Description of the HERMES Data on Hadronization in Nuclear Medium in Framework of the String Model

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• Double Hadron Attenuation
• Attenuation of Protons
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Theoretical Models

- Rescaling Model
  A.Accardi, V.Muccifora, H.J.Pirner et al.

- Gluon Bremsstrahlung Model
  B.Kopeliovich, J.Nemchik, B.Zakharov et al.

- Energy Loss Model
  X.Guo, M.Gyulassy, X.-N.Wang et al.

- FSI by means of BUU Transport Model
  W.Cassing, T.Falter, K.Gallmeister et al.

- String Model
  J.Ashman et al., Z.Phys. C52(1991)1
  N.Akopov, L.Grigoryan, Z.Akopov, hep-ph/0409359 (will be published soon in EPJ C)
Space-time structure of hadronization in the string model. The two constituents of the hadron are produced at different points. The constituents of the hadron $h$ are created at the points $P_2$ and $P_3$. They meet at $H_3$ to form the hadron.
The TSM and ITSM

The Two-Scale Model

J. Ashman et al., Z. Phys. C52 (1991) 1

The TSM is a purely absorption string model. Basic formula is:

\[ R_A = 2\pi \int_0^\infty db \int_{-\infty}^\infty dx \rho(b, x) \left[ 1 - \int_x^\infty dx' \sigma^{str}(\Delta x) \rho(b, x') \right]^{A-1} \]

\( b, x \) - coordinates of the DIS point,
\( \rho(b, x) \) - nuclear density function,
\( x' \) - longitudinal coordinate of the string-nucleon interaction point,
\( A \) - atomic mass number,
\( \sigma^{str}(\Delta x) \) - the string-nucleon cross section on distance \( \Delta x = x' - x \) from DIS point

\[ \sigma^{str}(\Delta x) = \theta(\tau_c - \Delta x)\sigma_q + \theta(\tau_h - \Delta x)\theta(\Delta x - \tau_c)\sigma_s + \theta(\Delta x - \tau_h)\sigma_h \] (1)

where \( \sigma_q, \sigma_s \) and \( \sigma_h \) are the cross sections for interaction with the nucleon of the initial string, open string and final hadron respectively.
\( \tau_c \) and \( \tau_h \) are constituent and yo-yo formation times.

\[
\tau_h - \tau_c = z \nu / \kappa,
\]

where \( z = E_h / \nu \), \( \kappa \) - string tension (string constant). Two expressions for \( \tau_c \) are using:

for hadrons containing leading quark

\[
\tau_c = (1 - z) \nu / \kappa \tag{2}
\]


in framework of the standard Lund model

\[
\tau_c = \left[ \frac{\ln(1/z^2) - 1 + z^2}{1 - z^2} \right] \frac{z \nu}{\kappa} \tag{3}
\]

In calculations instead of approximate expression (3) we use the precise expression for \( \tau_c \) from

A.Bialas, M.Gyulassy, Nucl.Phys. \textbf{B291}(1987) 793

In our calculations we also take into account absorption in deuterium, and use the ratio \( R_M^h = R_A / R_D \).
a) The behavior of the string-nucleon cross section as a function of distance in the TSM. b) The same as in a) for ITSM taking into account more realistic smoothly increasing string-nucleon cross section.
Improved Two-Scale Model

Some models for the shrinkage-expansion mechanism were applied. We used four versions for the definition of $\sigma^{str}$.


The first version of $\sigma^{str}$ definition is based on quantum diffusion:

$$\sigma^{str}(\Delta x) = \theta(\tau - \Delta x)[\sigma_q + (\sigma_h - \sigma_q)\Delta x/\tau] +$$

$$+ \theta(\Delta x - \tau)\sigma_h$$

where $\tau = \tau_c + c\Delta \tau$, $\Delta \tau = \tau_h - \tau_c$.

The second version follows from naive parton case:

$$\sigma^{str}(\Delta x) = \theta(\tau - \Delta x)[\sigma_q + (\sigma_h - \sigma_q)(\Delta x/\tau)^2] +$$

$$+ \theta(\Delta x - \tau)\sigma_h$$

Two other expressions for $\sigma^{str}$ were also used:

$$\sigma^{str}(\Delta x) = \sigma_h - (\sigma_h - \sigma_q)exp(-\frac{\Delta x}{\tau})$$

and:

$$\sigma^{str}(\Delta x) = \sigma_h - (\sigma_h - \sigma_q)exp(-\left(\frac{\Delta x}{\tau}\right)^2)$$
QCD predicts the $Q^2$-dependence of string-nucleon cross section in the form:

$$\sigma_q(Q^2) \sim 1/Q^2; \quad \sigma_s(Q^2_{\tau_c}) \sim 1/Q^2_{\tau_c}.$$ 

Using this prediction we can express the cross section for the initial string as

$$\sigma_q(Q^2) = (\hat{Q}^2/Q^2)\sigma_q(\hat{Q}^2).$$

In the same way the expression for the open string cross section can be written as:

$$\sigma_s(Q^2_{\tau_c}) = (\hat{Q}^2_{\tau_c}/Q^2_{\tau_c})\sigma_s(\hat{Q}^2_{\tau_c}),$$

where $Q^2_{\tau_c} = Q^2(\tau_c)$ is the virtuality of the string in the time interval $\tau_c$ after DIS. In order to estimate the ratio of $\hat{Q}^2_{\tau_c}/Q^2_{\tau_c}$ we adopt the scheme supposing that during time $t$ the quark decreases its virtuality from the initial $Q^2$ to the value $Q^2(t)$ as follows

$$Q^2(t) = \nu(t)\frac{Q^2}{\nu(t) + tQ^2},$$

where $\nu(t) = \nu - \kappa t$. 
Results for Fragmentation

For $\kappa$ value determined by the Regge trajectory slope was used:

$$\kappa = 1/(2\pi \alpha'_R) = 1 \text{GeV}/\text{fm}$$

The Nuclear Density Functions (NDF) were used as follows: for deuterium the Hard Core Deuteron Wave Functions were used

R.V.Reid, Annals of Physics 50 (1968) 411

For $^4$He and $^{14}$N, the Shell Model was used

L.Elton, ”Nuclear Sizes” Oxford University Press, 1961, p.34

$$\rho(r) = \rho_0 \left( \frac{4}{A} + \frac{2(A-4)}{3A} \frac{r^2}{r_A^2} \right) \exp \left( - \frac{r^2}{r_A^2} \right),$$

where $r_A=1.31$ fm for $^4$He and $r_A=1.67$ fm for $^{14}$N.

For $^{20}$Ne, $^{84}$Kr and $^{131}$Xe the Woods-Saxon distribution was used

$$\rho(r) = \rho_0 / (1 + \exp((r - r_A)/a)).$$
These three sets of NDF’s were used for the fitting with the following corresponding parameters:


\[
a = 0.54 \text{ fm} \quad r_A = (0.978 + 0.0206A^{1/3})A^{1/3} \text{ fm} \tag{8}
\]


\[
a = 0.54 \text{ fm} \quad r_A = \left(1.19A^{1/3} - \frac{1.61}{A^{1/3}}\right) \text{ fm} \tag{9}
\]


\[
a = 0.545 \text{ fm} \quad r_A = 1.14A^{1/3} \text{ fm}. \tag{10}
\]

where the values of \(\rho_0\) are determined from the normalization condition:

\[
\int d^3r \rho(r) = 1
\]
The Fit

The fit was performed based on published HERMES data for nuclear attenuation of $\pi^+$ and $\pi^-$ mesons on nitrogen and krypton nuclei:


For $\tau_c$ two expressions (2)-(3) were used. For $\sigma^{str}(\Delta x)$ one expression (1) in TSM and four different expressions (4)-(7) in ITSM were used. For NDF of krypton three different sets of parameters (8)-(10) were used. The values of $\sigma_h$ (hadron-nucleon inelastic cross section) used in the fit were set equal to: $\sigma_{\pi^+} = \sigma_{\pi^-} = 20$ mb. Two parameters were determined from the fit. In case of TSM and ITSM they are $\sigma_q$, $\sigma_s$ and $\sigma_q$, $c$, respectively. The values of $\sigma_h$ that were used in calculations are as follows: $\sigma_{\pi^0} = \sigma_{K^-} = 20$ mb, $\sigma_{K^+} = 14$ mb and $\sigma_{\bar{p}} = 42$ mb. Results of fit are presented in Table 1 for TSM version, and in Tables 2-3 for ITSM version. The curves correspond to the TSM and ITSM model calculations with the best set of parameters are presented on subsequent three figures.
Hadron multiplicity ratio $R_M^{π}$ of charged pions for $^{14}$N and $^{84}$Kr nuclei as a function of $ν$ (left panel) and $z$ (right panel). The solid curves correspond to the ITSM. Minimum value of $χ^2$ (best fit) is obtained for $σ^{str}$ in form (4) and $τ_c$ in form (2), at the values of parameters: $σ_q=0.46$ mb, $c=0.32$. The dashed curves correspond to the TSM. Now best fit correspond $τ_c$ in form (3), at the values of parameters: $σ_q=4.2$ mb, $σ_s=16.6$ mb. In both versions best fit is obtained for NDF (8) for $^{84}$Kr. These data were included in fit and curves were obtained in a result of fit.
Hadron multiplicity ratio $R_M^h$ of different species of hadrons produced on $^{84}$Kr target as a function of $\nu$ (left panel) and $z$ (right panel). These data did not included in fit. The curves are calculated with the values of parameters corresponding to the best fit.
The ratio $R_M^h$ for charged hadrons for $^{63}$Cu as a function of $\nu$ (upper panel) and $z$ (lower panel). The solid, dashed and dotted curves correspond to three sets of parameters with the minimal values of $\chi^2$/d.o.f. in case of ITSM (see Tables 2 and 3): solid - NDF (8), $\sigma_{str}^s$ (4), $\tau_c$ (2), $\sigma_q=0.46$ mb, $c=0.32$, $\chi^2$/d.o.f. =1.4; dashed - NDF (9), $\sigma_{str}^s$ (5), $\tau_c$ (3), $\sigma_q=1.0$ mb, $c=0.17$, $\chi^2$/d.o.f. =1.5; dotted - NDF (8), $\sigma_{str}^s$ (7), $\tau_c$ (3), $\sigma_q=1.5$ mb, $c=0.103$, $\chi^2$/d.o.f. =1.5. The dashed-dotted curves correspond to the best set of parameters in case of TSM (see Table 1): NDF (8), $\tau_c$ (3), $\sigma_q=4.2$ mb, $\sigma_s=16.6$ mb, $\chi^2$/d.o.f. =2.3.
### Table 1: The **TSM**: the best values for the fitted parameters and $\chi^2$/d.o.f. ($N_{exp} = 58$, $N_{par} = 2$).

<table>
<thead>
<tr>
<th></th>
<th>$\tau_c(2)$</th>
<th>$\tau_c(3)$</th>
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<tbody>
<tr>
<td>NDF</td>
<td>$\sigma_q$ (mb)</td>
<td>$\sigma_s$ (mb)</td>
</tr>
<tr>
<td>(8)</td>
<td>5.3±0.01</td>
<td>17.1±0.08</td>
</tr>
<tr>
<td>(9)</td>
<td>5.5±0.01</td>
<td>17.7±0.08</td>
</tr>
<tr>
<td>(10)</td>
<td>5.8±0.01</td>
<td>18.3±0.08</td>
</tr>
</tbody>
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### Table 2: The **ITSM**: $\tau_c(2)$. The best values for the fitted parameters and $\chi^2$/d.o.f. ($N_{exp} = 58$, $N_{par} = 2$).

<table>
<thead>
<tr>
<th></th>
<th>$\sigma^{str}(4)$</th>
<th>$\sigma^{str}(5)$</th>
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</thead>
<tbody>
<tr>
<td>NDF</td>
<td>$\sigma_q$ (mb)</td>
<td>$c$</td>
</tr>
<tr>
<td>(8)</td>
<td>0.46±0.02</td>
<td>0.32±0.03</td>
</tr>
<tr>
<td>(9)</td>
<td>0.62±0.01</td>
<td>0.31±0.01</td>
</tr>
<tr>
<td>(10)</td>
<td>0.78±0.02</td>
<td>0.30±0.03</td>
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### Table 3: The **ITSM**: $\tau_c(3)$. The best values for the fitted parameters and $\chi^2$/d.o.f. ($N_{exp} = 58$, $N_{par} = 2$).

<table>
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<tr>
<th></th>
<th>$\sigma^{str}(4)$</th>
<th>$\sigma^{str}(5)$</th>
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</thead>
<tbody>
<tr>
<td>NDF</td>
<td>$\sigma_q$ (mb)</td>
<td>$c$</td>
</tr>
<tr>
<td>(8)</td>
<td>0.0±0.001</td>
<td>0.56±0.02</td>
</tr>
<tr>
<td>(9)</td>
<td>0.0±0.002</td>
<td>0.53±0.02</td>
</tr>
<tr>
<td>(10)</td>
<td>0.0±0.002</td>
<td>0.49±0.006</td>
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<tr>
<th></th>
<th>$\sigma^{str}(6)$</th>
<th>$\sigma^{str}(7)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDF</td>
<td>$\sigma_q$ (mb)</td>
<td>$c$</td>
</tr>
<tr>
<td>(8)</td>
<td>0.0±0.001</td>
<td>0.24±0.02</td>
</tr>
<tr>
<td>(9)</td>
<td>0.0±0.002</td>
<td>0.21±0.02</td>
</tr>
<tr>
<td>(10)</td>
<td>0.0±0.002</td>
<td>0.18±0.02</td>
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Conclusions 1

- The HERMES data for $\nu$- and $z$ - dependencies of NA of $\pi^+$ and $\pi^-$ mesons on two nuclear targets ($^{14}$N and $^{84}$Kr) were used to perform the fit of the TSM and ITSM.
- The $\chi^2$ criterion was used for the first time for such kind of analysis, to perform comparison with the NA data.
- Two-parameter fit demonstrates satisfactory agreement with the HERMES data. Minimum $\chi^2$ (best fit) was obtained for the ITSM, including expressions (4) for $\sigma^{str}$ and (2) for $\tau_c$. The published HERMES data do not give the possibility to make a choice between expressions (4)-(7), as well as to make a distinct preference of definitions (2) or (3) for $\tau_c$, because they give close values of $\chi^2$. Preferable NDF’s are set (8) and (9).
- More precise data that is expected from HERMES will provide essentially better conditions for the choice of preferable version of the model and preferable expressions for $\sigma^{str}$ and $\tau_c$.
- In all versions we have obtained that $\sigma_q \ll \sigma_h$. This indicates that at early stage of hadronization process the Color Transparency takes place.
The semi-inclusive leptoproduction process of two hadrons on nucleus of atomic mass number A is:

\[ l_i + A \rightarrow l_f + h_1 + h_2 + X \]

The nuclear attenuation ratio for that process is defined as:

\[ R_{2h}^M = \frac{2d\sigma_A(\nu, Q^2, z_1, z_2)}{Ad\sigma_D(\nu, Q^2, z_1, z_2)}, \]

DIS take place at the point \((b, x)\). First constituents arises at the points \((b, x_1)\) and \((b, x_2)\). Second constituents at points \((b, x_{y1})\) and \((b, x_{y2})\).

There are simple connections between these points:

\[ x_{y1} - x_1 = z_1L \text{ and } x_{y2} - x_2 = z_2L \]

\( L \) is the full hadronization length, \( L = \nu / \kappa \), \( \kappa \) is string tension (string constant).
Leptoproduction of two-hadron system from a nuclear target
Double attenuation ratio can be expressed as

\[ R_{M}^{2h} \approx \frac{1}{2} \int d^2 b \int_{-\infty}^{\infty} dx \int_{x}^{\infty} dx_1 \int_{x_1}^{\infty} dx_2 \rho(b, x) \]

\[ [D(z_1, z_2, x_1 - x, x_2 - x)W_0(h_1, h_2; b, x, x_1, x_2) + \]

\[ + D(z_2, z_1, x_1 - x, x_2 - x)W_0(h_2, h_1; b, x, x_1, x_2)] \]

\[ W_0 \] is the probability that neither the hadrons \( h_1, h_2 \) nor intermediate state leading to their production (initial and open strings) interact inelastically in nuclear matter:

\[ W_0(h_1, h_2; b, x, x_1, x_2) = (1 - Q_1 - S_1 - (H_1 + Q_2 + S_2 + H_2 - \]

\[ - H_1(Q_2 + S_2 + H_2))^A) \]

The probabilities \( Q_1, Q_2, S_1, S_2, H_1, H_2 \) can be calculated using the general formulae:

\[ P(x_{\text{min}}, x_{\text{max}}) = \int_{x_{\text{min}}}^{x_{\text{max}}} \sigma_P \rho(b, x) dx, \]
Recently HERMES obtained for the first time the data on double hadron attenuation.


The following double ratio for leading and subleading hadrons has been considered:

\[ R_{2h}^{M}(z_2) = \frac{(d^2 N(z_1, z_2)/dN(z_1))_A}{(d^2 N(z_1, z_2)/dN(z_1))_D} \]
Double ratio $R_{M}^{2h}$ as a function of $z_2$ with $z_1 > 0.5$. Curves are results of calculations, points are preliminary experimental data of HERMES Collaboration. Only the charge combinations of leading and subleading hadrons: ++, -, +0, 0+, -0, 0-, 00 were included in experimental data. Curves on panels a), c), e) correspond the case of full attenuation of two-hadron system; while curves on panels b), d), f) obeys additional condition that only first produced hadron attenuates (maximal screening).
Conclusions 2

- String model gives natural and simple mechanism for description of two-hadron attenuation, which allows using the set of parameters obtained for single hadron attenuation, without additional fit satisfactory describe the available experimental data.

- In calculations we used pions only, supposing that contribution of other hadrons in multiplicity is considerably smaller. It will be very useful for us to have data for identified pions.

- Comparison with experimental data for $z_2$-dependence show that difference between versions is smaller than experimental errors, consequently, different versions of model can not be distinguished by means of comparison with these data.

- As follows from the results, double ratio has a weak sensitiveness to the mutual screening of hadrons for $z_2$-dependence.

- Theoretical curves satisfactory describes data for nitrogen. For krypton situation is more ambiguous. While three middle points describes satisfactory, two extreme points corresponding lower and higher values of $z_2$ describes worse. Possible cause is that model do not contain ingredients, necessary for description of these points. From our point of view, experimental point at $z_2=0.09$ is higher than unity, because in nucleus part of subleading hadrons are protons, which copiously produced at small $z$, and in this region have value of NA ratio larger than unity. Concerning point at $z_2=0.44$. We suppose considerable contribution from pairs of pions appeared in result of breaking of coherently produced diffractive $\omega$-mesons, which are proportional to $A^2$. In result, NA ratio for heavy nuclei raises.

- It is interesting to study also other aspects of two-hadron production in nuclear medium. For instance we propose to analyse the $\nu$ - dependence integrated over $z_1$ and $z_2$. 