Retardation effect for heavy parton collisional energy loss in QGP.

- Motivation
- Collisional energy loss in stationnary regime
- Retardation effect; main results
- Radiation
- Conclusion & Perspectives

Pol-Bernard Gossiaux
work done with Stéphane Peigné & Thierry Gousset
SUBATECH (Nantes)
hep-ph/0509185
Recent papers advocating the possible role of this mechanism for partons of moderate energy:


But not the main stream
... heavy parton collisional energy loss in QGP.

Hot topic; Avoid some complications due to colinear divergences

Usual QGP: no quasi bound state, at rest, static, « confined » over some extension $L_{\text{plasma}}$
... (Boring) heavy parton collisional energy loss in QGP.

Nearly everything is known; Braaten & Thoma (91)

\[ \frac{dE_{\text{coll}}(E)}{dx} \quad \text{in stationary regime} \quad \text{and then} \]

\[ \Delta E_{\text{coll}}(L) = \frac{dE_{\text{coll}}}{dx} \times L \]

stationary?

Figure taken from Mustafa et al (Phys.Rev. C72 (2005) 014905)
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What if the heavy quark is not produced at $t = -\infty$?

World I

World II

Physically reasonable, but HOW BIG?
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CEL in stationary regime.

Assume pQCD regime at high temperature: $g << 1$ and ordering

$$\frac{1}{M} << \frac{1}{r} = \frac{1}{T} << \frac{1}{q^*} << \frac{1}{gT} << \lambda \approx \frac{1}{g^2 T}$$

Heavy quark probes the medium via virtual gluon of momentum $k$

Zone I: $k > q^*$: hard; close collisions; individual; incoherent.

Zone II: $k < q^*$: soft; far collisions; collective; coherent; macroscopic.
CEL in stationary regime: close collisions.

Bjorken (82) evaluated this contribution using kinetic theory.

Reaction rate $\Gamma$:

$$\Gamma = \sum_{\{P',p,p'\}} n(p) (1-n(p')) \delta(P+p-P'-q')$$

From rate to energy loss:

$$\frac{dE}{dx} = \int \frac{d\Gamma(E',E)}{dE'} \times \frac{E'-E}{\nu} dE' = \frac{2}{3} \alpha m_0^2 \ln\left(\frac{\sqrt{ET}}{q^*}\right) \text{ for } \nu \approx 1.$$
CEL in stationary regime: far collisions.

Will embed most of the retardation effect.

First evaluated in stationary regime by Braaten & Thoma (82) following Weldon (83):

\[ \Gamma = -\frac{(1-n(E))}{2E} \times \text{tr}(P\gamma + M) \text{Im}(\Sigma) \]

Quite general relation: includes the soft collisions \((k < q^*)\) as well, provided one takes the full gluon propagator \(\Pi\):
CEL in stationary regime: far collisions (II).

$$\Sigma =$$

For $k>q^*$: through kinetic expressions

For $k<q^*$: evaluate the self energy $\Sigma$ with HTL resummation for $\Pi$ (consistent with collective polarization)... Imaginary time formalism.

Legitimate as both contributions are separately gauge invariant.
CEL in stationary regime: far collisions (III).

Results at the logarithmic accuracy:

\[ 0 < v < 1: \]

\[ dE_{\text{hard}} = \frac{2}{3} \alpha m_B \left[ \frac{1 - v^2}{v^2} \ln \left( \frac{1 + v}{1 - v} \right) \right] \ln \left( \frac{ET}{Mq^*} \right) \]

\[ dE_{\text{soft}} = \frac{2}{3} \alpha m_B \left[ \frac{1 - v^2}{v^2} \ln \left( \frac{1 + v}{1 - v} \right) \right] \ln \left( \frac{q^*}{m_D/\sqrt{3}} \right) \]

\[ \text{UV div} \rightarrow dE = \frac{2}{3} \alpha m_B \left[ \frac{1 - v^2}{v^2} \ln \left( \frac{1 + v}{1 - v} \right) \right] \ln \left( \frac{ET}{m_0/\sqrt{3}} \right) \]

\[ \text{V} \approx 1: \]

\[ dE_{\text{hard}} = \frac{2}{3} \alpha m_B \ln \left( \frac{\sqrt{ET}}{q^*} \right) \]

\[ dE_{\text{soft}} = \frac{2}{3} \alpha m_B \ln \left( \frac{q^*}{m_D/\sqrt{3}} \right) \]

Poor man’s prescription: take \( \frac{dE_{\text{soft}}}{dx} \) with some UV regulator \( k_{\text{cut}} \):

\[ k_{\text{cut}} = \min \left( \frac{ET}{M}, \sqrt{ET} \right) \]

Method followed by Thoma & Gyulassy (91)
Maxwell’s Equations with appropriate dielectric functions (collective response of the medium in linear response approx.) lead to the same result for Energy loss.

Easy strategy: evaluate induced chromoelectric field (medium polarization $E_{\text{ind}}$ and the Lorentz force of this field on the heavy parton / external current)

😊 Easy to generalize for non stationnary phenomena (heavy parton produced inside the QGP)… better suited to discuss physics.

😐 Misses some part of the hard dynamics… which plays little role for retardation 😊.

N.B.: L.R. ⇒ trivial color structure of dielectric functions.
Far collisions: alternative formulation (II)

Maxwell equations in Fourier space $(\omega, k) \Rightarrow$

$$\varepsilon_L \vec{E}_L + (\varepsilon_T - k^2/\omega^2) \vec{E}_T = \frac{4\pi}{i\omega} (\vec{j}_L + \vec{j}_T)$$

With $\varepsilon_L$ and $\varepsilon_T$: dielectric functions of the plasma

$$\varepsilon_L(k, \omega) = 1 + \Pi_L(\omega/k)/k^2; \varepsilon_T(k, \omega) = 1 - \Pi_T(\omega/k)/\omega^2$$

(Polarization $\Pi$ taken in H.T.L approximation)

$\vec{j}_L (\vec{j}_T)$: longitudinal (transverse) projection of the current on $\vec{k}$

$$\vec{E}_{\text{ind}}(k, \omega) = \frac{4\pi}{i} \left[ \frac{k^2}{k^2 + \Pi_L} \times \vec{j}_L + \frac{\omega}{\omega^2 - k^2 + \Pi_T} \times \vec{j}_T \right]_{\text{ind}} = 4\pi \left[ k^2 \Delta_L \times \frac{\vec{j}_L}{\omega} + \omega \Delta_T \times \vec{j}_T \right]_{\text{ind}}$$

Subtract the vacuum value

gluon propagator HTL
Fourier back and integrate the Lorentz force up to \( t = L/v \):

\[
-\Delta E(L) = i \sum_a \int \frac{d^3k}{4\pi^3} \int_{-\infty}^{+\infty} d\omega \left[ \frac{k^2}{k^2 + \Pi_L} \tilde{j}_{L,a}(K) + \frac{\omega}{\omega^2 - k^2 + \Pi_T} \tilde{j}_{T,a}(K) \right] \cdot \int_0^{L/v} dt \, e^{-iK\cdot \vec{v} t} q_a \tilde{\nu}(t)
\]

Special case of stationary current: \( j_a^\mu(t,\vec{x}) = q_a V^\mu \delta^3(\vec{x} - \vec{v} t) \) with \( V^\mu = (1, \vec{v}) \)

\[
\Rightarrow \quad j_a^\mu(K) = 2\pi q_a V^\mu \delta(K\cdot V) = 2\pi q_a V^\mu \delta(\omega - \vec{k} \cdot \vec{v}) = L/v
\]

\[
-\Delta E(L) = \frac{i}{v} \int \frac{d^3k}{2\pi^2} \int_{-\infty}^{+\infty} d\omega \left[ \frac{k^2}{k^2 + \Pi_L} \frac{v^2_L(K)}{\omega} + \frac{\omega}{\omega^2 - k^2 + \Pi_T} \frac{v^2_T(K)}{\omega} \right] \delta(\omega - \vec{k} \cdot \vec{v})
\]

Only imaginary part contributes

N.B.: « induced » not really needed
In stationary regime: Only the \( \text{Im}(\varepsilon) \) contributes (ok); collective modes do not get excited; no Cherenkov radiation.
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Retardation effect: model for partonic current

For partons produced at $t=0$

\[ j_0^\mu a(K) = i q^a \left( \frac{V_1^\mu}{K \cdot V_1 + i\eta} - \frac{V_2^\mu}{K \cdot V_2 + i\eta} \right) \]

with \( V_1 = (1, \vec{v}_1) \) and \( V_2 = (1, \vec{v}_2) \)

Satisfies current conservation \( K \cdot V = 0 \)

Simplifying assumption: \( v_2 = 0 \) \( \Rightarrow \) \( V_2 = (1,0) \) and \( V_1 = (1, \vec{v}) \)

Only one parton contributes to \( \vec{j} \), but the system is in fact color neutral.
Retardation effect: convergence and scales

Contour integration in lower half-(complex) plane:

\[
\frac{-\Delta E(L)}{C_R \alpha_s} = \int \frac{d^3 \vec{k}}{2\pi^2} \left\{ \frac{1 - e^{ikL \cos \theta}}{k^2 + m_D^2} + v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi \omega} \left[ k^2 \cos^2 \theta \rho_L + \omega^2 \sin^2 \theta \rho_T \right] \right\}
\times \mathcal{P} \left( \frac{1}{\omega - kv \cos \theta} \right) \frac{1 - e^{-i(\omega - kv \cos \theta) L/v}}{\omega - kv \cos \theta} \right\}_{\text{ind}}.
\]

With the spectral functions:

\[
\rho_s(\omega, k) \equiv 2 \text{Im} \Delta_s(\omega + i\eta, k)
= 2\pi \text{sgn}(\omega) z_s(k) \delta(\omega^2 - \omega_s^2(k)) + \beta_s(\omega, k) \theta(k^2 - \omega^2)
\]

\[
\text{Residue} \quad \text{Magnitude of the cut}
\]
Retardation effect: convergence and scales

Contour integration in lower half-(complex) plane:

\[
\frac{-\Delta E(L)}{C_R\alpha_s} = \int \frac{d^3k}{2\pi^2} \left\{ \frac{1 - e^{ikL\cos\theta}}{k^2 + m_D^2} + v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi\omega} \left[ k^2 \cos^2 \theta \rho_L + \omega^2 \sin^2 \theta \rho_T \right] \right\} \times \mathcal{P} \left( \frac{1}{\omega - kv \cos\theta} \right) \frac{1 - e^{-i(\omega - kv \cos\theta) L/v}}{\omega - kv \cos\theta} \bigg\}_{\text{ind}}.
\]

→ finite constant when \( L \rightarrow \infty \) (due to the induced prescription)
Retardation effect: convergence and scales

Contour integration in lower half-(complex) plane:

\[
-\frac{\Delta E(L)}{C_R \alpha_s} = \int \frac{d^3 \vec{k}}{2\pi^2} \left\{ 1 - \frac{e^{i k L \cos \theta}}{k^2 + m_D^2} \right\} + v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi \omega} \left[ k^2 \cos^2 \theta \rho_L + \omega^2 \sin^2 \theta \rho_T \right] \times \mathcal{P} \left( \frac{1}{\omega - kv \cos \theta} \right) \frac{1 - e^{-i(\omega - kv \cos \theta) k/L}}{\omega - kv \cos \theta} \right\} \}
\]

“\( \to \)" \( \frac{\pi L}{v} \delta(\omega - kv \cos \theta) \) when \( L \to \infty \) \( \Rightarrow \) add and remove:

\[
-\frac{\Delta E(L)}{C_R \alpha_s} = \int_{k<k_{cut}} \frac{d^3 \vec{k}}{2\pi^2} \left\{ 1 - \frac{e^{i k L \cos \theta}}{k^2 + m_D^2} \right\} + v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi \omega} \left[ k^2 \cos^2 \theta \rho_L + \omega^2 \sin^2 \theta \rho_T \right] \times \left[ \left( \frac{2 \sin^2 ((\omega - kv \cos \theta)L/(2v))}{(\omega - kv \cos \theta)^2} - \frac{\pi L}{v} \delta(\omega - kv \cos \theta) \right) + \frac{\pi L}{v} \delta(\omega - kv \cos \theta) \right] \}
\] \}

Brings a \( 1/k \) additional factor instead of \( L \Rightarrow \) UV convergent! Exactly the stationary E loss log divergent in UV
Conclusion: $-\Delta E(L) = \frac{dE_{\text{stat}}}{dx} \cdot L + B(L)$

With: $B(L)$ UV safe (as anticipated) and $\rightarrow$ cst when $L \rightarrow \infty$.

For small $L$, the hard scale $\ll 1/L$ would lead to $k_{\text{typ}}$ in $B(L)$ above $k_{\text{cut}} \Rightarrow$ Condition on $L$: must be typically larger then $1/m_D$.

Behaviour at small $L$ has been nevertheless studied, using sum rules; interesting cancelations were discovered.

$$-\Delta E(L) \bigg|_{L \ll k_{\text{cut}}^{-1}} = -\frac{C_R \alpha_s m_D^2 k_{\text{cut}}^3}{108 \pi v^2} L^4$$
Retardation effect: Results

\[ M = 1.5 \text{ GeV}; \alpha_s = 0.2; n_f = 2; T = 0.25 \text{ GeV}; m_D = 0.46 \text{ GeV} \]

Negative \( \Delta E(L) \) !!!

Large reduction of CEL due to retardation effect (Trento Tirami…)

\[ \Delta E_{\infty}/E \]
Retardation effect: Results (II)

\(-\Delta E_{\text{asymp}}(L,E,...) = \frac{dE_{\text{stat}}}{dx} * (L-\text{delay}(E,T))\)

Delay roughly \(\alpha 1/T\), but not of the order of \(r_D\) !!!
Retardation effect: Results (III)

Might have some phenomenological implications:

Jet absorption and corona effect at RHIC. Extracting collision geometry from experimental data

V.S. Pantuev
University at Stony Brook, Stony Brook, NY 11794-3800

Abstract

We propose a simple model based on Monte Carlo simulation of nucleus-nucleus collisions using a Glauber approach to explain experimental data on the angular dependence of the nuclear modification factor $R_{AA}$ at high transverse momentum in the reaction plane. The model has one free parameter $L \approx 2$ fm to describe the thickness of the corona area and was adjusted to fit the experimental data on AuAu collisions at centrality 50-60%. The model nicely describes the $R_{AA}$ dependence for all centrality classes. We extract the second Fourier component amplitude, $v_2$, for high pt particle azimuthal distribution and found $v_2$ should be at the level of 11-12% purely from the geometry of the collision with particle absorption in the core. We give a prediction for $R_{AA}$ in Cu+Cu collisions at 200 GeV. Our physical interpretation of the parameter $L$ is that it’s actually the formation time $T = L/c \approx 2$ fm/c.
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Retardation effect... and other related mechanisms.

\[ -\Delta E(L) = \frac{1}{C_R \alpha_s} \int \frac{d^3 k}{2\pi^2} \left\{ 1 - e^{ikL\cos \theta} \right\} \left[ \frac{1}{k^2 + m_D^2} + v^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi\omega} \left[ k^2 \cos^2 \theta \rho_L + \omega^2 \sin^2 \theta \rho_T \right] \right] \times P \left( \frac{1}{\omega - kv \cos \theta} \right) \frac{1 - e^{-i(\omega - kv \cos \theta) L/v}}{\omega - kv \cos \theta} \right\}_{\text{ind}}. \]

\[ \rightarrow \text{finite negative constant when } L \to \infty \text{ (dipole separation easier in QGP than in vacuum)} \]
Retardation effect… and other related mechanisms.

• If one just considers the force acting on the heavy parton: irrelevant question (It takes some time to reach its fully value, that’s all).

• Being less inclusive, one can identify another interesting mechanism...
Retardation effect... and radiation.

\[-\frac{\Delta E(L)}{C_R \alpha_s} = -i v^2 \int \frac{d^3 k}{4\pi^3} \int_{\infty}^{-\infty} \frac{d\omega}{\omega} \left[ k^2 \cos^2 \theta \Delta_L(\omega, k) + \omega^2 \sin^2 \theta \Delta_T(\omega, k) \right]_{\text{ind}} \times \left\{ 1 - e^{-i(\omega - ku \cos \theta) L/v} \right\} \frac{1}{(\omega - ku \cos \theta)(\omega - ku \cos \theta + i\eta)} \right\} \]

\[\delta(\omega - \omega_s(k)):\]
- Clear signature of some radiation (cf. identification of Cherenkov radiation in stationary Collisional E loss)
- Does not scale like L \(\Rightarrow\) Not Cherenkov (although emitted by the time dependent medium polarization), but ‘initial’ Brehmstrallung.
- \(\Delta E\) incorporates the difference of energies radiated in the medium (neglecting radiative rescattering) and in the vacuum.
In-vacuum radiation

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Radiative contribution $W(L)$ to the energy loss:

$$
\frac{dW(L)}{dk \, d\cos \theta} \bigg|_{vac} = C_R \alpha_s \sin^2 \theta \frac{\sin^2((k - kv \cos \theta) L/(2v))}{(\cos \theta - 1/v)^2} \quad \text{for } L \to \infty
$$

N.B.: For $v=1$, this is just the Z.O.L. result of GLV for $x<<1$

“Radiates like crazy”, but what should matter is the difference with the “In-medium” radiation.
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In-medium radiation

Radiation of collective modes, described at the level of the HTL resummation

Not considered before, to my knowledge

\[
\frac{dW(L)}{dk \, d\cos \theta} = \frac{C_R \alpha_s}{\pi} \left\{ z_L(k) \frac{k^2}{\omega_L^2(k)} \cos^2 \theta \frac{\sin^2((\omega_L(k) - k \nu \cos \theta) \frac{L}{2\nu})}{(\cos \theta - \omega_L(k)/(k \nu))^2} \right. \\
+ \left. z_T(k) \sin^2 \theta \frac{\sin^2((\omega_T(k) - k \nu \cos \theta) \frac{L}{2\nu})}{(\cos \theta - \omega_T(k)/(k \nu))^2} \right\} \\
\xrightarrow{L \to \infty} \frac{C_R \alpha_s}{2\pi} \left\{ z_L(k) \frac{k^2}{\omega_L^2(k)} \frac{\cos^2 \theta}{(\cos \theta - \omega_L(k)/(k \nu))^2} + z_T(k) \frac{\sin^2 \theta}{(\cos \theta - \omega_T(k)/(k \nu))^2} \right\}
\]

Long: suppressed at large \( k \)
Vacuum vs Infinite medium

- In all cases, the $\theta$–integrated In-medium radiation is weaker than in the vacuum (finite mass, residues $< 1$)

- Interplay of trans. and long. radiation for $\omega$ of the order of $m_D$
Infinite medium vs Finite (L=5fm) medium

- Depletion at small angle
- Gluon formed:

\[ \sin^2((\omega_L(k) - kv \cos \theta) L/(2v)) \] averages to \( \frac{1}{2} \)

Provides a criteria for formation time.
- Possible consequences on the understanding of far away hadrons.
Conclusions & Perspectives.

1. Extension of the formalism of Thoma and Guylassy in the case of partons produced inside the medium.

2. Collisional energy loss suppressed by large factor. Retardation time of the order of several fm/c.

3. In-vacuum gluon radiation is suppressed by the medium. Radiation of collective mode is treated using the correct dispersion relation.

4. Crucial implications on the phenomenology of jet quenching.

5. Consequences on the radiative energy loss induced by rescatterings?

6. Is $\frac{k}{P’}$ the splitting function of some in-medium evolution equation?