Energy Loss in Hot QGP

Sangyong Jeon – Presenting

Physics, McGill

RBRC

with S. Turbide, C. Gale and G. Moore
Where we want to go:

Au+Au @ $s^{1/2} = 200$ GeV

Central $\pi^0$ (0-10%)

$q(g) \rightarrow q + g \rightarrow \pi$

and much more!
How do we get there?
How do we get there?

- Calculate *local* energy loss rate within leading order Hot QCD
  - Done by Arnold, Moore and Yaffe (AMY).
How do we get there?

- Calculate local energy loss rate within leading order Hot QCD – Done by Arnold, Moore and Yaffe (AMY).

- Use Boltzmann-like kinetic equation to evolve parton momentum distributions.
How do we get there?

- Calculate local energy loss rate within leading order Hot QCD – Done by Arnold, Moore and Yaffe (AMY).

- Use Boltzmann-like kinetic equation to evolve parton momentum distributions.

  (Major) Differences with others
How do we get there?

- Calculate **local** energy loss rate within leading order Hot QCD
  – Done by Arnold, Moore and Yaffe (AMY).

- Use Boltzmann-like kinetic equation to evolve parton momentum distributions.

  **(Major) Differences with others**

- Medium is dynamic. – Absorption, $q\bar{q}$ annihilation included.
How do we get there?

- Calculate **local** energy loss rate within leading order Hot QCD
  – Done by Arnold, Moore and Yaffe (AMY).

- Use Boltzmann-like kinetic equation to evolve parton momentum distributions.

  **(Major) Differences with others**

- Medium is dynamic. – Absorption, $q\bar{q}$ annihilation included.

- Loss rate is good for all $p_T > T$. 

How do we get there?

- Calculate local energy loss rate within leading order Hot QCD – Done by Arnold, Moore and Yaffe (AMY).

- Use Boltzmann-like kinetic equation to evolve parton momentum distributions.

  (Major) Differences with others

- Medium is dynamic. – Absorption, $q\bar{q}$ annihilation included.

- Loss rate is good for all $p_T > T$.

- We solve the time evolution equation.
Little Detour

\[
\frac{E_d^3 \sigma_{AA/d_p}^3}{(N_{coll} E_d)^3 \sigma_{PP/d_p}^3}
\]

BH : \( \kappa = 3\% \)
BH : \( \kappa = 6.1\% \)
BH : \( \kappa = 10\% \)
LPM : \( E_{LPM} = 0.351 \text{ GeV} \)
Const. \( \epsilon = 0.3 \text{ GeV} \)

\[ s^{1/2} = 200 \text{ GeV} \]

S. Jeon, J. Jalilian-Marian and I. Sarcevic, QM02.
"High" $p_t$ Spectra

$\text{PHOBOS - } \frac{dN}{dp_t^2} \sim \# \text{ part.}'s$
$\not\text{ NOT } \# \text{ coll.}'s ?$

$\text{PHENIX: } R_{AA} \approx \text{ const.}$

$\text{P}t: 4 \rightarrow 8 \text{ GeV}$

$\frac{dN}{dp_t^2} \mid \text{ NLO} = \frac{\Delta E}{E} \sim 0.7/\text{scatter}.$

$\rightarrow \text{WRONG!}$

$\text{Right: } \Delta E \sim \sqrt{E}$

Jeon, Jalilian-Marian, Sareci, Baier, Dokshitzer, Mueller, Schiff

Pisarski, QM02 Summary.
Radiations in QED and QCD

Amplitude to radiate: Need to sum over all $N$ and all $M$ and all possible radiation points. Then square it to get the radiation rate. (But we all knew that. BDMPS told us so.)
Reason to resum:
1. Radiation angle is $g$. Hence the transverse speed is $g$.
2. $gT$ kick makes the size of the parton $1/gT$.
3. It takes $(1/gT)/g = 1/g^2T$ to get separated.
4. But that’s just the mean free path for the next $gT$ kick!
5. Another way of saying LPM matters.
Photon Radiation with scalar quarks

- Will need it later for photon production.
- Simpler problem to solve.
- Take care of spin dependent factors later.
Choose a frame such that \( K = (k, 0, 0, k) \).
\( k = O(T) \).

To estimate, use the HTL resummed propagator for \( K - L \) line but use free propagator for the \( L \) line. \( n_B(O(T')) \sim O(1) \).

\[
\Sigma_{I}^{\mu\nu} \sim \alpha_{EM} \int d^4L (2L - K)^\mu (2L - K)^\nu \delta(L^2 - m_T^2) \rho(L - K)
\]

\[
\sim \alpha_{EM} \int \frac{d^3l}{E_l} (2L - K)^\mu (2L - K)^\nu \rho(L - K)
\]

Must project with

\[
\hat{P}_T^{\mu\nu} = g^{\mu\nu} - K^\mu K^\nu / K^2
\]
Two regions dominate:

1. Hard momentum $L_\mu = O(T)$
   Every quantity is $O(T)$ except
   $$\rho(K-L) \sim \frac{\Sigma_I}{T^4} = O(g^2)$$
   $\Rightarrow \Sigma_{I,1} = O(\alpha_E \alpha_s T^2)$

2. Co-linear with $K$: $L = (O(T), O(gT), O(gT), O(T))$ – Pinching pole contribution

   $$\Sigma_{I,1} \sim \alpha_E g^2 T^2 (gT)^2 \left[ \frac{\Sigma_I (L-K)}{(L-K)^2 - m_T^2 |^2 + \Sigma_I (L-K)^2} \right]$$
   $$\sim \alpha_E g^2 T^2 (gT)^2 \left[ \frac{\Sigma_I (L-K)}{2L \cdot K |^2 + \Sigma_I (L-K)^2} \right]$$
   $$= O(\alpha_E \alpha_s T^2)$$
Consider a rung $Q = O(gT)$. Each vertex $gT$. The gluon propagator is $1/g^3T^2$ (HTL, Bose enhanced). So a rung is $O(1/g)$.

Phase space integral goes like

$$d^3qdq^0\delta((L + Q)^2 - m_{T}^2)G(L + Q - K) \sim \frac{g^3T^3}{g^2T^3} \sim g$$

Altogether, adding one more rung is $O(1)$. $\implies$ Must resum.
After much analysis, simplification: Final results

\[ 2p_\perp = i\delta E f(p_\perp; p_\parallel, k) + g^2 C_R \int_Q 2\pi\delta(q^0 - q_\parallel) \frac{m_D^2}{q_\perp^2(q_\perp^2 + m_D^2)} \times \left[ f(p_\perp; p_\parallel, k) - f(p_\perp - q_\perp; p_\parallel, k) \right] \]

with \( \delta E = k^0 + E_p - E_{p+k} \)
Photon Radiation Rate

\[
\frac{d\Gamma_\gamma}{d^3k} = \frac{d_F q^2 \alpha_{EM}}{4\pi^2 k} \int_{-\infty}^{\infty} \frac{dp||}{2\pi} \int \frac{dp_\perp}{(2\pi)^2} \left| \mathcal{J}_{p|| \leftarrow p|| + k} \right|^2 
\times \text{Re} \left\{ 2p_\perp \cdot f(p_\perp; p||, k) \theta(p||) \right\}
\]

with

\[
\left| \mathcal{J}_{p|| \leftarrow p|| + k} \right|^2 = \begin{cases} 
\frac{n_b(k+p||)[1+n_b(p||)]}{2p||(p||+k)} & \text{scalars} \\
\frac{n_f(k+p||)[1-n_b(p||)]}{2[p|| (p||+k)]^2} & \text{fermions}
\end{cases}
\]
Generalize to Gluon Radiation

SD Equation for Gluon Radiation

Must take care of:

- Gluon momentum $k$ can change now.
- Color factors.
- Must keep track of quarks and gluons.

\[
2h = i\delta E(h, p, k)F(h) + g^2 \int \frac{d^2q_\perp}{(2\pi)^2} C(q_\perp) \times \\
\times \left\{ (C_s - C_A/2)[F(h) - F(h-k q_\perp)] + (C_A/2)[F(h) - F(h+p q_\perp)] + (C_A/2)[F(h) - F(h-(p-k) q_\perp)] \right\},
\]

\[
\delta E(h, p, k) = \frac{h^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p-k)} - \frac{m_p^2}{2p}.
\]
Gluon – Cont.

Here $m^2$ are the medium induced thermal masses, equal to $m_D^2/2$ for a gluon and $C_f g_s^2 T^2/4 = g_s^2 T^2/3$ for a quark. For the case of $g \rightarrow qq$, the $(C_s - C_A/2)$ term is the one with $F(h - p q_{\perp})$ rather than $F(h - k q_{\perp})$. 
Gluon Radiation Rate

\[ \frac{d \Gamma_g(p, k)}{dk dt} = \frac{C_s g_s^2}{16 \pi p^7} \frac{1}{1 \pm e^{-k/T}} \frac{1}{1 \pm e^{-(p-k)/T}} \times \left\{ \begin{array}{ll}
\frac{1+(1-x)^2}{x^3(1-x)^2} & q \rightarrow qg \\
N_f \frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \rightarrow qq \\
\frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \rightarrow gg
\end{array} \right\}
\times \int \frac{d^2 h}{(2\pi)^2} 2h \cdot \text{Re} F(h, p, k), \]

where \( x \equiv k/p \) is the momentum fraction in the gluon (or either quark, for the case \( g \rightarrow qq \)).

\( h \equiv p \times k \): 2-D vector. \( O(gT^2) \)
Time evolution equation

\[
\frac{dP_{\bar{q}q}(p)}{dt} = \int_k P_{\bar{q}q}(p+k) \frac{d\Gamma_{qg}(p+k, k)}{dkdt} - P_{\bar{q}q}(p) \frac{d\Gamma_{qg}(p, k)}{dkdt} \\
+ 2P_g(p+k) \frac{d\Gamma_{qg}(p+k, k)}{dkdt},
\]

\[
\frac{dP_g(p)}{dt} = \int_k P_{\bar{q}q}(p+k) \frac{d\Gamma_{qg}(p+k, p)}{dpdt} + P_g(p+k) \frac{d\Gamma_{gg}(p+k, k)}{dkdt} \\
- P_g(p) \left( \frac{d\Gamma_{qg}(p, k)}{dkdt} + \frac{d\Gamma_{gg}(p, k)}{dkdt} \Theta(2k-p) \right),
\]

- \( k \) integrals range: \((-\infty, \infty)\).
- \( k < 0 \): Absorption of thermal gluons.
- \( k > p \): annihilation against and antiquark of energy \((k-p)\).
- \( \Theta(2k-p) \): To prevent double counting of final states.
Relationship to BDMPS

(only to the equation, but not the solution)

(c.f. Eq.(20) in BDMS NPB 531, 403 (1998).)

\[
\frac{\partial}{\partial t} f(U, V, t) = \frac{i(U - xV)^2 \mu^2}{2x(1-x)p} f(U, V, t)
\]

\[
+ \frac{\rho \sigma}{C_F} \int d^2 Q V(Q^2) \left[ \frac{N_c}{2} f(U-Q, V-Q, t) - \frac{1}{2N_c} f(U, V-Q, t) 
- \frac{N_c}{2} f(U, V, t) - \frac{C_F}{2} f(U, V, t) - \frac{C_F}{2} f(U, V, t) + \frac{N_c}{2} f(U-Q, V, t) \right]
\]

Can identify: \[ \int_{t_1}^{\infty} dt' f(U, V, t) \leftrightarrow F(p, k) \]

Except: BDMPS use (i) \[ V(Q^2) = 1/(Q^2 + m_D^2)^2 \] instead of HTL result \[ 1/[Q^2(Q^2 + m_D^2)] \]. (ii) \[ \delta E = \hbar^2 / 2pk(p - k) \] (mass terms are missing)
Relationship with BDMPS

(solutions)

• BDMPS typically solve the equation and calculate \( \frac{dI}{d\omega} \) in a large \( h \) approximation, valid for large \( p/T, k/T \) but unreliable for \( k \leq 10T \).

• Time (medium) evolution: Probability to lose energy \( p \)

\[
P(p) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \prod_{i=1}^{n} \int_0^{\infty} d\omega_i, \frac{dI(\omega_i)}{d\omega} \right] \delta(p - \sum_{i=1}^{n} \omega_i) \exp \left( - \int_0^{\infty} d\omega \frac{dI}{d\omega} \right)
\]

• This is the solution of

\[
\frac{dP}{dt} = \int d\omega \Gamma_{BDMPS}(\omega) P(p + \omega, t) - P(p, t) \int d\omega \Gamma_{BDMPS}(\omega)
\] (1)

We do (only displaying one component), with \( -\infty < \omega < \infty \).

\[
\frac{dP}{dt} = \int d\omega \Gamma_{p+\omega, \omega} P(p + \omega, t) - P(p, t) \int d\omega \Gamma(p, \omega)
\]  (2)
Results – Evolution

Lesson: Using just $dE/dx$ is dangerous.
Parton distribution ratios

This is for illustration only. Using $P(p, 0) = 1/(p^2 + p_0^2)^n$.

Lesson: Using just $dE/dx$ is dangerous!
Parton distribution ratios

This is for illustration only.

Extreme LPM limit is reached, but only at very high energies.
Understanding the ratio

Use BDMPS expression for the quenching factor for $1/p^n$ with large $n$ but with the energy range extended to $\omega < 0$:

$$R_{AA}(p) \approx \exp \left(- (1 - e^{-\omega n/p}) \int_{-\infty}^{\infty} d\omega \int_{0}^{t} dt' \Gamma(p, \omega, t) \right)$$  \hspace{1cm} (3)

For $\Gamma$, use simple estimates

$$\omega \frac{dI}{d\omega dt} \approx \frac{\alpha N_c}{\pi \lambda} \quad \text{for} \quad 0 < \omega < \lambda \mu^2$$

$$\omega \frac{dI}{d\omega dt} \approx \frac{\alpha N_c}{\pi} \sqrt{\frac{\mu^2}{\lambda \omega}} \quad \text{for} \quad \lambda \mu^2 < \omega < \lambda \mu^2 (L/\lambda)^2$$

$$\frac{dI}{d\omega dt} \approx \frac{\alpha N_c}{\pi |\omega|} e^{-|\omega|/T} \quad \text{for} \quad \omega < 0$$
• Features are roughly produced.
• Flat ratio due to energy-loss and thermal absorption
• BH part of the energy-loss is important.
Baseline calculation

$\pi^0$, p+p at $s^{1/2} = 200$ GeV

- PHENIX, [PRL 91, 241803 (2003)]
- $\pi^0$ from jet fragmentation

Using P. Aurenche et al.'s program.
Pion Production

\[
\frac{dN_{AA}}{dyd^2p_T} = \frac{\langle N_{\text{coll}} \rangle}{\sigma_{in}} \sum_{a,b,c,d} \int dx_a dx_b g_A(x_a, Q) g_A(x_b, Q) \\
\times K_{\text{jet}} \frac{d\sigma_{a+b\rightarrow c+d}}{dt} \tilde{D}_{\pi^0/c}^{0/c}(z, Q) \\
\times \frac{\pi z}{\mathcal{P}(r_\perp)}
\]

with

\[
\tilde{D}_{\pi^0/c}^{0/c}(z, Q) = \int d^2r_\perp \mathcal{P}(r_\perp) \tilde{D}_{\pi^0/c}^{0/c}(z, Q, r_\perp, n)
\]

and

\[
\tilde{D}_{\pi^0/c}^{0/c}(z, Q, r_\perp, n) = \\
\int dp_f \frac{z'}{z} \left( P_{qq/c}(p_f; p_i; \Delta t) D_{\pi^0/c}(z', Q) + P_{g/c}(p_f; p_i; \Delta t) D_{\pi^0/c}(z', Q) \right)
\]

with \( z = p_T/p_i \) and \( z' = p_T/p_f \).

\( \Delta t \) determined by the location of the production \( \mathbf{r} \) and the direction of the jet \( \mathbf{n} \).
Nuclear Modification Factor

Au+Au @ $s^{1/2} = 200$ GeV
Central $\pi^0$ (0-10%)

$q(g) \rightarrow q+g \rightarrow \pi$

$R_{AA}$

$p_T$ (GeV/c)

1-D expansion included.
Nuclear Modification Factor (LHC)

Pb+Pb @ $s^{1/2} = 5500$ GeV
Central $\pi^0$ (0-10%)

$T_i = 845$ MeV

$E_{i_{\text{max}}} = 400$ GeV

$\Delta E_{\text{eff}} = \Delta E(r,\phi)$

$R_{AA}$

$p_T$ (GeV/c)
High $p_T \nu_2$

**$V_2(p_T)$: Energy Loss Calculations (2)**

- AMY - BDMS-like energy loss calculation
  - w/ HTL screening masses
  - Additional diagrams beyond bremsstrahlung
- Calculation an extension of work in:

  - See plenary talk by C. Gale Monday morning.

Brian Cole’s QM05 plenary session slide. Mostly geometry.
\( \gamma \) from jets and QGP

Simon Turbide’s Ph.D. Thesis work (w/ Charles Gale).

Ref: Turbide, Gale, Jeon and Moore, PRC 72, 014906, 2005.

Photon sources:

- direct photons
- jet bremsstrahlung
- jet + thermal \( q\bar{q} \rightarrow g\gamma \) and \( gq \rightarrow \gamma q \)
- thermal + thermal \( q\bar{q} \rightarrow g\gamma \) and \( gq \rightarrow \gamma q \)
- jet fragmentation
\( \gamma - \text{Baseline} \)

- \( p+p \) at \( s^{1/2} = 200 \text{GeV} \)
- All scales = \( 1 \times p_T \)

Ed\( \gamma /d^3p \) (pb GeV\(^{-2}\))
\( \gamma \) – Effect of parton energy loss

![Graph showing photon production from jet-th for different collisions at RHIC and LHC with and without energy loss.](image)

- Au+Au at RHIC, E-loss
- Au+Au at RHIC, no E-loss
- Pb+Pb at LHC, E-loss
- Pb+Pb at LHC, no E-loss

Photon production from jet-th \( \gamma \) = 0
Photons at RHIC

\(\text{Au + Au } s^{1/2} = 200 \text{ GeV}\)

\(T_i = 370 \text{ MeV}\)

\[\frac{dN}{d\gamma p_T dy} \left[ \text{GeV}^2 \right]\]

\(y_\gamma = 0\)
$\gamma \rightarrow \text{PHENIX pre-QM05}$

Au+Au at RHIC
0- 10 % Central
$T_i = 370$ MeV

$dN_{\gamma}/d^2p_T dy$ [GeV$^2$]

- $pQCD + \text{QGP} + \text{HG}$
- $pQCD+\text{QGP (no jet-th)} + \text{HG}$
- $pQCD (\text{no E-loss})$
- PHENIX

$Y_{\gamma} = 0$
\( \gamma - \text{PHENIX QM05} \)

New (preliminary) PHENIX Data
(Bathe, Buesching)

\( y_{\gamma} = 0 \) Au+Au at RHIC
0-20% Central

\[
\frac{dN_{\gamma}}{d^2p_T dy} \text{[GeV}^{-2}]\]

- Red: Sum
- Purple: jet-th
- Purple dashed: jet-brem
- Cyan: pQCD
- Blue: Th-Th
- Green: HG
- Black: PHENIX

A prediction: all source sizes fixed prior to QM

\( rles \) Gale
McGill
$\gamma - \text{LHC prediction}$

Photons at LHC

$\text{Pb} + \text{Pb}, s^{1/2} = 5500 \text{ GeV}$

- $p\text{QCD+QGP+HG}$
- $p\text{QCD+QGP (no jet-th) + HG}$
- $p\text{QCD (no E-loss)}$

$dN_{\gamma}/d^2p_T dy [\text{GeV}^{-2}]$

$y_\gamma = 0$

$T_i = 845 \text{ MeV}$
$\gamma/\gamma$ Ratio – PHENIX

0-10% Central Au+Au

$s^{1/2} = 200$ GeV

$\gamma_{\text{Total}}/\gamma_{\text{background}}$

$P_T$ (GeV/c)

- Prompt + plasma $\gamma$, $T_i = 447$ MeV
- Prompt + plasma $\gamma$, $T_i = 370$ MeV
- Prompt $\gamma$, $T_i = 370$ MeV
$\gamma/\gamma$ Ratio – LHC prediction

0-10% Central Pb+Pb $s^{1/2} = 5500$ GeV

$T_i = 845$ MeV

$\frac{\gamma_{\text{Total}}}{\gamma_{\text{background}}}$ vs $P_T$ (GeV/c)

- Red: Prompt photons + plasma photons
- Blue: Prompt photons
Conclusions and Caveats

The Good

- Calculated and used the full leading order Hot QCD radiation rates for both gluons and photons.
- Important to use the full momentum distribution at any given time, not just $dE/dx$.
- Geometry and 1-D expansion included.
- Good description of existing data – pions and photons.
- For photons, jet-thermal interaction is crucial.
- LHC predictions – Should be better since pQCD should work better there.
The Bad

- Calculations consistent in the $g \ll 1$ limit for momenta $T < p$. Yet for quantitative calculations, we needed $\alpha_s \approx 1/3$ or $g \approx 2$! So in reality, one must sum all diagrams, not just pinching part of the ladder diagrams!
  - At this leading order, $\alpha_s$ is an overall factor. So one might hope that the structure of the solution is OK.
  - Right now, this is best we can do with perturbative calculation.
- $Q = p_T$?
  - Factor of 2 either way doesn’t change results too much.
  - But really it should be something like $\mu p_T$ (or may even be $Q_0^2 + \Delta k_T^2$).
The Ugly (Ducklings)

- Do better job with the medium evolution? – Need 3-D hydro code. Working on it.

- What about jet correlations? – Need to keep track of the evolution of the joint probability function of two jet energies. Single particle distribution was hard enough!

- Most energy-loss calculations these days do get $R_{AA}$ right. Is there an experimental way to distinguish? – Maybe. Photon bremsstrahlung due to acceleration should be able to distinguish few hard radiations and many soft radiations. How to fish that out of all others is another matter. Maybe some corner of momentum space? Will work on it.