  - determined by the consistency relations of large $N_c$ QCD
  - matrices $\tau^{(BB')}$ ($\sigma^{(BB')}$) act in isospin (spin) space, they are a generalization of Pauli matrices, which appear in the pion-nucleon-nucleon vertex

Feynman rules for propagators
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  - pion-baryon-baryon: $g_A \frac{n_{\pi}}{2f_{\pi}} \sqrt{\frac{2J(B')}{{J(B')}+1}} \tau_a^{(BB')} \sigma_i^{(BB')} p_i$
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- **Feynman rules for propagators**
  - pions: $\Delta_\pi(k) = \frac{i}{k^2 - m_{\pi}^2 + i\epsilon}$
    - fully relativistic propagators for pions, in the end $m_{\pi} \to 0$
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Feynman rules for propagators

- pions: $\Delta^\pi (k) = \frac{i}{k^2 - m_\pi^2 + i\epsilon}$
  - fully relativistic propagators for pions, in the end $m_\pi \to 0$
- baryons: $\Delta^N (k) = \frac{i}{k^0 + i\epsilon}$, $\Delta^\Delta (k) = \frac{i}{k^0 - \Delta + i\epsilon}$
  - non-relativistic propagators for baryons
diagrams to consider

- Isovector
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- two pion loop with either nucleon or delta in the intermediate state
- totally two diagrams to be taken into account
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- **Isovector**
  - two pion loop with either nucleon or delta in the intermediate state
  - totally two diagrams to be taken into account

- **Isoscalar**
  - three pion loop with nucleons and deltas in the intermediate states
  - totally four diagrams to be taken into account
result

- Position-space form factors
  - evaluating diagrams, Fourier transforming, setting \( m_\pi = 0 \), extracting longest distance part:
Position-space form factors

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$$\lim_{r \to \infty} \tilde{G}_E^{l=0} = \frac{3^3}{2^9 \pi^5} \frac{1}{f_\pi^3} \left( \frac{g_A}{f_\pi} \right)^3 \frac{1}{r^9}$$

$$\lim_{r \to \infty} \tilde{G}_M^{l=0} = \frac{3}{2^9 \pi^5} \frac{1}{f_\pi^3} \left( \frac{g_A}{f_\pi} \right)^3 \frac{\Delta}{r^7}$$

$$\lim_{r \to \infty} \tilde{G}_E^{l=1} = \frac{1}{2^4 \pi^2} \left( \frac{g_A}{f_\pi} \right)^2 \frac{\Delta}{r^4}$$

$$\lim_{r \to \infty} \tilde{G}_M^{l=1} = \frac{1}{2^5 \pi^2} \left( \frac{g_A}{f_\pi} \right)^2 \frac{1}{r^4}$$
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Model-independent relation

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\lim_{r \to \infty} \frac{r^2 \tilde{G}_E^{l=0} \tilde{G}_E^{l=1}}{\tilde{G}_M^{l=0} \tilde{G}_M^{l=1}} = 18
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$$\lim_{r \to \infty} \frac{r^2 \tilde{G}_E^{I=0} \tilde{G}_E^{I=1}}{\tilde{G}_M^{I=0} \tilde{G}_M^{I=1}} = 18$$ holds!

- as advertised, all low-energy constants canceled
comment about the presence of $\Delta s$

- For isovector form factors

\[
\begin{align*}
\tau_a^{(NN)} \tau_b^{(NN)} &= \delta_{ab} I_\tau + i \epsilon_{abc} \tau_c , \\
\tau_a^{(\Delta N)} \tau_b^{(N\Delta)} &= -\sqrt{2} \delta_{ab} I_\tau + \frac{i}{\sqrt{2}} \epsilon_{abc} \tau_c
\end{align*}
\]

- matrix structure for iso-space (same for the spin space matrices $\sigma$):
comment about the presence of $\Delta$s

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  - for example, look at isoscalar - vector amplitude
    \[ \approx 1 \times 1 + (-\sqrt{2} \times 1/\sqrt{2}) = 0 \]

  - cancel in the leading order in $1/N_c$ expansion, proportional to $\Delta$
comment about the presence of $\Delta$s

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\[ \approx 1 \times 1 + (-\sqrt{2} \times 1/\sqrt{2}) = 0 \]

- Cancel in the leading order in $1/N_c$ expansion, proportional to $\Delta$

- Isoscalar-vector ($\tilde{G}^l_{M=0} = 0$) and isovector-scalar ($\tilde{G}^l_{E=1}$) channels, the amplitudes subtract exactly in the leading order in $1/N_c$ (where $\Delta = 0$)

- For isoscalar-scalar ($\tilde{G}^l_{E=0}$) and isovector-vector ($\tilde{G}^l_{M=1}$) channels, $\Delta$ in the intermediate state only leads to a multiplicative factor
If chiral limit is taken prior to the large $N_c$ limit, only nucleons need to be considered in the intermediate states.
If chiral limit is taken prior to the large $N_c$ limit, only nucleons need to be considered in the intermediate states:

- cartoon picture (diagrams without $\Delta(s)$ + diagrams with $\Delta(s)$)

\[
\approx e^{-m_\pi r} + e^{-m_\pi r} e^{-\Delta r}
\]

limit $N_c \to \infty$

\[
\approx e^{-m_\pi r} + e^{-m_\pi r}
\]

limit $m_\pi \to 0$

\[
\approx 1 + 1
\]

limit $r \to \infty$

\[
\approx 1 + 1
\]
If chiral limit is taken prior to the large $N_c$ limit, only nucleons need to be considered in the intermediate states

- cartoon picture (diagrams without $\Delta(s)$ + diagrams with $\Delta(s)$)

\[
\lim_{N_c \to \infty} \approx e^{-m_\pi r} + e^{-m_\pi r} e^{-\Delta r}
\]

\[
\lim_{m_\pi \to 0} \approx e^{-m_\pi r} + e^{-m_\pi r}
\]

\[
\lim_{r \to \infty} \approx 1 + 1
\]

\[
\lim_{N_c \to \infty} \approx 1 + 0
\]
If chiral limit is taken prior to the large $N_c$ limit, only nucleons need to be considered in the intermediate states.

- cartoon picture (diagrams without $\Delta(s)$ + diagrams with $\Delta(s)$)

$$\approx e^{-m_\pi r} + e^{-m_\pi r} e^{-\Delta r}$$

- limit $N_c \to \infty$
  $$\approx e^{-m_\pi r} + e^{-m_\pi r}$$

- limit $m_\pi \to 0$
  $$\approx 1 + 1$$

- limit $r \to \infty$
  $$\approx 1 + 1$$

$$\lim_{N_c \to \infty} \lim_{r \to \infty} \lim_{m_\pi \to 0} \frac{r^2 \tilde{G}_E^{l=0} \tilde{G}_E^{l=1}}{\tilde{G}_M^{l=0} \tilde{G}_M^{l=1}} = 9$$
The relation \[ \lim_{r \to \infty} r^2 \frac{G_E^{I=0}}{G_M^{I=0}} \frac{G_E^{I=1}}{G_M^{I=1}} = 18 \] was proven in large $N_c \chi$PT

- provided that the large $N_c$ limit is taken at the outset of the problem.
The relation

\[ \lim_{r \to \infty} \frac{r^2 \, \tilde{G}_{E}^{I=0} \, \tilde{G}_{E}^{I=1}}{G_{M}^{I=0} \, G_{M}^{I=1}} = 18 \]

was proven in large \( N_c \chi\text{PT} \) provided that the large \( N_c \) limit is taken at the outset of the problem.

It may serve as an honest model-independent constrain on baryon models based on large \( N_c \) and chiral physics.

- it was shown to hold for:
  - Skyrme model\(^{(5)}\)
  - "bottom-up" holographic model\(^{(5)}\)
  - "top-down" holographic model (if treated properly)\(^{(6)}\)

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