Scaling properties of high-$p_T$ hadron production in heavy ion collisions

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Introduction

- Scale invariance and asymptotic freedom in QCD lead to power-like behavior of the inclusive cross section at high $p_T$:

$$p_T^n E \frac{d^3 \sigma(AB \rightarrow hX)}{d^3p} = F(x_T, y), \quad x_T = \frac{2p_T}{\sqrt{S}}. \quad (1)$$

This is the analog of Bjorken scaling in DIS.

- The power-law (instead of exponential) falloff of inclusive cross sections at large $p_T$ is evidence of pointlike hadron substructure.

- Knowledge of $n$ allows one to learn about production mechanism of high $p_T$ particles.

- Need to understand hadron production in $pp$ before investigating nuclear collisions.

- From nuclear modifications of the $p_T$ dependence, one can obtain information about energy loss.
Dimensional counting rules

• Assume that the hadronic cross section can be written in factorized form, even for higher twist processes:

\[
d\sigma(h_a h_b \rightarrow hX) = \sum_{abc} G_{a/h_a}(x_a) G_{b/h_b}(x_b) dx_a dx_b \frac{1}{2\hat{s}} |A_{fi}|^2 dX_f D_{h/c}(z_c) dz_c. \tag{2}
\]

• Dimensional analysis:
  - Normalize one-particle states to dimension length, \( \langle p | p' \rangle = 2E_p (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}') \).
  - Then, the (partonic) \( S \)-matrix has dimension length\(^{n_a} \), where \( n_a = n_{\text{in}} + n_{\text{out}} \) is the number of partons participating in the reaction
  - Because of

\[
S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^{(4)}(\sum p_{\text{in}} - \sum p_{\text{out}}) A_{fi}. \tag{3}
\]

The partonic matrix element \( A_{fi} \) has dimension length\(^{n_a-4} \), and the hard matrix element squared divided by the flux factor \( 2\hat{s} \) has dimension

\[
n = 2n_a - 4. \tag{4}
\]

• Intuitively, the larger the number of quarks that need to change direction, the steeper the cross section falls off.

• Idea: translate mass dimension of \( A_{fi} \) into power law for \( E d^3 \sigma / d^3 p \).

Blankenbecler, Brodsky, Gunion PRD18,900(1978)
The inclusive reaction $AB \rightarrow hX$ has 3 independent kinematic invariants, $S$, $T$ and $M_X^2$.

Investigate $Ed^3\sigma/d^3p$ at fixed values of $x_1 = -U/S$ and $x_2 = -T/S$, or alternatively fixed

$$y = \frac{1}{2} \ln \left( \frac{x_1}{x_2} \right),$$

$$x_R = x_1 + x_2 = 1 - \frac{M_X^2}{S} = \frac{2|\vec{p}_{cm}|}{\sqrt{S}}$$

This requires measurements at different energies.

Then only one dimensionful invariant exists so that

$$p_T^n E \frac{d^3\sigma(AB \rightarrow hX)}{d^3p} = F(x_R, y) \Leftrightarrow \sqrt{S}^n E \frac{d^3\sigma(AB \rightarrow hX)}{d^3p} = G(x_R, y)$$

with $n = 2n_a - 4$.

This is often referred to as $x_T$ scaling, because at $y = 0$ one has $x_R = x_T = 2p_T/\sqrt{S}$.

Note that $x_{1,2}$ are different from the momentum fractions $x_{a,b}$ in the factorization ansatz.
Examples of counting rules

• $2 \rightarrow 2$ partonic scattering $\Rightarrow n_a = 4$, $n = 2n_a - 4 = 4$ so that

$$E \frac{d^3\sigma(AB \rightarrow hX)}{d^3p} \propto \frac{F(x_R, y)}{p_T^4}$$  \hspace{1cm} (8)

• $uu \rightarrow p\bar{d}$, i.e. direct proton production with $n_a = 1 + 1 + 3 + 1 = 6$, $n = 8$

$$E \frac{d^3\sigma(AB \rightarrow pX)}{d^3p} \propto \frac{f_p^2}{p_T^8} F(x_R, y).$$  \hspace{1cm} (9)

The dimensionful factor $f_p$ reflects the physics of the proton distribution amplitude. The distribution amplitude of a hadron has dimension mass for mesons and mass squared for baryons, e.g. the pion distribution amplitude is normalized to $f_{\pi^+} = 130$ MeV.

• In the limit $x_R \rightarrow 1$, the missing mass approaches 0 and all fields participate, $n_a = 4 \cdot 3 = 12$,

$$E \frac{d^3\sigma(pp \rightarrow p (X = p))}{d^3p} \propto \frac{f_p^8}{p_T^{20}} f(y).$$  \hspace{1cm} (10)

Scaling laws for exclusive processes are experimentally well confirmed.
Relation to Bjorken scaling

- In the limit $Q^2 \to \infty$ at fixed $x = \frac{Q^2}{Q^2 + M_X^2}$, the DIS structure function depends only on $x$.

- However, for $Q^2 \to \infty$ and $M_X^2$ fixed (i.e. $x \to 1$), one has

$$F_2^p(M_X^2, Q^2) \propto (1 - x)^{2n_s - 1} = \left(\frac{M_X^2}{Q^2 + M_X^2}\right)^3 \sim \frac{1}{Q^6}. \quad (11)$$

That way, $F_2^p$ smoothly matches onto the elastic formfactor,

$$(1 - x)F_2^p(x, Q^2) \to G(Q^2) \propto \frac{1}{Q^8}. \quad (12)$$

This is still the case if QCD evolution is included. DGLAP evolution turns off at $x \to 1$.

- The analog of the Bjorken limit in inclusive hadron production is $p_T \to \infty$ at $x_R$ and $y$ fixed.

- At fixed hadronic cm. energy, large $p_T$ does not imply that higher twists disappear.

- Note, the $p_T$ dependence of $Ed^3\sigma/d^3p$ at fixed $\sqrt{S}$ is in general much steeper than at fixed $x_R$ and $y$, because structure and fragmentation functions are probed at different momentum fractions.

- $x_T$-scaling removes this effect and the $p_T$ dependence of the partonic process is unveiled.
Forward physics and the exclusive limit

• In the exclusive limit $\epsilon = 1 - x_R = M_X^2/S \to 0$, only valence quarks are important.

\[ \epsilon \sim 0 \text{ (exclusive limit)} \]

\begin{align*}
  x_T &= \frac{2p_T}{\sqrt{S}} = 2\sqrt{x_1x_2}, \\
  x_F &= \frac{2p_L}{\sqrt{S}} = x_1 - x_2, \\
  x_R &= \frac{2|\vec{p}|}{\sqrt{S}} = x_1 + x_2 = \sqrt{x_F^2 + x_T^2}.
\end{align*}

• At very large $\sqrt{S}$ the two kinematical domains of
  - $x_2 = -T/S \ll 1$ (Regge theory applies)
  - $p_T \gg \Lambda_{QCD}$ (pQCD applies)

overlap.

• Still, the gluon density approaches 0 like $(1 - x_R)^5$ toward the exclusive limit. This behavior is expected from QCD factorization.
The simple spectator counting rules \((1 - x)^{2n_s - 1}\) receive corrections from QCD evolution.

Gluon radiation off a constituent (quark, gluon, diquark, intrinsic meson . . .) changes the large \(x\) behavior of the distribution function to

\[
G_{(a/A)}(x_a, p_T) = (1 - x_a)^{2n_s - 1 + \xi(p_T)},
\]

\[
\xi(p_T) = \frac{C_R}{\pi} \int_{k_{x_a}^2}^{p_T^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \alpha_s(k_{\perp}^2) = \frac{4C_R}{\beta_0} \ln \left( \frac{p_T^2}{\Lambda_{QCD}^2} \right) \ln \left( \frac{k_{x_a}^2}{\Lambda_{QCD}^2} \right),
\]

where \(\beta_0 = 11 - 2N_F/3\) and \(C_R\) is the total color charge squared of the constituent, i.e. \(C_R = 4/3\) for quarks, \(C_R = 3\) for gluons, \(C_R = 10/3\) for sextet diquarks and \(C_R = 0\) for color-neutral objects.

Important: large \(x\) parton must be far off shell with virtuality

\[
k_{x_a}^2 = \frac{p_T^2}{1 - x_a}.
\]

Hence, QCD evolution turns itself off at large \(x\) and spectator counting rules become exact.

This “self-healing” of evolution is necessary to smoothly match the DIS structure function \(F_2(x, Q^2)\) onto the elastic formfactor \(G(Q^2) \propto 1/Q^8\) in the limit \(Q^2 \to \infty\) at fixed \(S_{\gamma^*p}\).
Scaling violations of dimensional counting rules

- QCD is only approximately scale invariant. Structure and fragmentation functions are logarithmically scale-dependent. So is the strong coupling constant $\alpha_s(p_T)$.

- Scaling violations lead to corrections to the nominal power laws. The effective exponent

$$n_{eff}(p_T) = -\frac{d \ln E \frac{d^3 \sigma(AB \rightarrow hX)}{d^3 p}}{d \ln (p_T)}$$

(16)

is now a slowly varying function of $p_T$.

- The invariant cross section behaves approximately as

$$E \frac{d^3 \sigma(AB \rightarrow hX)}{d^3 p} \sim \left[ \frac{\alpha_s(p_T^2)}{p_T^2} \right]^{n_a-2} \frac{(1 - x_R)^2}{x_R^{\lambda(p_T)}} \frac{n_s-1+3\xi(p_T)}{f(y)}$$

(17)
Experimental results: Fixed target energies

- The Chicago-Princeton collaboration measured high-$p_T$ inclusive hadron production in the SPS energy range PRD19,764(1979).
- Data for $p + p \rightarrow (\pi^+ + \pi^-)/2 + X$ behave as
  
  $$p_T^8 E \frac{d^3\sigma}{d^3p} \propto (1 - x_T)^9,$$

  suggesting a higher twist mechanism even at $p_T \sim 5$ GeV.
- Possible explanation: The hard subprocess
  
  $$q + (q\bar{q}) \rightarrow q + \pi$$

  has $n_a = 1 + 2 + 1 + 2 = 6$ and $n_s = 2 + 3 = 5$,
  yielding the observed scaling properties.
  Blankenbecler et al. PRD18,900(1978)
- Protons: $(p + p \rightarrow p + X)$
  
  $$p_T^{11.7} E \frac{d^3\sigma}{d^3p} \propto (1 - x_T)^{6.8}.$$

  Consistent with $q + (qq) \rightarrow M + p$ and $q + (qqq) \rightarrow q + p$. 
Experimental results: Collider energies

- PHENIX analysis of $x_T$-scaling between $\sqrt{S} = 130$ GeV and 200 GeV PRC69,034910(2004):

- Charged hadron and $\pi^0$ production go like
  
  \[
  E \frac{d^3N}{d^3p} \propto \frac{1}{p_T^{0.3 \pm 0.5}},
  \]

  which is slightly steeper than leading twist including scaling violations ($n_{eff} \approx 5$).

- Tevatron jet data: $n_{exp} = 4.45$ for $0.15 \leq x_T \leq 0.3$ between $\sqrt{S} = 630$ GeV and 1800 GeV.

- For $2 \to 1$ gluon fusion, one would expect $p_T^{-2}$. This is a possible signature of CGC at LHC.

- Conclusion: the mechanism of high-$p_T$ hadron production changes from fixed target to collider energies. \(\Rightarrow\) Nuclear effects are expected to change as well.
Nuclear effects

- Fixed target data suggest that pions are not produced by parton fragmentation.
- Instead, a small color neutral object of size $1/p_T^2$ is produced in the hard reaction, which then evolves into a pion.
- Color neutral objects are less affected by the medium than colored partons.
- The dipole interacts with inelastic cross section $\sigma_{in} \sim \pi/p_T^2 \approx 0.1$ mb. The mean free path in is thus of order
  \[ \lambda_{free} \approx \begin{cases} 
  450 \text{ fm } \rho_A = 0.16 \text{ fm}^{-3} \\
  45 \text{ fm } \rho_A = 1.6 \text{ fm}^{-3} 
\end{cases} \] (18)
- In this scenario, no quenching of pion spectra would occur at SPS. This mechanism can be verified directly by imposing exclusion cuts around a high $p_T$ pion.
- In addition, if protons are produced by such a mechanism (at SPS or RHIC), $R_{CP} \approx 1$ for protons. At RHIC, this results in an apparent proton enhancement in $AB$ collisions.
BDMPS-Z approach to medium induced gluon radiation


- LPM effect: The Bethe-Heitler (BH) formula for bremsstrahlung,

\[
\frac{dI_{BH}}{d\omega dz} \sim \alpha_s C_R \frac{1}{\omega \lambda_{free}},
\]

is modified for very energetic particles due to coherent rescattering.

- In QCD, in leading log(x) approximation, only the radiated gluon rescatters:

- The time needed to radiate a gluon (coherence time) can be estimated from the uncertainty relation, \( t_c \approx \omega / k_T^2 \).

- During that time, the gluon passes by \( N_{coh} = \sqrt{\omega / \omega_{BH}} \) scattering centers, which act as one single scattering center.

- Coherence effects are relevant for frequencies \( \omega > \omega_{BH} \approx \langle k_T^2 \rangle \lambda_{free} \sim \text{few-hundred MeV} \).

- Consequently, the gluon spectrum is modified,

\[
\frac{dI_{LPM}}{d\omega dz} \sim \frac{1}{N_{coh}} \frac{dI_{BH}}{d\omega dz} \sim \alpha_s C_R \frac{\sqrt{\hat{q}}}{\omega^{3/2}}, \quad \hat{q} = \frac{\omega_{BH}}{\lambda_{free}^2}.
\]
Estimate of the BDMPS transport coefficient

- The transport coefficient $\hat{q}$ and the dipole cross section $\sigma_{q\bar{q}}(r_T^2) = C r_T^2$, are both related to the average color-field strength $\langle F^2 \rangle$ in the medium JR, PLB557,184(2003),

$$C = \frac{\pi^2}{3} \alpha_s \langle F^2 \rangle$$

$$\hat{q} = 2 \rho_A \frac{\pi^2}{3} \alpha_s \langle F^2 \rangle$$

- The dipole approach has a highly developed and successful phenomenology in DIS, Drell-Yan, heavy flavor production, total hadronic cross sections, color transparency . . . .

- Use KST parameterization of $\sigma_{q\bar{q}}$ to determine $\hat{q}$. Kopeliovich et al. PRD62,054022(2000)

- Higher order corrections make $\hat{q}$ weakly energy dependent, $\hat{q} \propto E^{0.08}$.

- Note: this estimate is for cold nuclear matter and works only at high energies.
The transport coefficient in heavy ion collisions

- In HIC, a medium with high energy density is created. Bjorken's estimate of the initial energy density at RHIC yields

\[ \epsilon_{Bj} = \frac{\langle m_T \rangle}{\pi R_A^2 \tau_0} \left( \frac{dN}{dy} \right)_{y=0} \approx 10 \text{ GeV/ fm}^3 \approx 60 \epsilon_{\text{cold}} \] (23)

at initial time \( \tau_0 = 0.5 \) fm.

- Because of the expansion of the medium, the hard parton sees an averaged transport coefficient,

\[ \hat{q}^{med} = \frac{2\hat{q}}{L^2} \int_{\tau_0}^{\tau_0+L} d\tau (\tau - \tau_0) \frac{\tau_0}{\tau}. \] (24)

Salgado, Wiedemann, PRL89,092303(2002)

- The averaged transport coefficient is then

\[ \hat{q}^{med} \approx 10 \hat{q}^{\text{cold}} \approx 2 \text{ GeV/ fm}^2. \] (25)
Medium induced energy loss

- Distinguish 3 different regimes:
  1. $E < \omega_{BH} \sim$ few-hundred MeV: Bethe Heitler applies,

$$- \left( \frac{dE}{dz} \right)_{BH} \sim \int_{\omega_{BH}}^{E} d\omega \omega \frac{\alpha_s C_R}{\lambda_{free} \omega} \sim \frac{\alpha_s C_R E}{\lambda_{free}}. \quad (26)$$

2. $\omega_{BH} \ll E \ll \omega_{LPM} = \hat{q}L^2 \sim \begin{cases} 
5 \text{ GeV (cold)} \\
50 \text{ GeV (hot, expanding medium)}
\end{cases}$:

$$- \left( \frac{dE}{dz} \right)_{LPM_1} \sim \int_{\omega_{LPM}}^{E} d\omega \frac{\alpha_s C_R \sqrt{\hat{q}}}{\sqrt{\omega}} \sim \alpha_s C_R \sqrt{\hat{q}E}. \quad (27)$$

This is the same $E$ dependence as for the LPM effect in QED.

3. $\omega_{LPM} \ll E$:

$$- \left( \frac{dE}{dz} \right)_{LPM_2} \sim \int_{\omega_{LPM}}^{\hat{q}L} d\omega \frac{\alpha_s C_R \sqrt{\hat{q}}}{\sqrt{\omega}} \sim \alpha_s C_R \hat{q}L. \quad (28)$$

There is an additional contribution $\sim \alpha_s C_R E/L$, where the entire target acts as a single scattering center.
Energy loss and inclusive hadron production

- Discovery of RHIC: high-$p_T$ hadron spectra in central nucleus-nucleus collisions are strongly suppressed.
- Medium modified cross section for production of a high-$p_T$ parton,

\[
\frac{d\hat{\sigma}^{med}}{d\hat{p}_T^2} = \int d\epsilon P(\epsilon) \frac{d\hat{\sigma}^{vac}}{d\hat{p}_T^2}(\hat{p}_T + \epsilon). \tag{29}
\]
- With \(d\hat{\sigma}^{vac}/d\hat{p}_T^2 \propto \hat{p}_T^{-6}\), a small energy loss can result in a large suppression,

\[
\frac{1}{N_{coll}} \frac{d^2\sigma(AB \to hX)}{dydp_T^2} = \frac{d^2\sigma(pp \to hX)}{dydp_T^2} Q(p_T). \tag{30}
\]

In experiment (AuAu at \(\sqrt{S} = 200\) GeV): \(Q(p_T) \approx 0.2\) for \(h = \pi^0\) up to \(p_T = 20\) GeV.
- At fixed \(\sqrt{S}\), one has additional suppression due to structure and fragmentation functions,

\[
\frac{d\ln(Ed^3\sigma(\sqrt{S} = 200\text{ GeV})/d^3p)}{d\ln p_T} \approx -8. \tag{31}
\]

20% energy loss \(\Rightarrow 80\%\) suppression (i.e. \(Q(p_T) \approx 0.2\)).
Quenching weights

- Numerical results for quenching weights $Q(p_T)$ strongly depend on gluon radiation in the few-hundred MeV range.

- Idea: investigate $x_T$ scaling to find out if the medium has modified the $p_T$ dependence of the microscopic process,

$$n^{med} = n^{vac} - \frac{d \ln Q(p_T)}{d \ln p_T}$$  \hspace{1cm} (32)

- Following BDMS, JHEP09(2001)033, expand the logarithm of the cross section to obtain

$$Q(p_T) \approx \int d\epsilon P(\epsilon) \exp \left( -\frac{\epsilon}{p_T} n^{vac} \right) = \exp \left( -\frac{n^{vac}}{p_T} \int_0^\infty d\omega e^{-\frac{n^{vac} \omega}{p_T}} \int_{\omega'}^{\omega_{max}} \omega' \frac{dI}{d\omega'} \right)$$  \hspace{1cm} (33)

- Eventually, one arrives at the result

$$\Delta n \approx -\frac{d \ln Q(p_T)}{d \ln p_T} \approx \int d\omega \frac{dI}{d\omega} \sim \begin{cases} \alpha_s C_R \sqrt{\frac{\omega_{LPM}}{p_T}} & E \ll \omega_{LPM} \\ \alpha_s C_R & E \gg \omega_{LPM} \end{cases}$$  \hspace{1cm} (34)

Jörg Raufeisen, ECT* Workshop, September 2005
Experimental results

- PHENIX (PRC69,034910(2004)) found no centrality dependence of $x_T$ scaling for $\pi^0$.

- Apparently, only a small number of gluons is radiated and energy loss is proportional to energy within error bars. In the $\sqrt{qE}$ regime, this would require a considerable energy dependence of $\dot{q}$.

- For all charged hadrons, the $p_T$ dependence changes from $p_T^{-6.3\pm0.5}$ to $p_T^{-7.5\pm0.5}$.

- This observation suggests the proton production mechanism $uu \rightarrow p\bar{d}$, so that

\[
E \frac{d^3\sigma(AB \rightarrow pX)}{d^3p} \propto \frac{f_p^2}{p_T^8}.
\]

\[(35)\]

Pions are strongly quenched, protons do not lose energy in the medium.
Summary

• Scale invariance and asymptotic freedom in QCD lead to power-like behavior of the inclusive cross section at high $p_T$:

$$p_T^n E \frac{d^3 \sigma(AB \to hX)}{d^3 p} = F(x_R, y).$$

The power $n(y, x_R) = 2n_a - 4$ is related to the number of active fields.

• Leading twist pQCD: $n_a = n = 4$. Including scaling violations, one finds $n_{eff} \approx 4.5 \ldots 5$.

• Scaling properties of inclusive high-$p_T$ meson production change from SPS to RHIC
  − SPS: direct production of small color-neutral objects ($n = 8.5 \pm 0.5$) $\Rightarrow$ no jet quenching
  − RHIC: leading twist pQCD dominates in meson production ($n = 6.3 \pm 0.5$) $\Rightarrow$ strong medium effects

• Nuclear modifications of $x_T$ scaling suggest that a small number of gluons takes away a finite fraction of the projectiles momentum.

• Intermediate $p_T$ protons may be produced instantaneously as small colorless objects. This would explain the absence of nuclear effects in proton production at RHIC.

• Inclusive hadron production must approach its exclusive limit at $x_R \to 1 \Rightarrow n \to 20$.
  − Only valence quarks are important at the largest rapidities, high energy $\neq$ low $x$.
  − Coherence effects reach their maximum around $x_F \approx 0.5$. 