Collisional energy loss in the sQGP

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1. Bjorken’s estimate
2. Strongly coupled (Q)GP
3. $1 + 2$
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Now, for the first time since starting nuclear collisions at RHIC in the year 2000 and with plenty of data in hand, all four detector groups operating at the lab [BNL] . . . believe that the fireball is a liquid of strongly interacting quarks and gluons rather than a gas of weakly interacting quarks and gluons.
Why collisional energy-loss?

**paradigm:** radiative energy-loss [BDMPS] dominates collisional energy-loss

- How realistic are commonly used (pQCD based) input parameters for e-loss estimates?

- ‘observed’ at RHIC:
  - strongly coupled QGP (sQGP)
    - collective phenomena, in line with hydrodynamics
    - fast equilibration, low viscosity
    - large Xsections, large coupling

  \[ \downarrow \]

  dense liquid ⇔ collisional e-loss

- Fokker-Planck eqn. with drag and diffusion parameters related to \( dE_{\text{coll}}/dx \) [Mustafa, Thoma]

  \[ \Rightarrow \] compatible quenching factors
things are involved ⇒ simplify

- consider energy-loss of hard jet in static infinite thermalized medium (QGP)
- consider mostly quenched QCD; pQCD expectation *quarks and gluons differ by group factors* (coupling Casimirs, d.o.f.) seen also for large coupling (lattice)
- rescale:
  \[ T_c^{quench} = 260 \text{ MeV} \rightarrow 170 \text{ MeV} \]

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A. Peshier, Collisional energy loss

Bjorken’s estimate – p. 4
Bjorken's formula

[Bjorken '82] considers energy-loss per length due to elastic collisions

\[ \frac{dE}{dx} = \int k^3 \rho(k) \Phi \int dt \frac{d\sigma}{dt} \Delta E \]

\[ \int_{t_2}^{t_1} dt \frac{d\sigma}{dt} \Delta E = \frac{9}{4} \pi \alpha^2 \frac{1}{k(1 - \cos \theta)} \ln \frac{t_1}{t_2} \]

small \( t \) dominate: \( \frac{d\sigma}{dt} = 2\pi C_{gg} \frac{\alpha^2}{t^2} \)

\[ E, E' \gg k \sim T: \quad t = -2(1 - \cos \theta)k\Delta E \]
\[ \Phi = 1 - \cos \theta \]

divergences – cut-offs:

screening: \( t_2 = -\mu^2 \)

\[ \Delta E < \Delta E_{max}: \quad t_1 = -2(1 - \cos \theta)k\Delta E_{max} \]

jet persist: \( \Delta E_{max} \approx 0.5E \)
Bjorken’s formula

\[ \frac{dE}{dx} = \frac{9\pi}{4} \alpha^2 \int_k^3 \frac{\rho(k)}{k} \ln \left( \frac{1 - \cos \theta}{\mu^2} \right) k E \]

pragmatically: \((1 - \cos \theta) \rightarrow 2\)

\[ \int dk \ k \rho(k) \ln k \rightarrow \ln \langle k \rangle \int dk \ k \rho(k) \]

\[ \langle k \rangle \rightarrow 2T \]

with \(\rho(k) = 16n_b\):

\[ \frac{dE_B}{dx} = 3\pi \alpha^2 T^2 \ln \frac{4TE}{\mu^2} \quad \mu^2 \rightarrow m_D^2 = 4\pi \alpha T^2 \]

shortcomings:  
1. sloppy \(k\)-integral \(\leftrightarrow dE/dx < 0??\)
2. phenomenological IR cut-off
3. relevant scale for coupling?
Bjorken’s estimate

collisional energy-loss of hard gluons (+1) and quarks (−1)

\[
\frac{dE_B}{dx} = \left(\frac{3}{2}\right)^{\pm 1} (1 + \frac{1}{6} n_f) 2\pi\alpha^2 T^2 \ln \frac{4TE}{\mu^2}
\]

\begin{align*}
T &= 300 \text{ MeV} \\
\alpha &= 0.2 \\
\mu &= 0.5\ldots1 \text{ GeV} \\
\sim m_D = \sqrt{4\pi\alpha T}
\end{align*}

compare to

- ‘cold’ e-loss \( \sim 1 \text{ GeV/fm} \)
- radiative e-loss \( \Delta E_{rad} \sim E^\beta \), \( \beta = 0, \frac{1}{2}, 1 \)
orderly $t$ and $k$ integrals

\[ t_1 = -2(1 - \cos \theta)k\Delta E_{\text{max}} \]

$t_1 < -\mu^2$: $\Delta E > 0$
constrains $\int k^3$

limit $ET \gg \mu^2$:

\[ \frac{d\tilde{E}_B}{dx} = 3\pi\alpha^2 T^2 \ln \frac{0.64TE}{\mu^2} \]

screening systematically within HTL perturbation theory

\[ \frac{dE^*}{dx} = 3\pi\alpha^2 T^2 \ln \frac{1.27TE}{m_D^2} \]

[Braaten, Thoma]

$\mu_*^2 \approx 0.5m_D^2$

NB: conform with general form of collisional energy loss, in leading-log approximation, related to cut dressed 1-loop diagrams [Thoma]
Bjorken’s formula – improvements

- running coupling

\[ \alpha^{\text{pert}}(Q) = \frac{4\pi}{11 \ln(Q^2/\Lambda^2)} \]

often: \( Q \sim 2\pi T, \Lambda = T_c/1.14 \)

\[ T/ T_c \]

\[ m_D \text{ [GeV]} \]

\[ m_D > T \]

(pQCD: \( m_D = gT \ll T \))

what is relevant scale for \( \alpha \)?

how reliable is extrapolated pQCD?

NB: even conservative \( \alpha(T) \):
Quasiparticle perspective of s(Q)GP

- **2PI formalism**: thermodynamic potential in terms of *full* propagator

\[
\Omega = \frac{1}{2} \text{Tr} \left( \ln(-\Delta^{-1}) + \Pi \Delta \right) - \Phi, \quad \phi = \text{circles} + \text{dashed circles} + \text{double circles} + \ldots
\]

\[
\Pi = 2 \frac{\delta \phi}{\delta \Delta} = \text{circles} + \text{dashed circles} + \text{double circles} + \ldots
\]

- Truncation $\Rightarrow$ thermodynamically consistent resummed approximations

entropy functional of dressed (quasiparticle) propagator [Riedel, ...]

\[
s[\Delta] = - \sum_{i=T, L} \left( d_i \int_{p^4} \frac{\partial n_b}{\partial T} \left( \text{Im} \ln(-\Delta_i^{-1}) + \text{Im} \Pi_i \text{Re} \Delta_i \right) \right)
\]

(in Fermi liquid theory: *dynamical quasiparticle entropy*)
**dressed propagator** for momenta \( p \sim T \) (\( d_g = 16 \) transverse d.o.f.)

- **Ansatz**: Lorentzian spectral function corresponds to self-energy

\[
\Pi = m^2 - 2i\gamma\omega
\]

\( \gamma \rightarrow \) transport properties

- quasiparticle mass and (collisional) width parameterized in form of pQCD results (gauge invariant, momentum-independent)

\[
m^2 = 2\pi\alpha T^2, \quad \gamma = \frac{3}{2\pi} \alpha T \ln \frac{c}{\alpha}
\]

- to extrapolate to \( T \sim T_c \) use effective coupling

\[
\alpha(T) = \frac{4\pi}{11 \ln(\lambda(T - T_s)/T_c)^2}
\]
Quasiparticle perspective of s(Q)GP

- QP model [AP] vs. lattice data [Okamoto et al.]

- non-perturbative ‘quasiparticles’
  - \( m \sim T \) heavy excitations
  - \( \gamma \sim T \) short mean free path – except very near \( T_c \) (crit. slow down)
**s(Q)GP: coupling $\alpha(T)$**

- **pQCD**
  
  $$
  \alpha^{\text{pert}}(Q) = \frac{4\pi}{11 \ln(Q^2/\Lambda^2)}
  $$

  with $Q \sim 2\pi T$, $\Lambda \sim T_c$

- **analyze lattice data**
  
  1. entropy within QP model
  2. static $q\bar{q}$ free energy

  $$
  F(r, T) \to C_r \frac{\alpha}{r} \exp(-m_D r)
  $$

  [Kaczmarek et al.]

**for $T \sim \mathcal{O}(T_c)$**

IR enhancement of $\alpha$

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A. Peshier, Collisional energy loss
s(Q)GP: cross section

- pQCD & cut-off \( m_D^2 = 4\pi\alpha T^2 \)

\[
\sigma_{\text{pert}} = \int dt \frac{9\pi\alpha^2}{t^2} = \frac{9}{8} \frac{\alpha}{T^2}
\]

\( \rightarrow \quad \frac{9}{8} \frac{\alpha(2\pi T)}{T^2} \sim \mathcal{O}(1\text{ mb}) \)

- phenomenology \([\text{Molnar, Gyulassy}]\)

\( \sigma_{\text{RHIC}} \sim \mathcal{O}(10\text{ mb}) \)

- QP model, from \(2 \rightarrow m\) interaction rate \( d^4N/dx^4 \) \([\text{AP, Cassing}]\)

\[
\text{Tr}_{p_1,p_2} \left[ \frac{2\sqrt{\lambda}}{2\omega_1 2\omega_2} n_b(\omega_1)n_b(\omega_2) \sigma \Theta(P_1^2)\Theta(P_2^2) \right] = \text{Tr}_p \left[ \frac{1}{2\omega} \gamma n_b(\omega)\Theta(P^2) \right]
\]

average \(\sigma\) and \(\gamma\) \(\Rightarrow\) \(\sigma_{\text{eff}} = \gamma \frac{N_+}{l_2} \sim \mathcal{O}(10\text{ mb})\)
QP-model indicates an almost ideal liquid\(^\text{(*)}\) [AP, Cassing]

- large plasma parameter
  \[ \Gamma = 2 \frac{N_c \alpha}{N^{-1/3}} \frac{1}{\langle E_{\text{kin}} \rangle} \]

- large percolation measure
  \[ \kappa_2 = \sigma_{\text{eff}} N^{2/3} \]

- very low shear viscosity
  \[ \frac{\eta}{s} \approx 0.2 \text{ near } T_c \]

\(^\text{(*)}\) \(\neq\) ideal gas!!!
Interlude: quasiparticle model – implications

quantities relevant for radiative energy-loss

- mean free path $\lambda = \gamma^{-1}$
- transport coefficient $\hat{q} = m_D^2/\lambda$

**NB:** $\gamma^{-1}(p \sim T)$ as a lower estimate for mean free path of hard jet.
Non-perturbative parameterization: $\alpha(T)$

Can pQCD, by using appropriate scales, be extrapolated to ‘near’ $T_c$?

$$\alpha^\text{pert}(Q) = \frac{4\pi}{11 \ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

- pQCD: $\frac{Q}{\Lambda} \sim \frac{2\pi T}{T_c}$
- non-pert. parameterization (aim at $T \gtrsim 1.3T_c$):
  $$\frac{Q}{\Lambda} \rightarrow 1.7 \frac{T}{T_c}$$

IQCD data [Kaczmarek et al.]
Can pQCD be consistently extrapolated to ‘near’ $T_c$?

$$m_D^2 = 4\pi\alpha T^2$$

- **pQCD:**
  $$\alpha \rightarrow \alpha^{\text{pert}} (Q \sim 2\pi T)$$

- **non-pert. parameterization:**
  $$\alpha \rightarrow \alpha^{\text{NP}} (T)$$

- Observation within QP model:
  $$m_D \approx 2.7\gamma$$

IQCD data [Nakamura et al.], [Kaczmarek et al.]
Can pQCD be **consistently** extrapolated to ‘near’ $T_c$?

$$\sigma = \int_{-\mu^2}^{\mu^2} dt \frac{9\pi\alpha^2}{t^2}$$

- **pQCD:** $\sigma = \frac{9}{8} \frac{\alpha|Q|2\pi T}{T^2}$
- **running** $\alpha(t) = A/\ln(-t/\Lambda^2)$:

$$\sigma = \frac{9\pi}{2} \int_{-\mu^2}^{\mu^2} dt \left[ \frac{A}{t\ln(-t/\Lambda^2)} \right]^2$$

$$= \frac{9\pi}{2} \frac{A^2}{\Lambda^2} \left[ Ei(-\ln(\mu^2/\Lambda^2)) + \frac{\Lambda^2/\mu^2}{\ln(\mu^2/\Lambda^2)} \right]$$

$$\rightarrow \frac{9\pi}{2} \frac{\alpha^2(\mu)}{\mu^2} [1 + \ldots]$$

$\mu_{NP} = 0.6 m_D$

$\Lambda_{NP} = 420 \text{ MeV}$
Indeed, pQCD can consistently be extrapolated to ‘near’ $T_c$.

- lattice results for $\alpha(T)$, $m_D(T)$, for $T/T_c \in [1.3, 4]$, consistent with

$$\alpha^{NP}(T) = \frac{4\pi}{11 \ln(1.7 T/T_c)^2}$$

- cross section ($\times 10$) enhancement consistent with

pert. Xsection
$$d\sigma/dt \sim \alpha^2/t^2$$

running coupling
$$\alpha(t) = \frac{4\pi}{11 \ln(-t/\Lambda^2_{NP})}, \ \Lambda_{NP} = 420 \text{ MeV}$$

cut-off
$$\mu = 0.6 m_D$$ (compare to $\mu_\star = 0.7 m_D$)

- relation between $\alpha^{NP}(T)$ and $\alpha(t)$, assuming $\sqrt{|t|} = \kappa T$,

$$\kappa = 1.7 \frac{\Lambda_{NP}}{T_c} \approx 2.74$$ (compare to $\langle k \rangle = \frac{\int k^3 k \rho(k)}{\int k^3 \rho(k)} \approx 2.70 T$)
Interlude: Cut-off and running coupling

\[
\frac{\alpha_0}{P^2} \quad \rightarrow \quad \frac{\alpha}{P^2 - \Pi}
\]

(resummation, renormalization)

vacuum:

\[
\Pi = \Pi_{\text{ren}} \sim \alpha P^2 \ln(-P^2/\mu^2)
\]

\[
\alpha(P^2) = \frac{4\pi}{11 \ln(-P^2/\Lambda^2)}
\]

medium:

\[
\Pi = \Pi_{\text{ren}} + \Pi_{\text{mat}}
\]

\[
\Pi_{\text{mat}} \sim \alpha T^2 \sim m_D^2
\]

IR cut-off: \( P \gtrsim m_D \)

running important when \( m_D \sim gT \sim \Lambda \) (non-pert. regime)
Collisional e-loss with running coupling

Bjorken:

\[ \frac{dE}{dx} = \int_{k^3} \rho(k) \Phi \int dt \frac{d\sigma}{dt} \Delta E \]

t-integral, with \( \alpha(t) = A / \ln(-t/\Lambda^2) \)

\[ \Phi \int_{t_1}^{t_2} dt \frac{d\sigma}{dt} \Delta E = -\frac{9}{4} \pi A^2 \int_{t_1}^{t_2} dt \frac{1}{k} \frac{1}{t \ln^2(-t/\Lambda^2)} \]

\[ = \left\{ \begin{array}{l}
\frac{9}{4} \pi A \left[ \alpha(\mu^2) - \alpha((1 - \cos \theta)kE) \right] \\
\text{constraint } (1 - \cos \theta)k \geq \mu^2/E
\end{array} \right. \]

\( \theta \)-integration leads to logarithmic integrals, \( \text{li}(x) = \mathcal{P} \int_0^x dt / \ln(t) \)

\[ \frac{dE}{dx} = \frac{9A^2}{8\pi} \int_{\bar{k}}^\infty dk k \rho(k) \left[ \frac{1 - \mu^2/(2E\bar{k})}{\ln(\mu^2/\Lambda^2)} + \frac{\Lambda^2}{2E\bar{k}} \left( \text{li} \frac{\mu^2}{\Lambda^2} - \text{li} \frac{2E\bar{k}}{\Lambda^2} \right) \right], \quad \bar{k} = \frac{\mu^2}{2E} \]

\[ = T^2 F\left( \frac{\mu^2}{2TE}, \frac{\mu^2}{\Lambda^2} \right) \rightarrow \left. \frac{dE_B}{dx} \right|_{\alpha(\mu)} \left[ 1 + \mathcal{O}(\alpha) \right] \]
Collisional e-loss in s(Q)GP – numerical results

- non-perturbative parameterization \( T \gtrsim 1.3 T_c \)

\[
dE/dx = T^2 F\left(\frac{\mu^2}{2TE}, \frac{\mu^2}{\Lambda^2}\right)
\]

\[
\Lambda \rightarrow \Lambda_{NP}
\]

\[
\mu^2 = 0.6m_D^2
\]

\[
m_D^2 \rightarrow 4\pi\alpha^{NP}(T)T^2
\]

A. Peshier, Collisional energy loss
IQCD/QP parameterization of Debye mass: \( m_D \approx 2.7\gamma \)

Small \( m_D(T_c) \) (\( \sim \) phase transition) \( \rightarrow \) increased energy loss

\[
\frac{dE}{dx} = T^2 F\left(\frac{\mu^2}{2TE}, \frac{\mu^2}{\Lambda^2}\right)
\]

Parameters:

\[
\Lambda \rightarrow \Lambda_{NP}
\]
\[
\mu^2 = 0.6m_D^2
\]
\[
m_D^2 \rightarrow m_{D,\text{eff}}^2
\]
Collisional e-loss in s(Q)GP – numerical results

- unquenching: energy-loss of a quark in the sQGP

\[ \frac{dE}{dx} = T^2 F\left(\frac{\mu^2}{2TE}, \frac{\mu^2}{\Lambda^2}\right) \]

parameters

\[ T_c \rightarrow 170 \text{ MeV} \]
\[ C_{gg} \rightarrow C_q(1 + n_f / 6) \]
Collisional e-loss in s(Q)GP – numerical results

Is \( \frac{dE}{dx} \sim 1 \text{ GeV/fm} \) enough?

**assume**
- constant \( \frac{dE}{dx} \)
- Bjorken dynamics
- quenching factor

\[
\frac{dN}{d^2 p_T} = Q(p_T) \frac{dN_0}{d^2 p_T} = \frac{1}{2\pi R^2} \int_0^{2\pi} d\phi \int_0^R dr^2 \frac{dN(p_T + \Delta p_T)}{d^2 p_T}
\]

comparable to [Müller]: BDMPS + transv. profile + Bjorken dynamics
realistic parameters for sQGP
⇒ enhanced $\frac{dE_{coll}}{dx}$

(quasi) critical screening
⇒ $T_c$ quenching

does sQGP\(^\star\) quench too much?
- far/near side jets vs. geometry+delay
  (talk Cassing)
- retardation (talk Gossiaux)

\(^\star\)strongly Quenching (Q)GP