Hadronization via Coalescence

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- Introduction
- Quark coalescence
  - Baryon/meson ratio
  - Hadron elliptic flows and quark number scaling
  - Effect of resonance decays
  - Higher Fock states
  - Charm flow
  - Higher-order anisotropic flows
- Coalescence in transport model
- Entropy problem
- Summary
Puzzle: Large proton/meson ratio

\[ \pi^0 \text{ suppression: evidence of jet quenching before fragmentation} \]

- Fragmentation leads to \( p/\pi \sim 0.2 \)
- Jet quenching affects both
- Fragmentation is not the dominant mechanism of hadronization at \( p_T < 4-6\text{GeV} \)
Coalescence vs. Fragmentation

**Fragmentation**
Leading parton with $p_T$ leads to hadrons of $p_h = z p_T$ with a probability $D_h(z)$, where $z < 1$

**Colascence**
- partons are already there
- $p_h = n p_T$, $n = 2, 3$
- Need to be close in phase space
- Partonic hydro behavior is shifted to higher $p_T$

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**Parton spectrum**

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<th>$p_T$ (GeV)</th>
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Surprise: quark number scaling of hadron elliptic flow

Except pions, $v_{2,M}(p_T) \sim 2 \, v_{2,q}(p_T/2)$ and $v_{2,B}(p_T) \sim 3 \, v_{2,q}(p_T/3)$ consistent with hadronization via quark recombination
Coalescence model in heavy ion collisions

- Extensively used for light clusters production
- First used for describing hadronization of QGP by Budapest group
- Currently pursued by
  - Oregon: Hwa, Yang (PRC 66 (02) 025205), ........
  - Duke-Minnesota: Bass, Nonaka, Meuller, Fries (PRL 90 (03) 202303; PRC 68 (03) 044902 )
  - Ohio and Wayne States: Molnar, Voloshin (PRL 91 (03) 092301; PRC 68 (03) 044901)
  - Texas A&M: Greco, Levai, Rapp, Chen, Ko (PRL (03) 202302; PRC 68 (03) 034904)
- Most studies are schematic, based on parameterized QGP parton distributions
- Study based on parton distributions from transport models has been developed by TAMU group (PRL 89 (2002) 152301; PRC 65 (2002) 034904 ) and is now also pursued by D. Molnar (nucl-th/0406066)
Coalescence model  

Number of hadrons with $n$ quarks and/or antiquarks

\[
N_n = g \int \prod_{i=1}^{n} p_i d\sigma_i \frac{d^3 p_i}{(2\pi)^3 E_i} f_{q,i}(x_i, p_i) f_n(x_1, \ldots, x_n; p_1, \ldots, p_n)
\]

**Spin-color statistical factor**

g_M  
e.g.  
g_\pi = g_{K} = 1/36  \quad g_{\rho} = g_{K^*} = 1/12  
g_p = g_{\bar{p}} = 1/108,  \quad g_\Delta = g_{\bar{\Delta}} = 1/54

**Quark distribution function**

\[
f_q(x, p) = \int p \cdot d\sigma \frac{d^3 p}{(2\pi)^3 E} f_q(x, p) = N_q
\]

**Coalescence probability function**

\[
f_M(x_1, x_2; p_1, p_2) = f_2(x_1 - x_2; p_1 - p_2)
\]

\[
= \exp[(x_1 - x_2)^2 / 2 \Delta_x^2]
\]

\[
\times \exp\{[(p_1 - p_2)^2 - (m_1 - m_2)^2] / 2 \Delta_p^2\}
\]

For baryons, Jacobi coordinates for three-body system are used.
Monte-Carlo method

Introduce quark probabilities $P_q(i)$ according to their transverse momentum and spatial distributions

$$\frac{dN}{d^2 \vec{p}_T}^M = g_M \prod_{i,j} P_q(i) P_{\bar{q}}(j) \delta^{(2)}(\vec{p}_T - \vec{p}_{iT} - \vec{p}_{jT}) \times f_M(x_i, x_j; p_i, p_j)$$

$$\frac{dN}{d^2 \vec{p}_T}^B = g_B \sum_{i \neq j \neq k} P_q(i) P_q(j) P_q(k) \delta^{(2)}(\vec{p}_T - \vec{p}_{iT} - \vec{p}_{jT} - \vec{p}_{kT}) \times f_B(x_i, x_j, x_k; p_i, p_j, p_k)$$

Allow to treat all quarks on same footing
Parton transverse momentum distributions

- Thermal QGP \( p_T \leq 2 \text{GeV} \)
- Power-law minijets \( p_T \geq 2 \text{GeV} \)
- Choose \( R = 8 \text{ fm} \)
  \[ \tau = 5 \text{ fm}, \quad |y| \leq 0.5 \]
  \[ \Rightarrow V \approx 1100 \text{ fm}^3 \]
  \[ N_u = N_d \approx 245, \quad N_s \approx 149 \]
  \[ \left. \frac{dE_T}{dy} \right|_{|y| \leq 0.5} \approx 788 \text{ GeV} \]

Consistent with data (PHENIX)

P. Levai et al., NPA 698 (02) 631
Other inputs and assumptions

- **Minijet fragmentation** via KKP fragmentation functions (Kniehl, Krammer, Potter, NPB 582, 514 (2000))

\[
\frac{dN}{d^2\vec{p}_{\text{had}}} = \sum_{\text{jet}} \int dz \frac{dN}{d^2\vec{p}_{\text{jet}}} \frac{D_{\text{had/jet}} (z, Q^2)}{z^2}, \quad z = \frac{p_{\text{had}}}{p_{\text{jet}}}
\]

- **Gluons** are converted to quark-antiquark pairs with equal probabilities in all flavors.

- Quark-gluon plasma is given a transverse collective flow velocity of \(\beta=0.5\ c\), so partons have an additional velocity \(v(r)=\beta(r/R)\).

- Minijet partons have current quark masses \(m_{u,d}=10\ \text{MeV}\) and \(m_s=175\ \text{MeV}\), while QGP partons have constituent quark masses \(m_{u,d}=300\ \text{MeV}\), \(m_s=475\ \text{MeV}\) (Non-perturbative effects, Levai & Heinz, PRC 57, 1879 (1998))

- Use **coalescence radii** \(\Delta p=0.24\ \text{GeV}\) for mesons and 0.36 GeV for baryons
Pion and proton spectra

\[ \rho \rightarrow \pi \pi \]

Similar results from other groups
Oregon: parton distributions extracted from pion spectrum
Duke group: no resonances and s+h but use harder parton spectrum
Baryon/Meson ratio

\[ \frac{p}{\pi} \text{ ratio} \]

Au+Au@200AGeV
(central)

TAMU

OROGEN

DUKE

\[ p \rightarrow \pi \pi \]

\[ p_{T} (GeV) \]

\[ p/\pi \text{ Au+Au 130 GeV} \]

\[ p/\pi \text{ Au+Au 200 GeV} \]
Baryon/meson ratio at lower energy

Greco, Ko & Vitev, PRC 71, 041901(R) (2005)

$p/\pi$ increases by 20% while $p\overline{p}/\pi$ decreases slightly
Elliptic flow

Quark $v_2$ extracted from pion and kaon $v_2$ using coalescence model.
Naïve quark coalescence model

Only quarks of same momentum can coalescence, i.e., $\Delta p = 0$

Quark transverse momentum distribution

$$f_q(p_T) \propto 1 + 2v_{2,q}(p_T)\cos(2\phi)$$

Meson elliptic flow

$$v_{2,M}(p_T) = \frac{2v_{2,q}(p_T/2)}{1 + 2v_{2,q}^2(p_T/2)} \approx 2v_{2,q}(p_T/2)$$

Baryon elliptic flow

$$v_{2,B}(p_T) = \frac{3v_{2,q}(p_T/3)}{1 + 6v_{2,q}^2(p_T/3)} \approx 3v_{2,q}(p_T/3)$$

Quark number scaling of hadron $v_2$ (except pions):

$$\frac{1}{n}v_2(p_T/n)$$

same for mesons and baryons
Effects due to wave function and resonance decays

Wave function effect

Effect of resonance decays

Wave fun. + res. decays

Effect of resonance decays
Elliptic flow of resonances

Nonaka et al, PRC 69, 31902 (04)

- $K^*$ produced during hadronization has $v_2$ given by $v_{2,q} + v_{2,s}$

- $K^*$ produced from $K\pi$ scattering has $v_2$ given by $v_{2,\pi} + v_{2,K} \sim 3v_{2,q} + v_{2,s}$

- Observed $K^*$ has $v_2$ given by $v_{2}^{\text{full}} = r(P_T) v_{2}^{\text{QGP}} + (1-r(P_T)) v_{2}^{\text{HG}}$

with $r(p_T)$ depending on $K^*$ width and $K\pi$ scattering cross section
Higher Fock States

Meuller, Fries & Bass, PLB 618, 77 (05)

\[ |M\rangle = c_1 |q_\alpha \bar{q}_\beta \rangle + c_2 |q_\alpha \bar{q}_\beta g \rangle + c_3 |q_\alpha \bar{q}_\beta q_\gamma \bar{q}_\gamma \rangle + \ldots \]

Spectra are also not affected (at least for \( p_T >> m \))

\[ v_2^M (p) = \sum_{\nu} c_{\nu}^{(M)} n_{\nu}^{(M)} v_2 \left( \frac{p}{n_{\nu}^{(M)}} \right) \]

\[ v_2^B (p) = \sum_{\nu} c_{\nu}^{(B)} n_{\nu}^{(B)} v_2 \left( \frac{p}{n_{\nu}^{(B)}} \right) \]

\( \nu \): Fock state, \( n_{\nu} \) = # of partons

\( C_2 = 0.3 \)
Charm spectra

Charm quark

\[ \frac{dN_c}{dy} = 2.5 \]

\[ \frac{dN_c}{dp_T} = \text{d}N_c / \text{d}p_T \]

\( \text{Au+Au @ 200 A GeV} \)

D meson

J/ψ

Bands correspond to flow velocities between 0.5 and 0.65

\[ T = 0.72 \text{ GeV} \]

\[ T = 0.35-0.50 \text{ GeV} \]

\( N_{J/ψ} = 2.7 \cdot 10^{-3} \)

\( N_{J/ψ} = 0.9 \cdot 10^{-3} \)

Greco, Rapp & Ko, PLB595 (04) 202
Charmed meson elliptic flow

Data consistent with thermalized charm quark with same $v_2$ as light quarks

Smaller charm $v_2$ than light quark $V_2$ at low $p_T$ due to mass effect

Greco, Rapp & Ko, PLB595 (04) 202
Effect of higher-order parton anisotropic flows

Including 4th order quark flow  

\[ f_q(p_T) \propto 1 + 2v_{2,q}(p_T)\cos(2\varphi) + 2v_{4,q}(p_T)\cos(4\varphi) \]

Meson elliptic flow

\[ v_{2,M} = \frac{2v_{2,q} + 2v_{2,q}v_{4,q}}{1 + 2(v_{2,q}^2 + v_{4,q}^2)} \quad v_{4,M} = \frac{2v_{4,q} + v_{2,q}^2}{1 + 2(v_{2,q}^2 + v_{4,q}^2)} \]

Baryon elliptic flow

\[ v_{2,B} = \frac{3v_{2,q} + 6v_{2,q}v_{4,q} + 3v_{2,q}^3 + 6v_{2,q}^2v_{4,q}^2}{1 + 6(v_{2,q}^2 + v_{4,q}^2 + v_{2,q}^2v_{4,q}^2)} \quad v_{4,B} = \frac{3v_{4,q} + 3v_{2,q}^2 + 6v_{2,q}v_{4,q} + 3v_{4,q}^3}{1 + 6(v_{2,q}^2 + v_{4,q}^2 + v_{2,q}^2v_{4,q}^2)} \]

\[ \Rightarrow \quad \frac{v_{4,M}}{v_{2,M}^2} = \frac{1}{4} + \frac{1}{2} \frac{v_{4,q}}{v_{2,q}^2}, \quad \frac{v_{4,B}}{v_{2,B}^2} = \frac{1}{3} + \frac{1}{3} \frac{v_{4,q}}{v_{2,q}^2} \]
Higher-order anisotropic flows

Data can be described by a multiphase transport (AMPT) model

Data

\[ \frac{v_4}{v_2} \approx 1.2 \Rightarrow v_{4,q} \approx 2v_{2,q} \]

Parton cascade

\[ v_{4,q} \approx v_{2,q}^2 \]

Hydro gives a ratio of \( \frac{1}{2} \)

(Borghini & Ollitrault, nucl-th/0506045)
A multiphase transport model

Default: Lin, PaL, Zhang, Li & Ko, PRC 61, 067901 (00); 64, 041901 (01)

- Initial conditions: HIJING (soft strings and hard minijets)
- Parton evolution: ZPC
- Hadronization: Lund string model for default AMPT
  Coalescence model for string melting scenario
- Hadronic scattering: ART

String melting: PRC 65, 034904 (02); PRL 89, 152301 (02)

- Convert hadrons from string fragmentation into quarks and antiquarks
- Evolve quarks and antiquarks in ZPC
- When stop interacting, combine nearest quark and antiquark to meson, and nearest three quarks to baryon,
- Hadron flavors are determined by quarks’ invariant mass
Zhang’s parton cascade (ZPC)


$$p^\mu \partial_\mu f_1(x, p, t) \propto \int dp_2 d\Omega \mid \vec{v}_1 - \vec{v}_2 \mid (d\sigma/d\Omega)(f'_1 f'_2 - f_1 f_2)$$

$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha_s^2}{2(t-\mu^2)^2}, \quad \sigma = \frac{9\pi\alpha_s^2}{2\mu^2} \frac{1}{1 + \mu^2/s}$$

- Using $\alpha_s=0.5$ and screening mass $\mu=gT\approx0.6$ GeV at $T\approx0.25$ GeV, then $<s>^{1/2} \approx 4.2T \approx 1$ GeV, and pQCD gives $\sigma \approx 2.5$ mb and a transport cross section

$$\sigma_t \equiv \int d\Omega \frac{d\sigma}{d\Omega} (1 - \cos \theta) \approx 1.5 \text{mb}$$

- $\sigma=6$ mb $\rightarrow \mu \approx 0.44$ GeV, $\sigma_t \approx 2.7$ mb
- $\sigma=10$ mb $\rightarrow \mu \approx 0.35$ GeV, $\sigma_t \approx 3.6$ mb
Transverse momentum and rapidity distribution from default AMPT

BRAHMS Au+Au @ 200 GeV
Transverse momentum spectra from AMPT with string melting

- Spectra are softer than in default AMPT as current quark masses are used, whose spectra are less affected by collective radial flow.
Quark elliptic flows from AMPT

- $p_T$ dependence of charm quark $v_2$ is different from that of light quarks
- At high $p_T$, charm quark has similar $v_2$ as light quarks
- Charm elliptic flow is also sensitive to parton cross sections
Elliptic flow from AMPT  

Lin & Ko, PRC 65, 034904 (2002)

Need string melting and large parton scattering cross section

\[ \sigma_p = 6 \text{ mb} \]
Pseudorapidity dependence of $v_1$ and $v_2$

Chen, Greco, Ko & Koch, PLB 605, 95 (2005)

- String melting describes data near mid-rapidity ($|\eta|<1.5$)
- At large rapidity ($|\eta|>3$), hadronic picture works better
System size dependence of elliptic flow

Chen & Ko, nucl-th/0505044

Ratio of elliptic flow is \( \sim \frac{1}{3} \) and scales with the size of colliding systems (\( \sim \) product of ratios of initial eccentricity (\( \sim \frac{1}{2} \)) and energy density \( \sim \frac{2}{3} \))
Charmed meson elliptic flow from AMPT

Zhang, Chen & Ko, PRC 72, 024906 (05)

Current light quark masses are used in AMPT. With constituent masses will enhance the charmed meson elliptic flow.
Entropy

For non-relativistic system

\[
S = N \left[ \frac{5}{2} - \frac{\mu}{T} \right] = N \left[ \frac{5}{2} + \frac{m}{T} - \log \left( \frac{N}{N_{th}} \right) \right]
\]

For \( g \rightarrow \pi \) in duality picture with \( m_g = m_\pi = 0 \) and gluon in equilibrium

\[
S_g = 2.5 N_g \quad S_\pi = N_g \left[ \frac{5}{2} - \log \left( \frac{g_g}{g_\pi} \right) \right] \approx 0.8 N_g \quad 70\% \text{ decrease}
\]

Coalescence model

\[
S_{\text{QGP}} \approx 4870 \quad S_{\text{Had}} \approx 4080 \quad 16\% \text{ decrease}
\]

But energy is not conserved \( \Delta E/E \approx 18\% \)

Need to take into account binding effect such as treated approximately in AMPT by considering invariant mass of coalescence quarks
Summary

- Quark coalescence can explain observed large baryon/meson ratio at \( p_T \sim 3\text{GeV} \)
  - Quark number scaling of hadron \( v_2 \)
    - Signature of deconfinement?
- Coalescence of minijet partons with thermal partons is significant
  - Medium modification of minijet fragmentation.
- Scaling violation of pion \( v_2 \) can be explained by resonance decays.
- Coalescence of thermalized charm quarks can explain preliminary charmed meson spectrum and \( v_2 \) as well as \( J/\psi \) yield.
- Required quark \( v_2 \) is consistent with that from parton cascade with large parton cross section (~10 mb).
- Appreciable parton \( v_4 \) is seen in parton cascade.
- Entropy violation (~16%) is not as large as one naively thinks and is related to energy violation of similar magnitude.